Workshop
"Robust Design Optimization with optiSLang"

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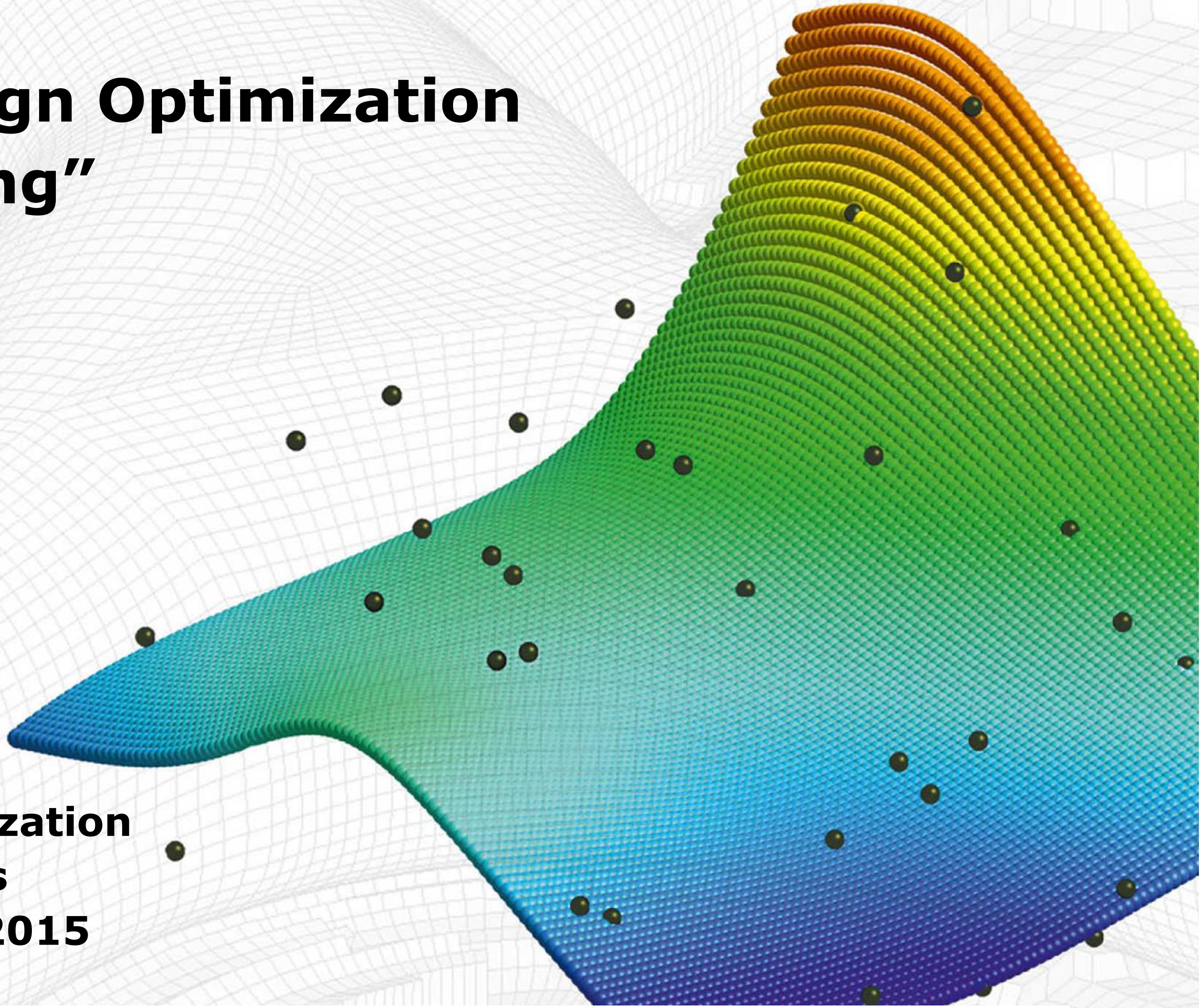
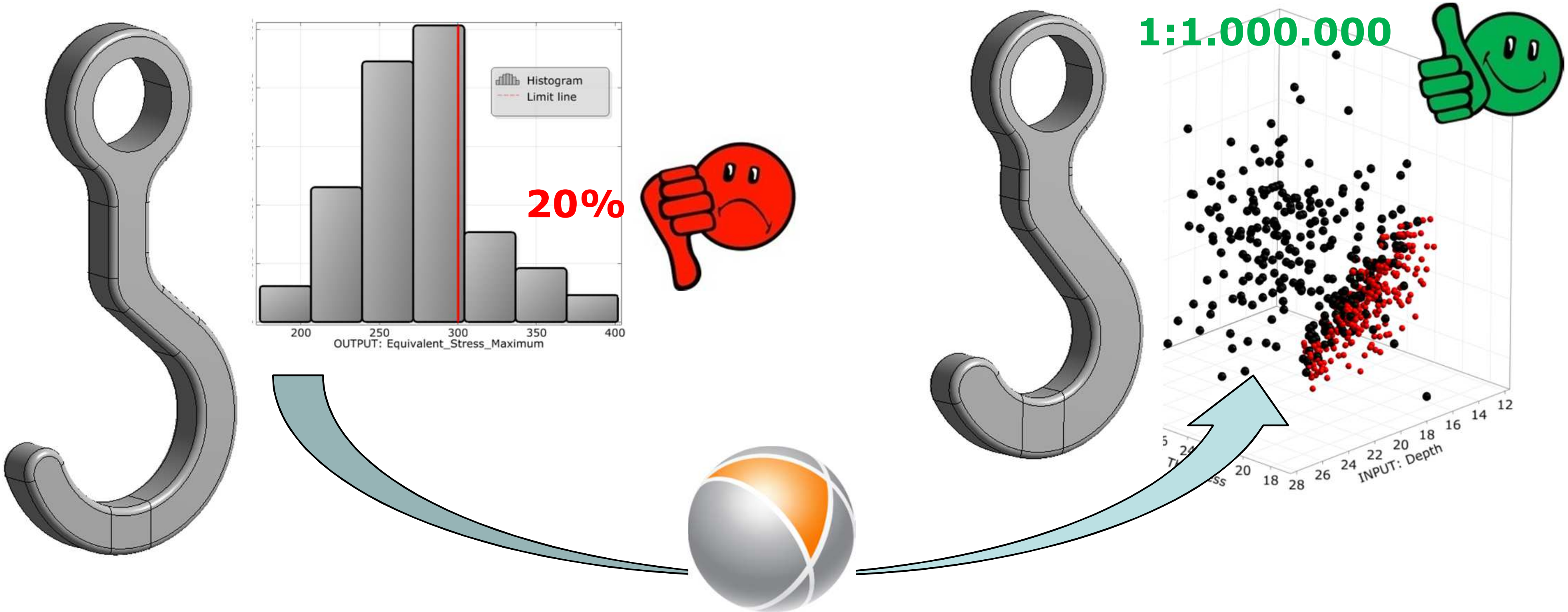


12th Weimar Optimization and Stochastic Days
05-06th November 2015

Outline

- Robustness Analysis
- Reliability Analysis
- Robust Design Optimization
- Illustrative Example: Robust Design Optimization of a Steel Hook



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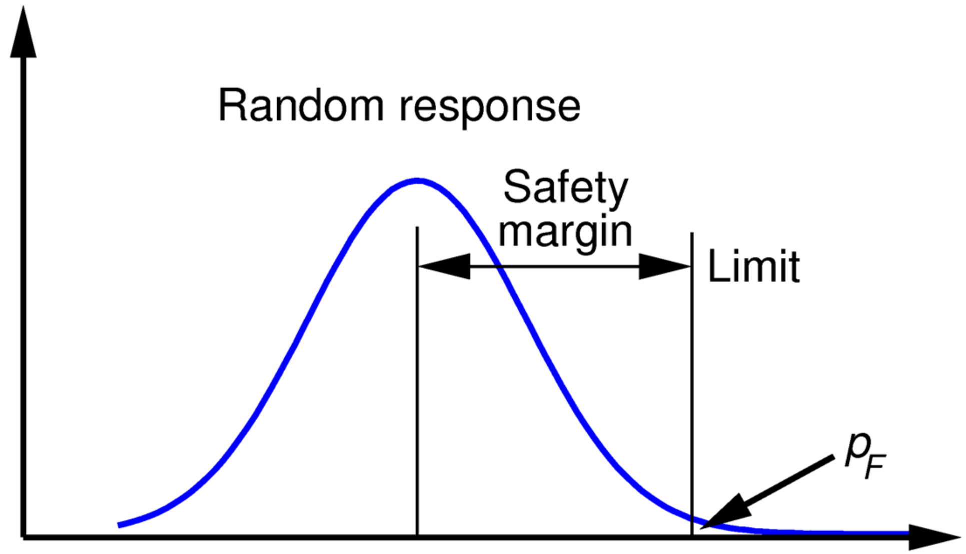
Robustness Analysis

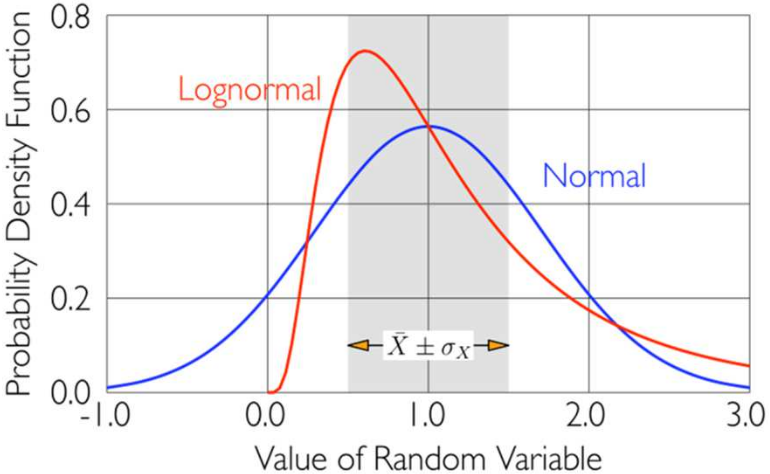
How to Define the Robustness of a Design?

- **Intuitively**: The performance of a robust design is largely unaffected by random perturbations
- **Variance indicator**: The coefficient of variation (CV) of the objective function and/or constraint values is not greater than the CV of the input variables
- **Sigma level**:
  The interval mean+/- sigma level does not reach an undesired performance
  (e.g. design for six-sigma)
- **Probability indicator**:
  The probability of reaching undesired performance is smaller than an acceptable value

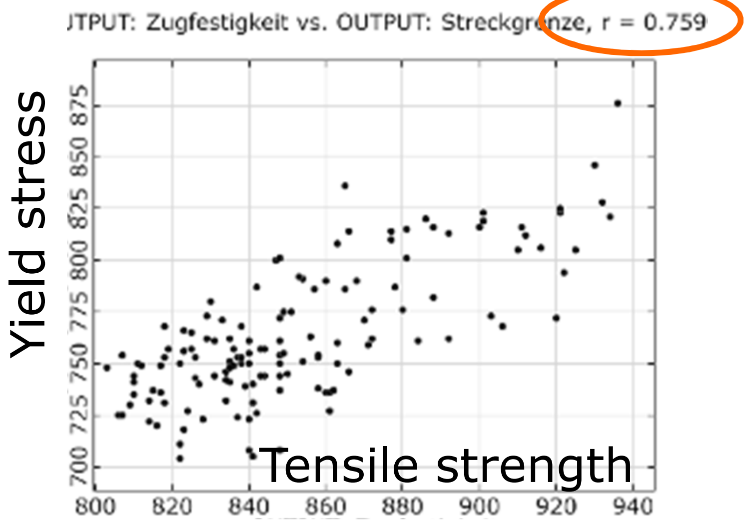


Definition of Uncertainties

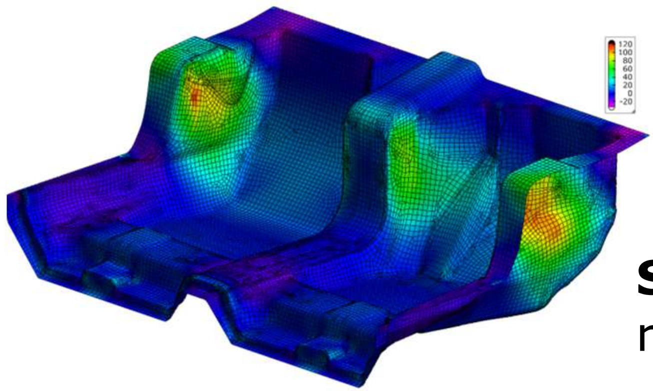
- Translate know-how about uncertainties into proper scatter definition



Distribution functions define variable scatter



Correlation is an important characteristic of stochastic variables



Spatial Correlation = random fields

Definition of Input Scatter in optiSLang

- The random variable properties are defined in the Parameter table.
- Defaults: mean values are the reference values, 10% CoV, NORMAL distribution type
- Probability density functions for all random variables are plotted corresponding to the defined variable properties
- Standard deviation, Coefficient of Variation (CoV), Distribution parameters can be specified
- A nominal design (mean values) can be imported from arbitrary flows or result files

	Name	Parameter type	Reference value	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	m	Opt.+Stoch.	1		NORMAL	1	0.02	2 %	1; 0.02
2	k	Opt.+Stoch.	20		NORMAL	20	1	5 %	20; 1
3	D	Stochastic	0.02		NORMAL	0.02	0.002	10 %	0.02; 0.002
4	Ekin	Stochastic	10		NORMAL	10	1	10 %	10; 1

Definition of Input Correlations in optiSLang

- The definition of linear input correlations is possible
- Pairwise selection or definition of complete correlation matrix

	Name	Parameter type	Reference value	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	m	Opt.+Stoch.	1		NORMAL	1	0.02	2 %	1; 0.02
2	k	Opt.+Stoch.	20		NORMAL	20	1	5 %	20; 1
3	D	Stochastic	0.02		NORMAL	0.02	0.002	10 %	0.02; 0.002
4	Ekin	Stochastic	10		NORMAL	10	1	10 %	10; 1

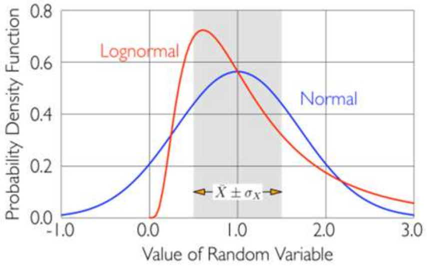
	k	m
m	0.67	1
k	1	0.67

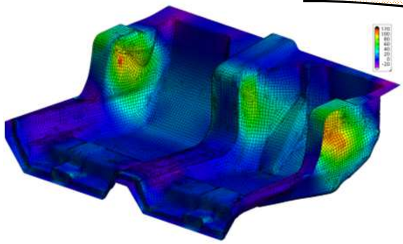
Restore Defaults | OK | Cancel | Apply

- Positive definiteness is checked automatically

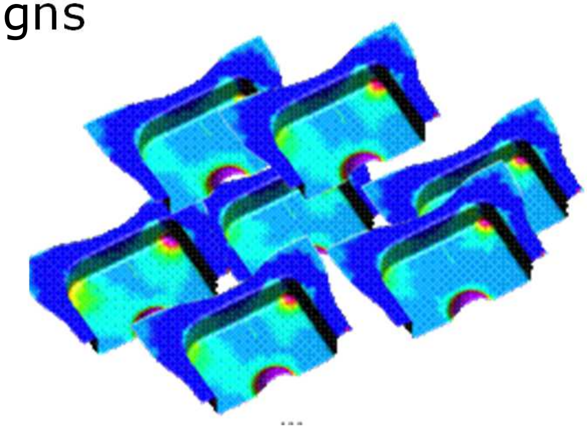
Variance based Robustness Analysis

1) Define the robustness space using scatter range, distribution and correlation

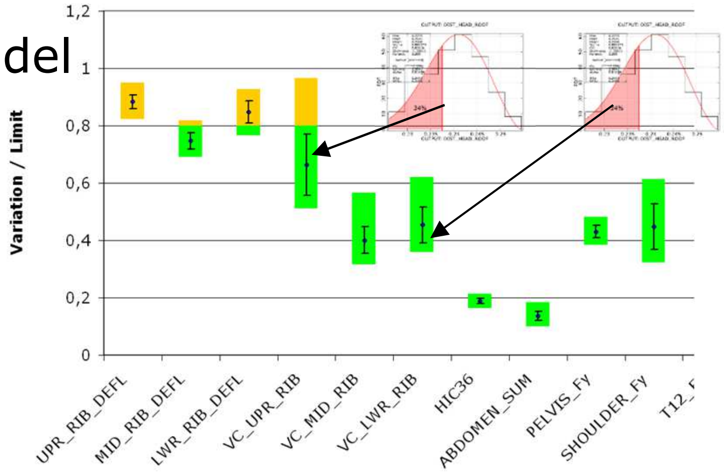




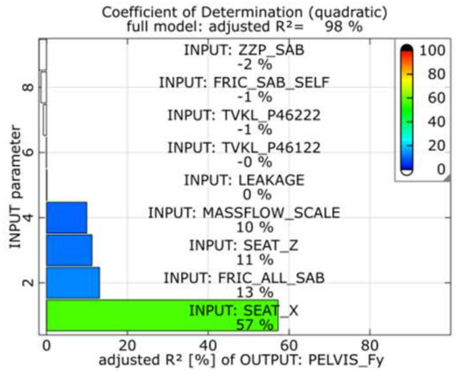
2) Scan the robustness space by producing and evaluating *n* designs



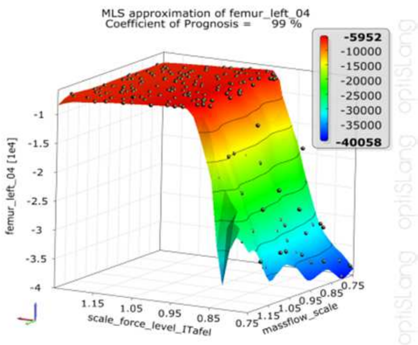
3) Check the variation



4) Check the explainability of the model

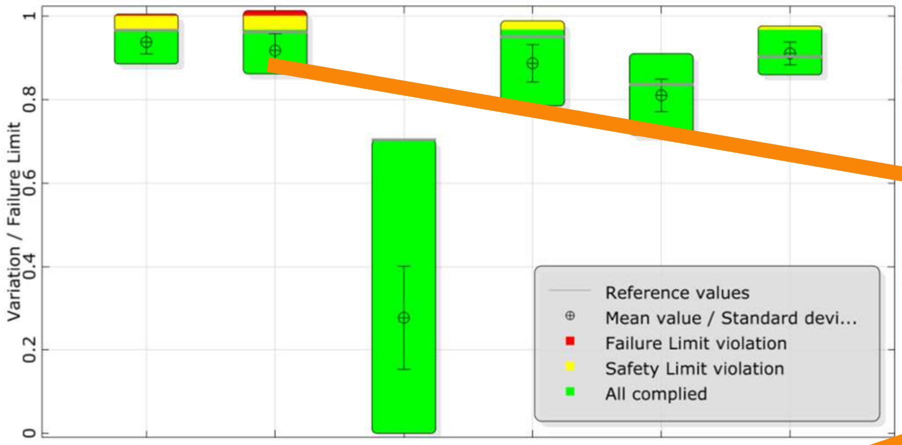


5) Identify the most important scattering variables

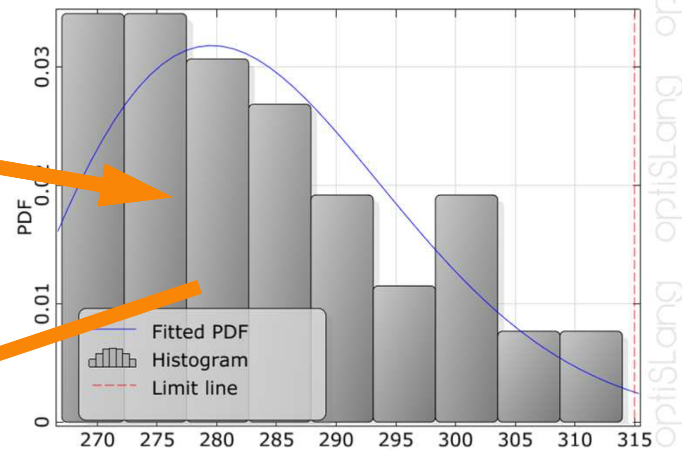


Robustness Measures

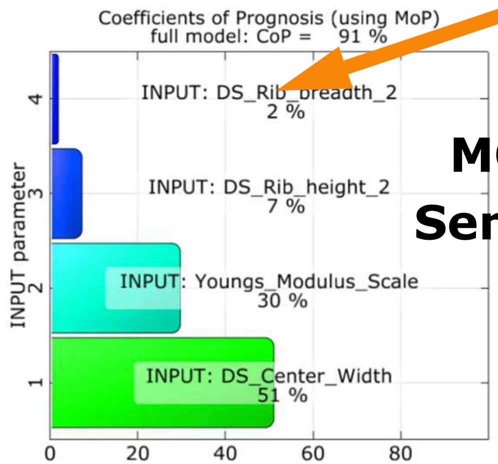
Traffic light plot



Histogram & Statistical Data



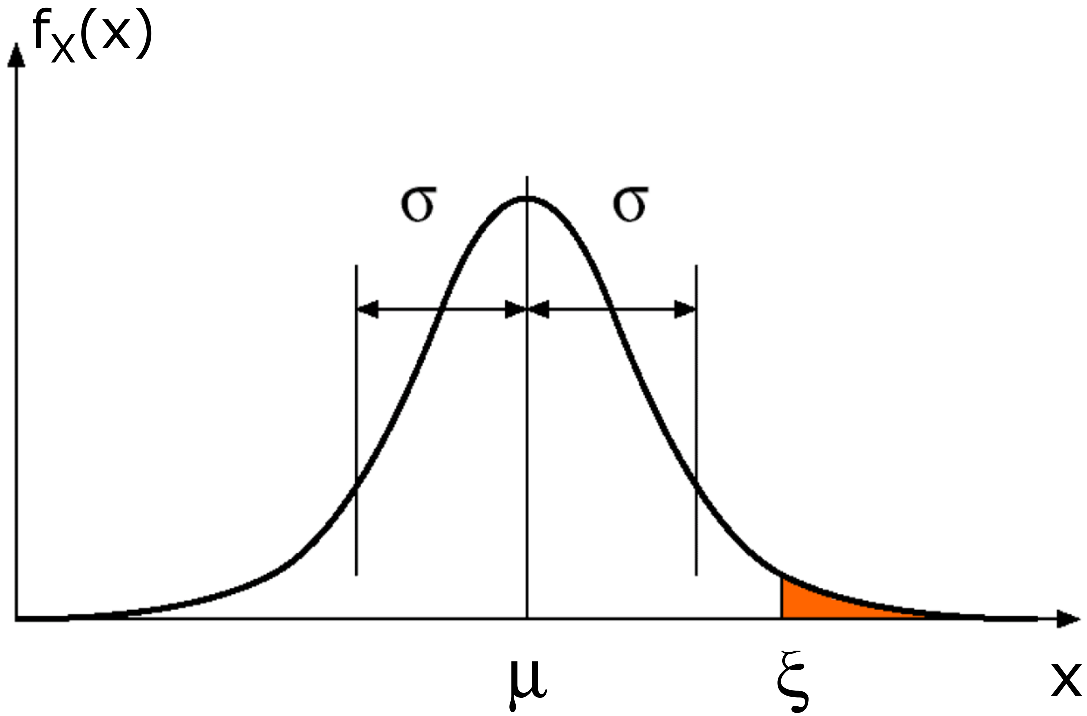
Statistic data			
Min:	267	Max:	314
Mean:	284.4	Sigma:	12.49
CV:	0.04393		
Skewness:	0.5633	Kurtosis:	2.338
Fitted PDF: Rayleigh			
Mean:	284.4	Sigma:	12.49
Limit x = 315			
P_rel:	1	1 - P_rel:	0
P_fit:	0.983168	1 - P_fit:	0.016832
Sigma-Level:	2.4496		



MOP/CoP Sensitivities

Exceedance Probability

- Probability of reaching values above a limit, for Normal distribution:



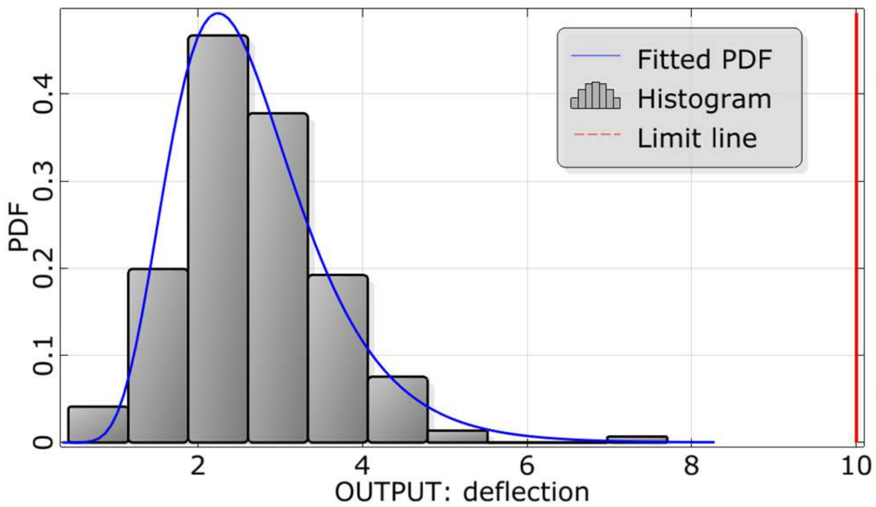
$$P_\xi = P[X \geq \xi]$$

ξ	μ	$\mu + \sigma$	$\mu + 2\sigma$	$\mu + 3\sigma$	$\mu + 4\sigma$	$\mu + 5\sigma$
P_ξ	$5.0 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-2}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-5}$	$2.9 \cdot 10^{-7}$

Variance based Robustness Analysis

- Sufficient estimates of **mean** and **variance** with 50 to 100 samples
- Distribution fit and extrapolation of **small event probabilities** may be very inaccurate

- More precise **reliability methods** should be applied to verify small probabilities



