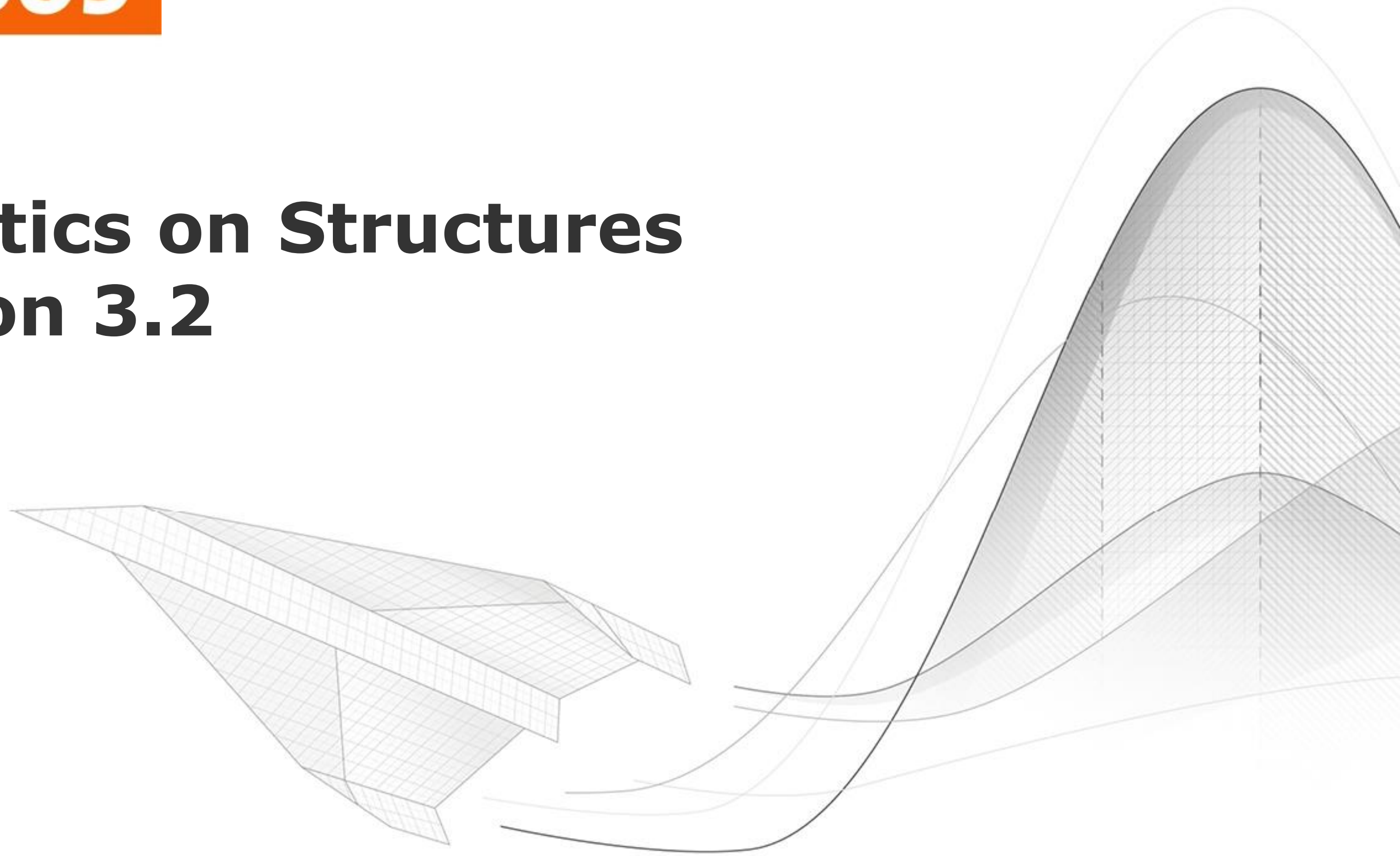
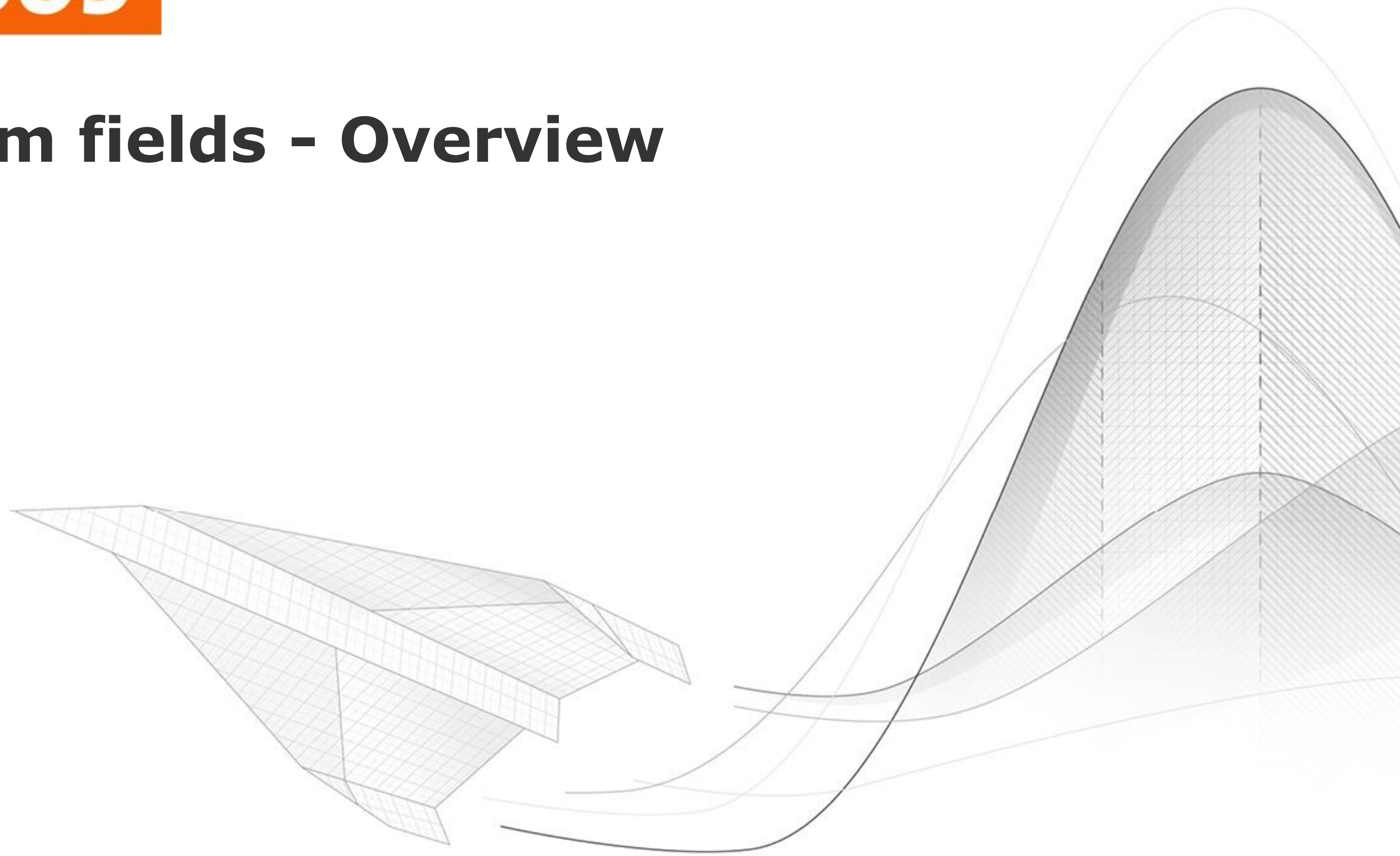
Statistics on Structures Version 3.2



Outline

1. Overview on random fields
2. New features
    1. New mesh mappers
    2. Synthetic random field models
    3. Direct export of geometries
    4. Analysis of signals
    5. Model calibration with F-MOP

Random fields - Overview



Uncertainties in design and optimisation

- Observed randomness/uncertainties in
  - Design variables (e.g. manufacturing tolerances)
  - Objective function (e.g. design variables, tolerances, external factors)
  - Constraints (e.g. design variables, tolerances, external factors)

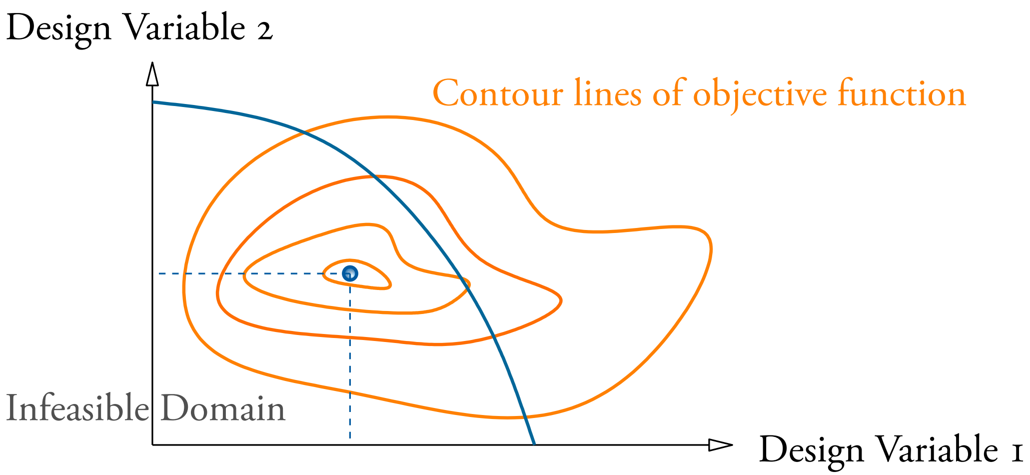
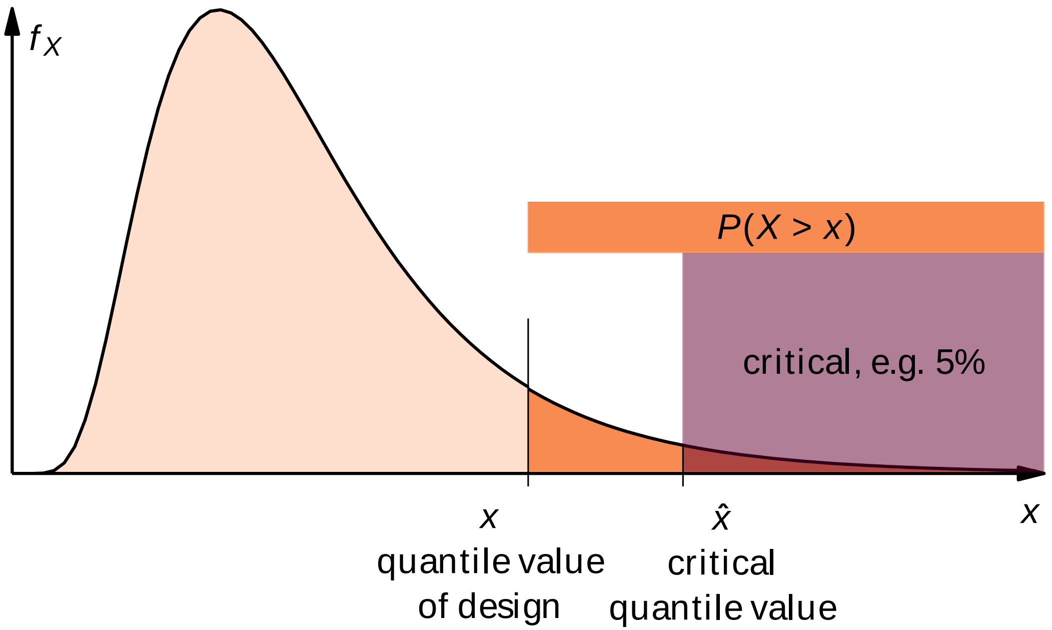


