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Challenges and potentials of random fields in robustness analysis

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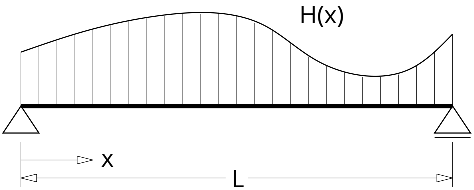
Basic Concept

- Location in structure defined by $\mathbf{x} = (x_1, x_2, \ldots, x_n)$
- Random variable H(**x**) at **x** (e.g. cross section or vertical coordinate)
- Different value H(**y**) at a different location $\mathbf{y} = (y_1, y_2, \ldots, y_n)$
- Expect high correlation if **x** and **y** are close to each other



Formal definition

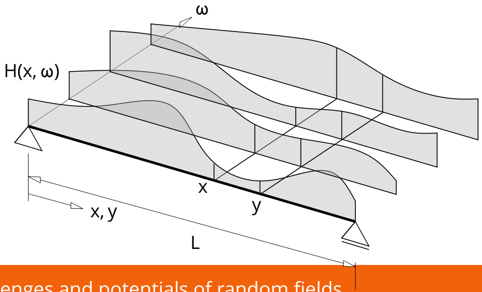
- A random field H(**x**) is a real-valued random variable whose statistics can be different for each value of **x**

$$H \in \mathbb{R}; \quad \mathbf{x} = [x_1, x_2, \ldots x_n]^T \in \mathcal{D} \subset \mathbb{R}^n$$

- Mean value function

$$\bar{H}(\mathbf{x}) = \mathbf{E}[H(\mathbf{x})]$$

- Expected value **E** is taken at an arbitrary, but fixed location **x** across the ensemble



Spatial correlation structure

- Autocovariance function

$$C_{HH}(\mathbf{x}, \mathbf{y}) = \mathbf{E}[\{H(\mathbf{x}) - \bar{H}(\mathbf{x})\}\{H(\mathbf{y}) - \bar{H}(\mathbf{y})\}]$$

- A random field H(**x**) is called *weakly homogeneous* if

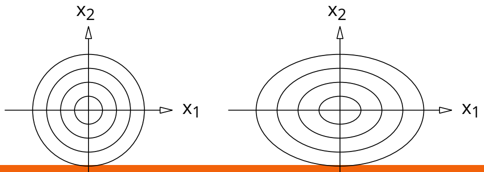
$$\bar{H}(\mathbf{x}) = \text{const.} \quad \forall \mathbf{x} \in \mathcal{D}$$

$$C_{HH}(\mathbf{x}, \mathbf{x} + \boldsymbol{\xi}) = C_{HH}(\boldsymbol{\xi}) \quad \forall \mathbf{x} \in \mathcal{D}$$

- A homogeneous field is *isotropic* if

$$C_{HH}(\mathbf{x}, \mathbf{x} + \boldsymbol{\xi}) = C_{HH}(\|\boldsymbol{\xi}\|) \quad \forall \mathbf{x} \in \mathcal{D}$$

- Contours of constant correlation



Spectral decomposition

- Represent a continuous random field $H(\mathbf{x})$ in terms of discrete random variables c_k; $k = 1 \ldots \infty$:

$$H(\mathbf{x}) = \sum_{k=1}^{\infty} c_k \phi_k(\mathbf{x}), \quad \mathbf{x} \in \mathcal{D} \subset \mathbb{R}^n; c_k, \phi_k \in \mathbb{R}$$

- $\phi_k(\mathbf{x})$ are deterministic spatial shape functions forming an orthonormal basis on $\mathcal{D}$, c_k are uncorrelated random variables
- Decomposition of the covariance function

$$C_{HH}(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^{\infty} \lambda_k \phi_k(\mathbf{x}) \phi_k(\mathbf{y})$$

- λ_k and $\phi_k(\mathbf{x})$ are the eigenvalues and eigenfunctions respectively.

