Simulation and Optimization Methods for Reliability Analysis

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Innsbruck/Natters, Austria

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Plan of Presentation

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- Description of the model
  - Brute force Monte Carlo benchmarking
  - Quality assessment of estimators by resampling and Bayes

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  - Turning the random field on
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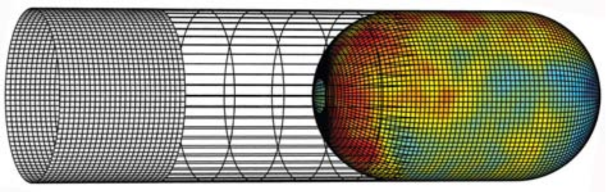
Thanks to: Dynardo GmbH, INTALES, CTU Prague, Astrium Ottobrunn



Small launcher model:

FE-model, ABAQUS, 18.000 elements, shell elements and beam elements for stiffeners. 91.000 DoF.

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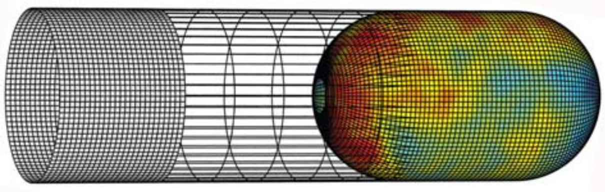

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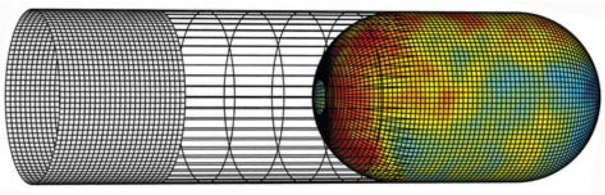

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Input statistics: Uniform distributions with spread $\pm 15\%$ around nominal value.

Failure Criterion

Output: Failure probability

$$p_f = \mathrm{P}(\Phi(\mathbf{x}) > 1)$$

defined by the the critical demand-to-capacity ratio (CDCR)

$$\Phi(\mathbf{x}) = \max\left\{\frac{PEEQ(\mathbf{x})}{0.07}, \frac{SP(\mathbf{x})}{180}, \frac{0.001}{EV(\mathbf{x})}\right\},$$

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Result of benchmark simulation:

$$p_f = 0.0116.$$

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First analysis: Bootstrap resampling. Drawing from the original sample of size $N = 5000$ with replacement, $B = 10000$ samples with the same (empirical) distribution and corresponding p_f are obtained. Result: an estimate of the statistical variation of p_f.

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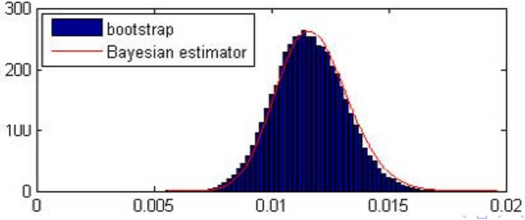

Variation of p_f around its estimated value of 0.0116.

Reliability Analysis: Faster Methods (1)

Subset Simulation:

$$\mathrm{P}(F) = \mathrm{P}(F_m \big| F_{m-1})\mathrm{P}(F_{m-1} \big| F_{m-2}) \ldots \mathrm{P}(F_1 \big| F_0)\mathrm{P}(F_0)$$

where $F = F_m$ and F_0 is the starting region.

$$F = \{\mathbf{x} : \Phi(\mathbf{x}) > 1\}, \qquad F_i = \{\mathbf{x} : \Phi(\mathbf{x}) > \alpha_i\}$$

and α_i is chosen so that $\mathrm{P}(F_i \big| F_{i-1}) = 0.2$, say.

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Asymptotic sampling: $p_f = \Phi(-\beta)$, $\beta = \Phi^{-1}(1 - p_f)$. Transformation to normal probability space with $\sigma = 1$. Instead of simulating $\beta = \beta(1)$, one simulates $\beta(v)$ for smaller values of $v = 1/\sigma$, which is easy, and sets up a regression

$$\beta(v) = A + Bv + C/v + \ldots$$

Best model chosen by data analysis.

Reliability Analysis: Faster Methods (2)

Importance sampling:

$$p_f = \int \mathbb{1}_F(\mathbf{x})\rho(\mathbf{x})\,\mathrm{d}\mathbf{x} = \int \mathbb{1}_F(\mathbf{x})\frac{\rho(\mathbf{x})}{g(\mathbf{x})}g(\mathbf{x})\,\mathrm{d}\mathbf{x}.$$

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How to choose $g(\mathbf{x})$? Start with cheap sensitivity analysis.