Image source: Christian Bucher: "Basic concepts of stochastic analysis", 10th Weimar Optimization and Stochastic Days, 2013, Weimar, lecture notes

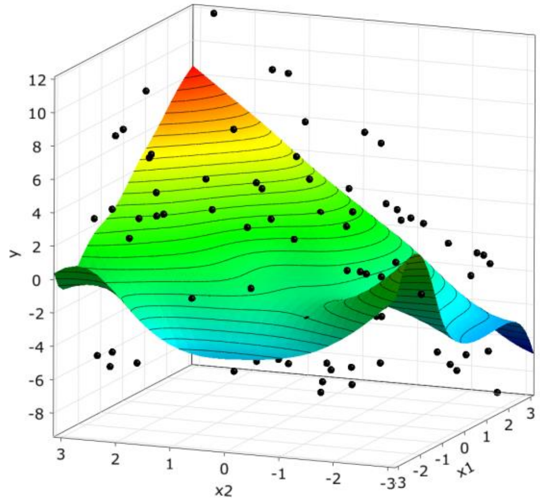
Robustness in terms of constraints

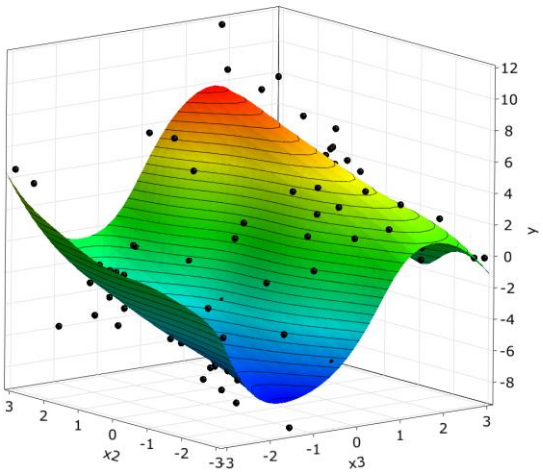
- Example: Determine robustness in terms of
  - Exceedance probabilities
  - Quantiles
  - 6-sigma design
  - Standard deviation
  - QCS statistics...



How to determine failure sources ?
Sensitivity analysis based on meta modelling

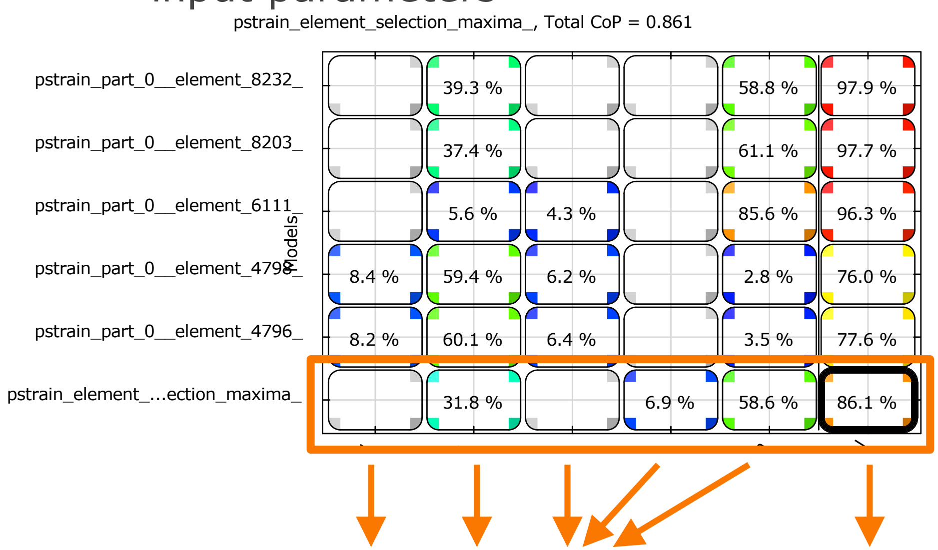
- Meta model:
  - "Surrogate", "Response surface"
  - Based on a design of experiments (DoE)
  - Approximates CAE solution by simple algebraic functions (fast evaluation)
    - Polynomial regression
    - Neural networks
    - Support vector machines
    - Moving Least Squares
    - Kriging
    - Radial basis functions
    - …
  - Combinations with nonlinear transformations (e.g. Box Cox)

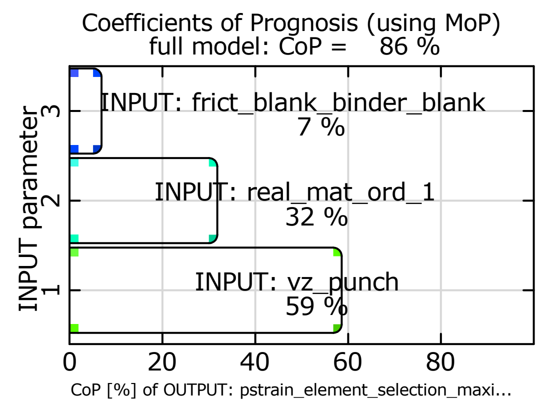


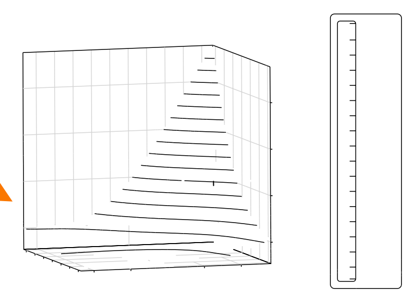


How to determine failure sources ?
Metamodel of Optimal Prognosis

- Software optiSLang: Metamodel of Optimal Prognosis (MOP)
  - Compare prognosis quality of a large set of meta model types
  - Identify the meta model with best prognosis quality and use it to rank input parameters







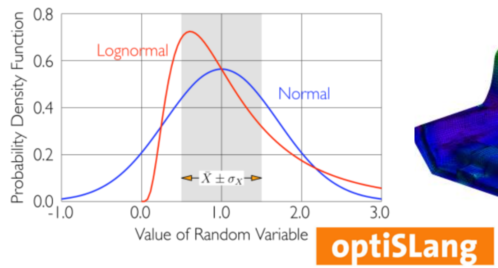
Influence of input parameters

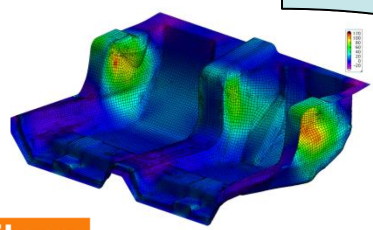
Accuracy of model

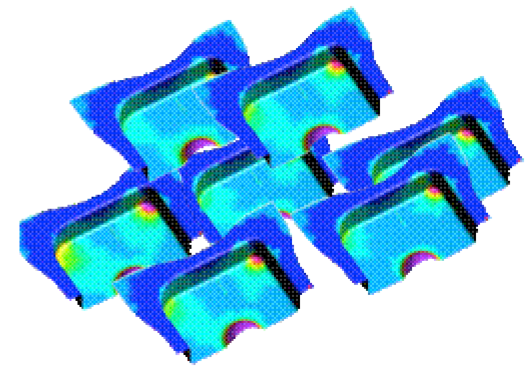
Variance based robustness analysis

1) Define the robustness space using scatter range, distribution and correlation

2) Sampling: Scan the robustness space by producing and evaluating n designs



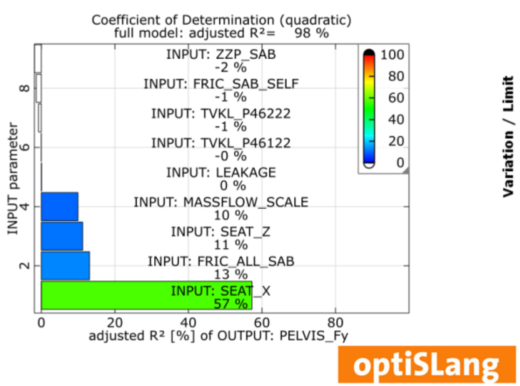


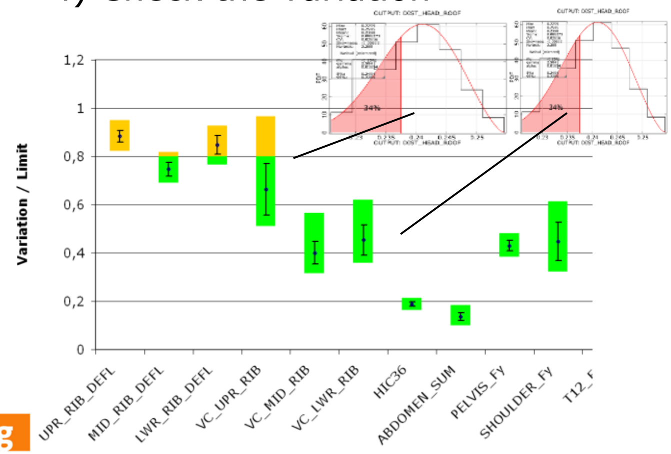


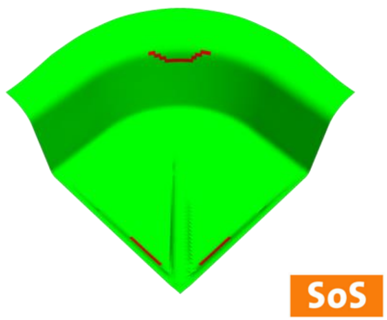
5) Identify the most important scattering variables