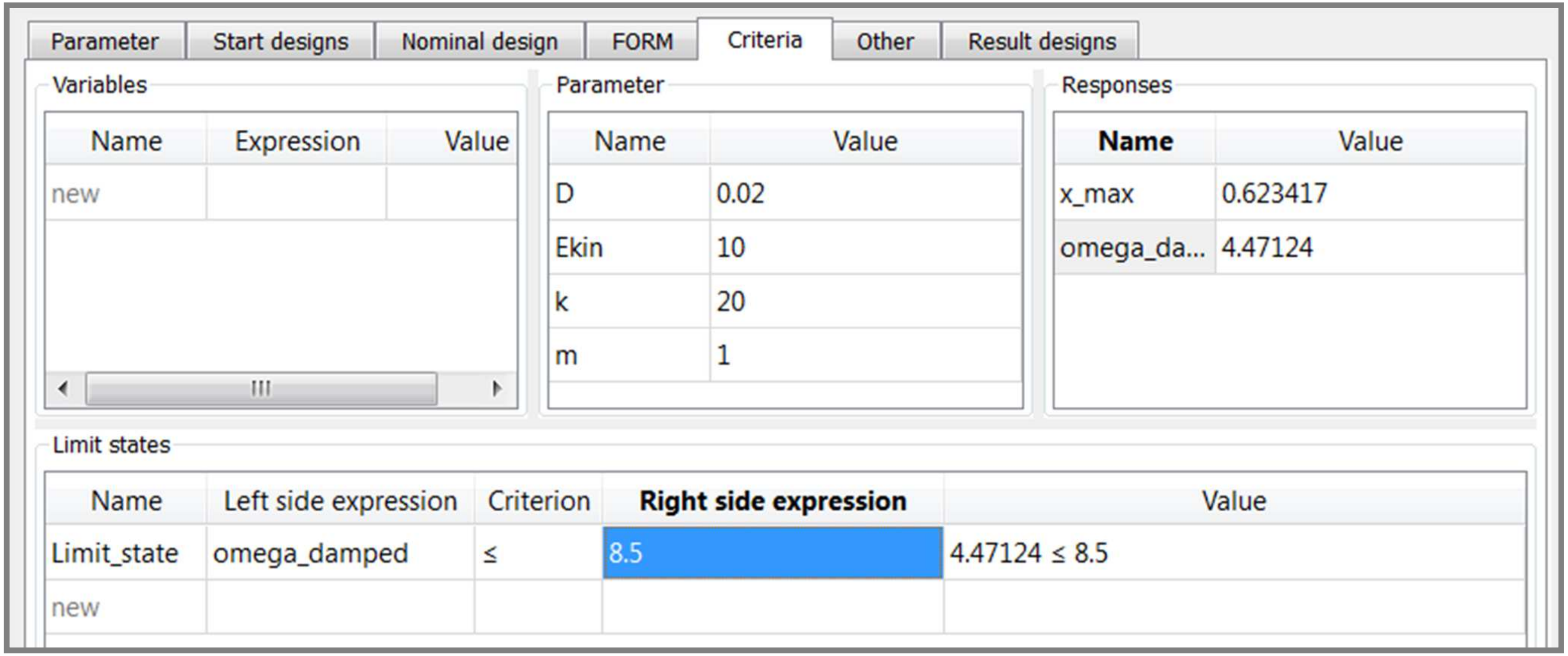
Statistic data			
Min:	0.4254	Max:	7.704
Mean:	2.67	Sigma:	0.9357
CV:	0.3505		
Skewness:	1.017	Kurtosis:	6.465

Fitted PDF: Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	1	1 - P_fit:	2.33147e-015
Sigma-Level:	7.83397		

Fitted PDF: Log-Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	0.999974	1 - P_fit:	2.56303e-005
Sigma-Level:	7.83397		

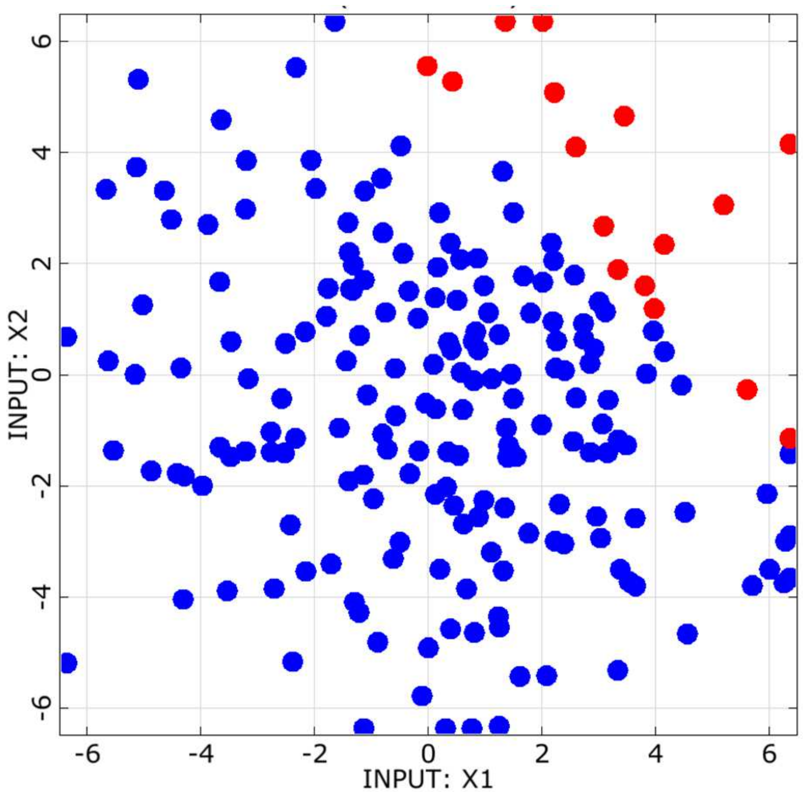
Reliability Analysis

Definition of Limit State Functions in optiSLang



- The “positive”, i.e. non-failed case is expressed
- Several criteria are automatically interpreted as series system (failure is assumed, if at least one LSF is violated)

Monte Carlo Simulation



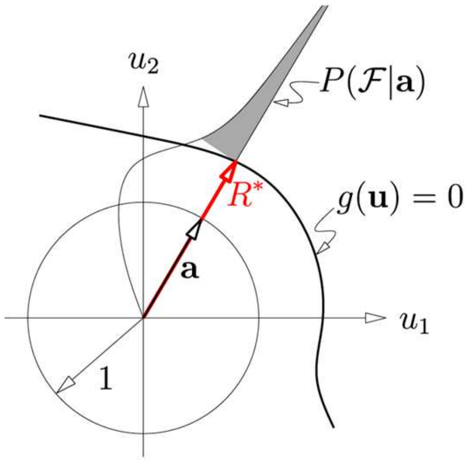
Sigma level	Number of required samples (σ / P_F = 20 %)
$\pm 2\sigma$	1.100
$\pm 3\sigma$	18.500
$\pm 4.5\sigma$	7.300.000

$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} I\left(g(\mathbf{x}_i)\right),$$

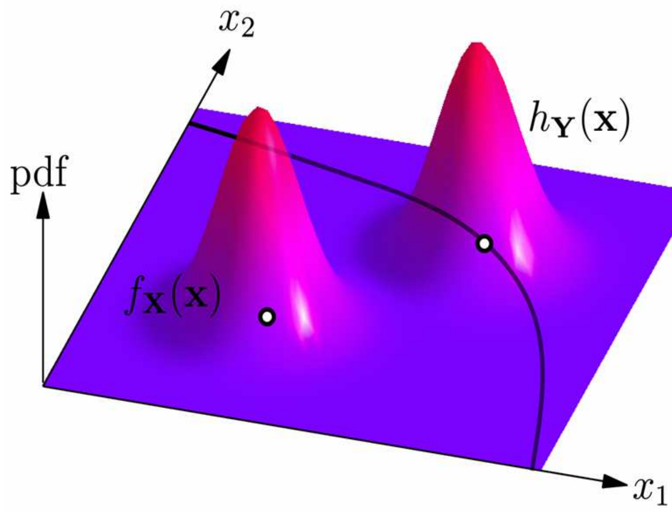
- Robust for arbitrary limit state functions
- Independent of number of random variables
- Huge effort for small failure probabilities
- ➢ Should be applied only for benchmarking

Advanced Methods for Reliability Analysis

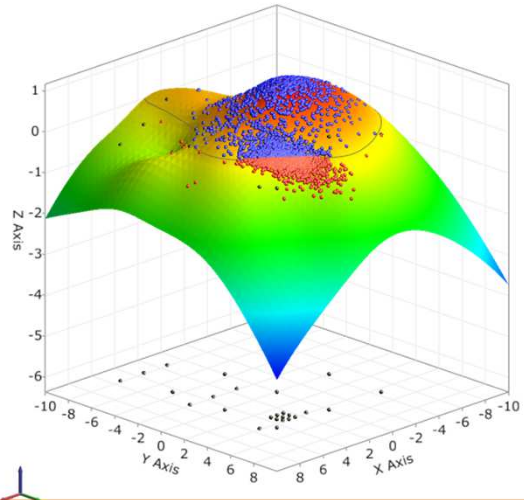
Directional Sampling



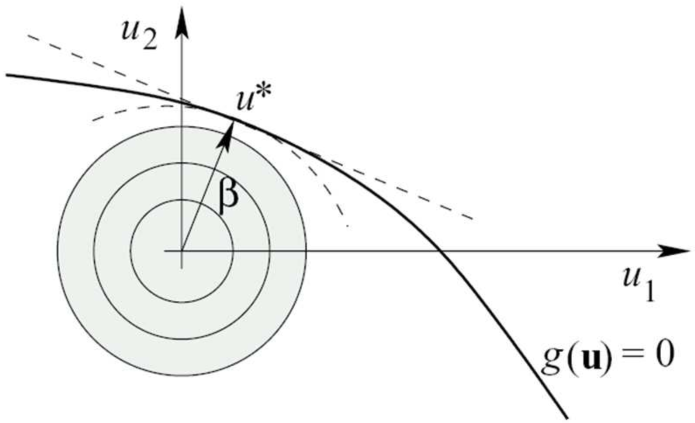
Adaptive Importance Sampling



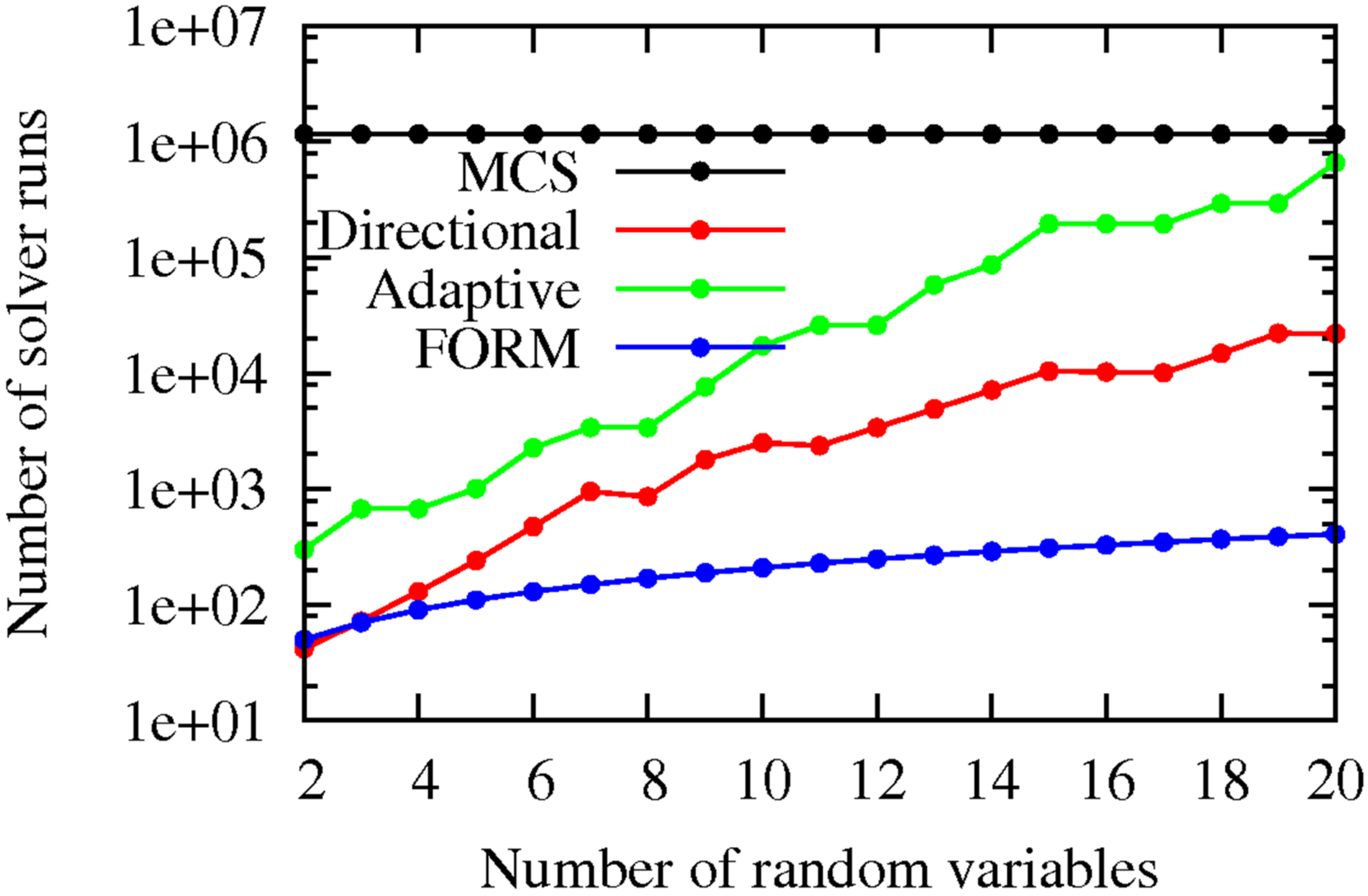
Adaptive Response Surface Method



First Order Reliability Method



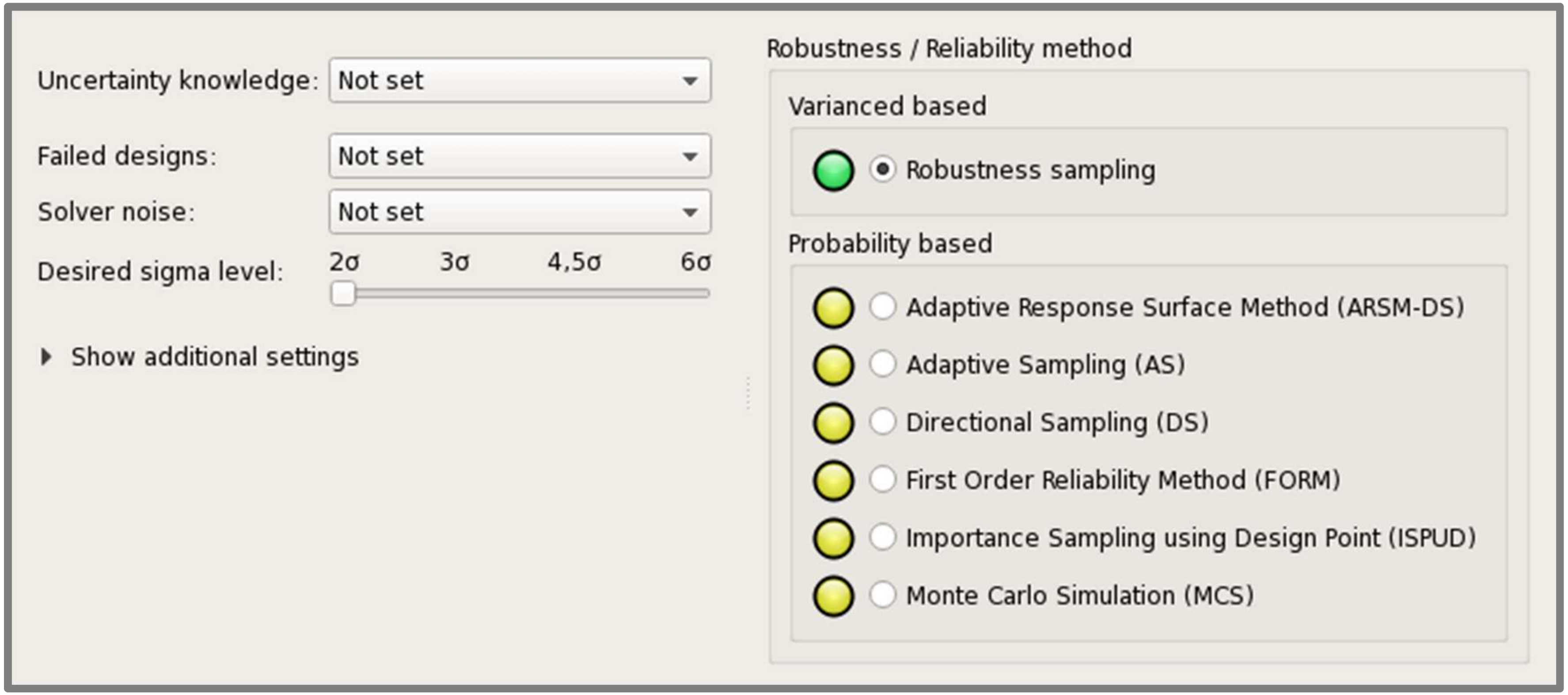
Performance



- Approximation methods are much more efficient

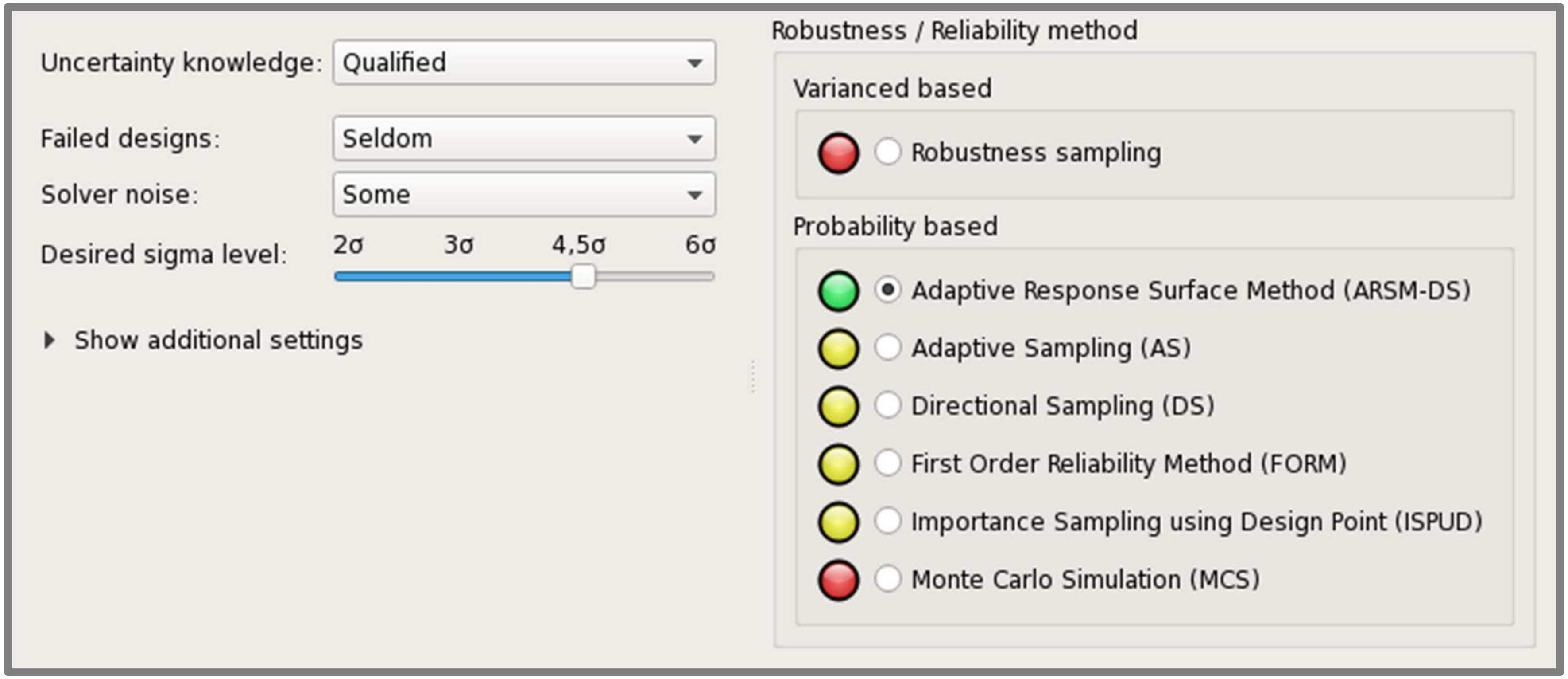
Robustness-Reliability Wizard

Robustness/Reliability Wizzard



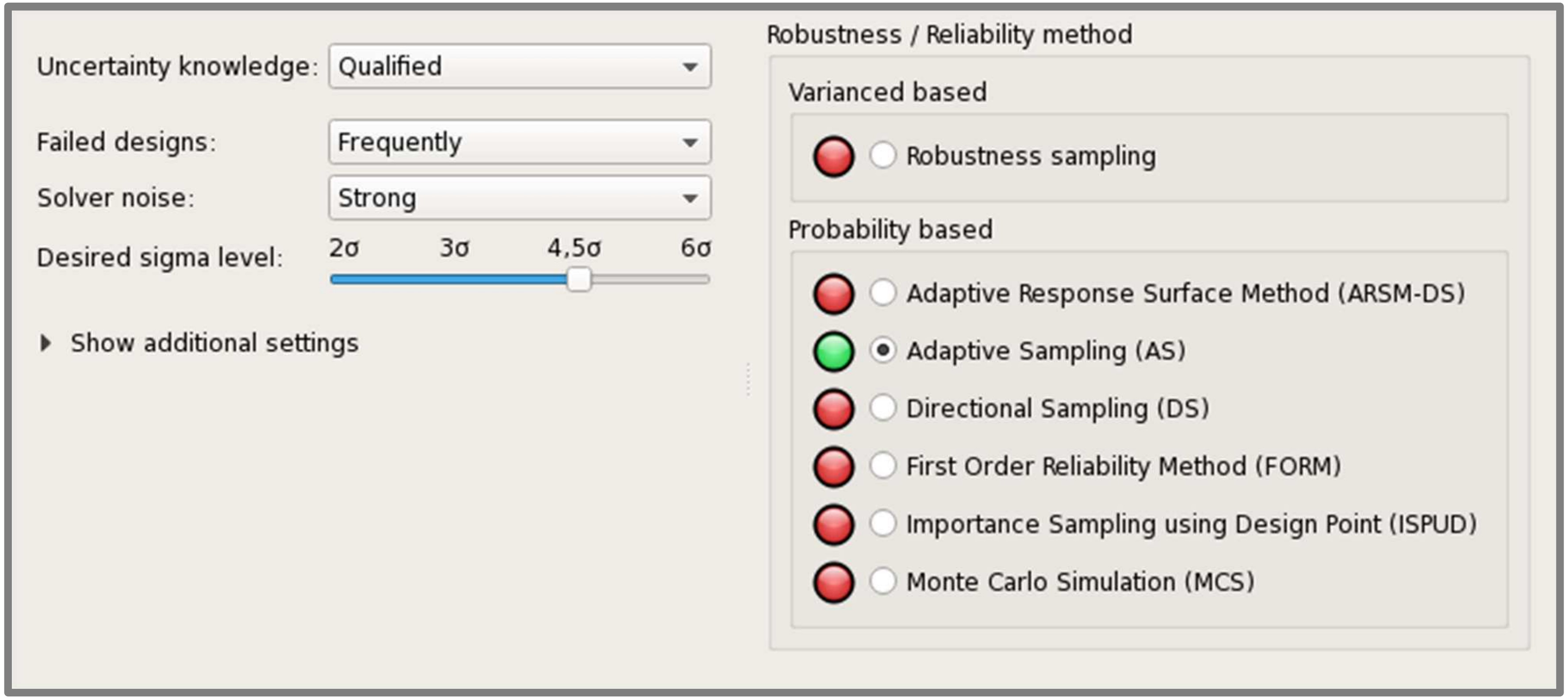
- If **no limit state** is defined or
- If the uncertainty knowledge is **not qualified**
- ➢ Robustness sampling is recommended
- ➢ However, an extrapolation for more than 3 sigma is difficult

Robustness/Reliability Wizzard



- **Seldom failed designs** and **some solver noise** can be handled by all reliability methods
- For up to 15 variables, the ARSM-DS is the best compromise between accuracy and efficiency

Robustness/Reliability Wizzard

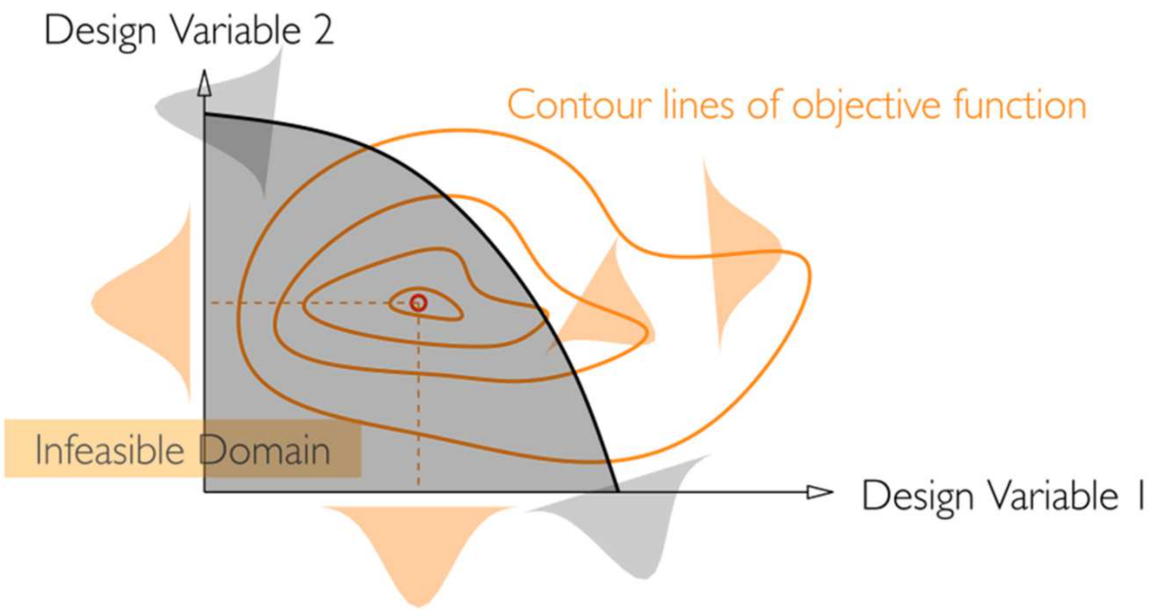


- In case of **frequently failed designs** and **strong solver noise** ARSM-DS, FORM and directional sampling may be not robust enough
- ➢ Adaptive sampling is the best compromise between accuracy and efficiency up to 15 variables