Employing all tricks of the trade (Latin hypercube sampling, correlation control), a sample size around 100 – 200 suffices.

Determine the most relevant input parameters.

Distort their distribution according to the degree of correlation with the output (assuming monotone dependence).

Reliability Analysis: Comparison

	MC	SS	AS	IS
Sample Size	5000	780	800	780
Estimated p_f	0.0116	0.0155	0.0093	0.0084
Bootstrap 95%-CI	0.0088 — 0.0146	0.0104 — 0.0217	0.0028 — 0.0219	0.0055 — 0.0118
Bayesian 95%-CI	0.0090 — 0.0150	0.0112 — 0.0225		

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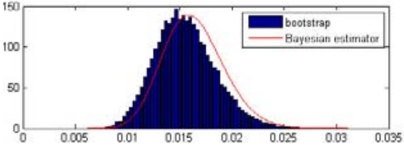
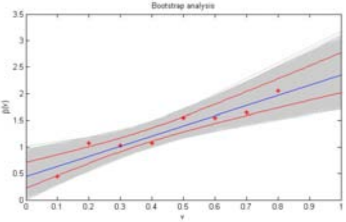
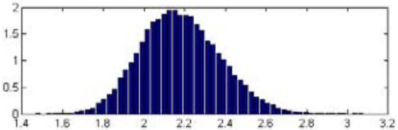


Subset simulation: Bootstrap and Bayesian estimate of the variability of p_f.



Asymptotic sampling: Bootstrap regression and corresponding distribution of β.



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In all cases, the implementation of bounds (on input) and constraints (on output) requires additional rules.

First application: In reliability analysis, the location of the failure region and the most critical points are of interest.

Worst Case Scenarios

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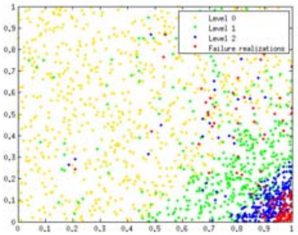
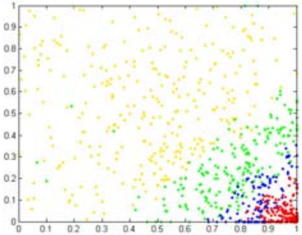
All algorithms compute clouds of points that can be ordered according to their Φ-values and used for further analysis.

Subset Simulation (top) and genetic algorithm (bottom), pressure load sphere 2 versus yield stress cylinder 3.





Legend

yellow	0 — 0.9543
green	0.9544 — 0.9885
blue	0.9886 — 1
red	> 1

- GRADE algorithm with CERAF restart strategy (CTU Prague)
- Nelder-Mead with probabilistic restart (in-house)
- Genetic algorithm optiSLang
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Mass Optimization under Constraint

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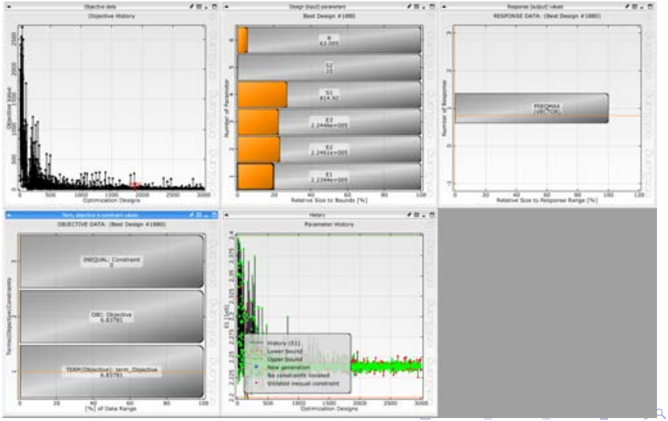
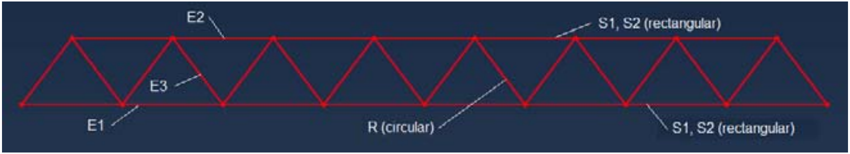
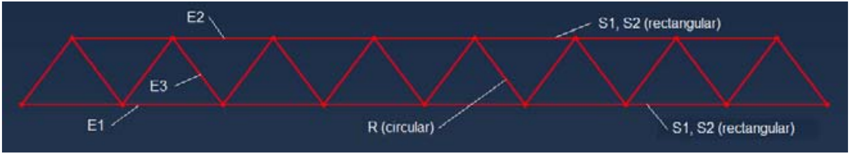
optiSLAng-Gui particle swarm