4) Check the variation

3) Identify hot spots







Robustness evaluation
Uncertainties of scalar parameters

- Many tools for process integration and automatisation - easy to use flows
- Requires the simulation of a design of experiments (DoE)
  - Number of designs depends on degree of nonlinearity and number of scattering parameters
  - Statistics of input parameters must be known
- Robustness evaluation based on statistical means
- Sensitivity analysis identifies most significant scattering parameters
  - Parameters whose input variations influence the response variations at most
  - Changes to these parameters affects robustness at most
  - Approximate model response by meta models to speed up robustness and reliability analysis in Robust Design Optimization
- **Limitations**:
  - Most FEM responses are field data (varying value AND varying location!)
  - Robustness often affected by distributed quantities (i.e. random fields)

Robustness example
Deep drawing

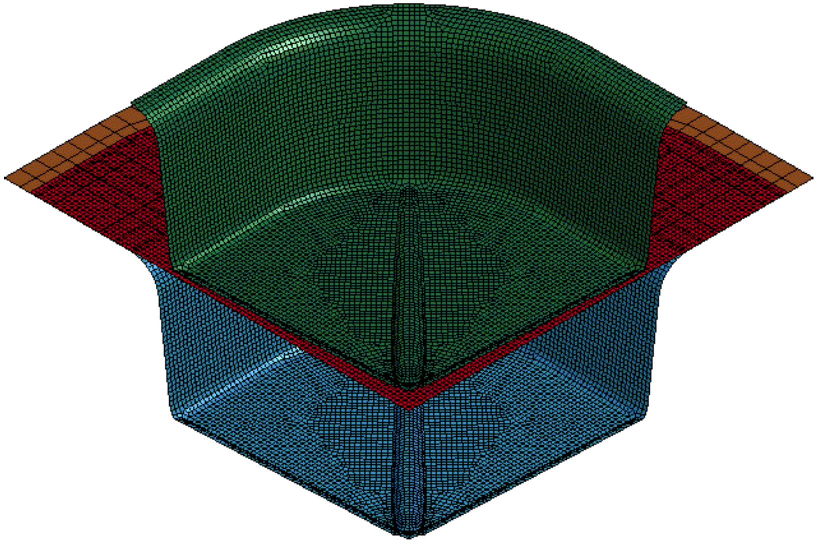
- Simulation of production process
- Analysis of random variations of production parameters

- Robustness goals: max. pstrain and max. thinning exceed critical thresholds by max. probability p

- Solution using scalar parameters: Analyse statistics of maximum values

- Problem: **Varying position of maximum plastic strain** and maximum thinning

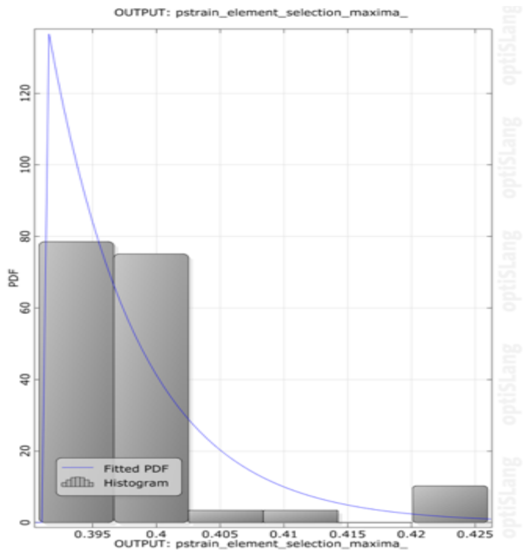
- optiSLang + LS-Dyna, 100 designs

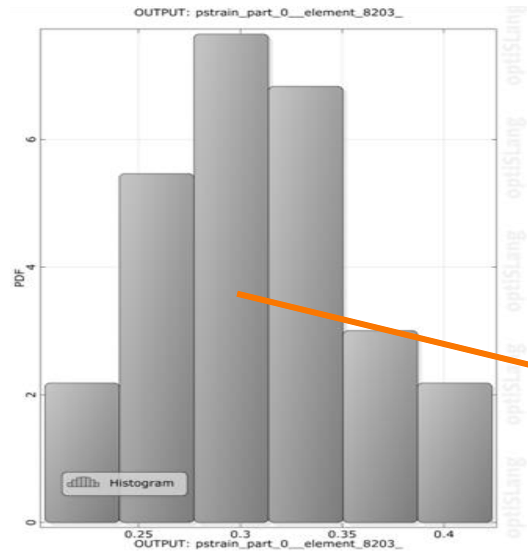


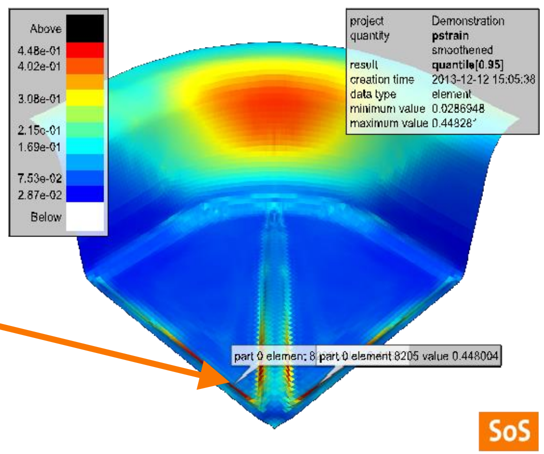
Random fields
Effect of varying location

- Statistical analysis of

  maximum plastic strain vs. plastic strain at hot spots







SoS

- Sensitivity analysis: Meta model of Optimal Prognosis in optiSLang

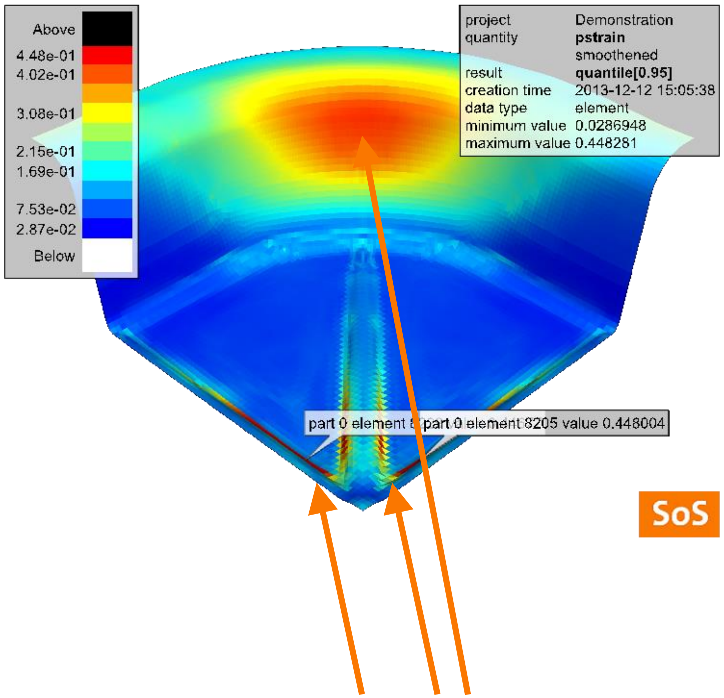
  CoP(Total)=86% CoP(Total)=98%

- **Improved accuracy at hot spot instead of maximum value!**

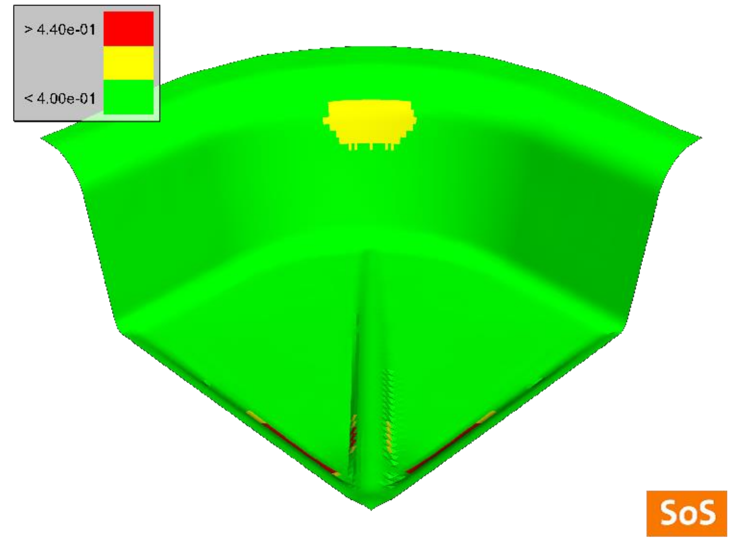
Random fields

Hot spot detection and robustness evaluation

- 95%-Quantile values of plastic strain



Hot spots
(Local maxima)



