Workshop
"Robust Design Optimization in Industrial Virtual Product Development"

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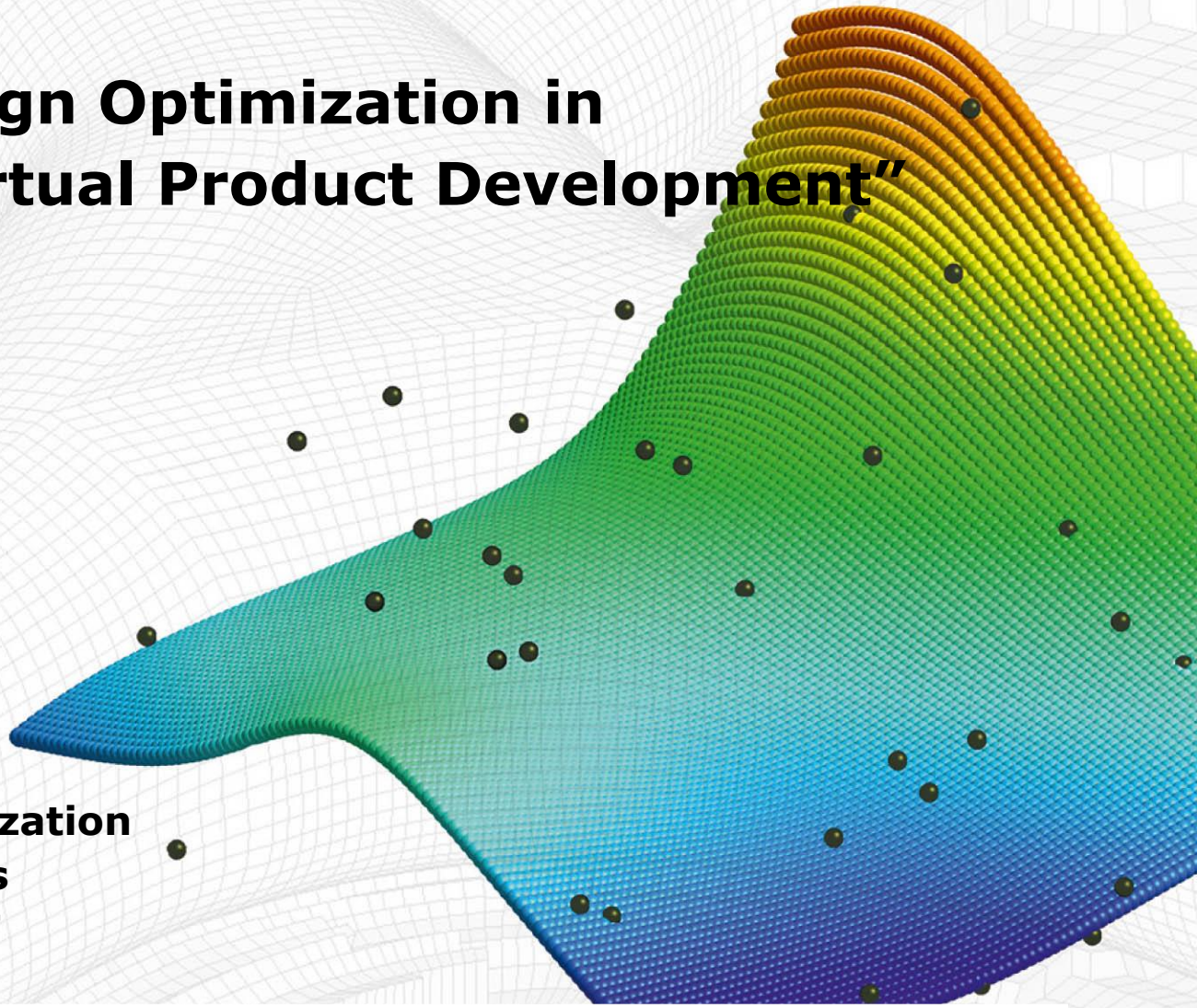
Dynardo GmbH