Discrete version

- Evaluate random field at discrete locations $\mathbf{x_i}$

$$H_i = H(\mathbf{x_i}); \quad i = 1 \ldots N$$

- Spectral representation

$$H_i = \sum_{k=1}^{N} \phi_k(\mathbf{x_i}) c_k = \sum_{k=1}^{N} \phi_{ik} c_k; \ \mathbf{H} = \mathbf{\Phi c}$$

- Orthogonality condition for the columns of $\mathbf{\Phi}$

$$\mathbf{\Phi}^T \mathbf{\Phi} = \mathbf{I}$$

- Covariance matrix of the components of the coefficient vector $\mathbf{c}$

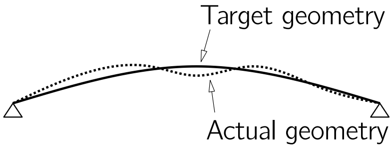
$$\mathbf{C_{cc}} = \mathrm{diag}(\sigma^2_{c_k})$$

- Both conditions can be met if the columns $\boldsymbol{\phi}_k$ of the matrix $\mathbf{\Phi}$ solve the following eigenvalue problem

$$\mathbf{C_{HH}} \boldsymbol{\phi}_k = \sigma^2_{c_k} \boldsymbol{\phi}_k; \quad k = 1 \ldots N$$

Geometrical imperfections

- Manufacturing tolerances → deviations of the actual geometrical shapes from those designed
- Differences between the target geometry and the actual random geometry can be expressed as random field
- Certain regularity or ”waviness” of the geometrical imperfections can be modeled by choosing a suitable spatial correlation function.

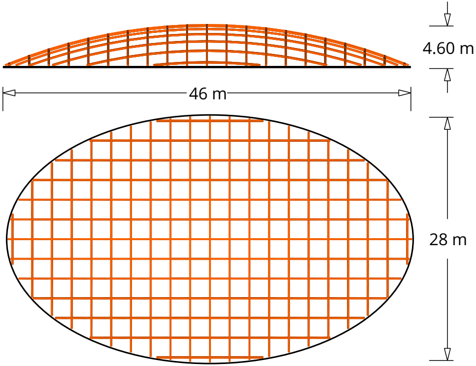


Example: dome structure

- Structure consists of slender beams connected rigidly, forming a doubly curved dome.
- Uniform vertical loading
- Geometrical imperfections are represented in terms of random deviations $\hat{z}$ of the vertical coordinates z
- Spatial autocovariance function

$$C_{\hat{z}\hat{z}} = \sigma_{\hat{z}}^2 \exp\left(-\frac{r}{L_c}\right)$$

r denotes the spatial separation and $L_c = 6$ m .



Robustness analysis

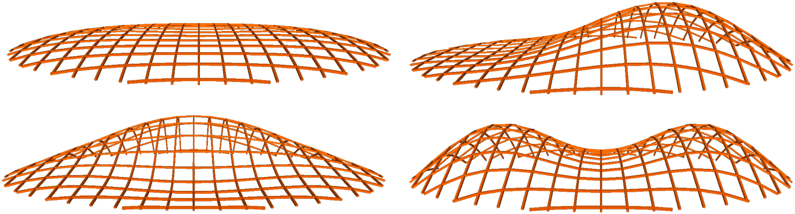
- Assume random perturbation $\hat{z}$ of nodal z-coordinates.
- Describe $\hat{z}$ by a conditional random field (modeled as a homogeneous background field with superimposed conditions of zero variability at the structural supports.
- Zero mean value, standard deviation $\sigma_{\hat{z}}$ varied from 1 mm to 100 mm.
- Correlation length fixed at L_c = 6 m
- Compute samples for the geometry deviation
- Compute samples for the critical load factor (static analysis)
- Compute samples for the displacement field