Highlight locations where a critical value is exceeded

Random fields:
Typical quantities being random fields

- geometric perturbations
  - node coordinates
  - shell thickness
  - thickness of composite layers
- material properties
  - concrete: mortar, admixtures (gravel)
  - ceramics: porosity
  - contact friction
- damage
  - plastic strain, cracks
- loading
- state variables
  - stresses, strains
  - displacements
- signals (random fields in time)

Random fields
Illustration

- Different realizations of a one-dimensional field

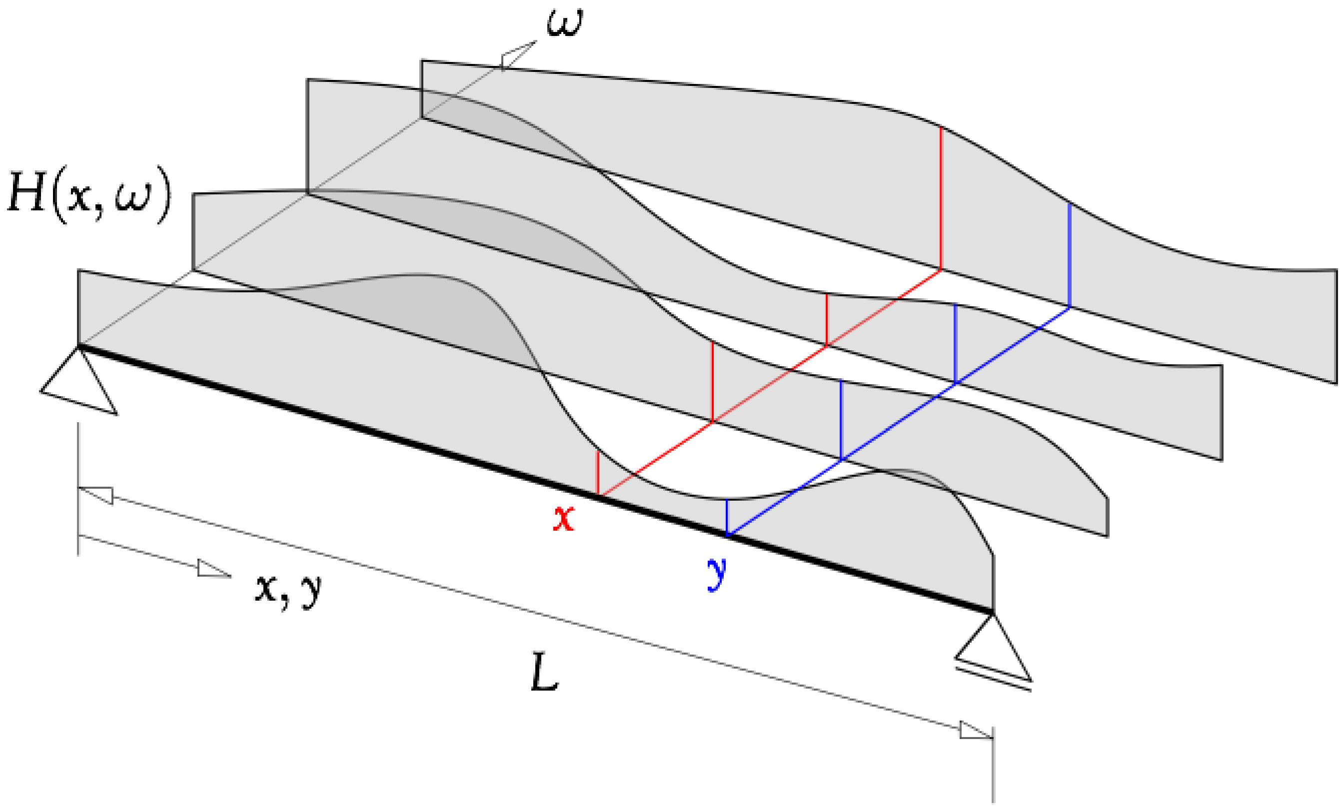
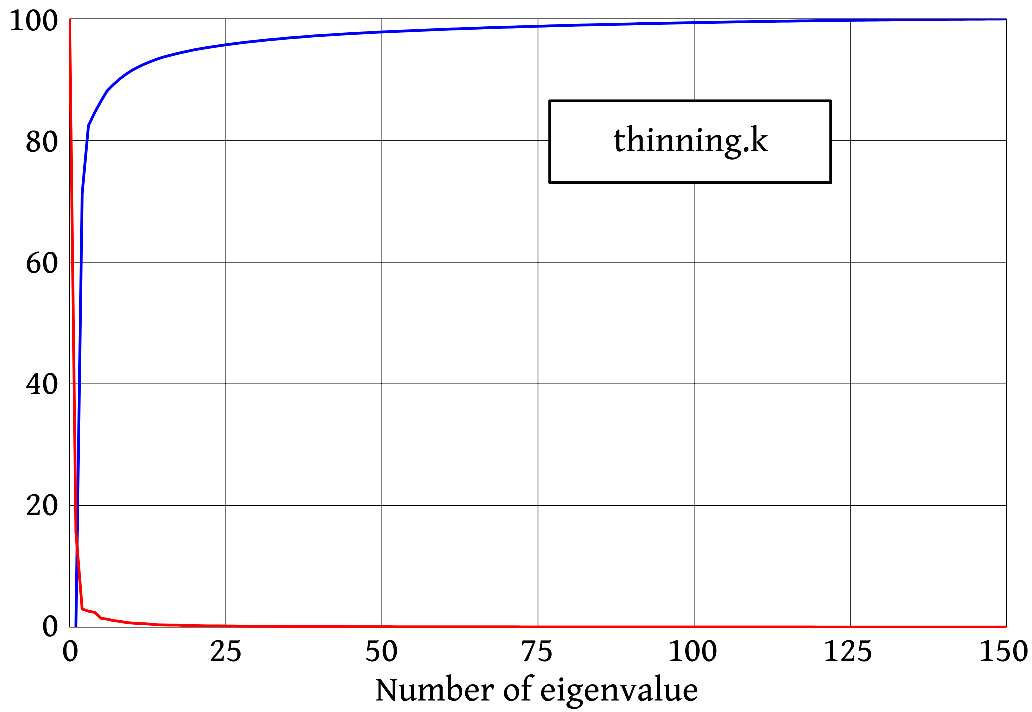


Image source: Christian Bucher: "Computational Analysis of Randomness in Structural Mechanics". Taylor and Francis. 2009

Random fields
Modal analysis of covariance matrix

- Eigenvalues (red) and their cumulative sum (blue) for a random vector
- 3 eigenvalues: >90% total variability



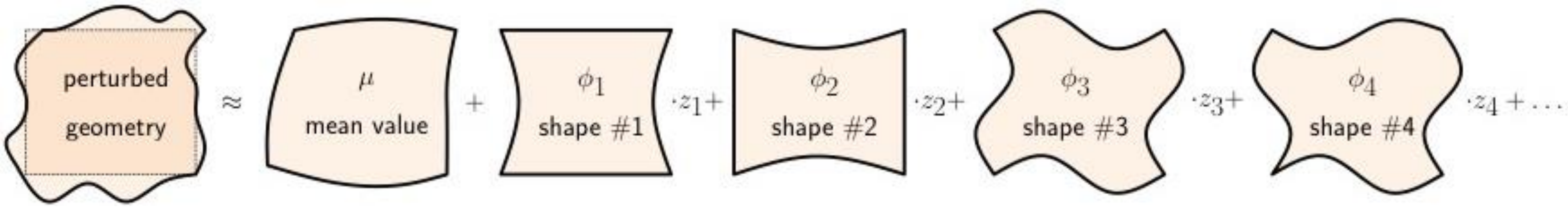
Random fields

Karhunen-Loeve expansion for discrete fields

- Karhunen-Loeve expansion: Modal analysis of the covariance matrix
- Spectral representation

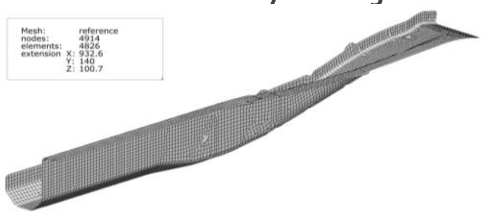
$$\mathbf{H} = \mathbf{\Phi z} + \mu_{\mathbf{H}}$$

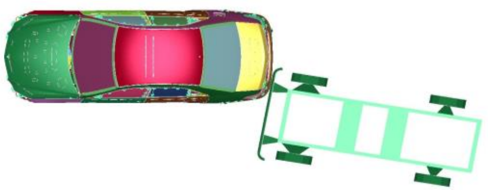
- Phi: Eigen vectors of covariance matrix ("mode shapes", "scatter shapes")
- z: vector of reduced set of uncorrelated random numbers ("amplitudes")



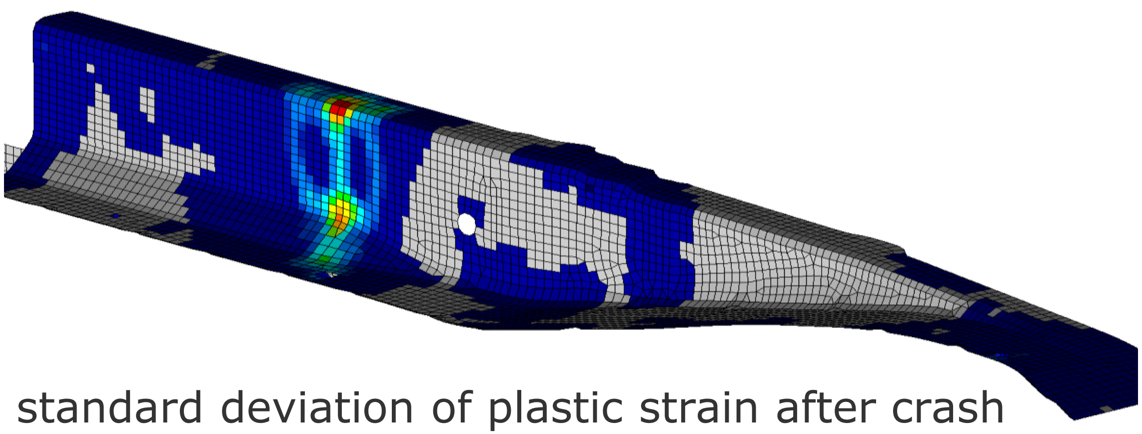
Analysis of distributed effects
Example: Nonlinear buckling shapes