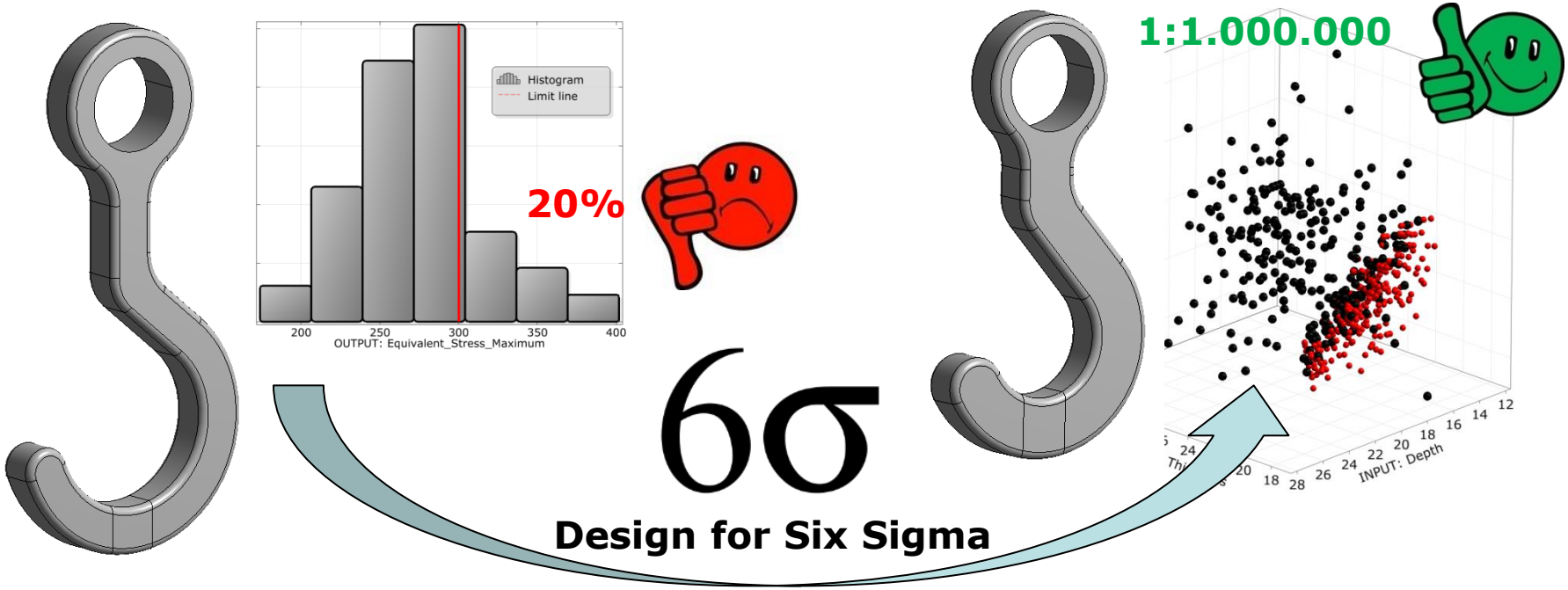


14th Weimar Optimization and Stochastic Days
01st-02nd June 2017

Motivation

- Robustness analysis and Robust Design are accepted, well-known but rarely applied methods in quality management