Purpose of analysis

- Assess influence of geometrical imperfections on the stability of the structure (global or local buckling)
- Identify relevant imperfection shapes for critical load factor from robustness/sensitivity analysis
- Determine stochastic variability of deflections due to geometrical imperfections
- Identify relevant imperfection shapes for maximum deflections from robustness analysis
- **Input random field: geometry**
- **Output random field: deflection**
- **Describe relation between input and output random fields**

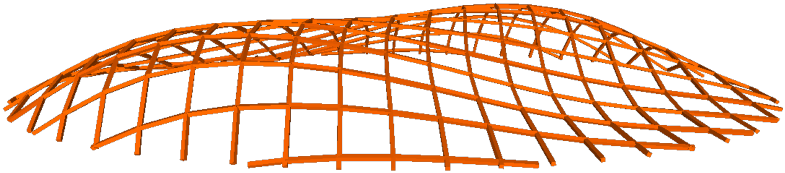
Karhunen-Loève decomposition

- Shapes belonging to the largest eigenvalues (magnified)
- 20 shapes define the random field up to 60% accuracy (in terms of variance)
- 100 shapes define the random field up to 87% accuracy
- 200 shapes define the random field up to 99.9% accuracy
- Field samples can be generated based on first N shapes



Deterministic buckling analysis

- Classical stability analysis based on 2nd order theory (linearized buckling analysis
- Critical buckling mode indicates a divergence-type instability (unsymmetric buckling mode for symmetric structure under symmetric loading).
- Critical load factor λ_c = 3.66



Stochastic buckling analysis

- Monte Carlo simulation with 500 samples of imperfect geometry
- Only N = 20 shapes are required to obtain statistically stable result (comparison with N = 100 has been done)
- First step: Compute mean values $\bar{\lambda}$ and standard deviations σ_λ of λ_c for different magnitudes of geometry variations

$\sigma_{\hat{z}}$	Mean value $\bar{\lambda}$	Standard deviation σ_λ
1 mm	3.66	0.001
10 mm	3.66	0.018
50 mm	3.65	0.051
100 mm	3.59	0.166

Establish Metamodel of optimal prognosis

- Quadratic model describing dependence between magnitude of individual imperfection shapes and critical load factor λ_c
- Coefficients of prognosis for different magnitudes of geometry variations

$\sigma_{\hat{z}}$	CoP
1 mm	1.000
10 mm	0.999
50 mm	0.998
100 mm	0.969

Ranking of input variables

- Assess relevance α_i of individual imperfection amplitudes c_i on the variation of the buckling load
- Ranking based on α_i = CoP(c_i)

$\sigma_{\hat{z}}$	CoP	α_4	α_7	α_{13}
1 mm	1.000	0.34	0.61	0.17
10 mm	0.999	0.28	0.64	0.15
50 mm	0.998	0.28	0.70	0.18
100 mm	0.969	0.23	0.56	0.18

- All other values α_i are very close to zero.

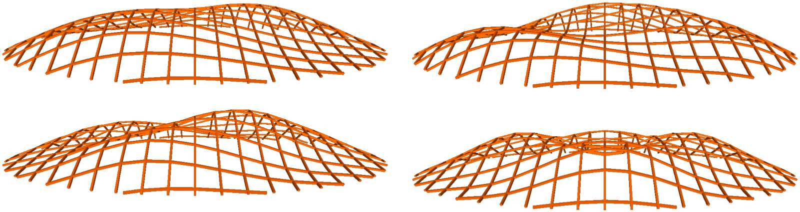
Stochastic deflection analysis

- Monte Carlo simulation with 500 samples of imperfect geometry
- Compute mean values $\bar{w}_{max}$ and standard deviations $\sigma_{w_{max}}$ of maximum deflection for different magnitudes of geometry variations
- Compute CoP values
- CoP's are rather bad even for small variability of geometry
- Need to take into account random field description of deflection

$\sigma_{\hat{z}}$	Mean value $\bar{w}_{max}$	Standard deviation $\sigma_{w_{max}}$	CoP
1 mm	2.0 mm	0.006 mm	1.000
10 mm	2.3 mm	0.35 mm	0.734
50 mm	4.9 mm	1.2 mm	0.717
100 mm	15.3 mm	4.8 mm	0.586