- Stringer in a car body subject to crash simulation





- Hardware experiment: observation of plastic buckling
- Deterministic simulation: plastic effects not reproducible
- Stochastic simulation: DoE and robustness evaluation with optiSLang

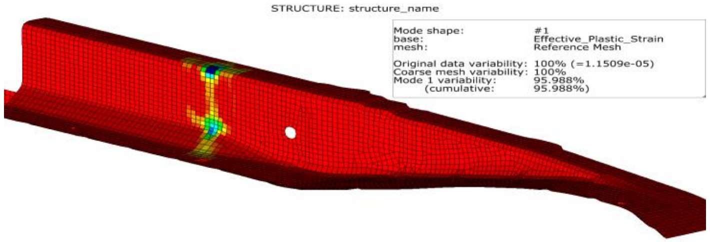


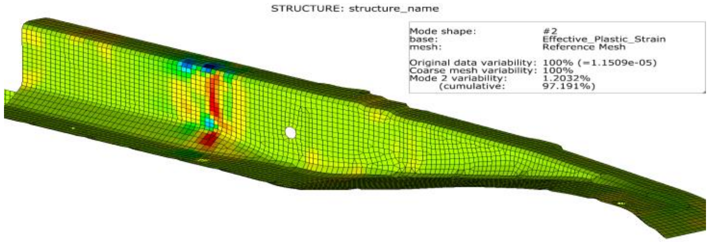
standard deviation of plastic strain after crash

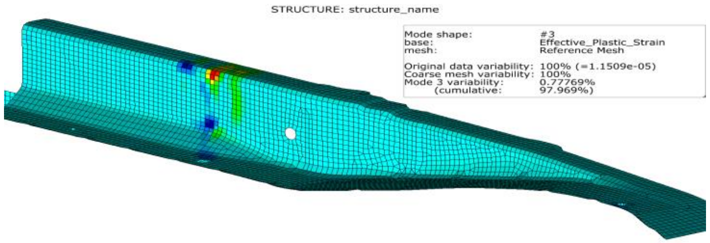
Analysis of distributed effects

Example: Nonlinear buckling shapes

- How to analyse failure sources ?
  Buckling shape is distributed!
- Decomposition of plastic strain random field into uncorrelated scatter shapes
- Right: 1st three scatter shapes
- Only 3 parameters explain 98% of total variation (compared with original 4826 parameters, one for each element)
- 1st shape (96% of total scatter) explains most of the effects observed in standard deviation
- This strategy is always interesting since typically input parameters affect several locations at same time



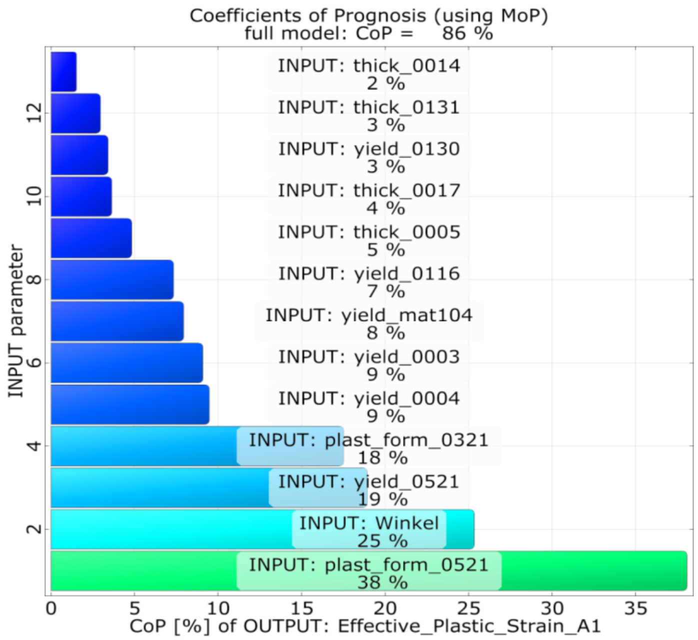




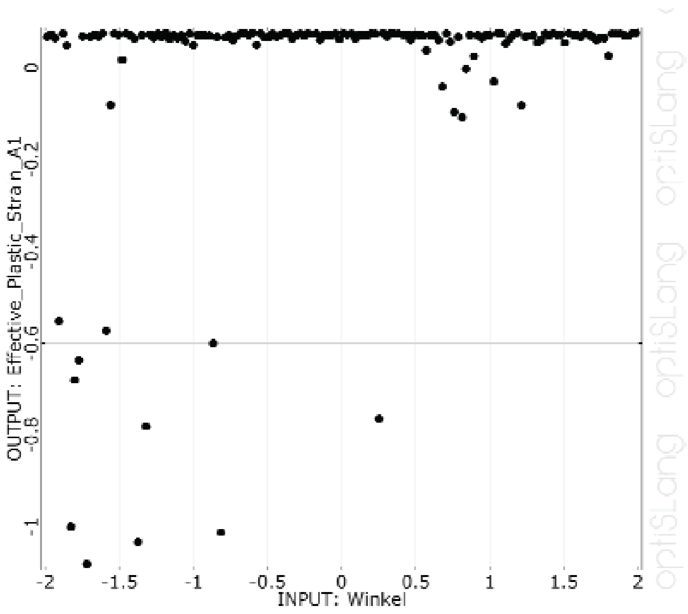
Analysis of distributed effects

Example: Nonlinear buckling shapes

- Solution: 13 of 55 input variables with significant effect on 1st amplitude; These are the initial plastic strains due to manufacturing (forming), the barrier angle and the yield stress



CoP for 1st amplitude



Scatter plot of plastic strain (vert.) over barrier angle (horiz.)

Cause analysis

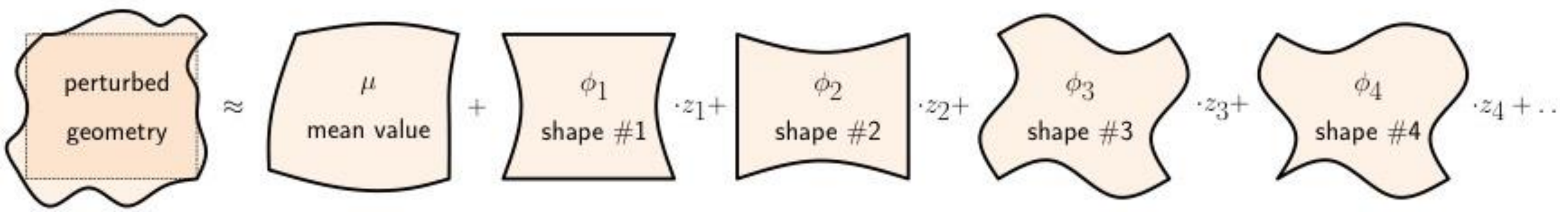
How large is the influence of some input parameter onto the outcome compared with other inputs ?

What is the amount of explainable variation of our response ?

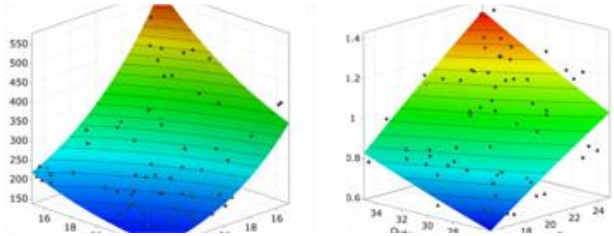
Fiele meta model

Combine Random field with MOP: „Field MOP“

- 1st layer: Represent spatial variations of field responses in terms of scalar parameters z_i



- 2nd layer: Represent these parameters in terms of the inputs by MOP



- Black box model (F-MOP):

Inputs → **F-MOP** → **Response field**

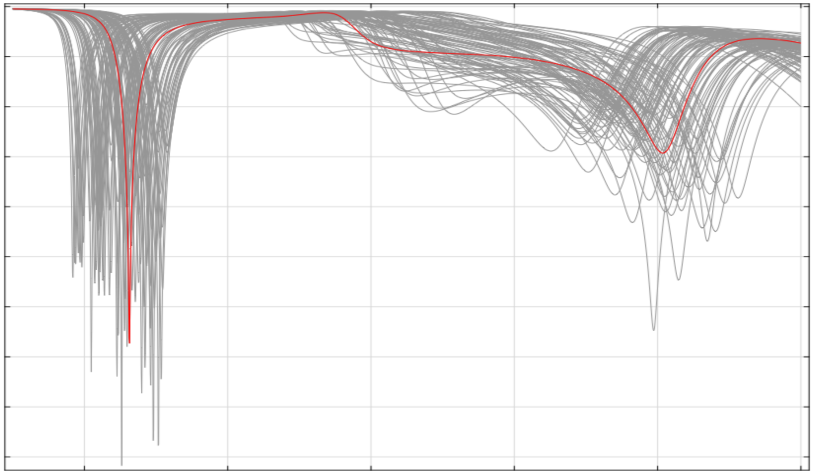
Field meta model
Sensitivity analysis

- Field Coefficient of Optimal Prognosis (F-CoP):
  - A global F-CoP value: integral value of the CoP for the entire field
  - F-CoP plot: approx. CoP at the respective position
- F-CoP of whole model:
  - Explainable variation at specific position
- F-CoP for individual input parameters:
  - Explainable variation at specific position through respective parameter

- **Identifies and orders important input parameters**
- **Simplifies presentation and interpretation !**