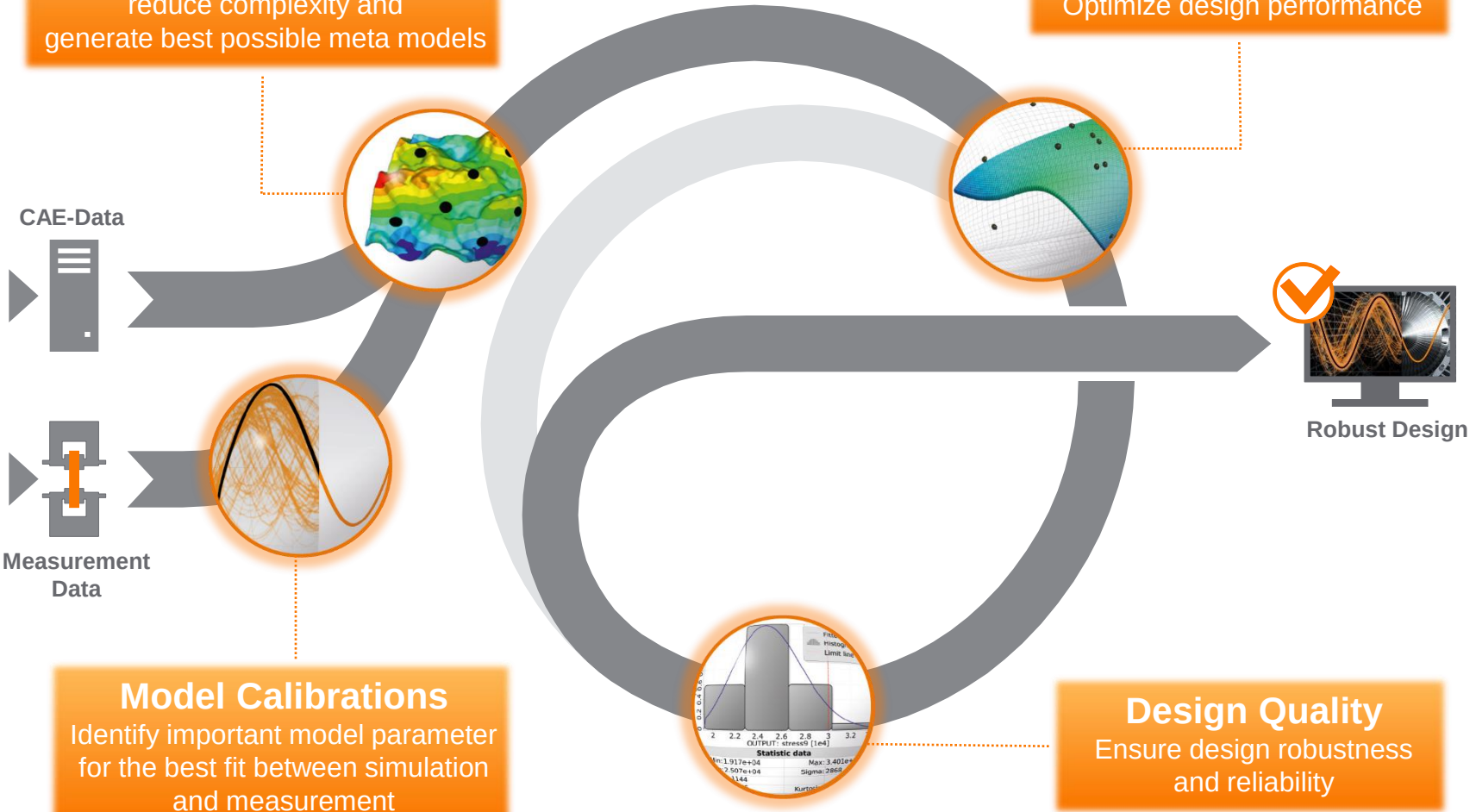
➢ Mainly the quantification of the input scatter and the computational burden are the critical points



DMAIC - Define – Measure – Analyze – Improve – Control
DMADV – Define – Measure – Analyze – Design – Verify

Design Understanding
Investigate parameter sensitivities, reduce complexity and generate best possible meta models

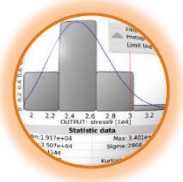
Design Improvement
Optimize design performance



Model Calibrations
Identify important model parameter for the best fit between simulation and measurement