Robust Design Optimization

Robust Design Optimization

- Robust Design Optimization (RDO) optimizes the design performance while taking into account scatter of design (optimization) variables *and* other tolerances or uncertainties
- As a consequence of input scatter the location of the optima as well as the contour lines of constraints may vary



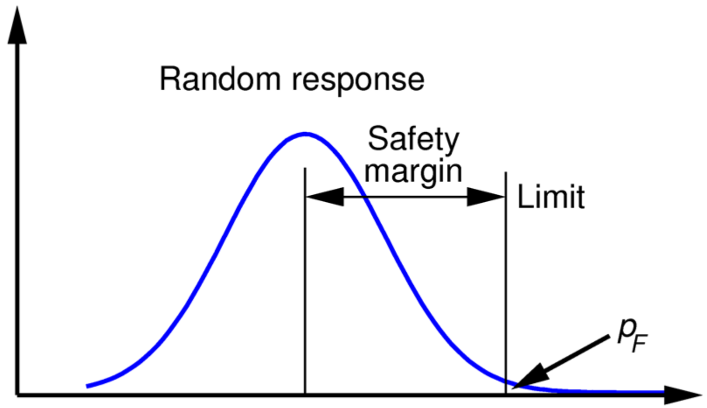
- To proof Robust Designs, safety distances are quantified with variance or probability measures using stochastic analysis

Methods for Robust Design Optimization

Variance-based RDO

- Safety margins of all critical responses are larger than a specified sigma level (e.g. Design for Six Sigma)

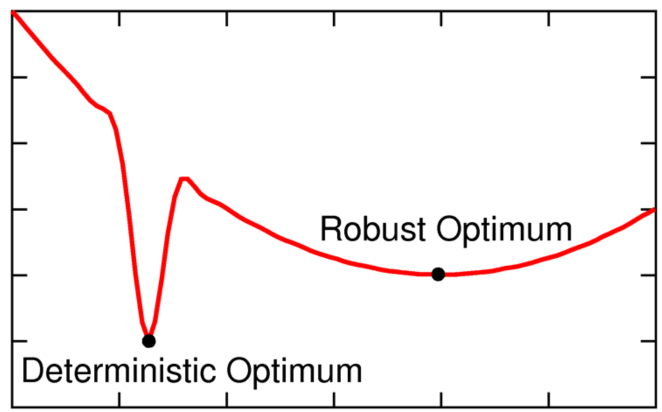
$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$



Reliability-based RDO

- Failure probability with respect to given limit states is smaller as required value

$$p_F \leq p_F^{target}$$

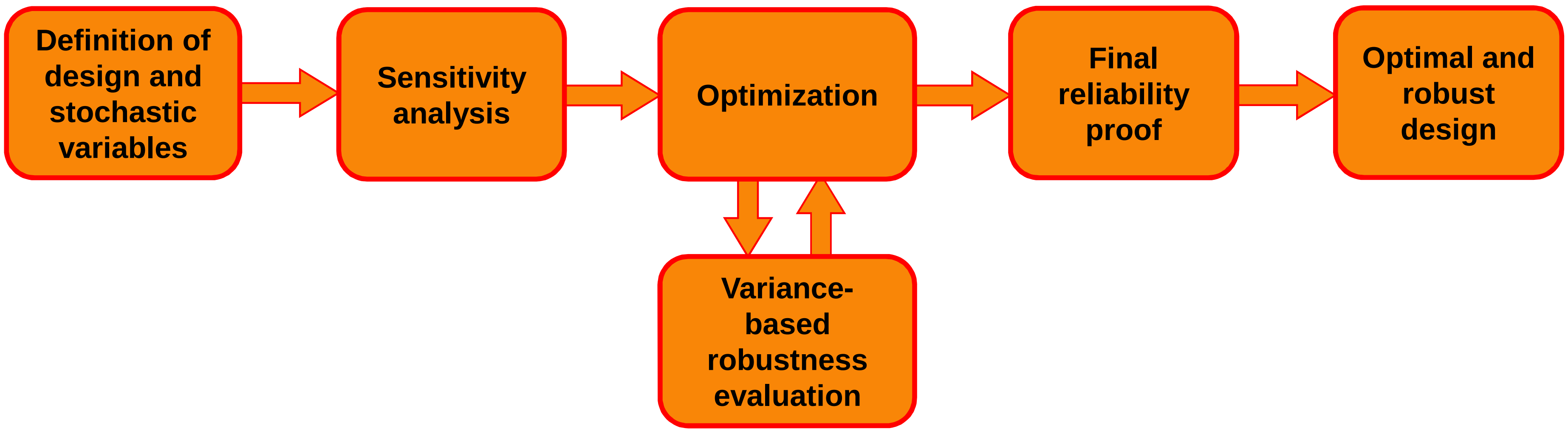


Taguchi-based RDO

- Taguchi loss functions
- Modified objective function

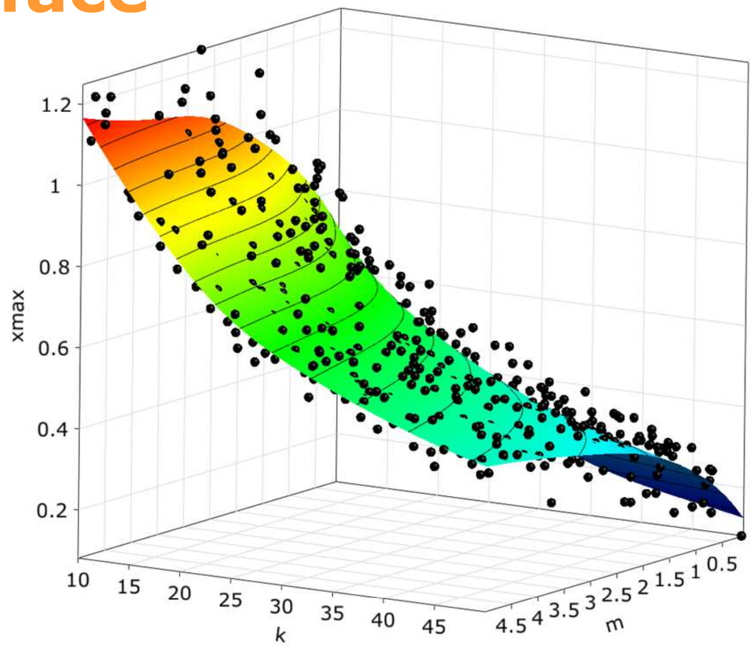
Coupled Robust Design Optimization

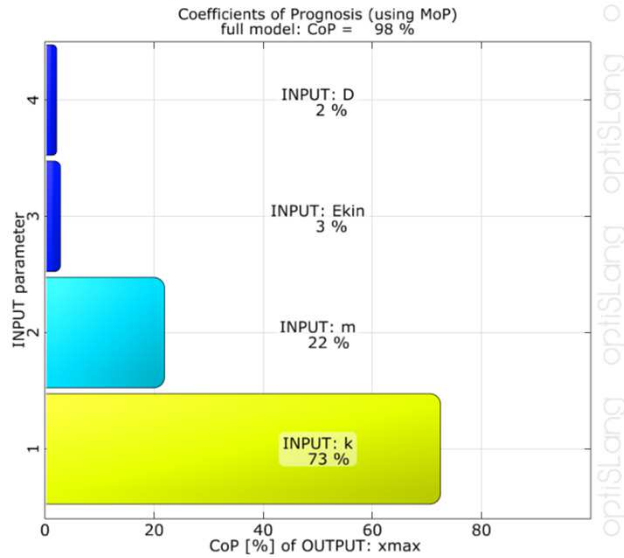
- Fully coupled optimization and robustness/reliability analysis
- For each design during the optimization procedure (nominal design), the robustness/reliability analysis is performed
- Applicable to variance-, reliability- and Taguchi-based RDO
- ➢ Our efficient implementation uses small sample variance-based robustness measures during the optimization and a final (more accurate) reliability proof
- ➢ But still the procedure is often not applicable to complex CAE models



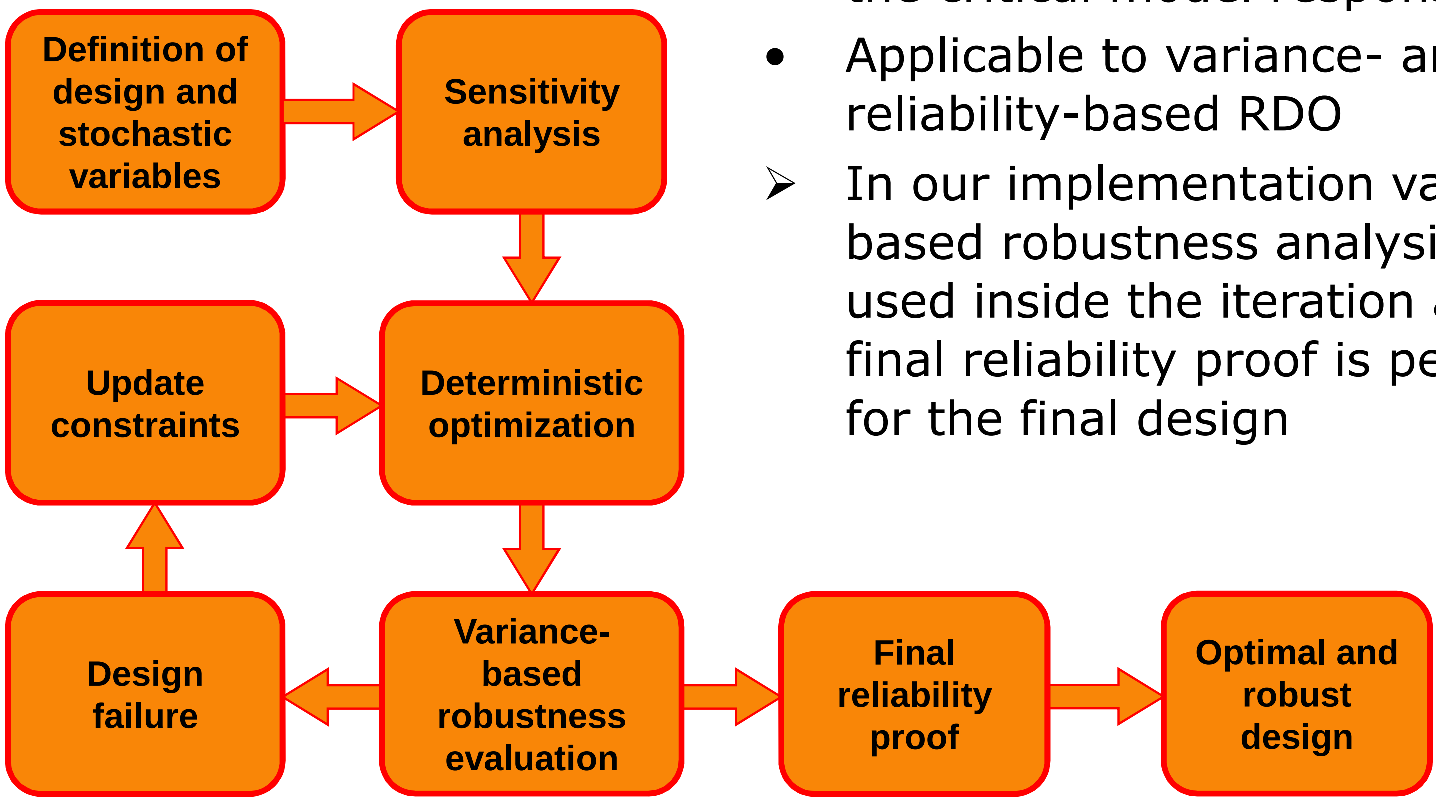
RDO on Global Response Surface

- Approximation of model responses in mixed optimization/stochastic space
- Simultaneous RDO is performed on a global response surface
- Applicable to variance-, reliability- and Taguchi-based RDO
- Approximation quality significantly influences RDO results
- ➢ Final robustness/reliability proof is required
- Pure stochastic variables have small influence compared to design variables
- ➢ Important local effects in the stochastic space may be not represented





Iterative Robust Design Optimization



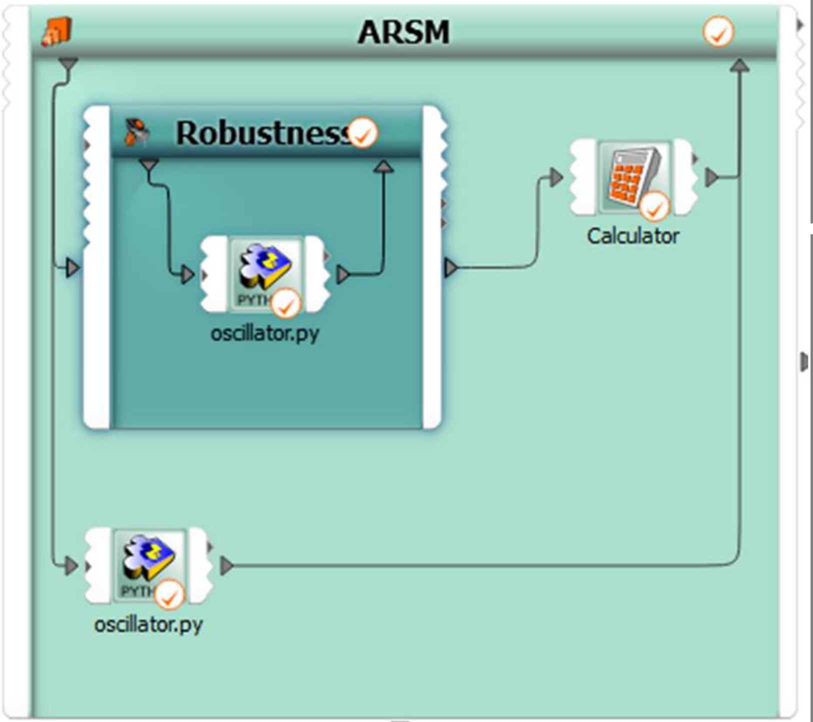
- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety margins are adjusted for the critical model responses
- Applicable to variance- and reliability-based RDO

➢ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design

Coupled RDO in optiSLang

- Nested loop enables the fully coupled RDO
- Optimizer has to handle statistical errors of inner robustness analysis
- Sigma level as constraint

	ID	Type	Value	Expression
1	mean_omega	UNINITIALIZED		mean(omega_damped)
2	std_omega	UNINITIALIZED		stddev(omega_damped)
3	mean_xmax	UNINITIALIZED		mean(x_max)
4	std_xmax	UNINITIALIZED		stddev(x_max)
5	sigma_level_omega	UNINITIALIZED		(8.5-mean_omega)/std_omega



Objectives

Name	Criterion	Expression	Value
Objective	MIN	mean_xmax	0
new			

Constraints

Name	Left side expression	Criterion	Right side expression	Value
Constraint	sigma_level_omega	≥	4.5	0 ≥ 4.5
new				

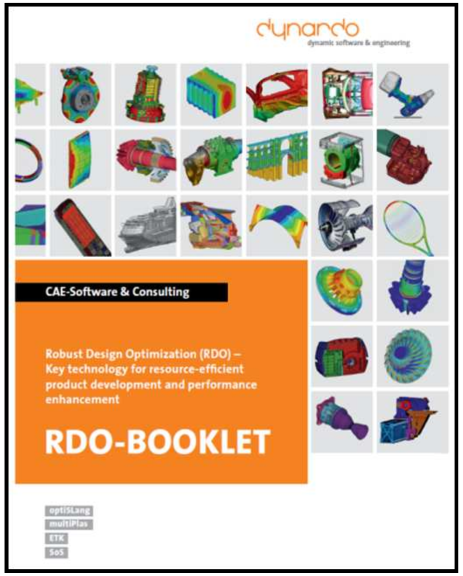
Iterative Robust Design Optimization of a Steel Hook

Summary

- Highly optimized structures tend to loose robustness
- Variance-based robustness analysis can estimate small sigma levels
- Reliability analysis is necessary to proof small failure probabilities
- Fully coupled optimization and reliability analysis is often not applicable to real world problems
- Iterative optimization/variance-based analysis with final reliability proof is applicable to industrial tasks

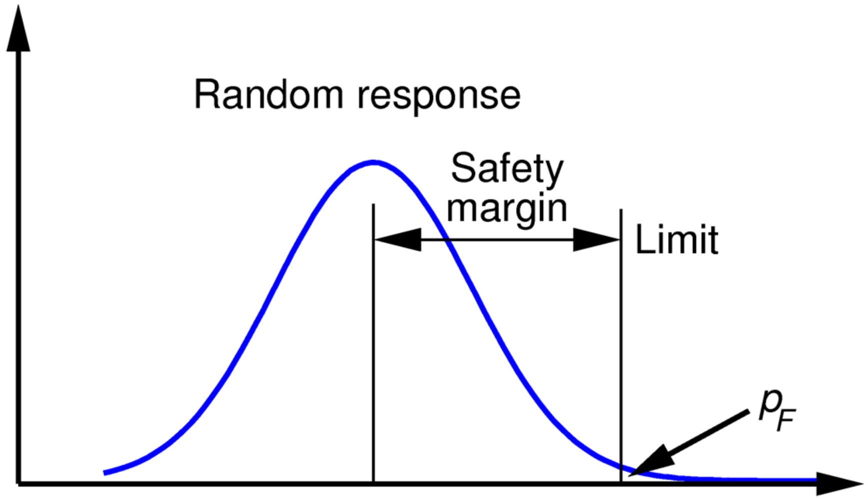
Further Information

- For hook RDO example contact support@dynardo.com
- One day special seminar "Robust Design & Reliability" Weimar, 3rd December 2015
- RDO booklet in Your conference material
- **Discuss with us at the WOST conference!**



Appendix

Robustness in terms of constraints



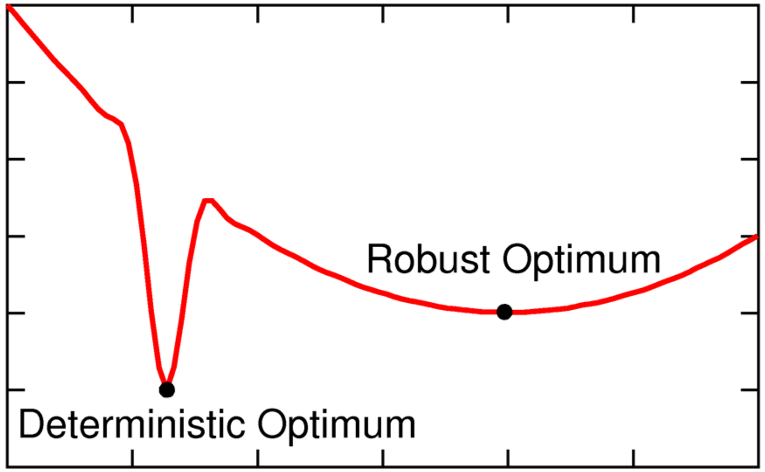
- Safety margin (sigma level) of one or more responses *y*:

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective



- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Simulation of Input Correlations

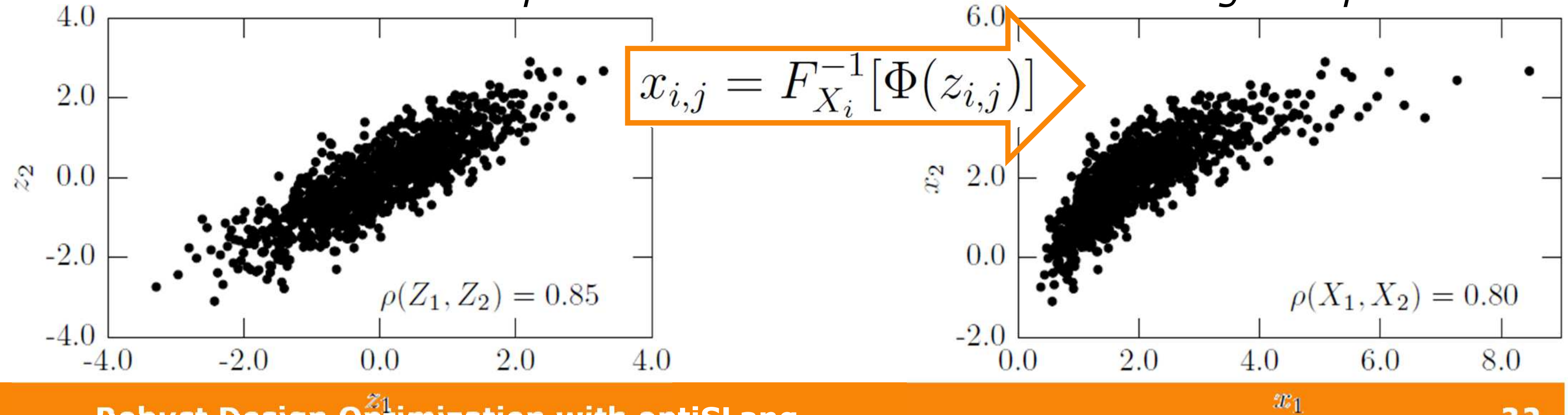
The Nataf Model

- Samples are generated according to a multi-dimensional standard normal distribution

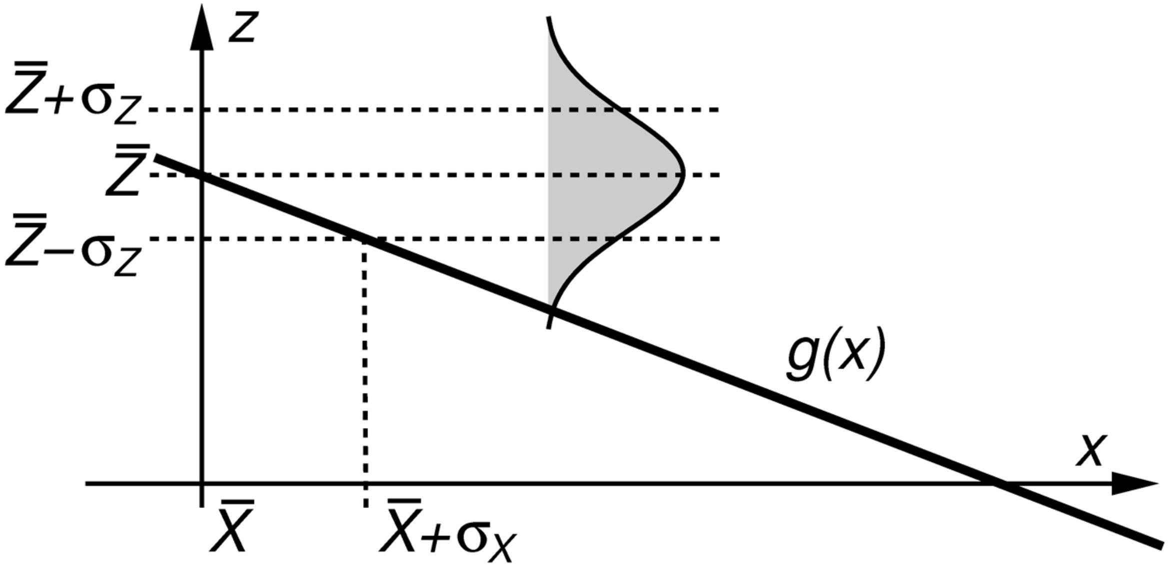
$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{C_{XX}}|}} \exp\left[-\frac{1}{2}(\mathbf{x}-\bar{\mathbf{X}})^T \mathbf{C}_{\mathbf{XX}}^{-1}(\mathbf{x}-\bar{\mathbf{X}})\right]$$

- For each random variable the original marginal distribution is obtained by using the inverse distribution function
- Required linear correlation coefficients in standard normal space are iteratively obtained from correlations in original space

Standard normal space *Original space*

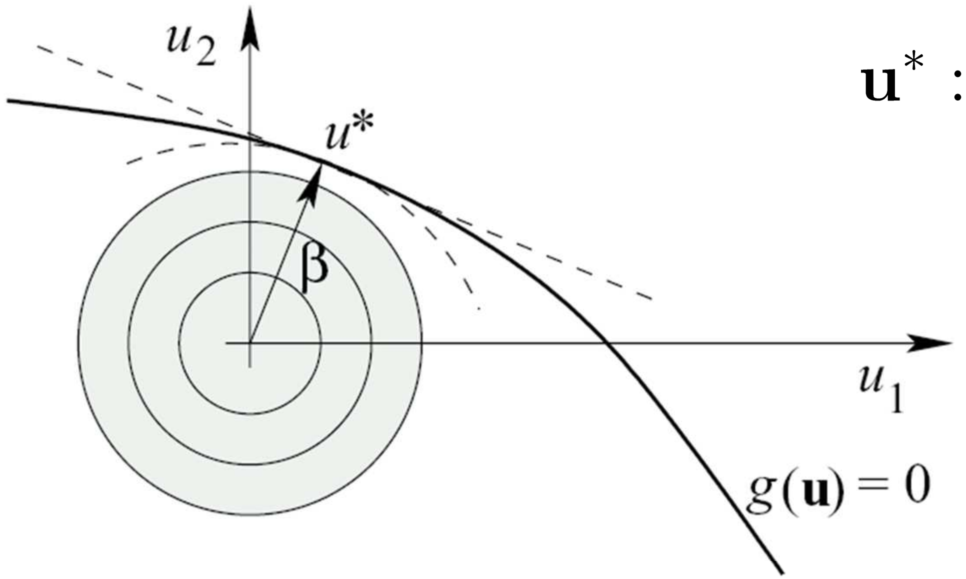


First Order Second Moment Concept (FOSM)



- Linearization of limit state function at mean value vector
  $$g(\mathbf{x}) \approx g(\mathbf{x}_0) + \nabla g(\mathbf{x})^T(\mathbf{x} - \mathbf{x}_0), \quad \mathbf{x}_0 = \bar{\mathbf{X}}$$
- Approximation of safety margin with normal distribution yields
  $$\bar{Z} = g(\bar{\mathbf{X}}), \quad \sigma_Z^2 = \nabla g(\mathbf{x})^T \mathbf{C}_{\mathbf{XX}} \nabla g(\mathbf{x}), \quad P_F = \Phi(-\frac{\bar{Z}}{\sigma_Z})$$
- Equivalent to sigma level approach
- Not available in optiSLang!