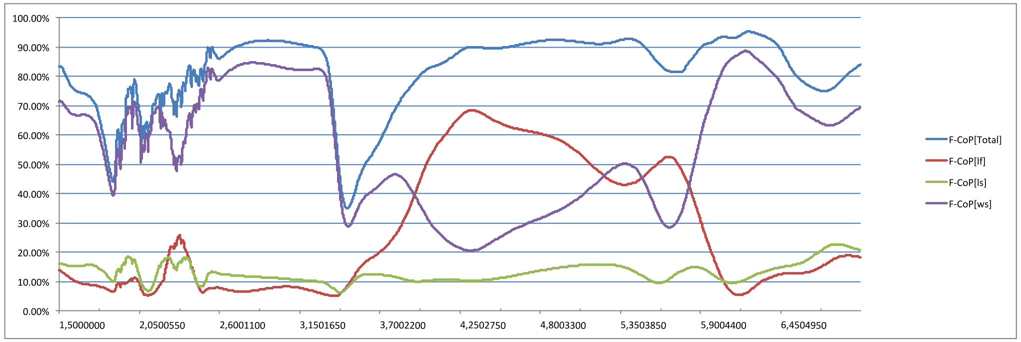
Field meta model
Sensitivity analysis of signal data

Signal data: „return loss" of a dual band antenna in frequency domain



Top:
Signals in DoE

Bottom:
F-CoP of field meta model and
F-CoP of individual input parameters onto signal variation for each frequency



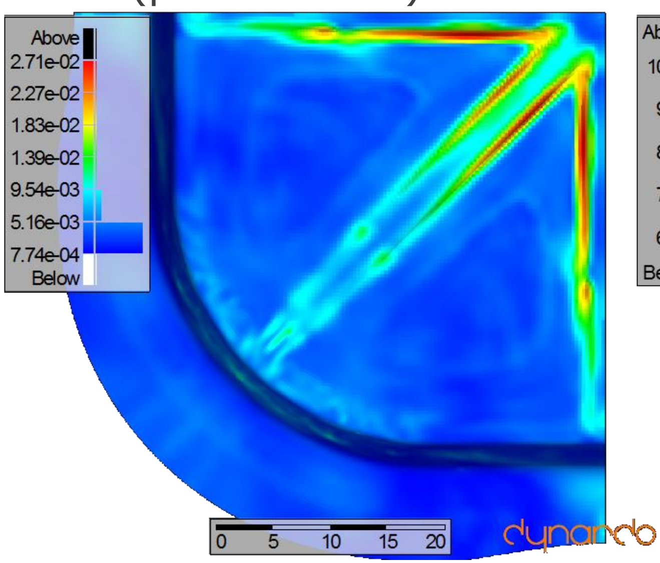
With courtesy of CADFEM

Field meta model

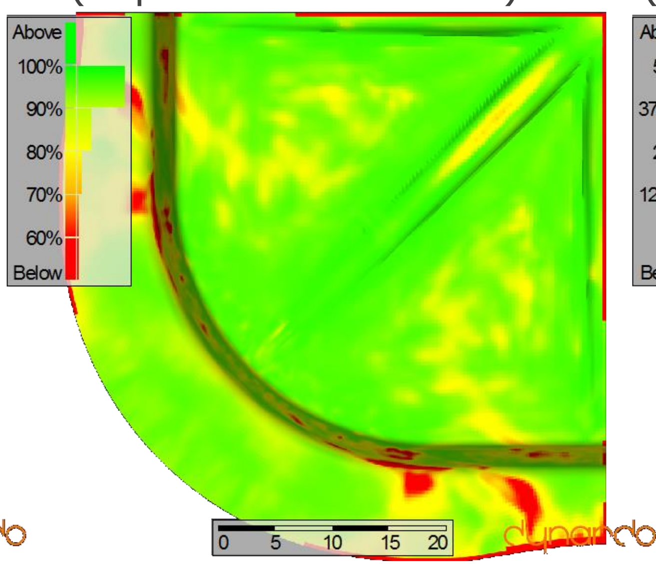
Sensitivity analysis of 3D data

- Visualize accuracy of meta model (center) and sensitivity of individual input parameters (right) with respect to different locations of field variations
- Use: critical location is not yet known; model validation, multiple hot spots ...

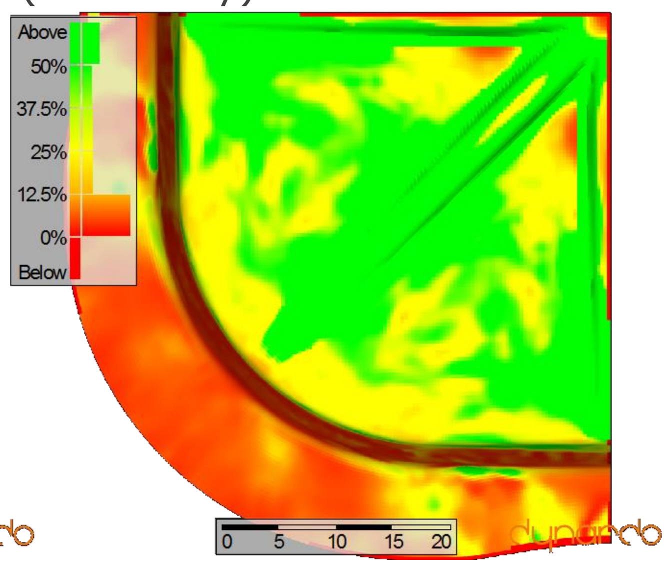
std.deviation
(plastic strain)



F-CoP (Total)
(explainable variation)



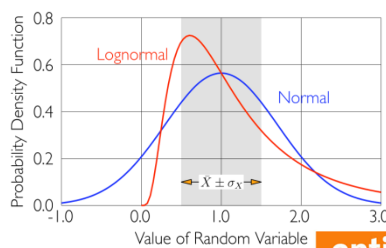
F-CoP (input vz_punch)
(sensitivity)

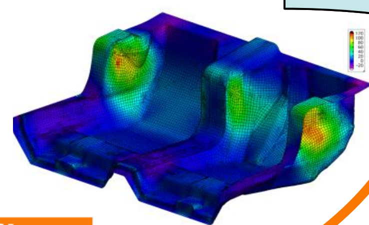


Generation of random designs

Variance based robustness analysis

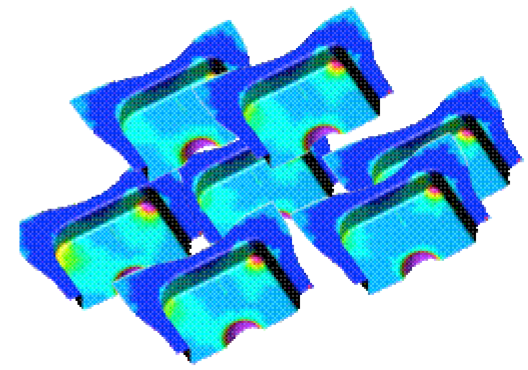
1) Define the robustness space using scatter range, distribution and correlation



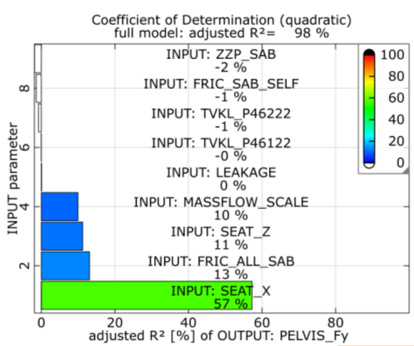


optiSLang

2) Sampling: Scan the robustness space by producing and evaluating n designs

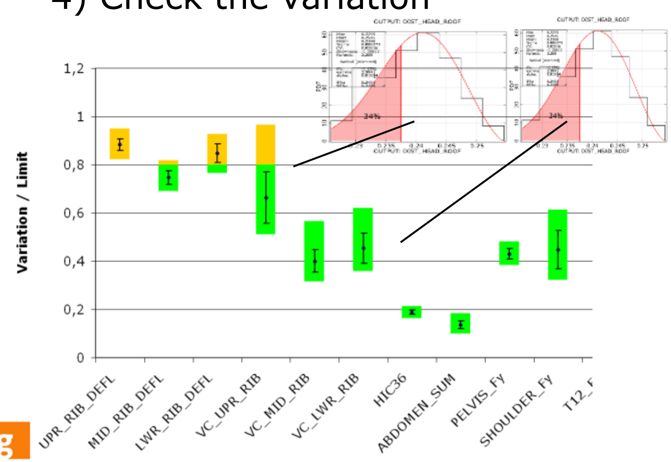


5) Identify the most important scattering variables

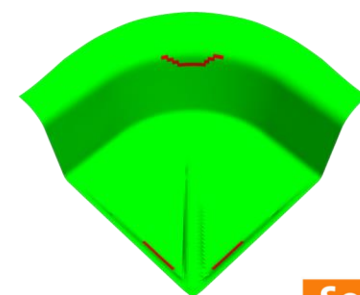


optiSLang

4) Check the variation



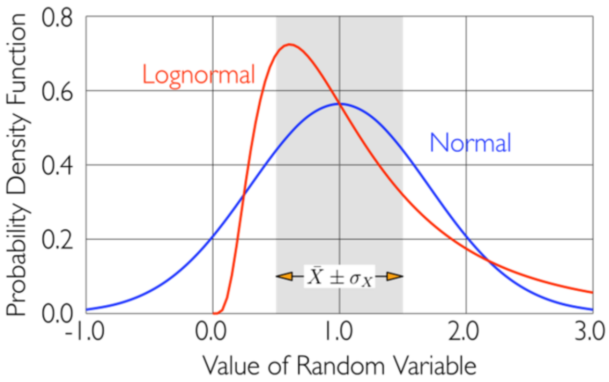
3) Identify hot spots



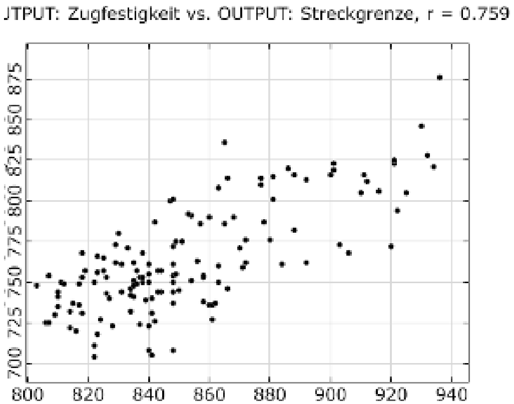
SoS

Generation of random designs
Definition of uncertainties

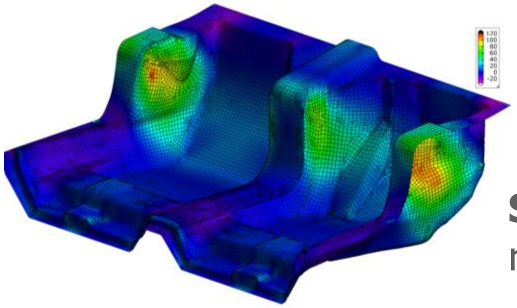
- Translate know-how about uncertainties into proper scatter definition