Random field analysis of deflection

- Create empirical random field from 500 Monte Carlo samples
- Compute discrete Karhunen-Loève decomposition
- Deflection field is represented by 10 shapes with good accuracy
- Compute deflection field amplitudes based on empirical random field



Accuracy of KL decomposition of displacement field

- Accuracy depends on variability of geometry

$\sigma_{\hat{z}}$	Accuracy (variance)
1 mm	1.000
10 mm	0.999
50 mm	0.998
100 mm	0.969

Field CoP values 1

- Combine CoP values of random field amplitudes with their contribution to the total variance
- $\sigma_{\hat{Z}}$ = 10 mm

Field amplitude k	CoP	Variation
1	1.000	0.360
2	1.000	0.196
3	1.000	0.166
4	1.000	0.089
5	1.000	0.061
6	1.000	0.040
7	1.000	0.035
8	1.000	0.026
9	0.999	0.010
10	0.998	0.007

- Excellent field CoP value F-CoP = 0.989

Field CoP values 2

- $\sigma_{\hat{z}}$ = 100 mm

Field amplitude k	CoP	Variation
1	0.988	0.395
2	0.990	0.171
3	0.991	0.156
4	0.982	0.088
5	0.980	0.052
6	0.988	0.042
7	0.973	0.031
8	0.953	0.023
9	0.956	0.010
10	0.861	0.008

- Very good field CoP value F-CoP = 0.961

Ranking of input variables

- Assess relevance α_i of individual imperfection amplitudes c_i on the amplitudes d_k of deflection random field
- $\sigma_{\hat{z}}$ = 10 mm

k	CoP	α_3	α_5	α_7	α_8	α_{10}	α_{13}	α_{14}
1	0.988	0.00	0.99	0.00	0.00	0.00	0.00	0.00
2	0.990	0.17	0.00	0.00	0.60	0.18	0.00	0.13
3	0.991	0.00	0.00	0.00	0.00	0.93	0.00	0.00
4	0.981	0.00	0.00	0.34	0.00	0.00	0.70	0.00

- $\sigma_{\hat{z}}$ = 100 mm

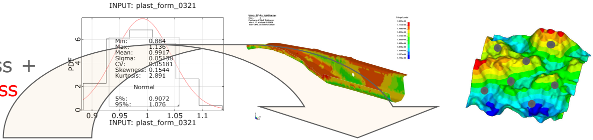
k	CoP	α_3	α_5	α_7	α_8	α_{10}	α_{13}	α_{14}
1	0.988	0.00	0.94	0.00	0.00	0.00	0.00	0.00
2	0.990	0.10	0.00	0.00	0.51	0.18	0.00	0.09
3	0.991	0.00	0.00	0.00	0.12	0.85	0.00	0.00
4	0.981	0.00	0.00	0.29	0.00	0.00	0.76	0.00

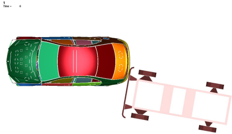
- All other values α_i are very close to zero.

Robustness Evaluation Insurance Crash 1

J. Will & T. Frank, WOST 2008

Scatter of sheet thickness + forming process scatter cov_{max}=0.05, yield strength, friction, test conditions,





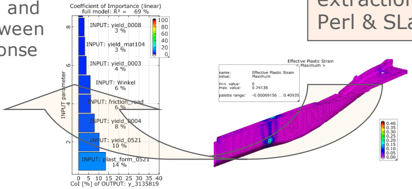
Introduction of spatial correlated forming process scatter