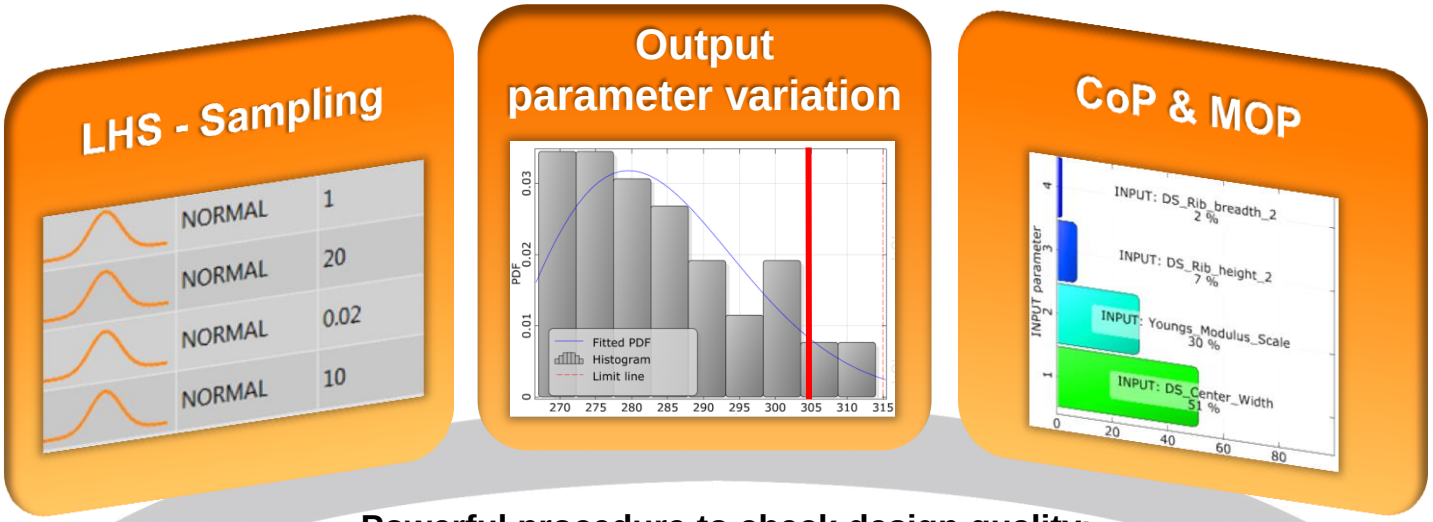
Design Quality
Ensure design robustness and reliability

Robustness Analysis



Robustness Evaluation

Ensure your product quality!



Powerful procedure to check design quality:

- Robustness evaluation with optimized Latin Hypercube Sampling
- **Proof of** Reliability with leading edge algorithms
- check variation interval limits and probabilities of overstepping
- Identify the most important scattering variables
- **decision tree for robustness algorithms**

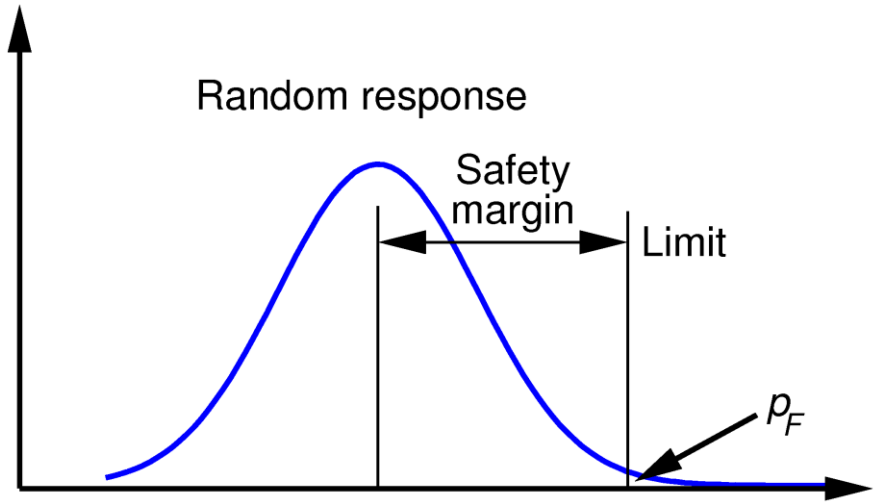
How to Define the Robustness of a Design?