Distribution functions define variable scatter



Correlation is an important characteristic of stochastic variables

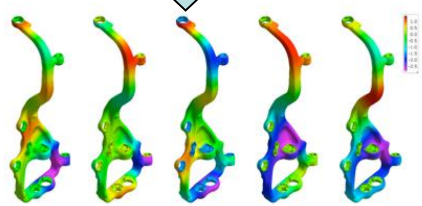


Spatial Correlation = random fields

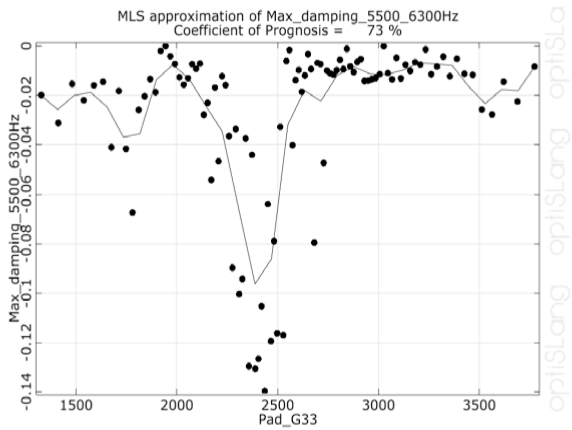
Generation of random designs
Synthetic random fields

- Synthetic random fields: Based on a single or only few measurements
- Spatial correlation defined by numerical model
- Example: Analysis of influence of geometric variations of a knuckle onto frequency of brake squeal noise

difference between measured and modeled geometry of a knuckle



simulation of imperfect geometries

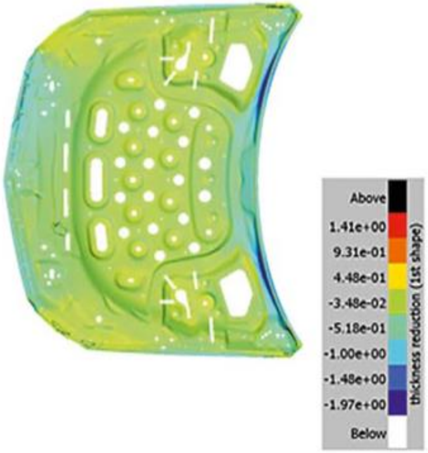


Predict change of Eigen frequencies due to uncertain geometric perturbations

Generation of random designs

Empirical random field models

- Accurate stochastic model for variations including inhomogeneity, anisotropy, non-Gaussian distribution and cross-correlation among vector-valued fields
- E.g. geometry, pre-stress, plastic strains, shell thickness, surface friction, loading conditions

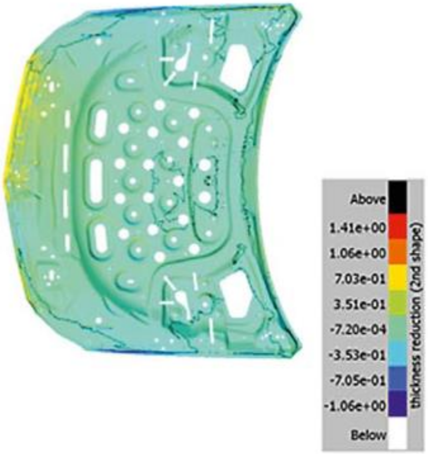


Industrial example: Simulate random geometries of a deep drawing process in a joining simulation

Step 1: DoE of deep drawing process with spring back, Analyse random field of resulting geometries

Step 2: Simulate new random geometries in a joining simulation

Step 3: Analyse influence of deep drawing uncertainties in subsequent joining process

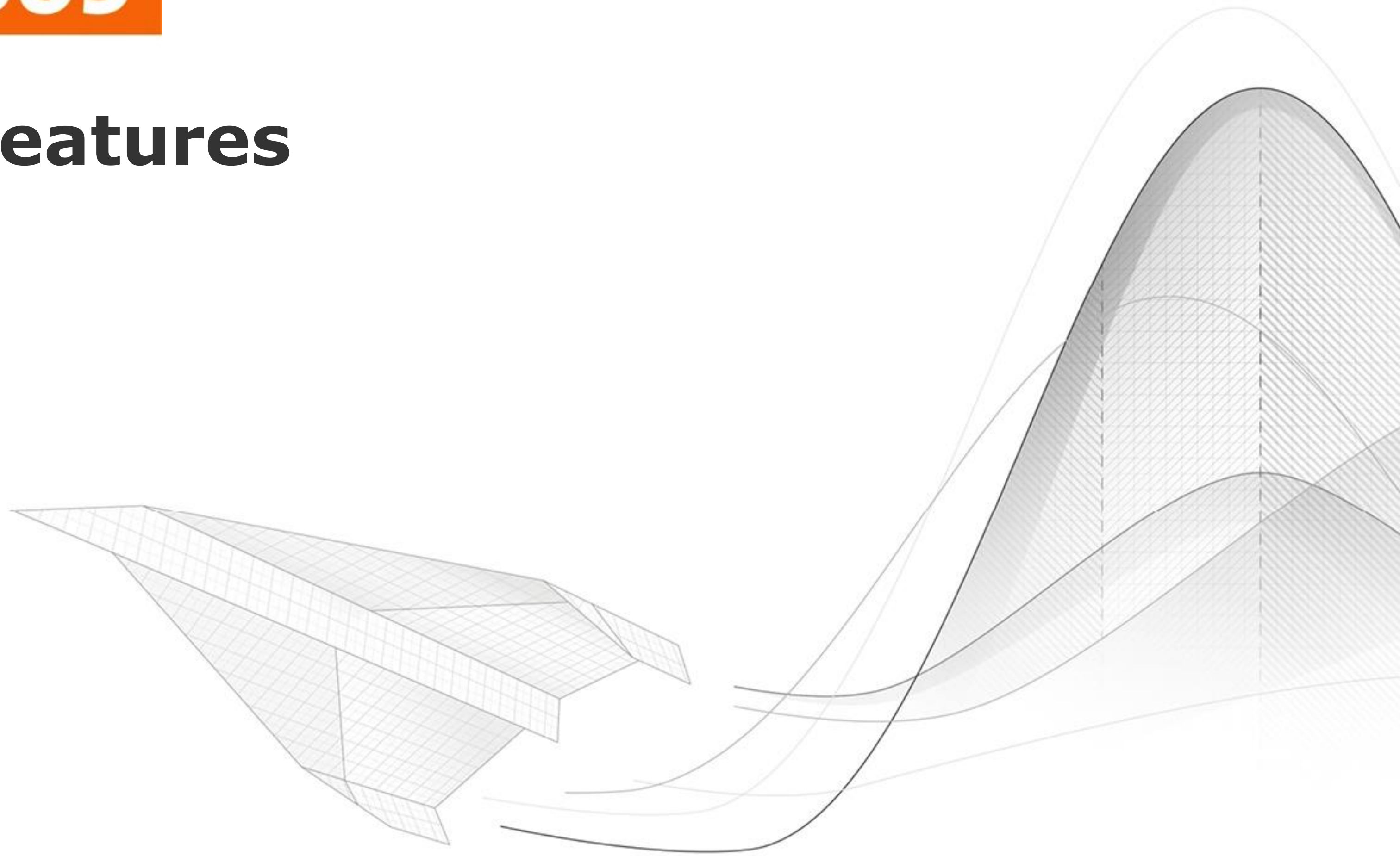


Summary

- **Hot spot detecton**: Identify most likely failure points based on statistical means
- **Random field modelling**: If hot spot detection is not applicable (too many hot spots or distributed "smeared" effects). Finds a "parametric" to represent fields in optiSLang
- **Sensitivity analysis** based on field meta models: Visualize sensitivities for all model locations in space or time; Analyse DoE before actual hot spot is known
- **Generation of random designs** based on statistical models:
  - Accurate empirical models (many measurements)
  - Synthetic models with empirical mean/stddev (3-5 measurements)
  - Synthetic models (based on assumptions)
- **Model calibration**: If reference is a field quantity (e.g. measured signal or geometry)

SoS

New features



New features in version 3.2

- New mesh mapper for data transfer between meshes: User specified search direction
- New: Synthetic random field models
  - Linear and squared exponential autocorrelation, isotropic
  - Using an empirical mean or standard deviation (e.g. >3 measurements)
  - Using constant mean and standard deviation (e.g. 1 or no measurements)
- Visualization: Instantly show random field variations
- Export: Direct export of FEM meshes, e.g. export quantile limits of geometries
- Analysis of Signals
  - Convert your signals to SoS format
  - Compute F-CoP values for signals in sensitivity analysis
- Model calibration based on field references and meta modelling (e.g. target geometries, target signals)
- New file formats