150 LS-DYNA simulation

extraction via LS-PREPOST, Perl & SLang

optiSLang

result variation and correlation between input and response scatter

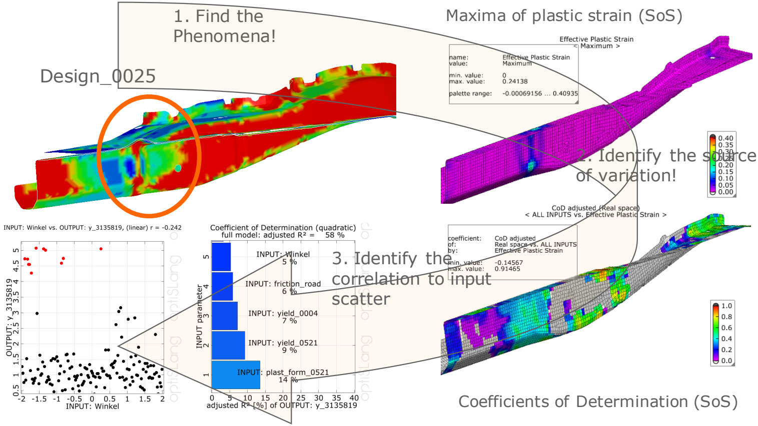


SoS Post processing

Robustness Evaluation Insurance Crash 2

J. Will & T. Frank, WOST 2008

11 design (from 149 =7%) show the test phenomena. Scatter of barrier angle and scatter of thickness from forming show highest correlation.

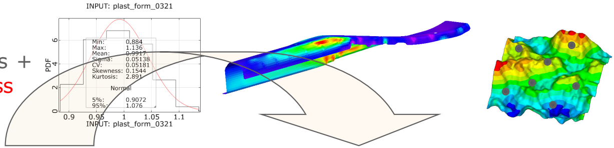


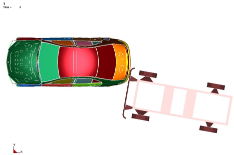
Robustness Evaluation Insurance Crash 3

J. Will & T. Frank, WOST 2008

Scatter of sheet thickness + forming process scatter cov_{max}=0.03, yield strength, friction, test conditions,

optiSLang





Introduction of spatial correlated forming process scatter

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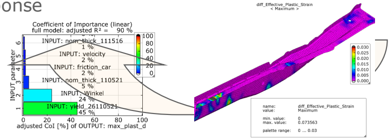
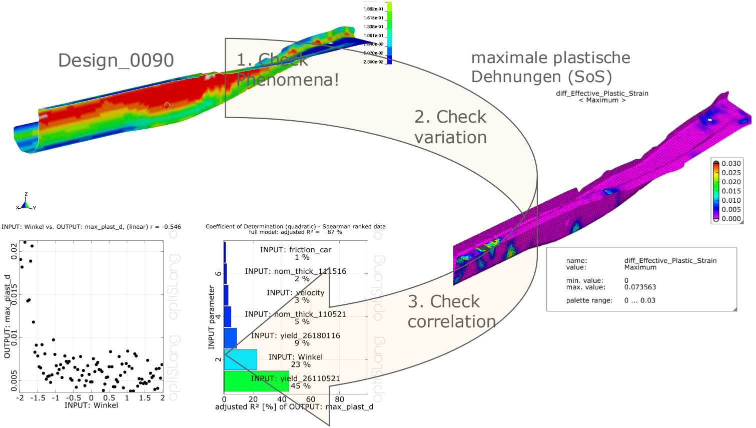


SoS Post processing

Robustness Evaluation Insurance Crash 4

J. Will & T. Frank, WOST 2008

Plastic strains now within the limits. Due to moving the stiffness jump scatter of forming simulation no longer sensitive.



Why random fields in robustness analysis?

- Allows to pinpoint sources for local variabiity
- Achieve high level of prognosis quality in metamodels
- Reduce spatial variability of input and output quantities to a small number of relevant random effects
- Random effects represented by deterministic shapes and corresponding random amplitudes
- Substantially increases prognosis capability of metamodels
- Essential for the efficient meta-modeling of locally nonlinear effects
- Significantly widens application area of robustness analysis and RDO

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