First Order Reliability Method (FORM)

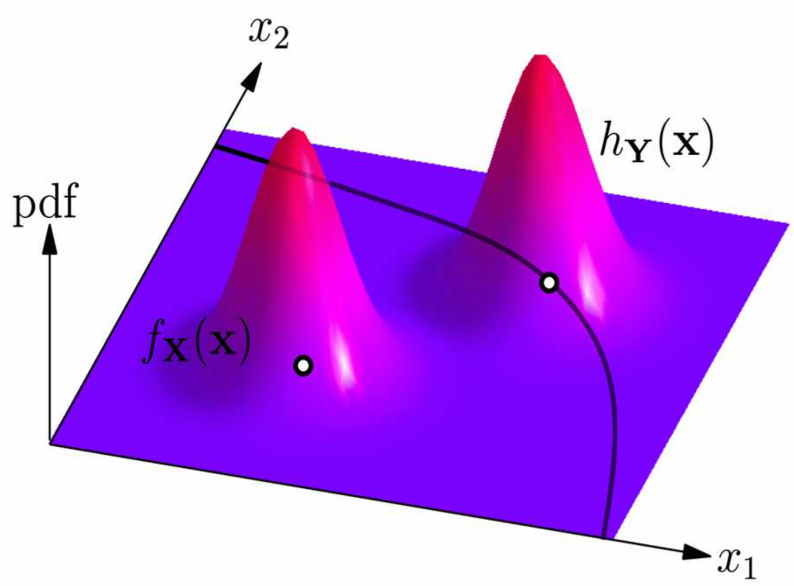


$$\mathbf{u}^* : \frac{1}{2}\mathbf{u}^T\mathbf{u} \rightarrow \min, \quad g(\mathbf{u}) = 0$$

$$\hat{P}_F = \Phi(-\beta)$$

- Search for failure point with maximum probability (MPP)
- Limit state function is linearized around design point
- Default algorithm is gradient-based minimization of distance to mean (in standard normal space)
- ➢ Requires continuously differentiable limit state function
- ➢ Multiple design points (local minima) are not supported
- ➢ Independent search for each limit state may be more robust

Importance Sampling

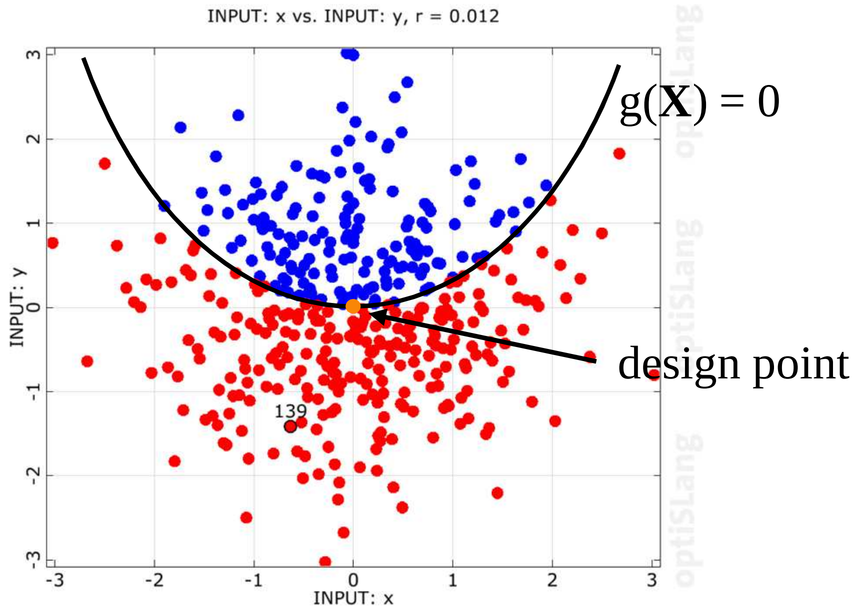


$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{h_{\mathbf{Y}}(\mathbf{x}_i)} I\left(g(\mathbf{x}_i)\right)$$

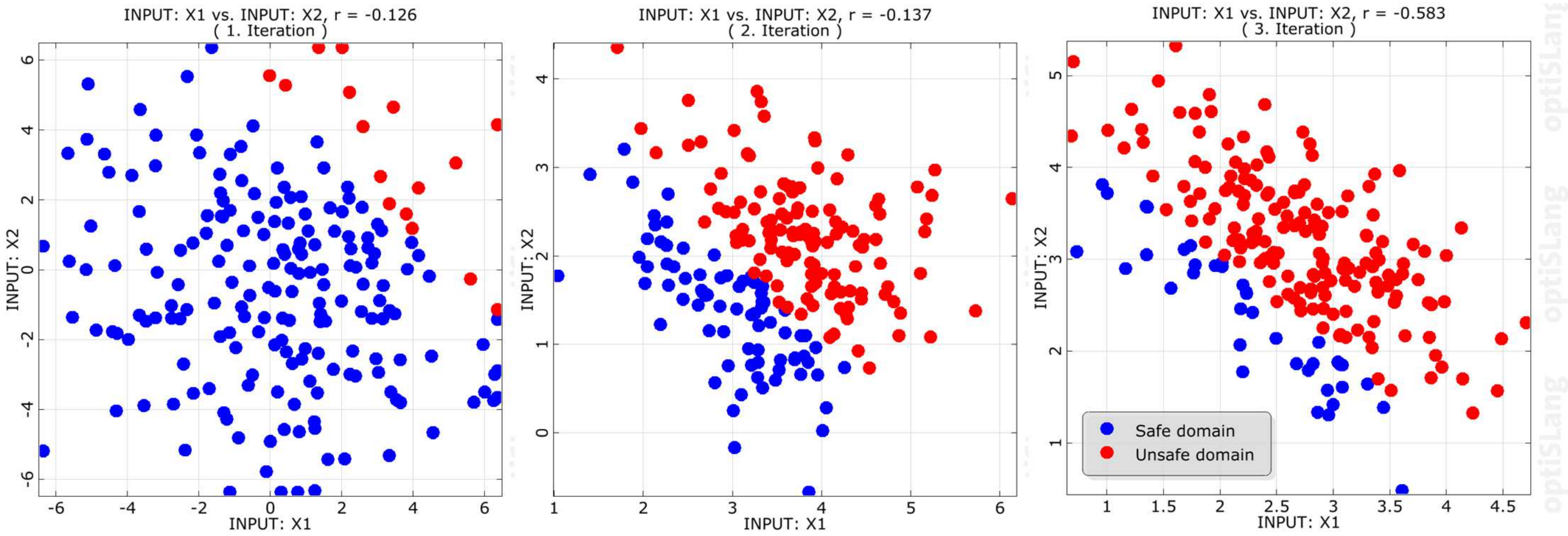
- Sampling around design point to capture nonlinear LSF
- Indicator function is weighted by relation between original and modified sampling density
- Different strategies exist to estimate an "optimal" sampling density
- Applicable for noisy limit state functions with significant global trends
- Applicable for smooth and even discontinuous limit state functions

Importance Sampling Using Desing Point (ISPUD)

- Based on FORM
- Sampling density is centered at the design point
- ➢ Requires continuously differentiable limit state function
- ➢ Multiple design points (local minima) are not supported
- ➢ May be able to mitigate error due to linearization in FORM (oscillating limit state surface)
- ➢ Moderate number of random variables

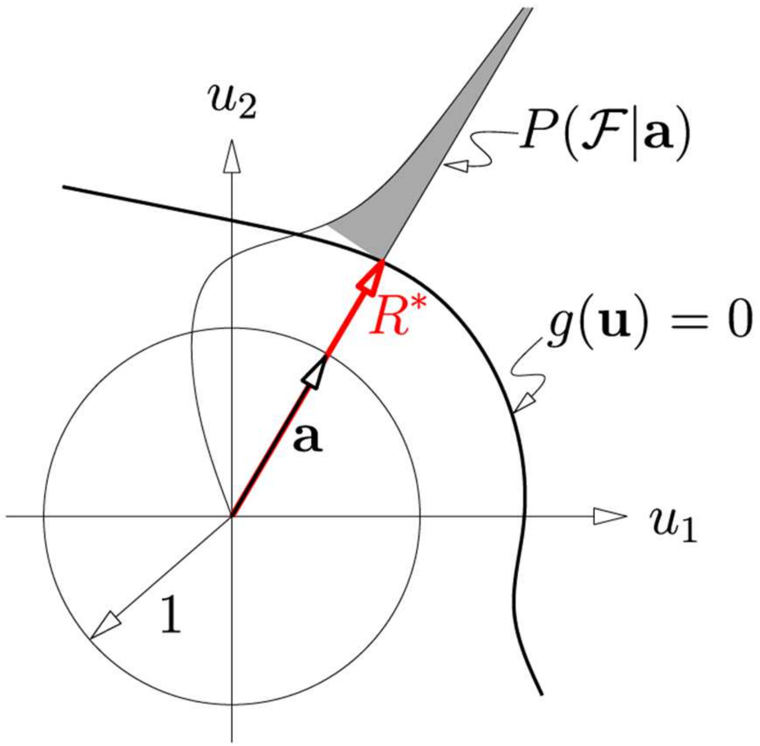


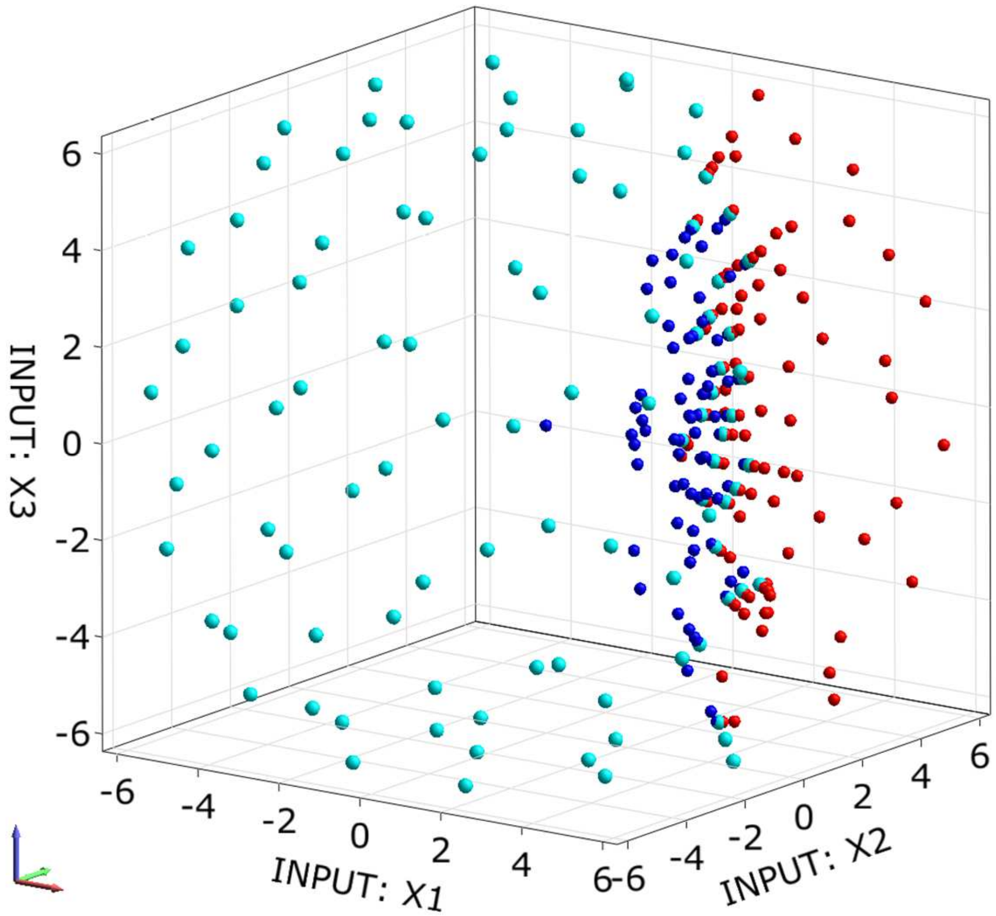
Adaptive Importance Sampling



- Importance sampling approach
- Search for dominant failure region by 2-3 sampling iterations
- No design point required
- Applicable for smooth and even discontinuous limit state functions
- Limited for small number of random variables

Directional Sampling





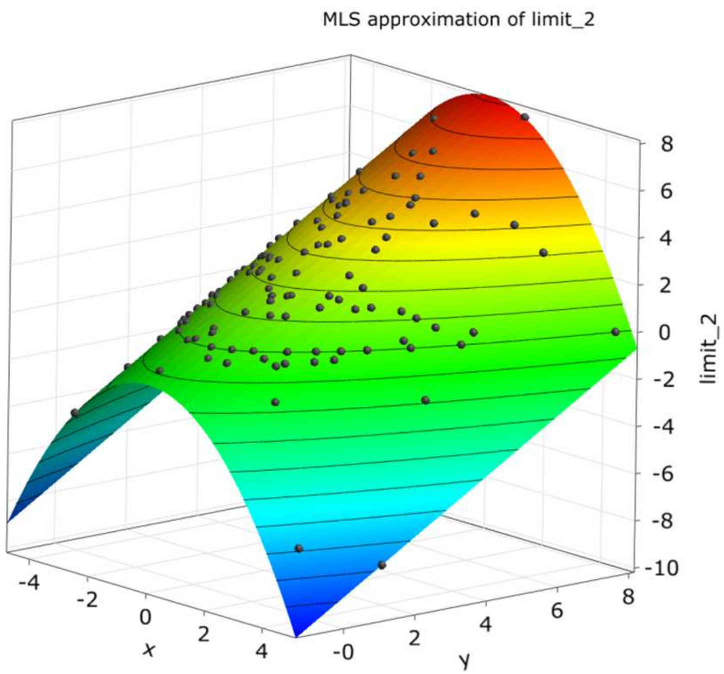
- Radial search for multiple failure regions
- Applicable for smooth and even discontinuous limit state functions
- Limited to small number of random variables

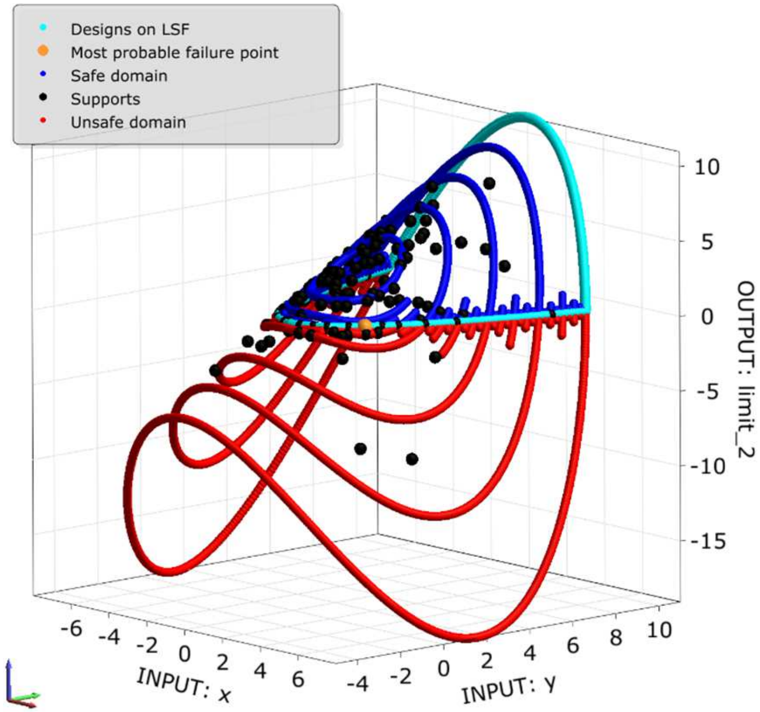
Adaptive Response Surface Method

- The limit state function is approximated by an Adaptive Response Surface Method using a Moving Least Squares model
- Directional Sampling is performed on the Response Surface
- Additional supports are added near the limit state surface in regions of high probability density

➢ Applicable to a wide range of limit state functions

➢ Efficient for a moderately high number of random variables





Methods for Robust Design Optimization

Taguchi-based RDO

- Target value is optimal

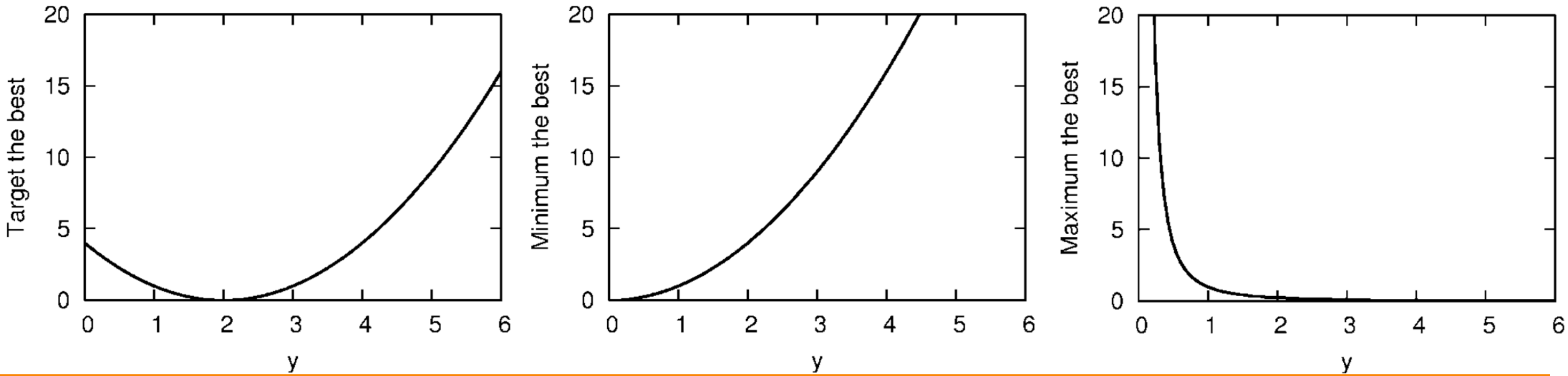
$$f(y) = \frac{k}{N} \sum (y_i - y_{target})^2$$

- Minimum is optimal (requires positive objective)

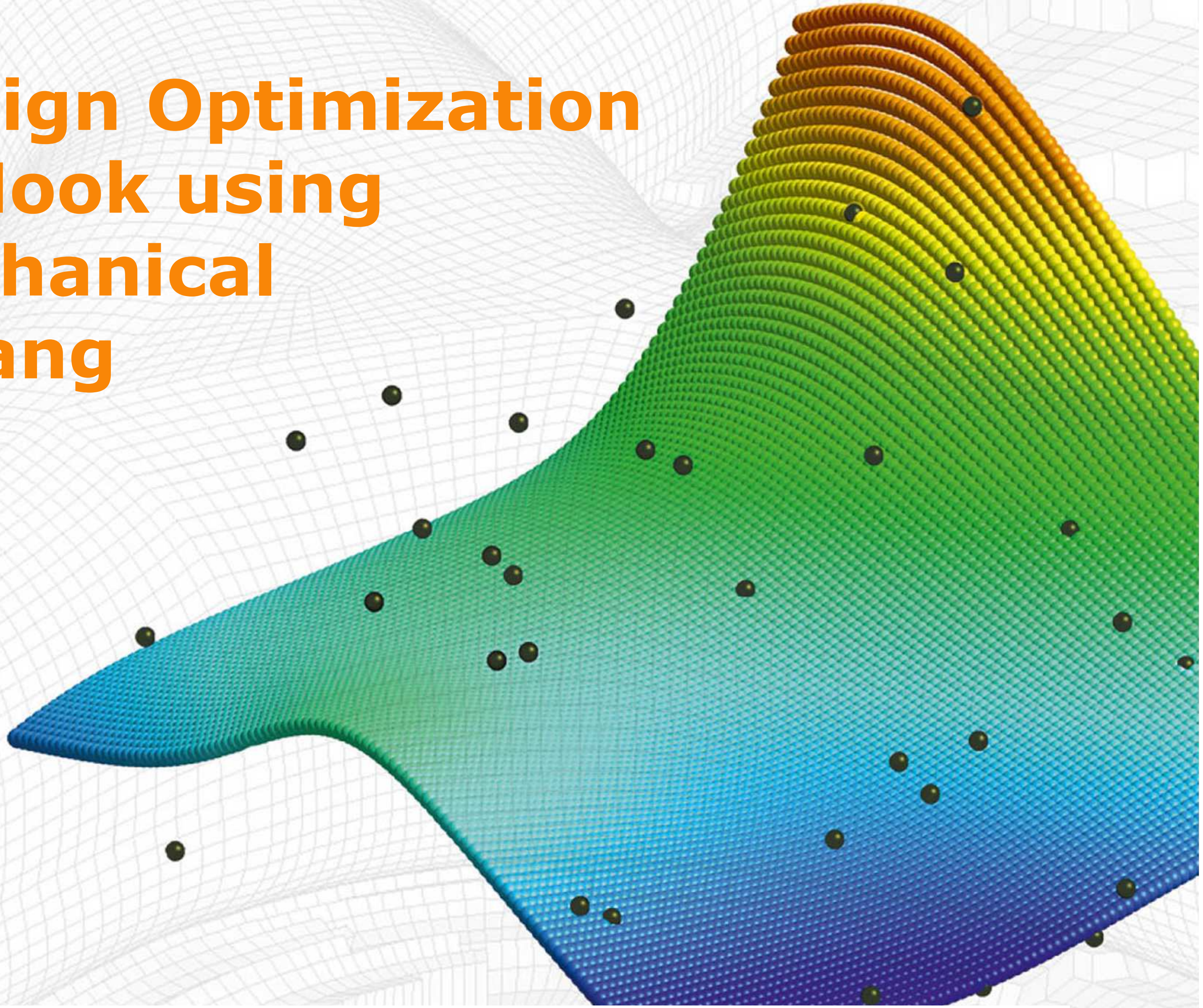
$$f(y) = \frac{k}{N} \sum y_i^2 = k(\bar{y}^2 + \sigma_y^2)$$

- Maximum is optimal (requires positive objective)

$$f(y) = \frac{k}{N} \sum \frac{1}{y_i^2}$$



Robust Design Optimization of a Steel Hook using ANSYS Mechanical and optiSLang



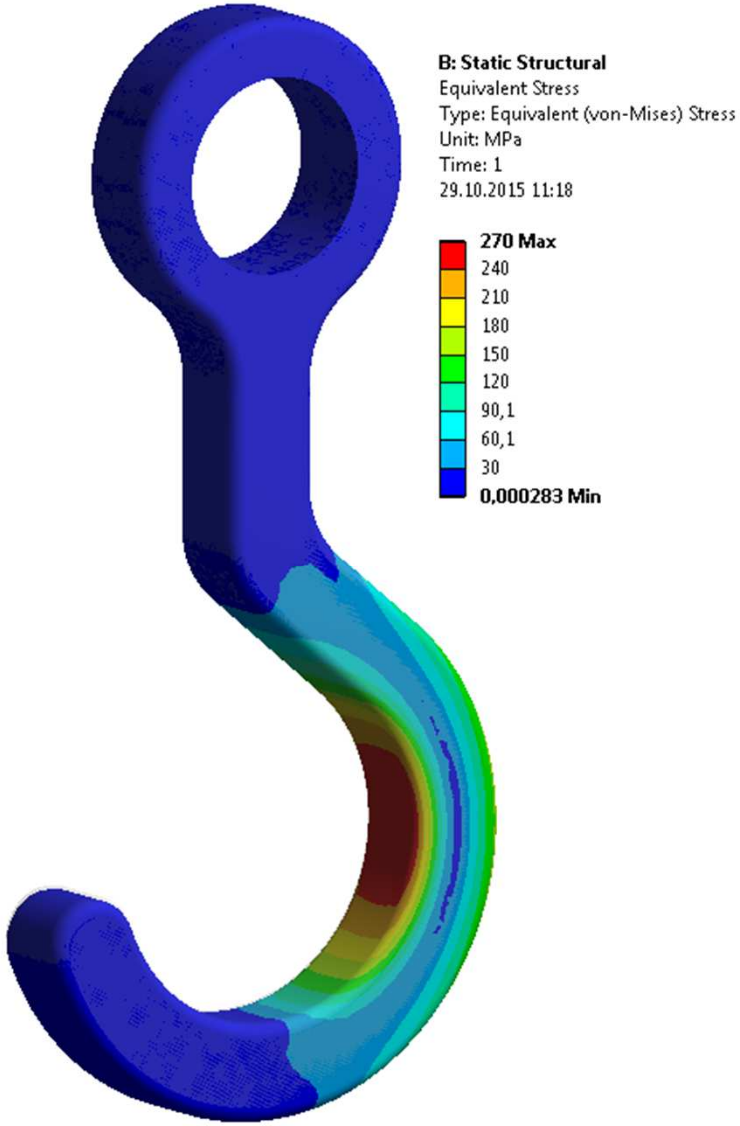
The Robust Design Optimization Task

Deterministic Optimization

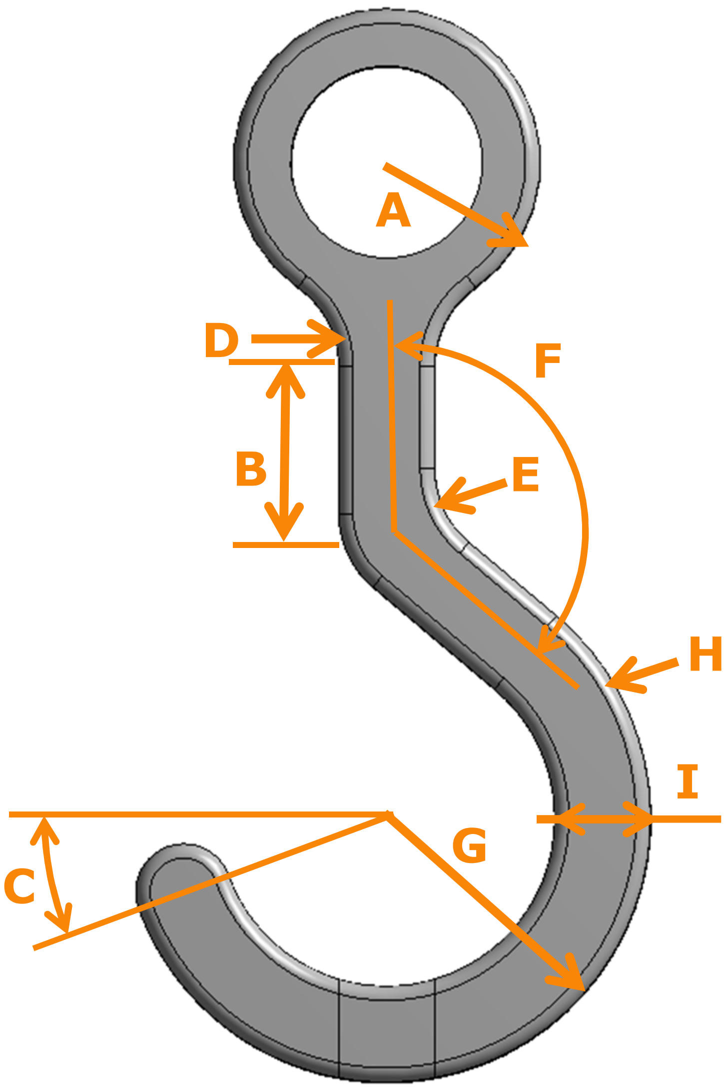
- Minimize the **mass**
- The **maximum stress** should not exceed 300 MPa
- **10 geometry parameters** are varied for the design variation