Import data to SoS

Software demonstration

Random field models

Software demonstration

Export geometries

Software demonstration

Analysis of signals

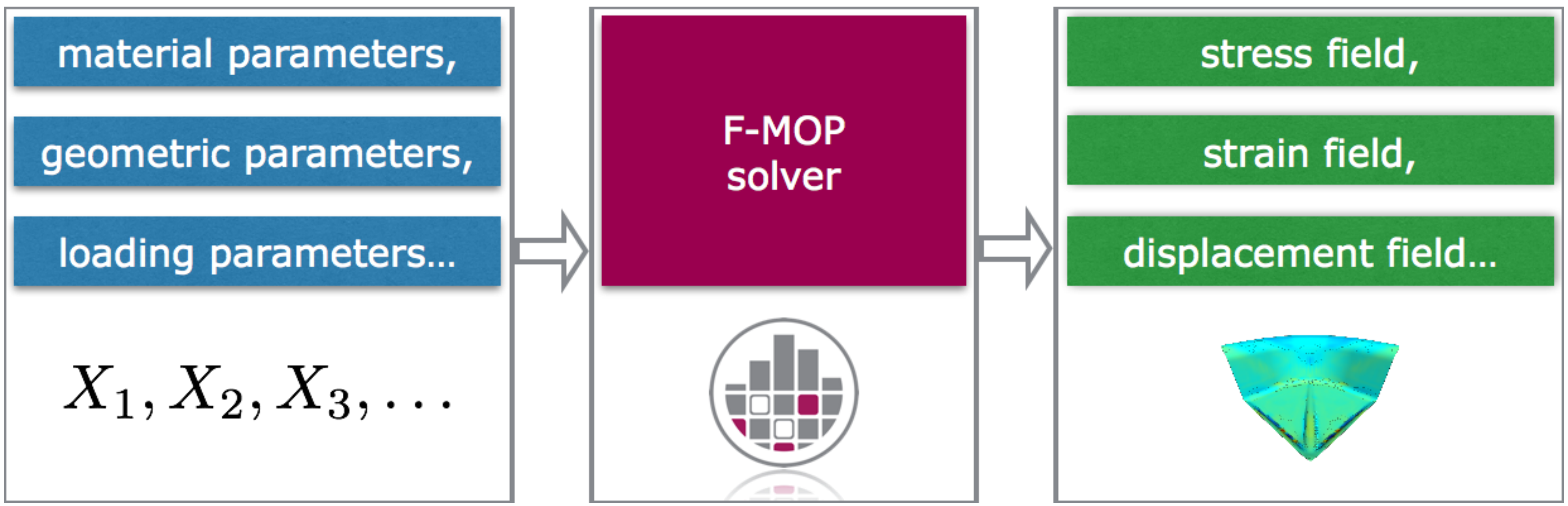
Software demonstration

Model calibration with F-MOP

Software demonstration

F-MOP Solver
Overview

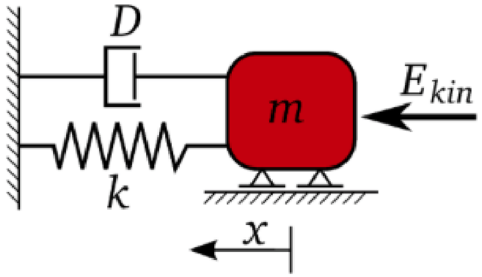
- Approximate field data by field meta modelling
- Replace the complete CAE solution by a meta model, fast evaluation
- Used in optimisation, model calibration, etc.



F-MOP Solver

Example: Parameter identification using signals

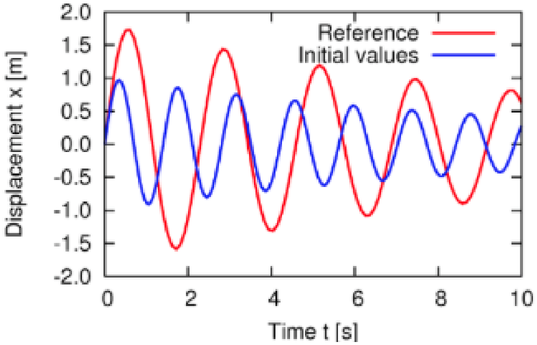
- Linear mechanical SDOF oscillator
- Task: Identify parameters m, k, D, Ekin such that displacement in time matches a measured reference signal



- DoE with 100 designs
- **optiSLang: squared error norm between response signal and reference signal with CoP of 57% only**

$$m \in [0.1;\ 5.0\ \mathrm{kg}]$$
$$k \in [10;\ 50\ \mathrm{N/m}]$$
$$D \in [0.01;\ 0.05]$$
$$E_{kin} \in [10;\ 100\ \mathrm{Nm}]$$

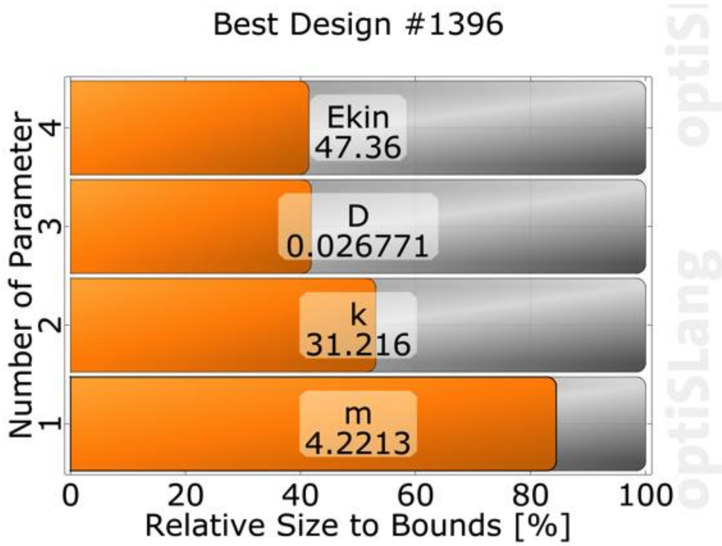
- SoS: create F-MOP for signal based on same DoE
- Minimize error norm based on approximated signals

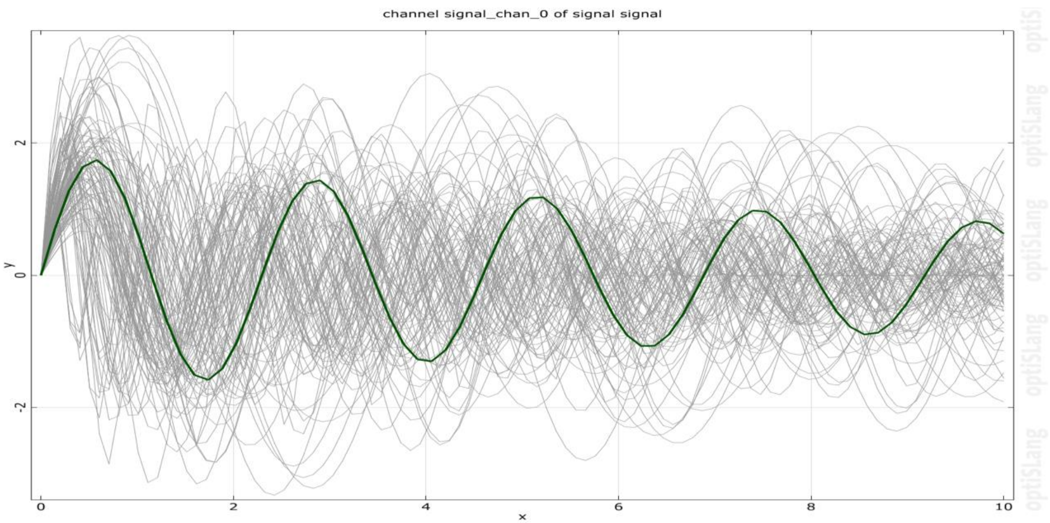


F-MOP Solver

Example: Parameter identification using signals

- Solution on field meta model



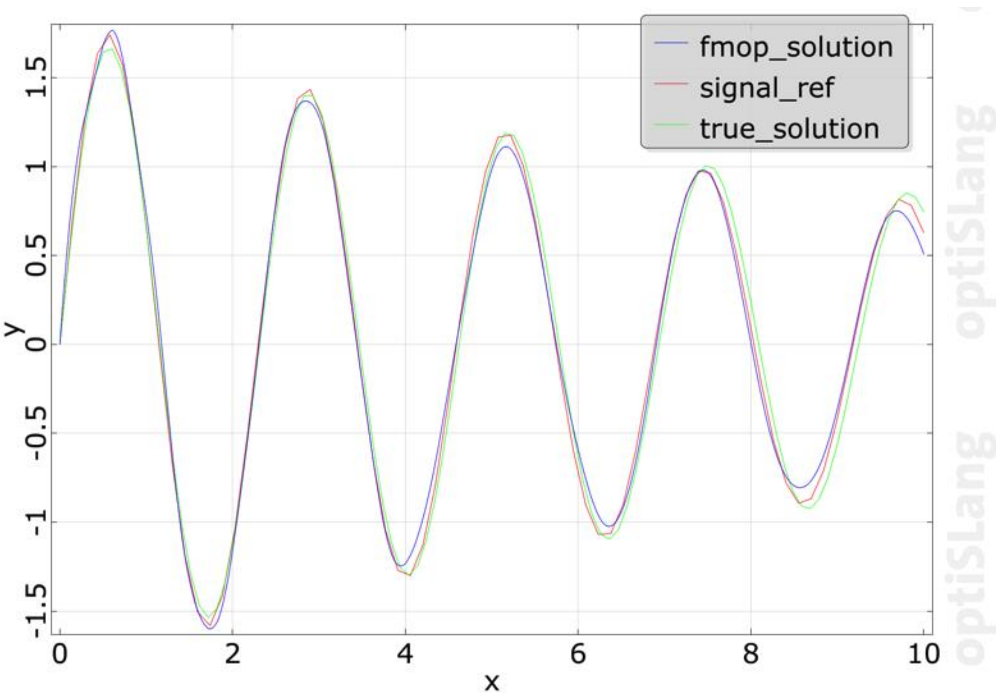


- Evolutionary Algorithm (global optimization problem, non-unique solutions): Minimize error between measured signal and approximated signal
- Signals in SoS: 1000 support points, reference signal: 70 support points

F-MOP Solver

Example: Parameter identification using signals

- Solution on field meta model (reduced error norm from **14** to **2.08**)
- True solution at best design of pre-optimization: error norm = **2.64**
- Accept approximation or use it as start point for optimization on true CAE process



Contact us at the DYNARDO booth in the conference lobby

Send an email to
contact@dynardo.at
Dr. Sebastian Wolff