Truss Structure – Comparison of Results

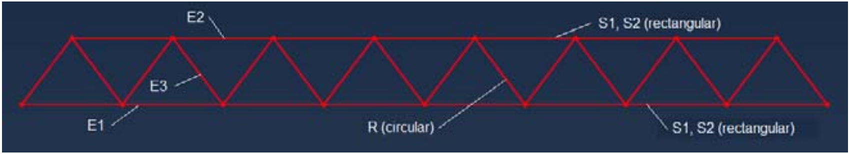




Objective value: MASS
Constraint: PEEQ = 0

Bounds:

S1	S2	R	E1	E2	E3
10	10	10	220000	220000	220000
3000	3000	1000	240001	240001	240001



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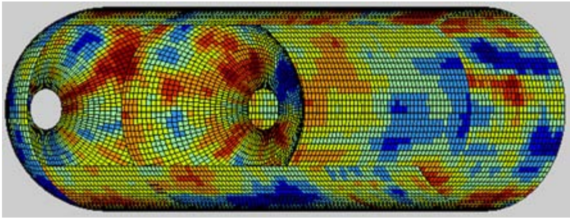
	Function calls	Mass	S1	S2	R	E1	E2	E3
GA - GRADE	1620	6.98346	828.30	10.00	63.78	240001	222901	220000
Nelder-Mead	349	6.80596	799.27	10.11	62.95	223330	238207	228351
GA - optiSLang	1528	6.85535	817.40	10.00	63.07	229650	220620	220960
PS - optiSLang	1996	6.08253	730.28	10.00	59.26	223280	229500	224290

Random Field Modelling (1)

A random field on the small launcher model (material properties)



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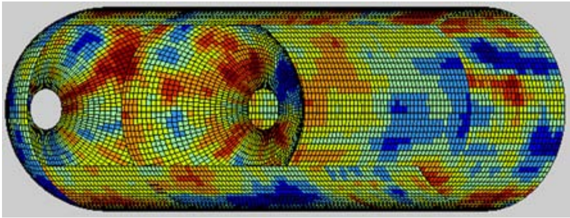

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- spectral decomposition of covariance matrix
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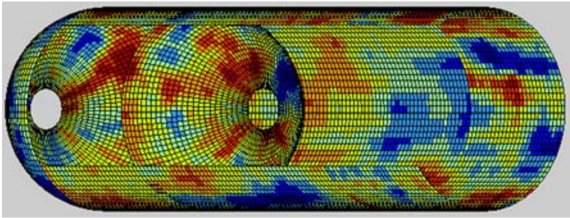

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Typical autocovariance function

$$\mathrm{COV}(X_P, X_Q) = \sigma^2 \exp(-\mathrm{dist}_1(P,Q)/\ell_1) \exp(-\mathrm{dist}_2(P,Q)/\ell_2)$$

Random Field Modelling (2)

Exemplary application:

- Loads as random variables as before
- Material properties as random fields
- **Question:** Change of output with/without random field

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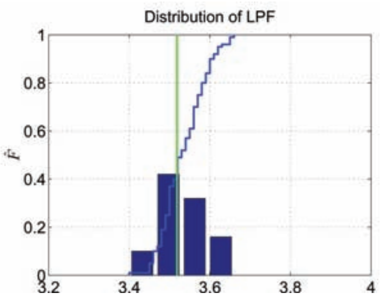
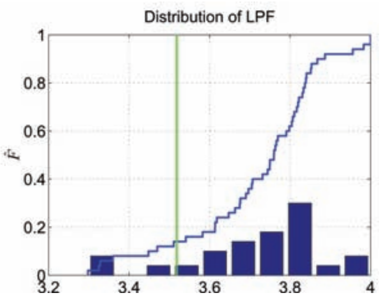
Example – change of distribution of load proportionality factor LPF without random field (left) and with random field (right):





Thank you for your attention!