Robustness requirement

- Proof for the optimal design that the failure stress limit is not exceeded with a **4.5 sigma** safety margin
- **15 scattering parameters** are considered (geometry and material properties and the load components)



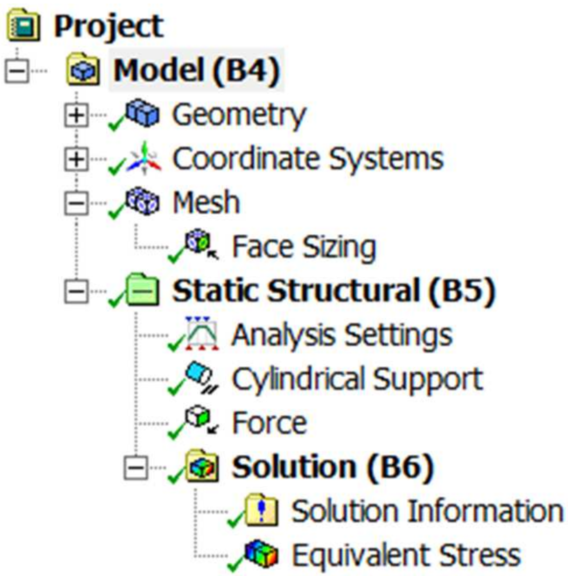
The Geometry Parameters

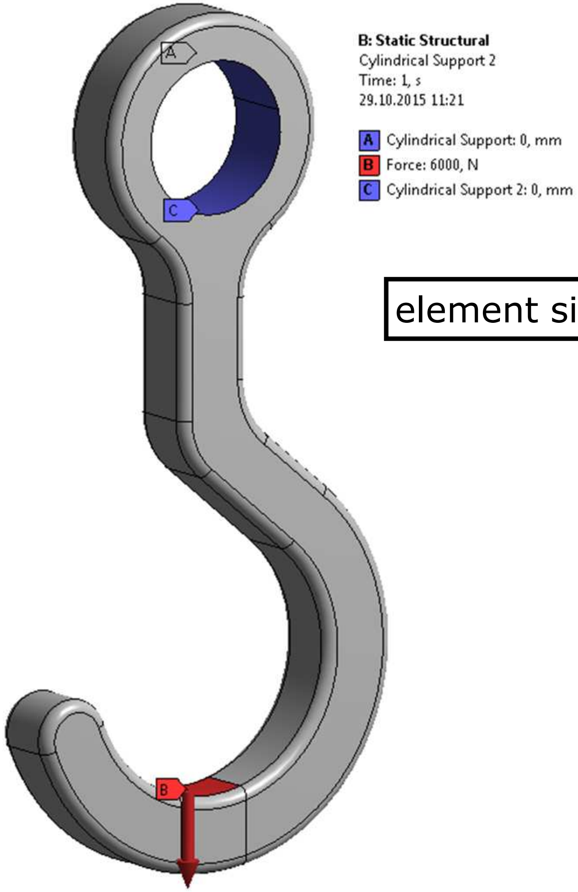


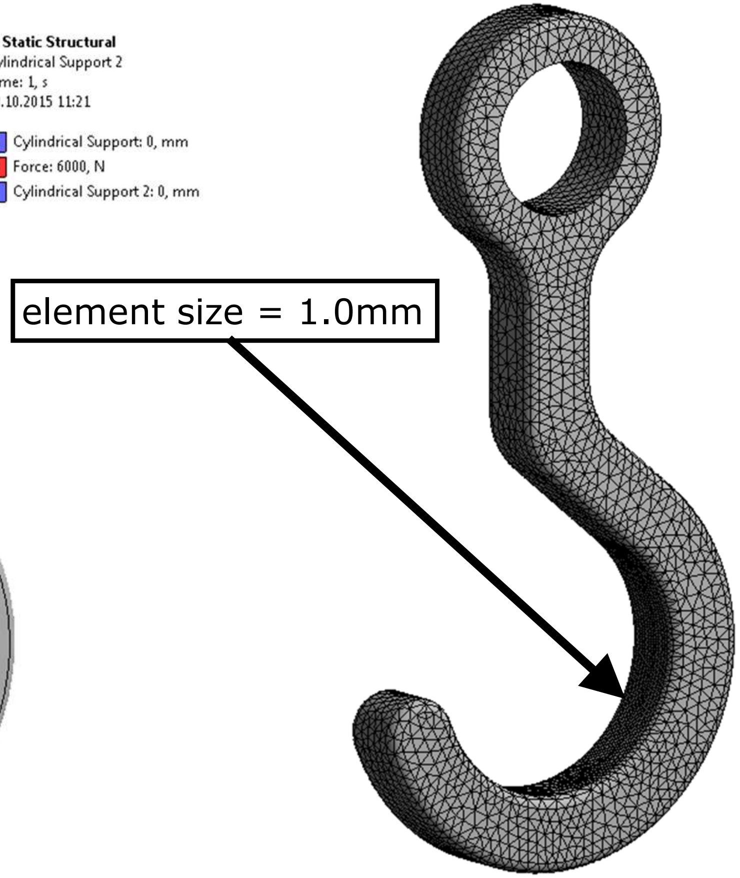
A	Outer_Diameter	28-35 mm
B	Connection_Length	20-50 mm
C	Opening_Angle	10-30 °
D	Upper_Blend_Radius	18-22 mm
E	Lower_Blend_Radius	18-22 mm
F	Connection_Angle	120-150 °
G	Lower_Radius	45-55 mm
H	Fillet_Radius	2-4 mm
I	Thickness	15-25 mm
	Depth	15-25 mm

Boundary Conditions

- Load F=6000 N
- Cylindrical support, tangential direction is free
- Small elements in region with maximum stresses

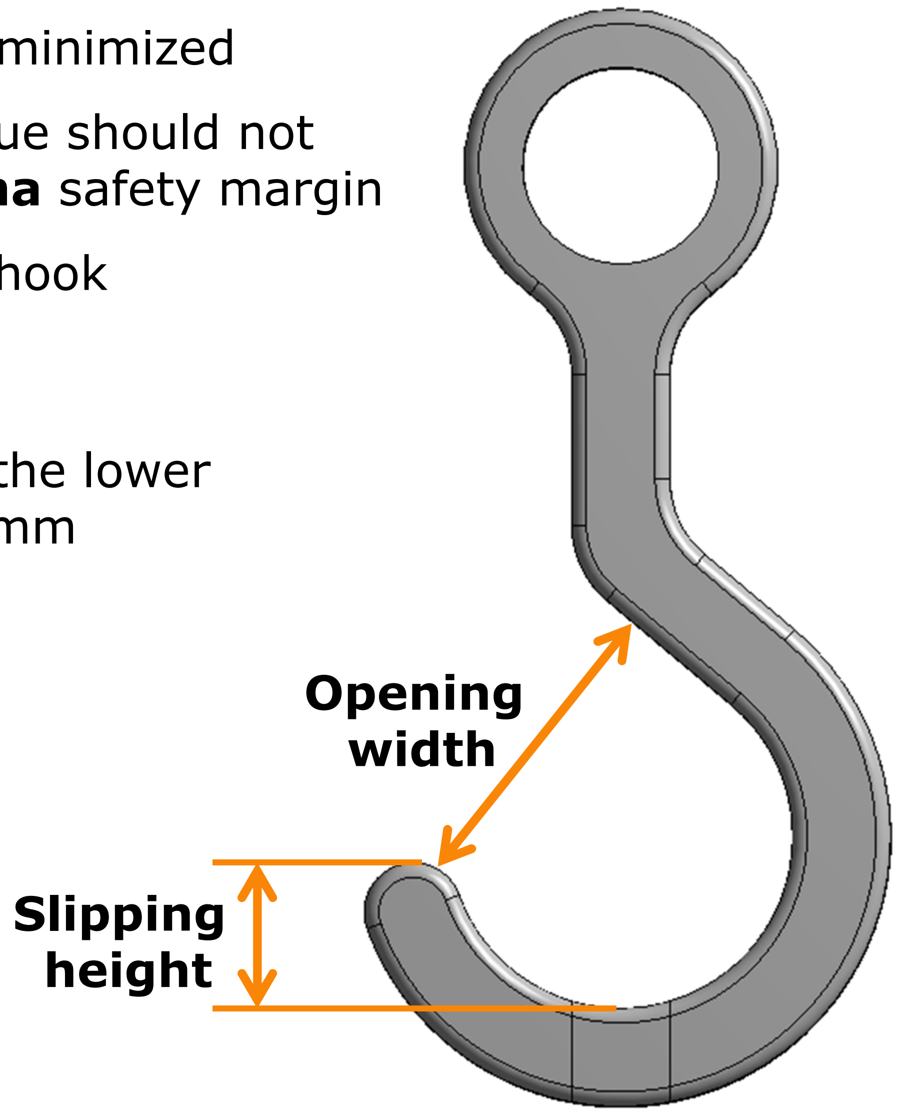






Responses and Criteria

- Total **mass** of the hook should be minimized
- **Maximum equivalent stress** value should not exceed 300 MPa within a **4.5 sigma** safety margin
- **Slipping height** of the deformed hook should be larger than 5 mm within a **4.5 sigma** safety margin
- **Opening width** (undeformed) of the lower half circle should be minimum 50 mm in the nominal design

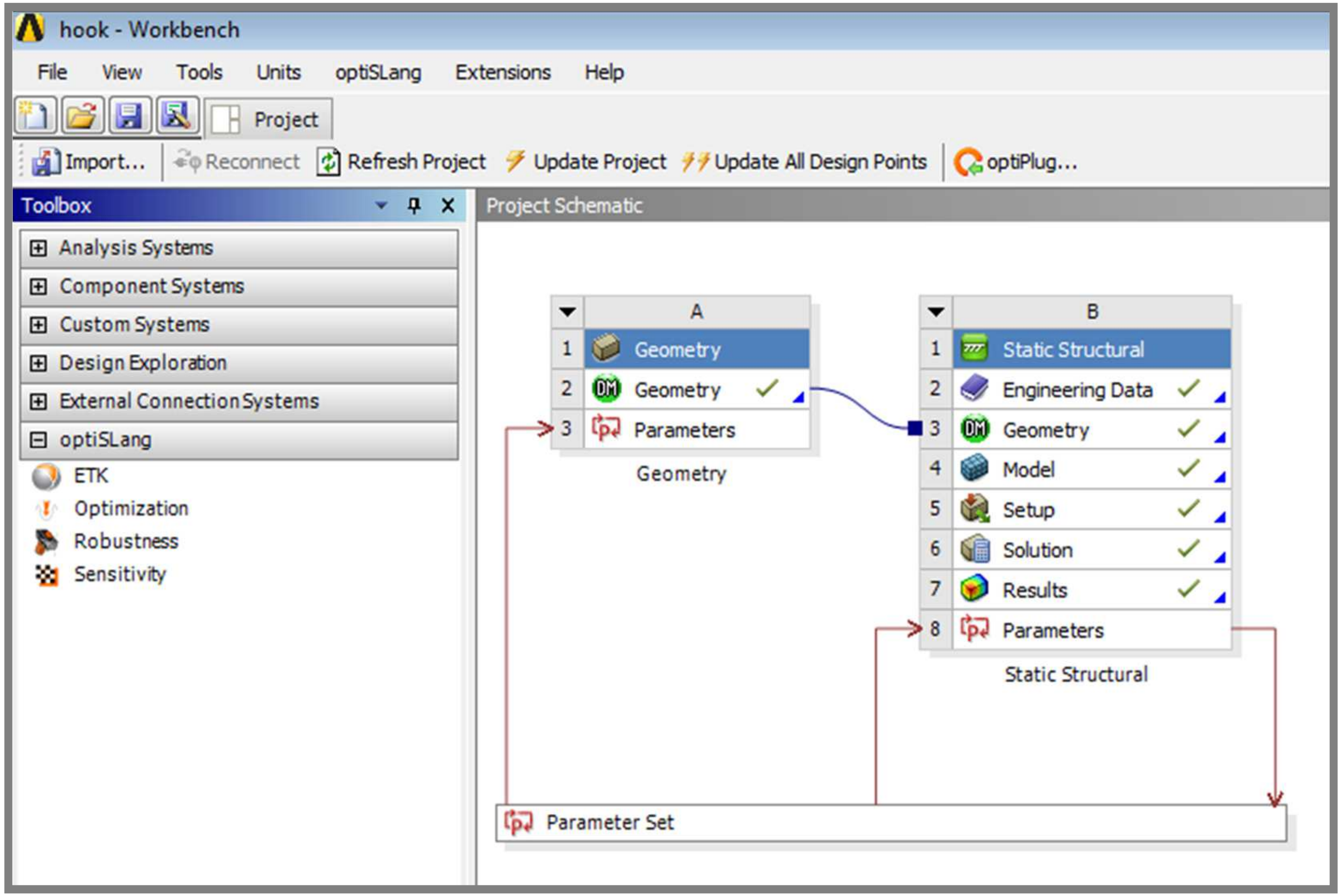


Initial nominal values

• Mass	1100 g
• Maximum stress	270 MPa
• Slipping height	28 mm
• Opening width	64 mm

Solver: ANSYS Mechanical

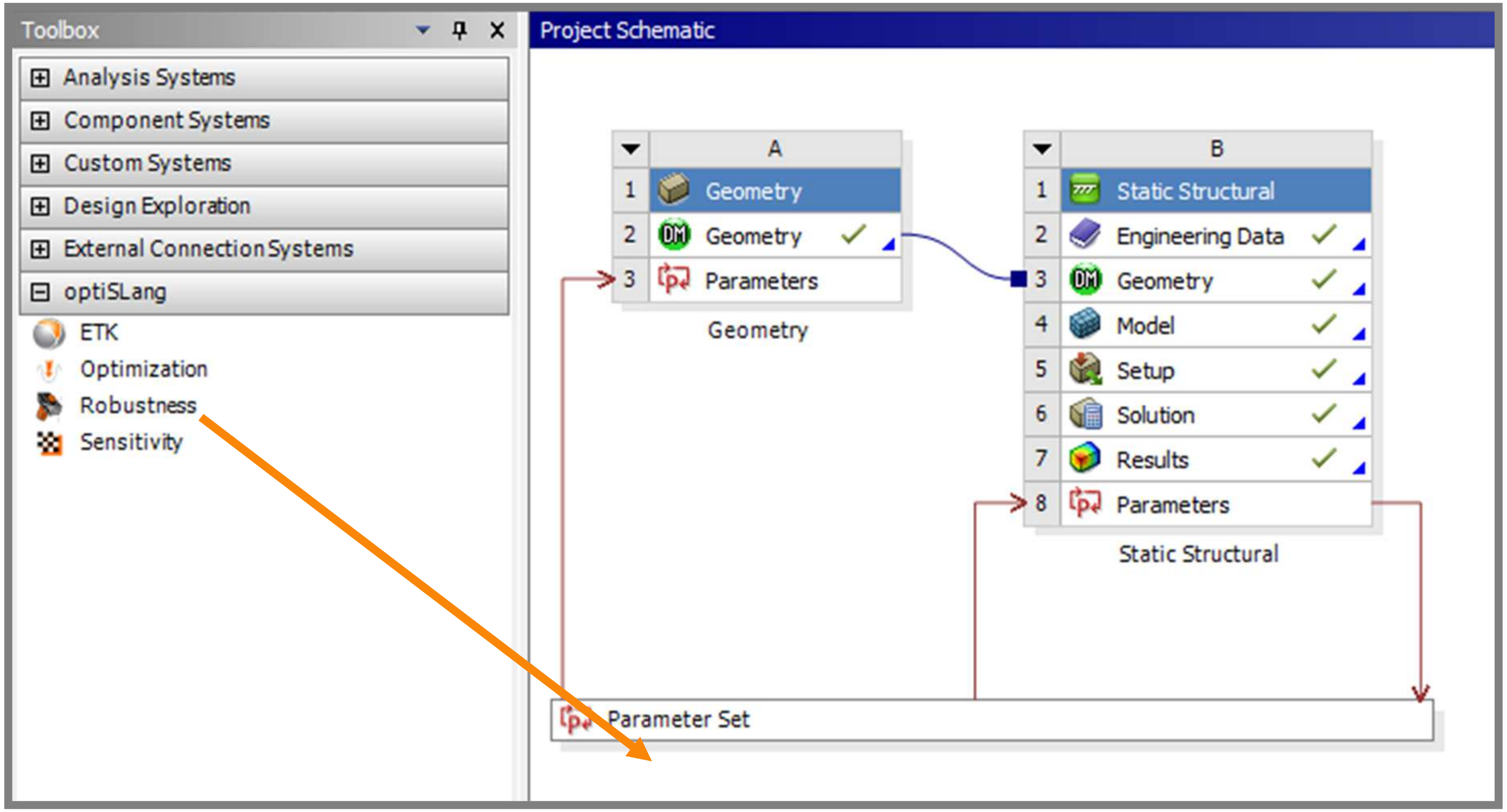
- Open the ready to use Workbench project hook_rdo.wbpz
- In ANSYS Workbench ANSYS Mechanical is used as solver



Robustness Evaluation of the Initial Design

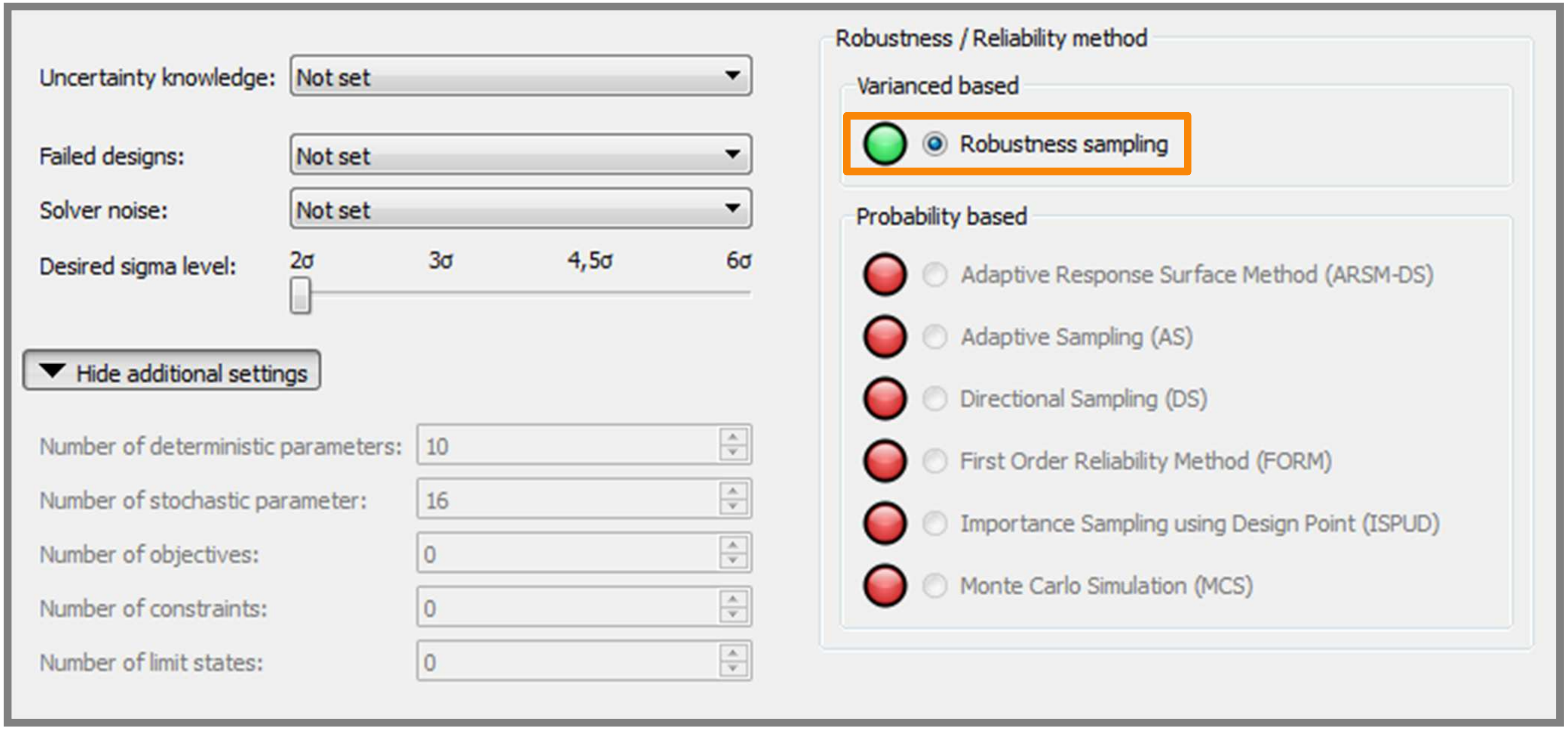
Robustness Analysis

- Create a new robustness system



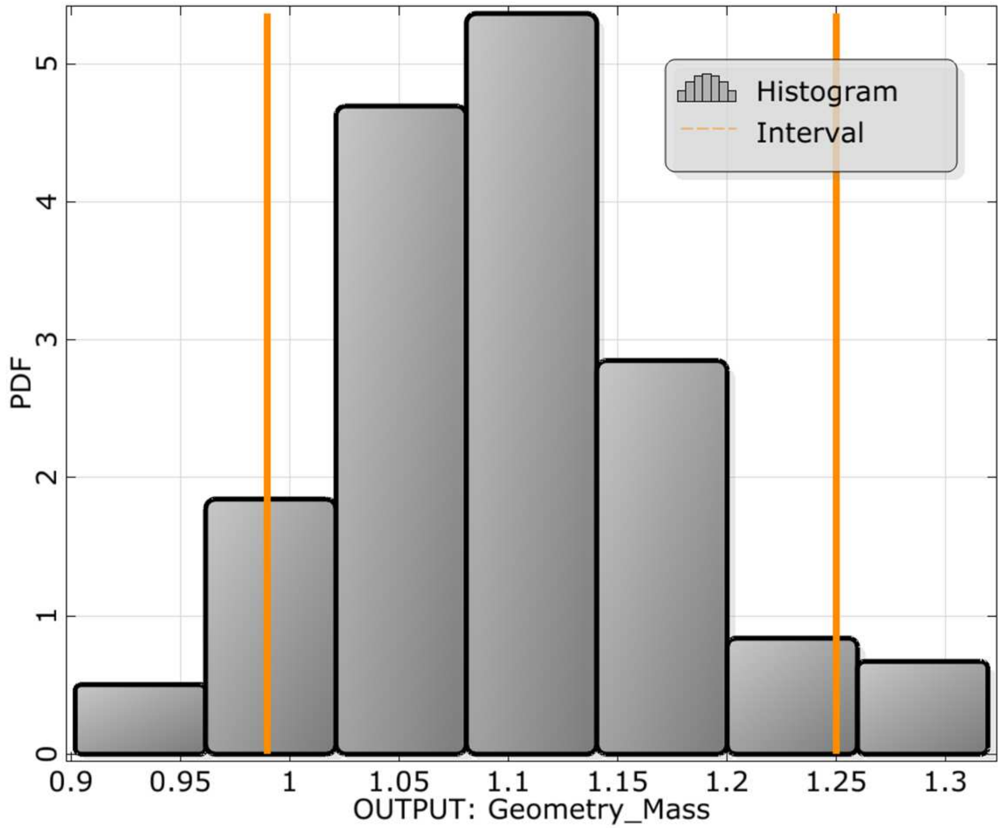
Robustness Analysis

- A variance-based robustness evaluation is performed
- Limit state functions are not necessary
- Keep wizard settings and continue with robustness sampling



Robustness Evaluation

- Evaluation of 100 Latin Hypercube samples
- **Statistical Evaluation of the Mass**
  - Range of 990g - 1250g within 90 % quantile
  - Scatter of the mass is not relevant for the safety assessment

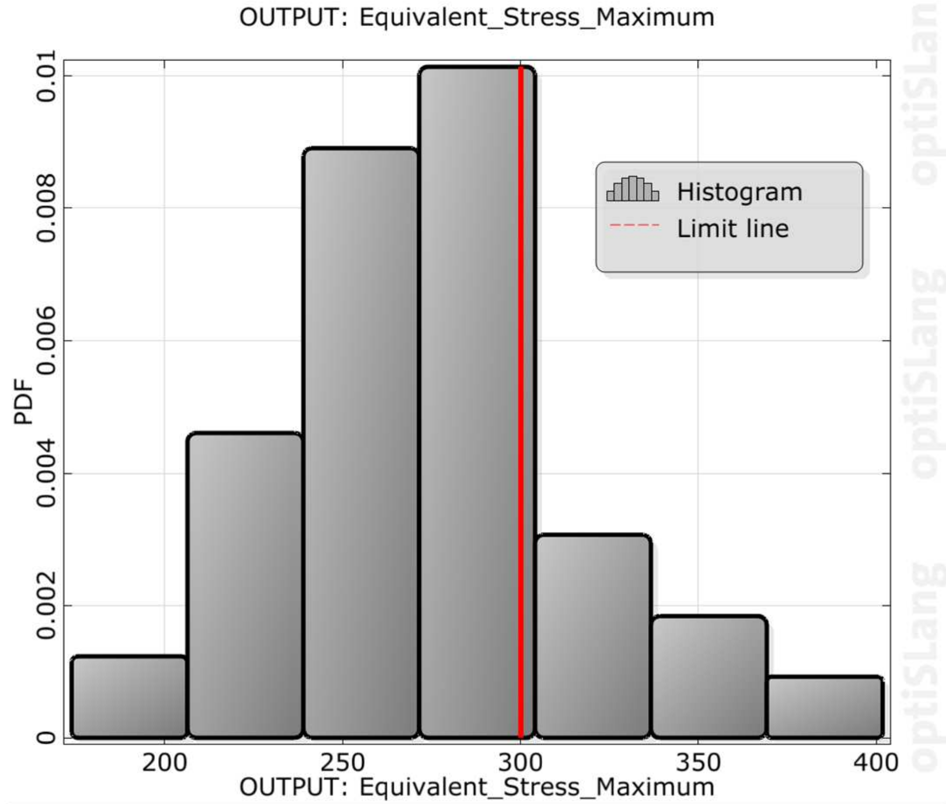


Robustness Evaluation

Statistical Evaluation of the Maximum Stress:

- Failure stress of 300MPa is exceeded with a probability of about **23%**
- Far away from 4.5 sigma

➢ Further reliability analysis to verify this result is not necessary



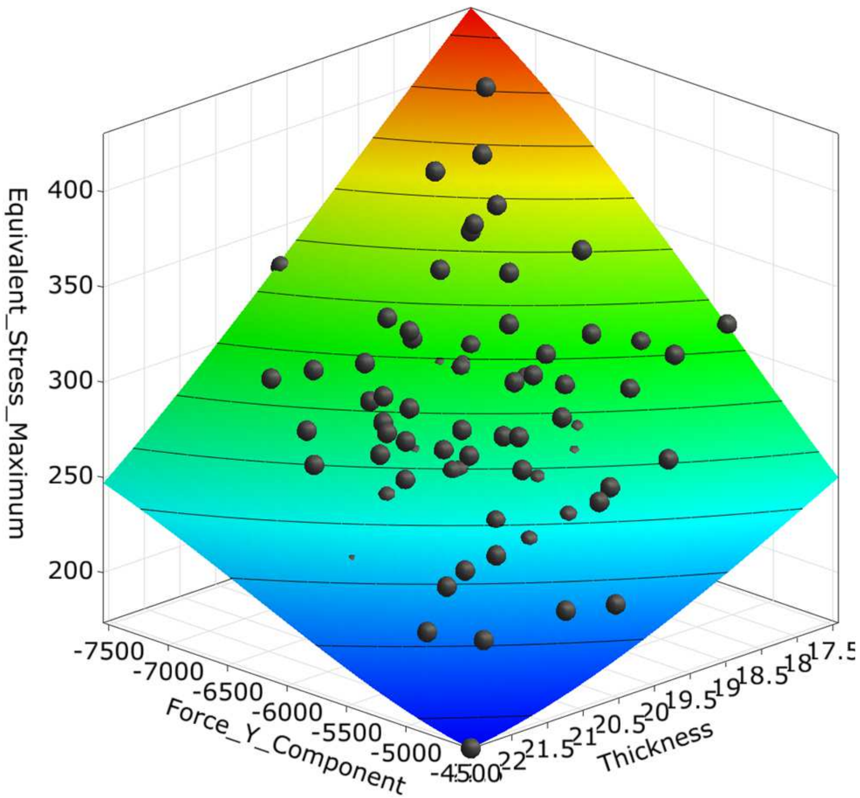
Statistic data			
Min:	173.8	Max:	401.8
Mean:	274.1	Sigma:	42.09
CV:	0.1536		
Skewness:	0.4233	Kurtosis:	3.334
Limit x = 300			
P_rel:	0.77	1 - P_rel:	0.23
Sigma-Level:	0.615076		

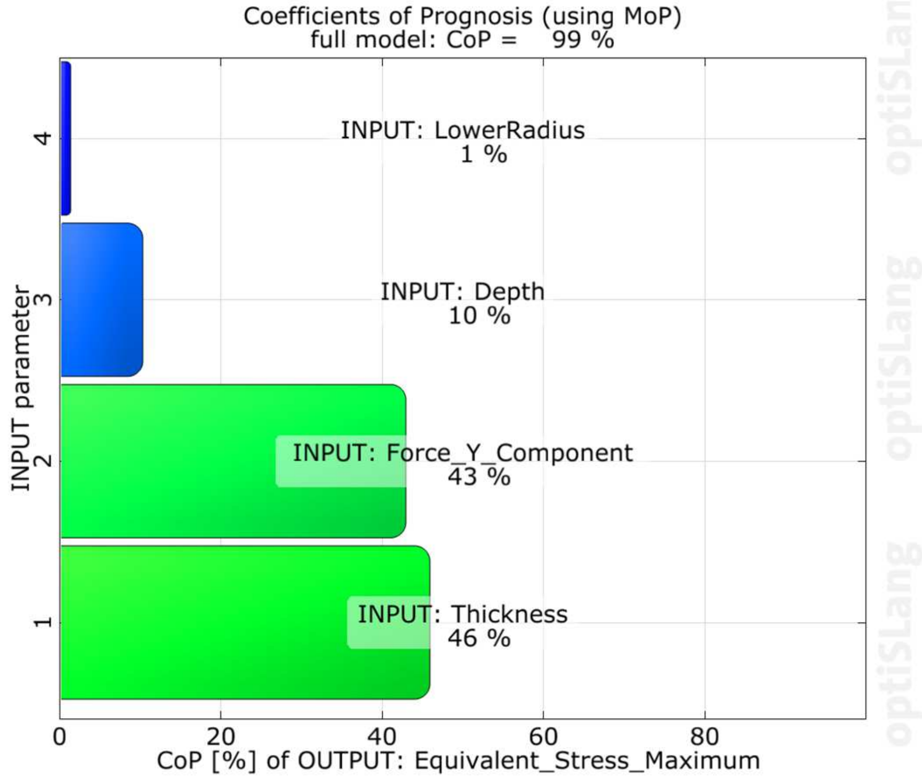
Robustness Evaluation

- Force in main direction and thickness are the most important input parameters for the maximum stress

➢ Attention: Scatter of force uncertainty is difficult to be reduced

➢ Therefore, the design has to be changed to reduce the mean value of maximum stress and to fulfill the robustness requirement

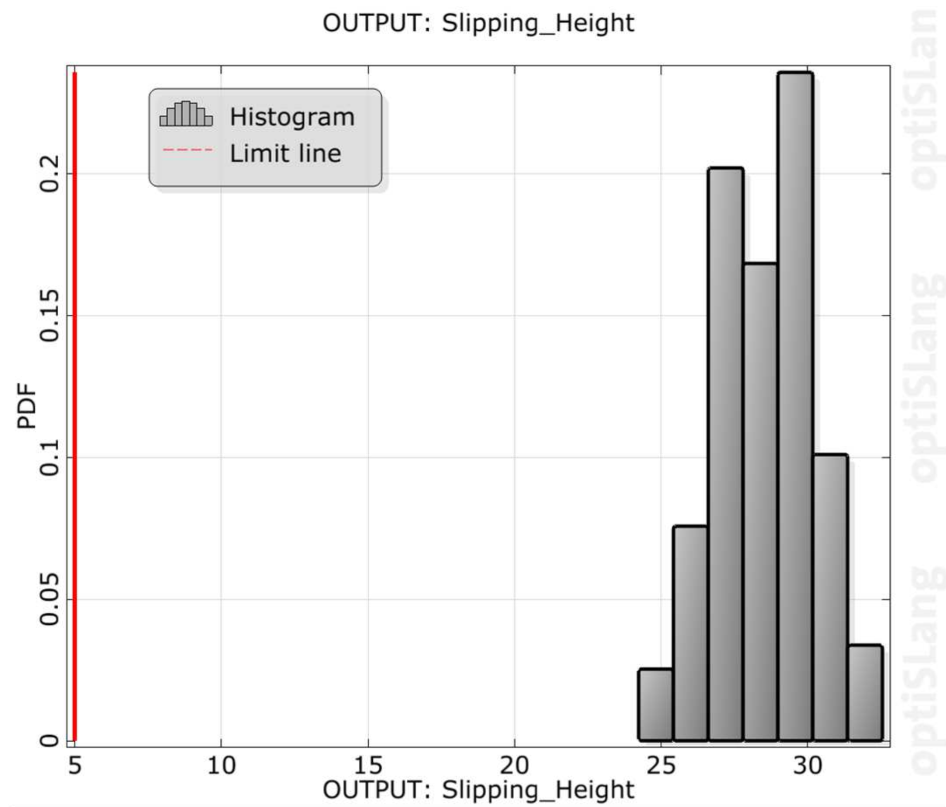




Robustness Evaluation

Statistical Evaluation of the Slipping height:

- High safety margin between minimum value of 5 mm and the observed variation
- ➢ Robustness criteria for slipping is fulfilled



Statistic data			
Min:	24.23	Max:	32.55
Mean:	28.55	Sigma:	1.704
CV:	0.0597		
Skewness:	-0.04769	Kurtosis:	2.549
Limit x = 5			
P_rel:	0	1 - P_rel:	1
Sigma-Level:	13.8163		

Robustness Evaluation – Summary Initial Design

- **Varianced-based robustness** evaluation has observed:
  - Probability of exceeding the stress limit is much to large
  - Significant reduction of the input scatter seems not possible
  - Safety margin of slipping height seems sufficient

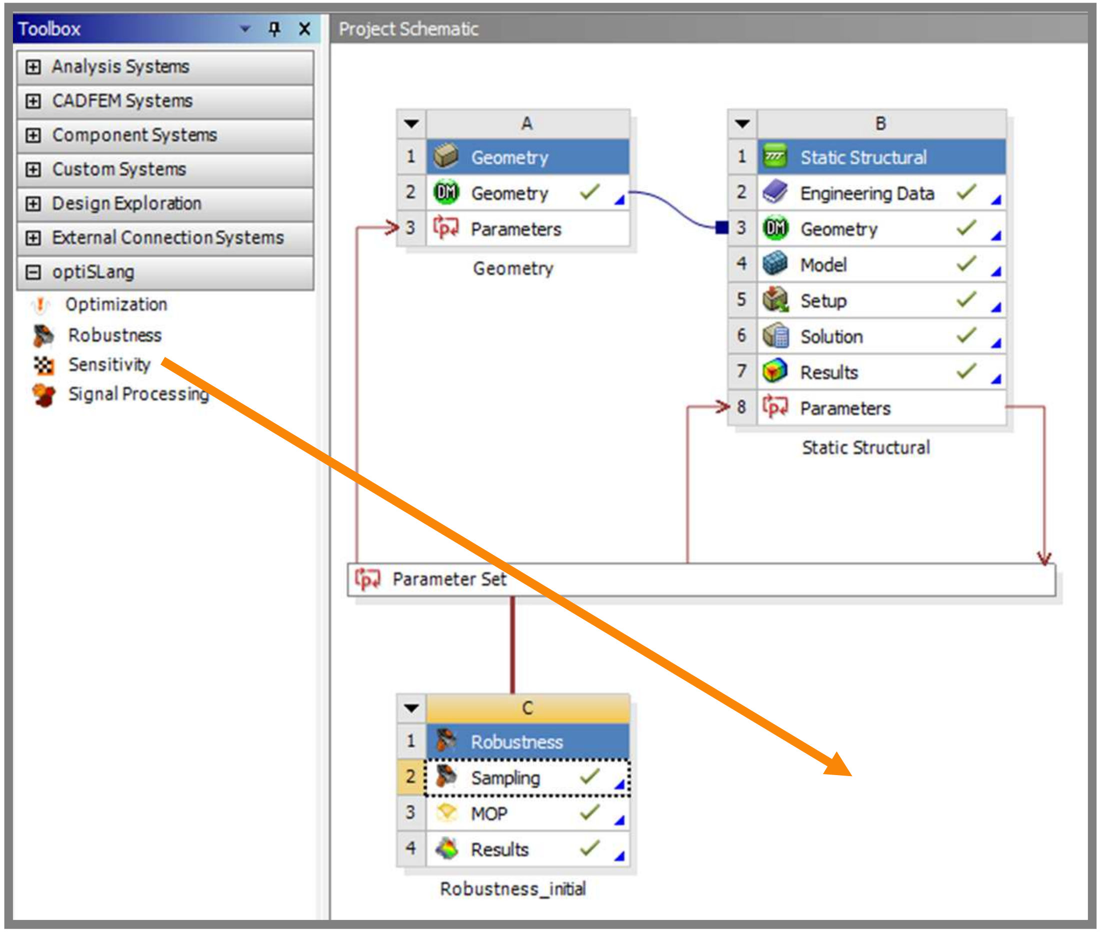
- **Design improvement** is done in next step:
  - Iterative Robust Design Optimization
  - We modify the design be reducing the mean of the maximum stress using deterministic optimization
  - We check the robustness again

- **Deterministic constraints** for first optimization step
  - Mean stress + 4.5 * mean stress * CV ≤ 300
  - ➢ **Mean stress ≤ 180**
  - **Mean slipping height ≤ 10**

First Robust Design Optimization Step

Sensitivity Analysis

- A sensitivity analysis is performed to quantify the importance of the optimization parameters
- The parameter properties are imported from the robustness system



Definition of the Objective and Constraints

- Minimize the mass
- Maximum stress < 180 Mpa
- Slipping height > 10 mm
- Opening width > 50 mm

Objectives

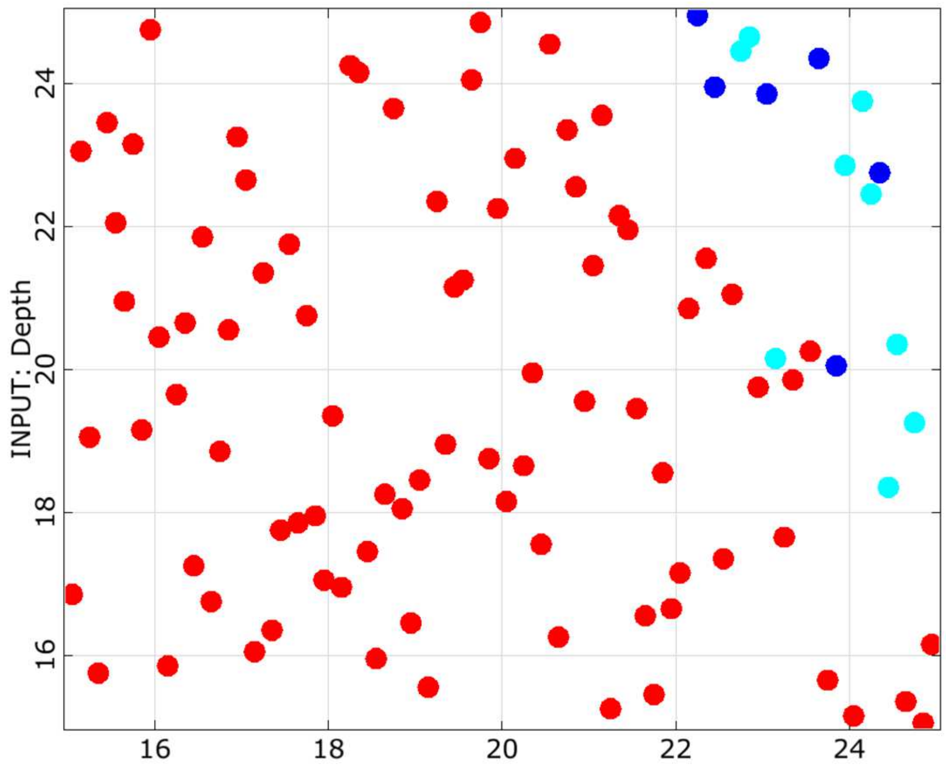
Name	Criterion	Expression	Value
Objective_mass	MIN	Geometry_Mass	1.09976
new			

Constraints

Name	Left side expression	Criterion	Right side expression	Value
Constraint_stress	Equivalent_Stress_Maximum	≤	180	270.434 ≤ 180
Constraint_slipping	Slipping_Height	≥	10	28.5589 ≥ 10
Constraint_opening	Opening_Width	≥	50	64.3124 ≥ 50
new				

Design of Experiments

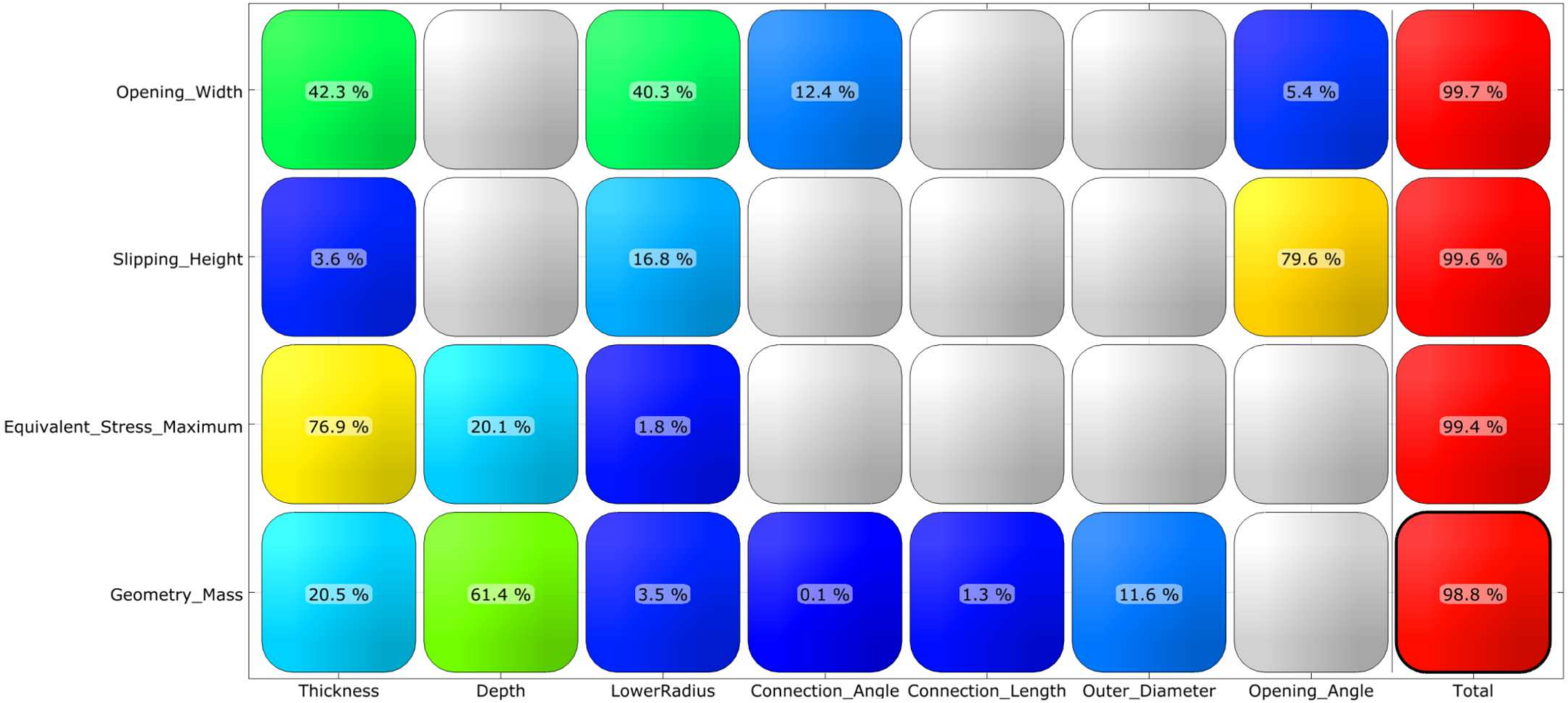
- 100 designs are evaluated with the ANSYS workbench model
- **84 designs** violate the stress constraint
- Further **10 designs** violate only the opening width constraint
- All designs fulfill the slipping constraint
- Only small subdomain of the parameter space is **feasible**



Feasible
Stress violation
Opening violation

Sensitivity Analysis using MOP

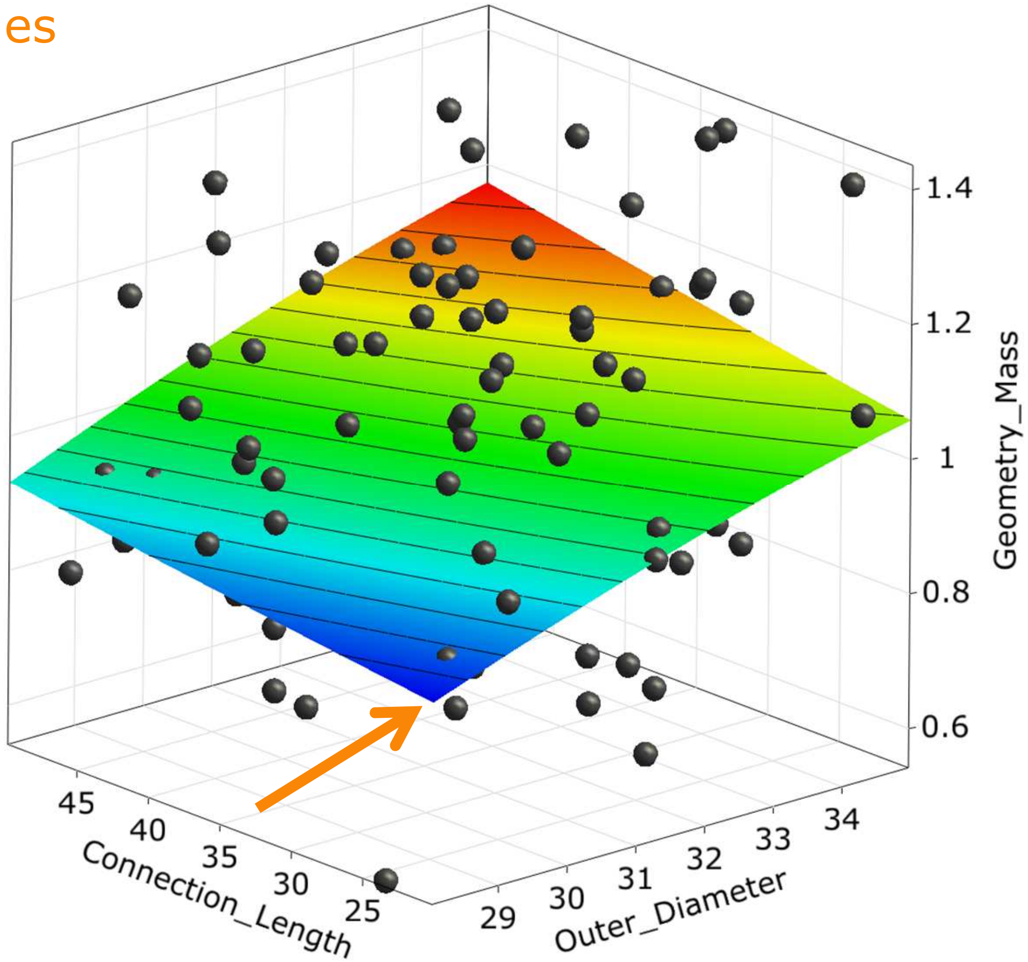
- All responses can be explained very well with the MOP
- Connection length and outer diameter are only important for the mass
- Lower and upper blend radii as well as fillet radius are not important for any response and can be neglected in the optimization



Sensitivity Analysis using MOP

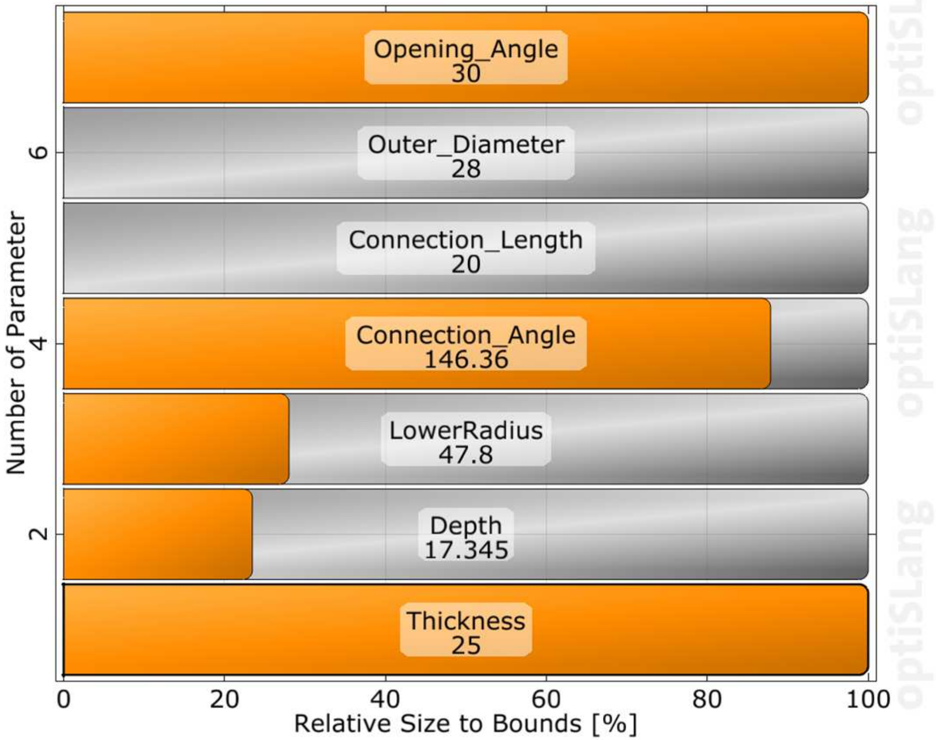
- Connection length and outer diameter are only important for the mass
- Their minimum values lead to minimum mass
- ➢ They can be set to minimum values without interference to stress

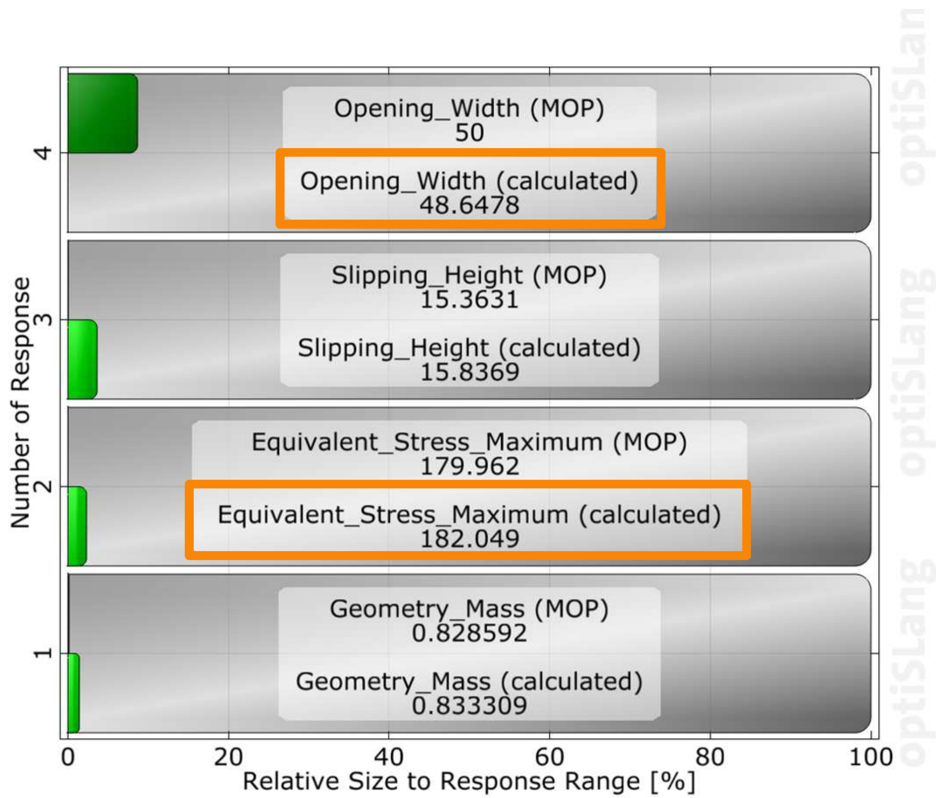
- ➢ Following optimization has to consider only 5 parameters
- ➢ Due to the excellent CoP values an optimization on the MOP is applied to get a good start solution



Optimization using the MOP

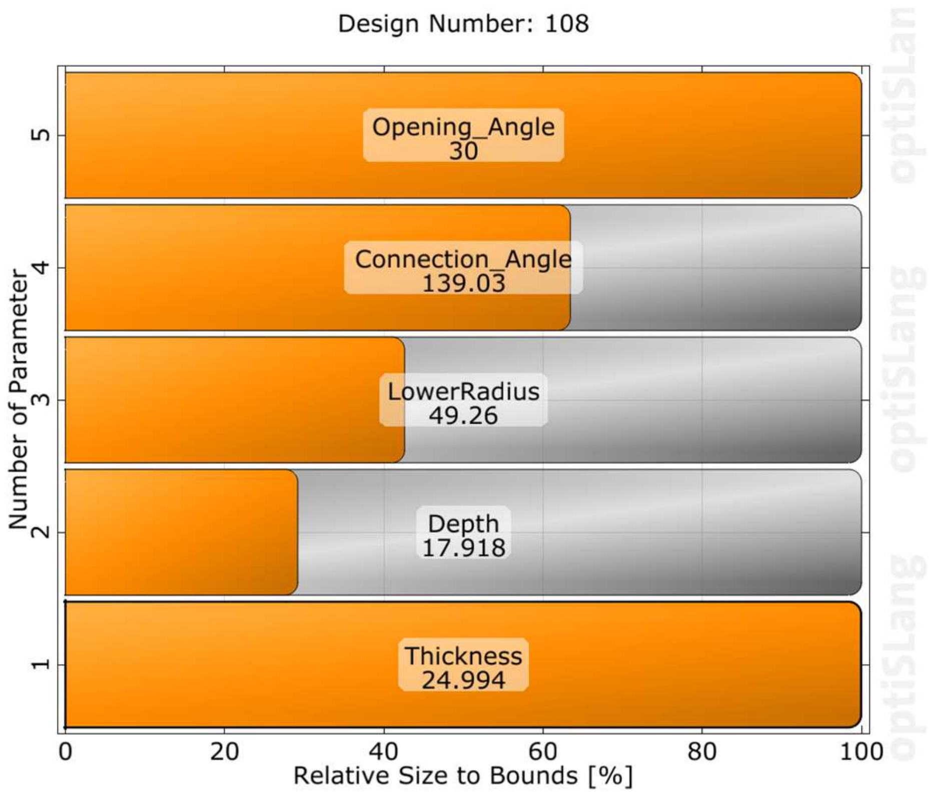
- The NLPQL optimizer converges within a few iterations
- The responses and objective/constraints of the best design are verified
- Due to the global approximation the constraints of the best design are slightly violated in the verification
- ➢ The best design is used as start design for a local optimization with direct solver call

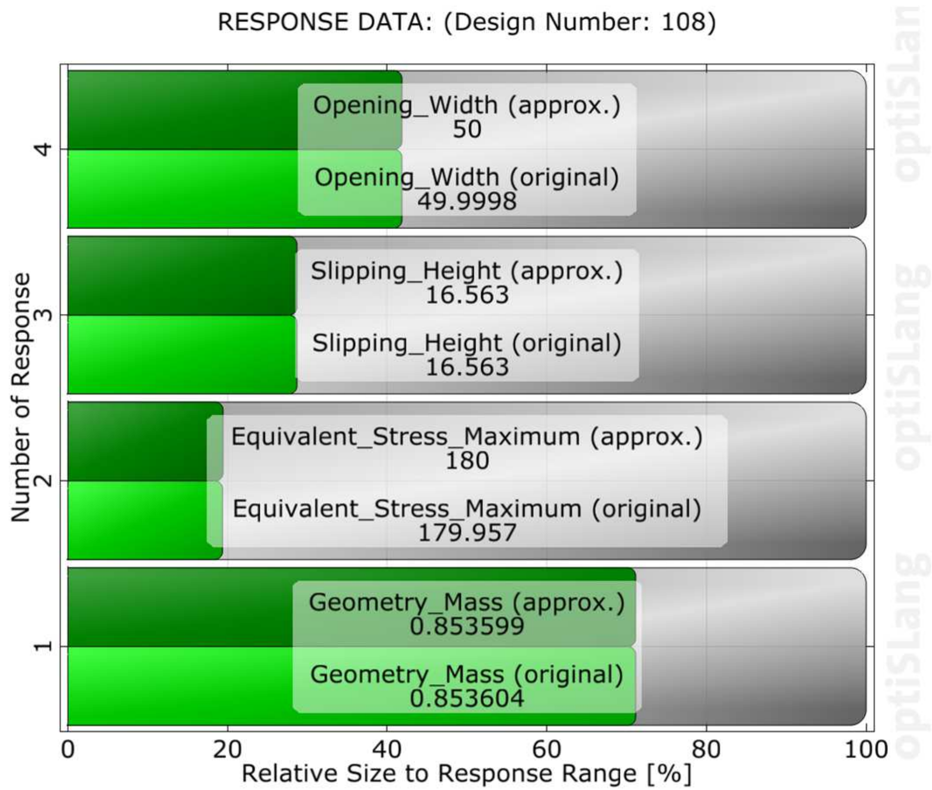




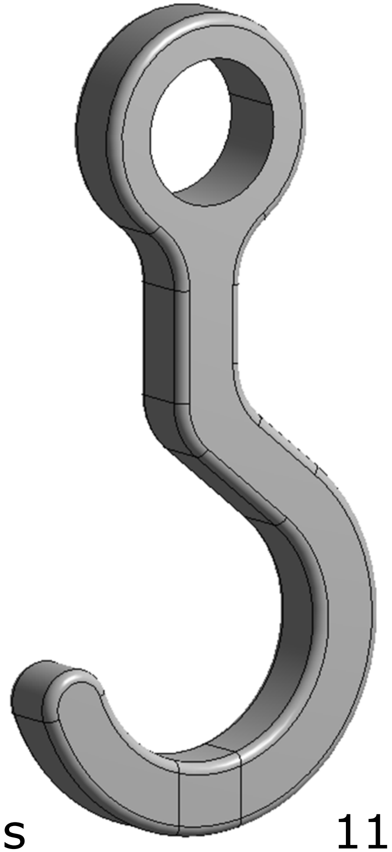
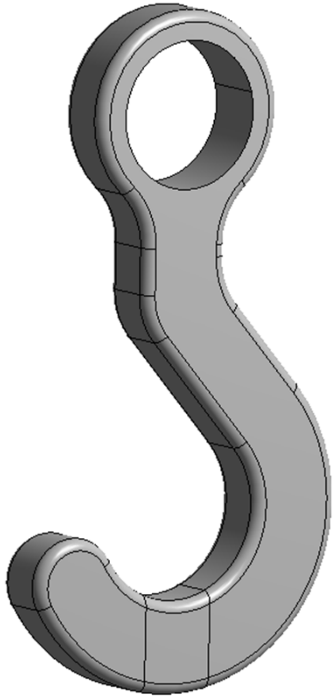
Optimization with Adaptive Response Surfaces

- Optimizer obtains an optimal design within 10 iterations fulfilling all constraint conditions



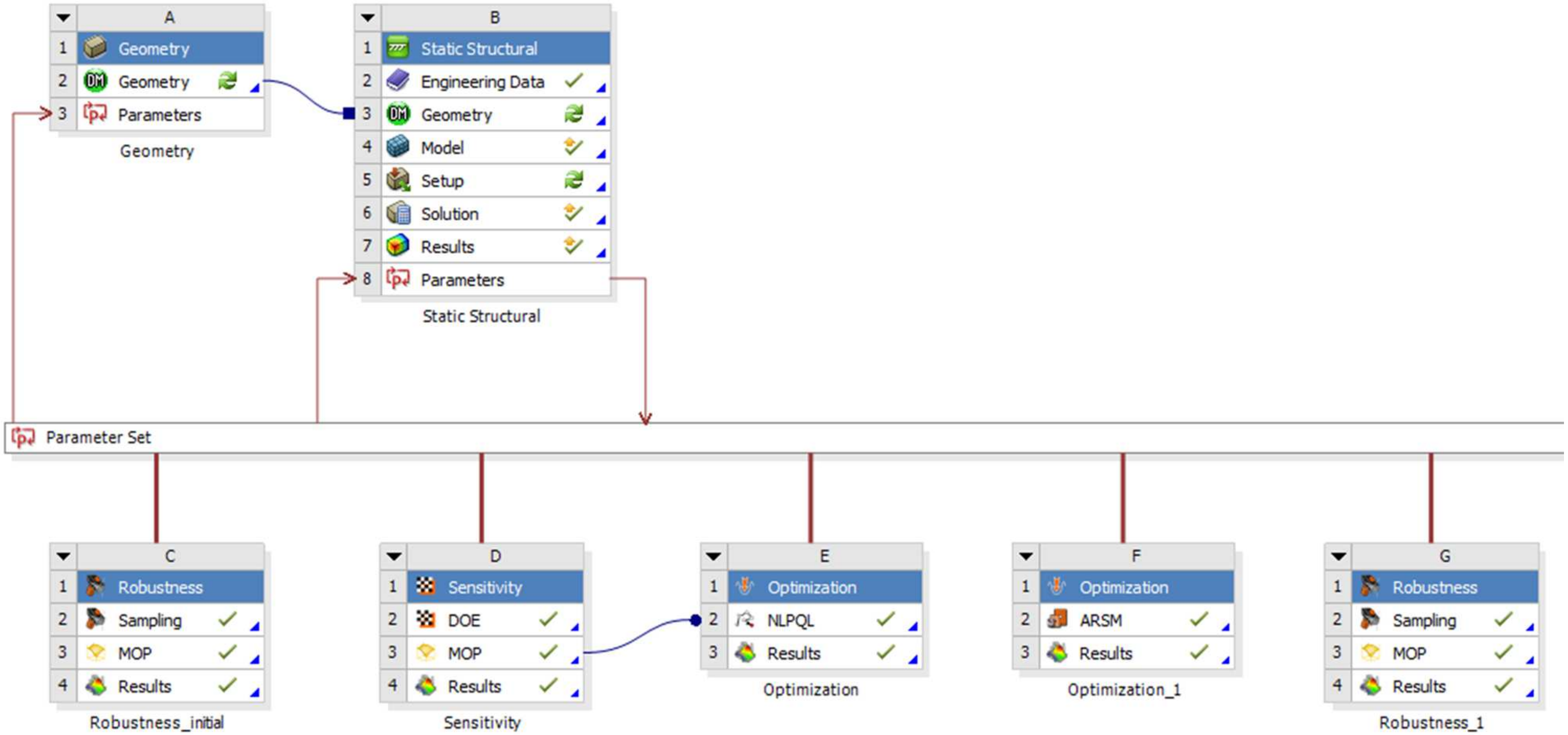


Initial vs. Optimal Design

Initial Design		Optimal Design	
			
Mass	1100 g	Mass	854 g
Maximum stress	270 MPa	Maximum stress	180 MPa
Slipping height	28 mm	Slipping height	16 mm
Opening width	64 mm	Opening width	50 mm

Robustness Evaluation of First Optimization Step

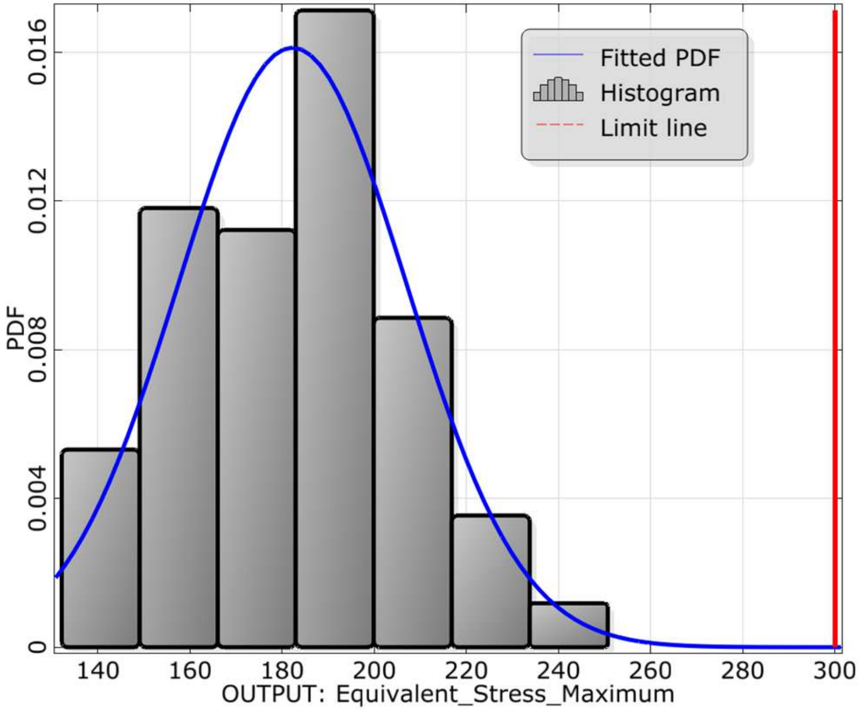
- Robustness evaluation is performed again for new optimal design:
- Import parameters from initial robustness analysis and use best design of optimization as nominal design
- Run the default robustness sampling with 100 samples



Robustness Evaluation of First Optimization Step

Statistical Evaluation of the Maximum Stress:

- Safety margin to failure limit of 300MPa is estimated with a sigma level of 4.75, which corresponds to a failure probability of 10^{-6}, if the response would be perfectly normally distributed
- Attention: Since the real distribution is not known and 100 LHS samples are far too less to proof such a small probability, a reliability analysis is necessary to proof this safety level!

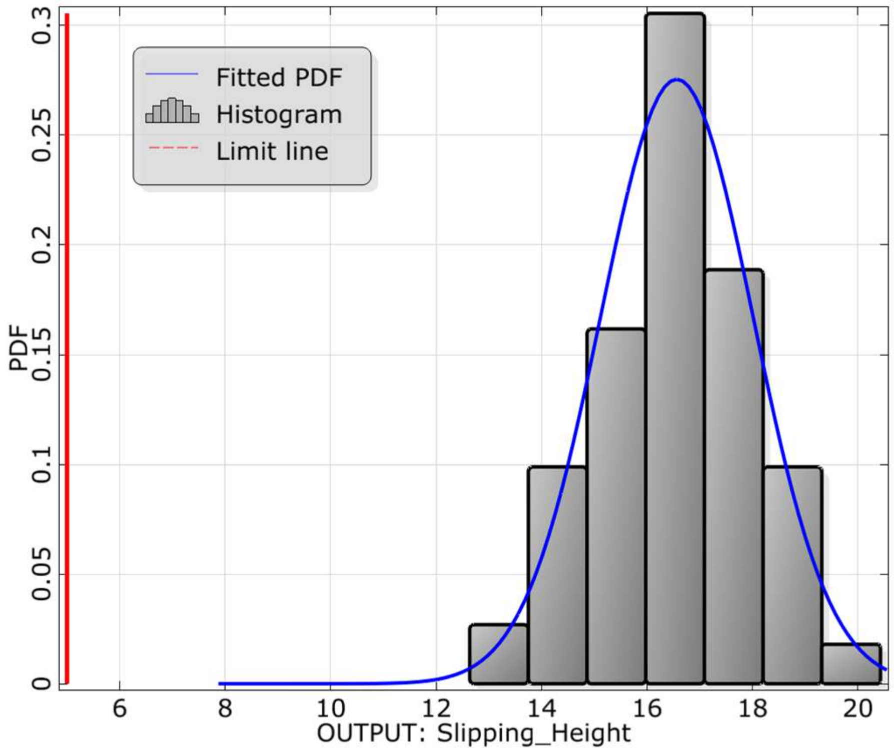


Statistic data			
Min:	132.2	Max:	250.7
Mean:	182.3	Sigma:	24.75
CV:	0.1358		
Skewness:	0.2399	Kurtosis:	2.591
Fitted PDF: Normal			
Mean:	182.3	Sigma:	24.75
Limit x = 300			
P_rel:	1	1 - P_rel:	0
P_fit:	0.999999	1 - P_fit:	9.80206e-007
Sigma-Level:	4.75746		

Robustness Evaluation of First Optimization Step

Statistical Evaluation of the Slipping height:

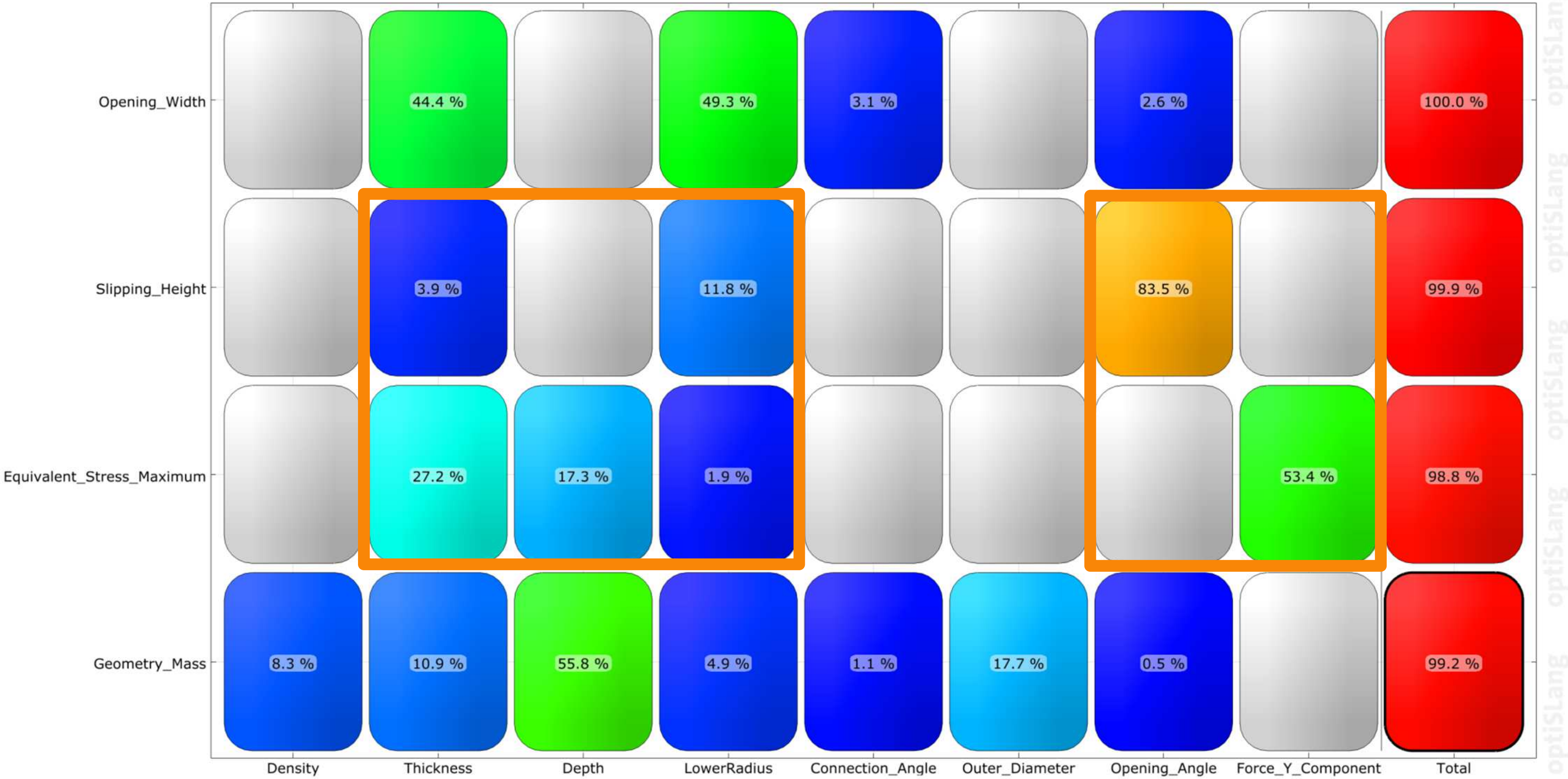
- A safety margin of 8 sigma is estimated
- ➢ Robustness criteria for slipping seems to be fulfilled
- ➢ Again a reliability analysis is required to proof the small probability



Statistic data			
Min:	12.64	Max:	20.43
Mean:	16.56	Sigma:	1.448
CV:	0.08742		
Skewness:	-0.04145	Kurtosis:	3.039
Fitted PDF: Normal			
Mean:	16.56	Sigma:	1.448
Limit x = 5			
P_rel:	0	1 - P_rel:	1
P_fit:	6.66134e-016	1 - P_fit:	1
Sigma-Level:	7.98603		

Robustness Evaluation of First Optimization Step

- Only 5 parameters are important for the slipping height and maximum stress which are considered in the following reliability analysis

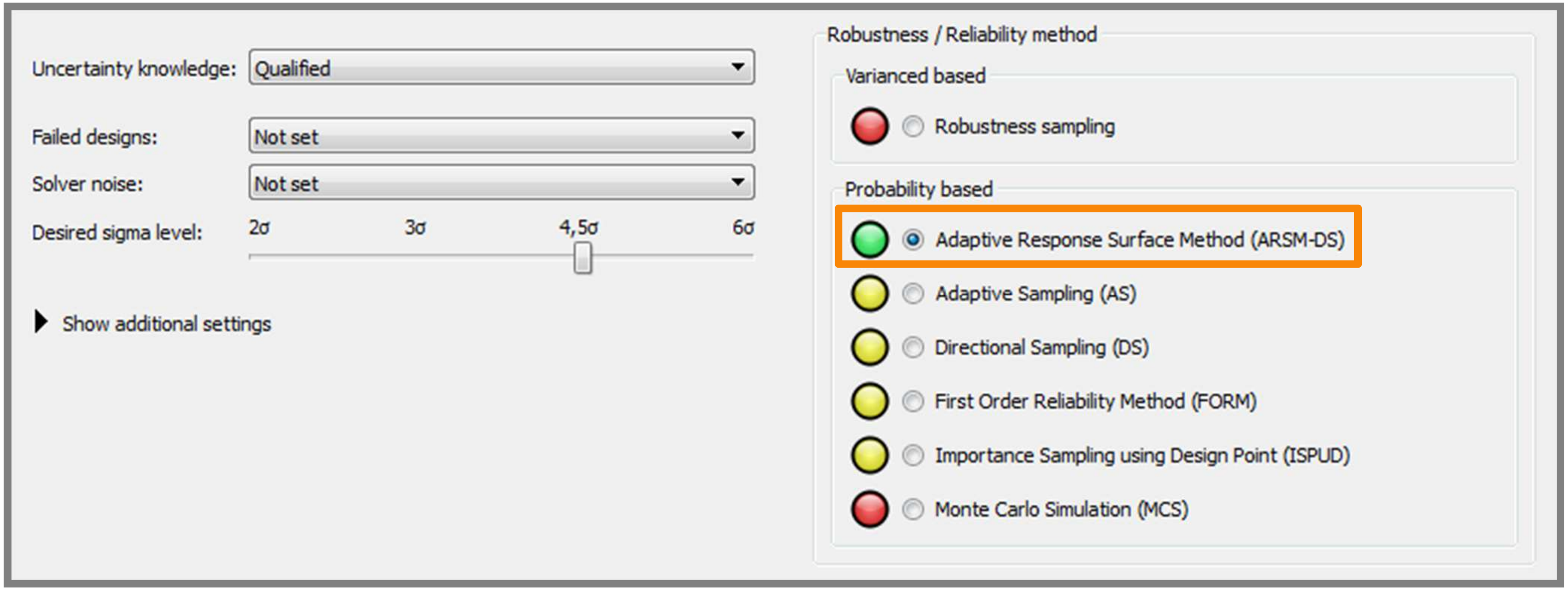


Reliability Analysis of First Optimization Step

- ➢ The Adaptive Response Surface Method is suggested

Limit states

Name	Left side expression	Criterion	Right side expression	Value
Limit_stress	Equivalent_Stress_Maximum	≤	300	270.434 ≤ 300
Limit_slipping	Slipping_Height	≥	5	28.5589 ≥ 5



Reliability Analysis of First Optimization Step

- Failure region at small thickness and depth and large force component
- Failure probability is much larger as allowed
- Corresponding reliability index is 3.8 instead of 4.5

Method : Directional Sampling on Adaptive Response Surfaces (ARSM-DS)

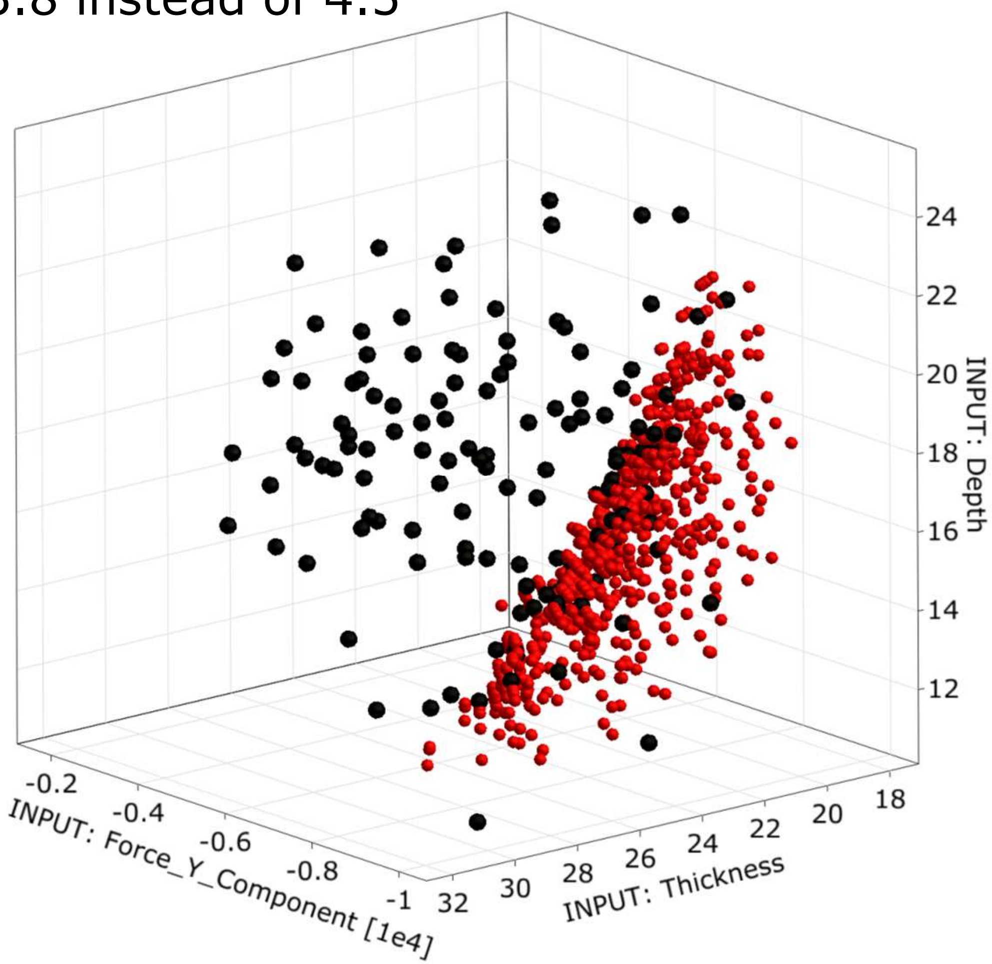
Selected data : 3. Approximation

Number of designs : 140 (0 failed)

Approximation errors : R^2 / R^2pred
Limit_stress : 1 / 0.983792
Limit_slipping : 1 / 0.998662

Complete directions : 1000 / 1000

Probability of failure : 8.12e-005
Standard deviation error : 1.917e-005
Reliability index : 3.7713



Robustness Evaluation – Summary First Step

- **Varianced-based robustness** evaluation has observed:
  - Safety margin of stress limit seems sufficient (4.7 sigma)
  - Safety margin of slipping height seems sufficient (8 sigma)

  - Reliability analysis has estimated a failure probability of **10^{-4}** which corresponds to a **reliability index of 3.8**
  - ➢ A further design modification is necessary

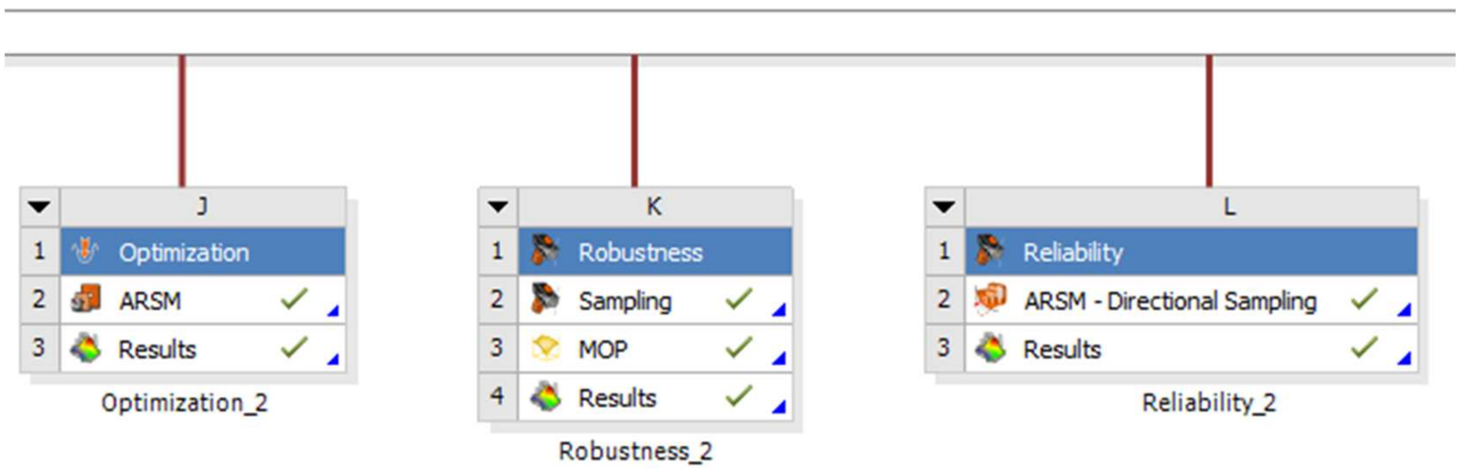
- **Deterministic constraints** for second optimization step
  - Initial design has a sigma level of 0.6 at 270 MPa mean stress
  - Optimized design has a reliability index of 3.77 at 180 MPa
  - Linear extrapolation for 4.5 sigma:
  - ➢ **Mean stress ≤ 160**
  - **Mean slipping height ≤ 10**

Second Robust Design Optimization Step

Second Robust Design Optimization Step

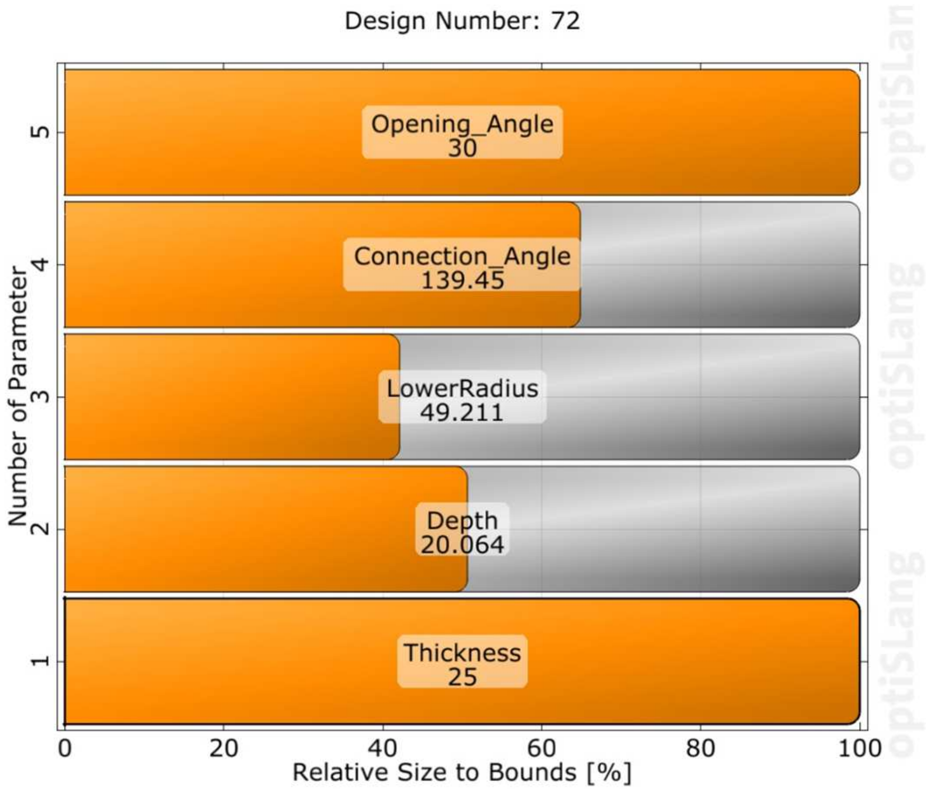
- Optimization is performed again with new constraints
- Start design, which fulfills constraints, is selected from sensitivity designs
- ARSM optimizers is applied as suggested in the decision tree

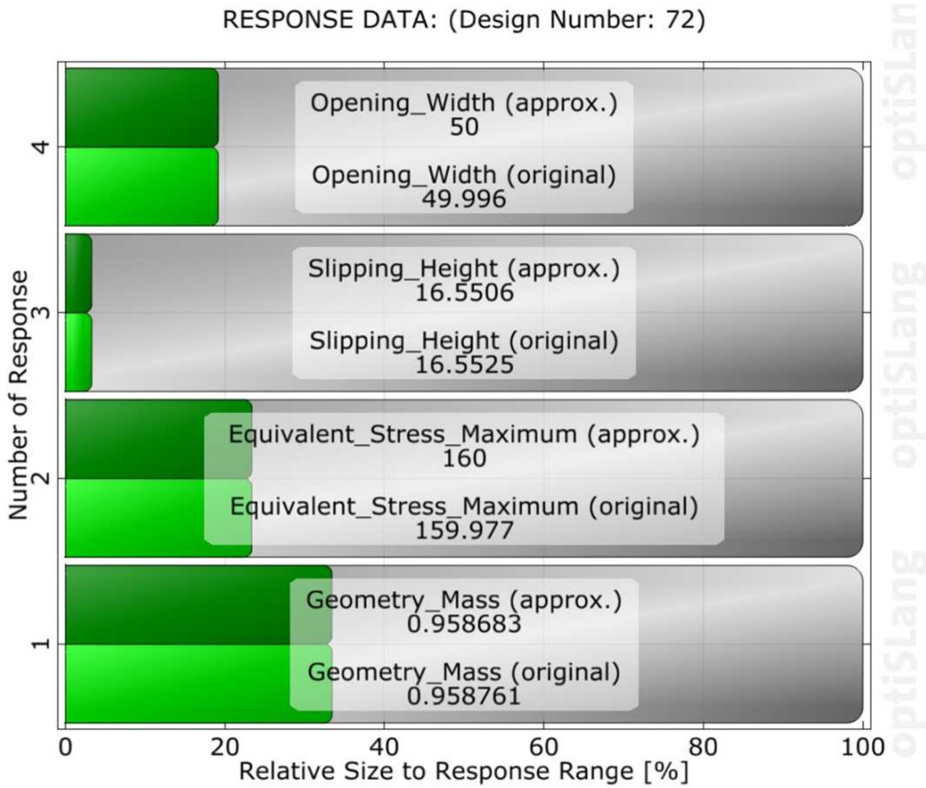
- Robustness sampling is performed again at new optimal design (parameters from second robustness analysis are imported and mean values are taken from the best design of optimization)
- Reliability analysis is performed again to verify sampling estimates



Second Robust Design Optimization Step

- ARSM optimizer obtains an optimal design within 6 iterations which fulfills the constraint conditions
- Mass is 10% larger as in the first optimization step
- Slipping height and opening width are almost the same





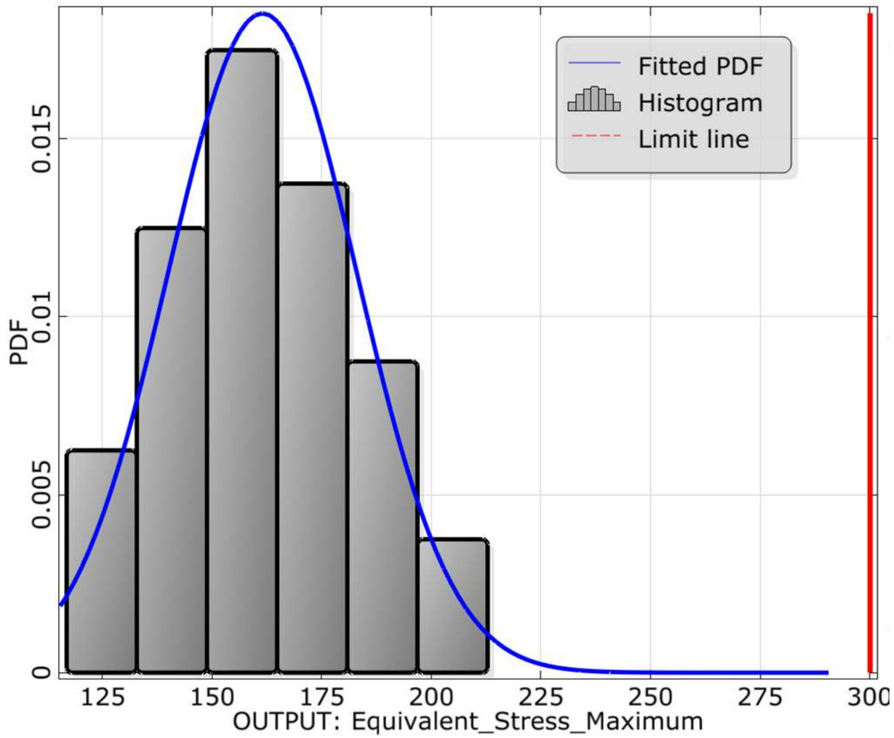
Second Robust Design Optimization Step

Statistical Evaluation of the Maximum Stress:

- 100 Latin Hypercube samples are computed
- Safety margin to failure limit of 300MPa is estimated with a sigma level of 6.4 assuming a normal distribution

➢ Significant increase w.r.t. first RDO step

➢ Again reliability analysis is required to proof this result

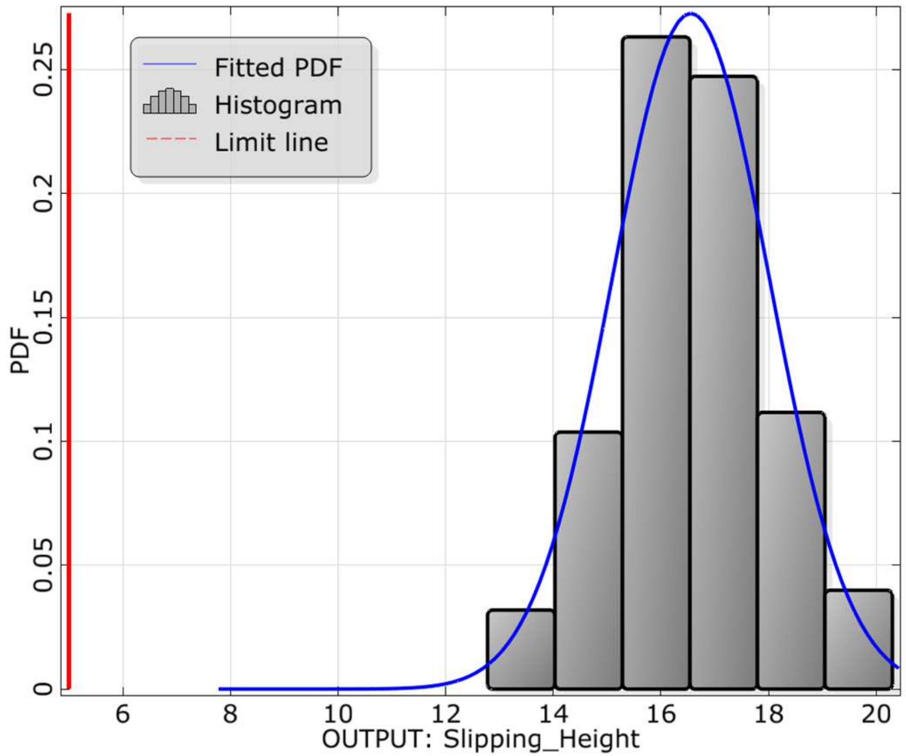


Statistic data			
Min:	116.8	Max:	212.8
Mean:	161.5	Sigma:	21.53
CV:	0.1333		
Skewness:	0.09242	Kurtosis:	2.439
Fitted PDF: Normal			
Mean:	161.5	Sigma:	21.53
Limit x = 300			
P_rel:	1	1 - P_rel:	0
P_fit:	1	1 - P_fit:	6.23184e-011
Sigma-Level:	6.43358		

Second Robust Design Optimization Step

Statistical Evaluation of the Slipping height:

- The safety margin is slightly smaller as is the first RDO step
- Robustness criteria for slipping seems to be fulfilled



Statistic data			
Min:	12.78	Max:	20.3
Mean:	16.56	Sigma:	1.463
CV:	0.08837		
Skewness:	0.1695	Kurtosis:	3.024
Fitted PDF: Normal			
Mean:	16.56	Sigma:	1.463
Limit x = 5			
P_rel:	0	1 - P_rel:	1
P_fit:	1.44329e-015	1 - P_fit:	1
Sigma-Level:	7.89865		

Final Reliability Proof

- Reliability analysis is performed again with default ARSM
- Failure probability is smaller than 10^{-6}
- Corresponding reliability index is about 4.8 which fulfills the robustness requirements

Method : Directional Sampling on Adaptive Response Surfaces (ARSM-DS)

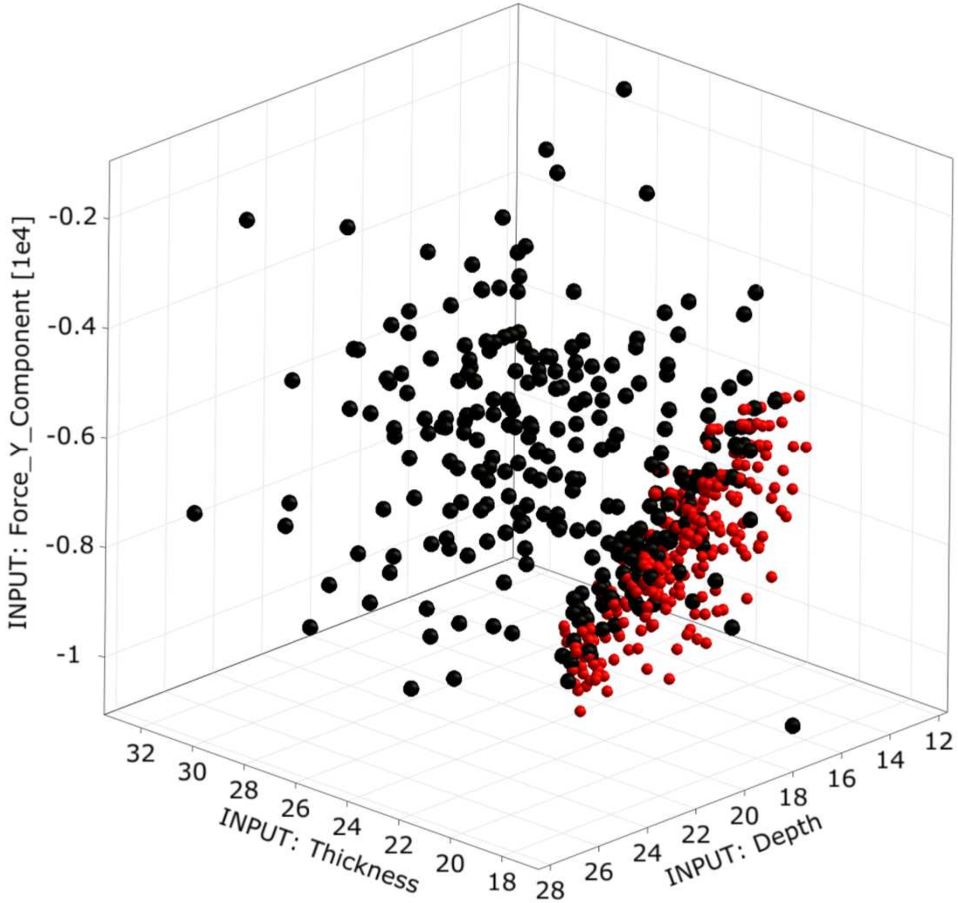
Selected data : 3. Approximation

Number of designs : 300 (0 failed)

Approximation errors : R^2 / R^2pred
Limit_stress : 1 / 0.983288
Limit_slipping : 1 / 0.99892

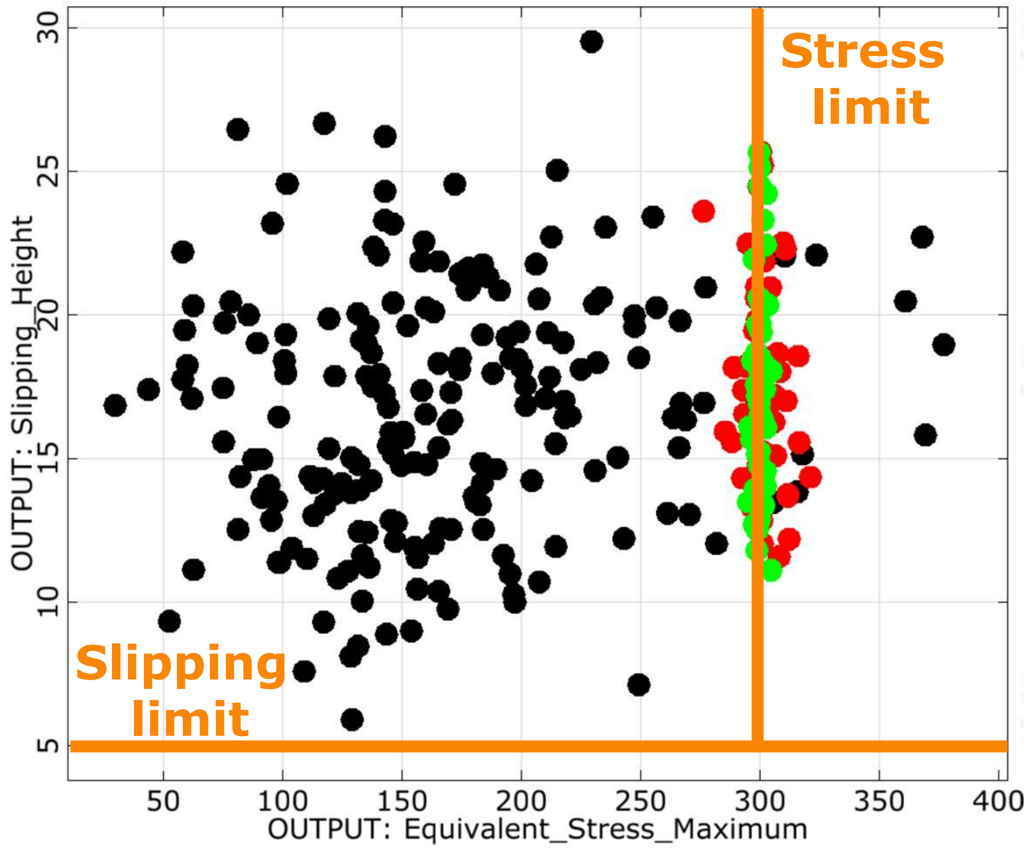
Complete directions : 1000 / 1000

$\boxed{\text{Probability of failure : 8.293e-007}}$
Standard deviation error : 2.799e-007
$\boxed{\text{Reliability index : 4.7911}}$



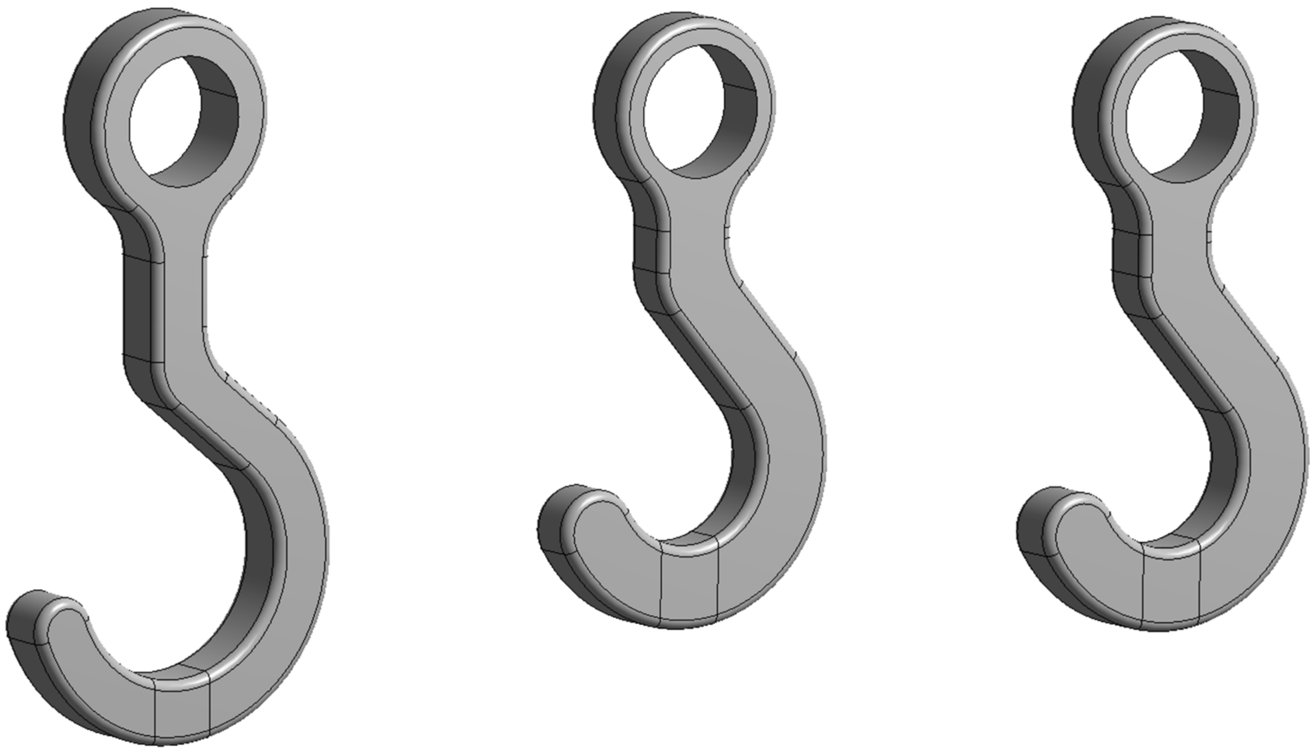
Final Reliability Proof

- ARSM generates new support points only in the region of the stress limit
- ➢ Limit of slipping height is much less important
- After the 3rd iteration the stress limit is represented quite accurately



1. Iteration
2. Iteration
3. Iteration

Overview Robust Design Optimization



	Initial Design	1st RDO Step	2nd RDO Step
Mass	1100 g	854 g	959 g
Mean stress	270 MPa	180 MPa	160 MPa
Failure probability	23%	10^{-4}	10^{-6}
Reliability index	0.6	3.8	4.8