- **Intuitively**: The performance of a robust design is largely unaffected by random perturbations

- **Variance indicator**: The coefficient of variation (CV) of the objective function and/or constraint values is not greater than the CV of the input variables

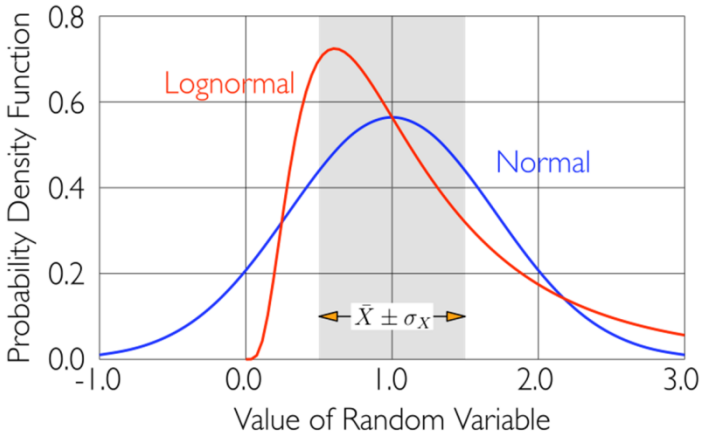
- **Sigma level**:
  The interval mean+/- sigma level does not reach an undesired performance (e.g. design for six-sigma)

- **Probability indicator**:
  The probability of reaching undesired performance is smaller than an acceptable value

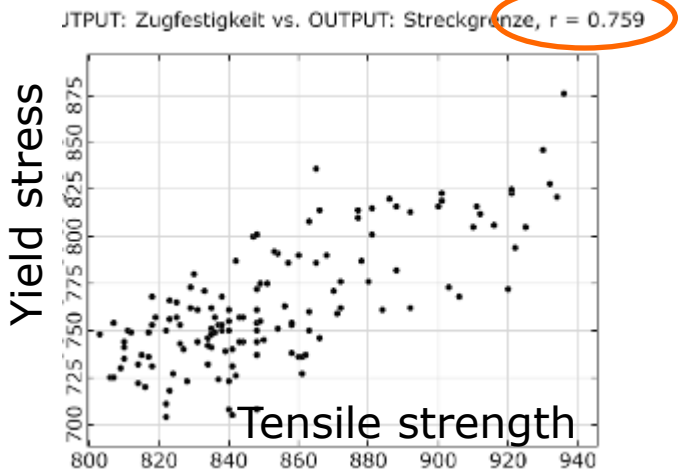


Definition of Uncertainties

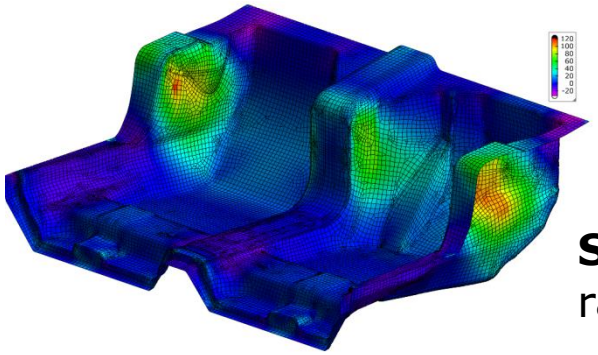
- Translate know-how about uncertainties into proper scatter definition



Distribution functions
define variable scatter



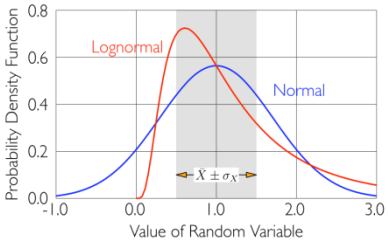
Correlation is an important characteristic of stochastic variables

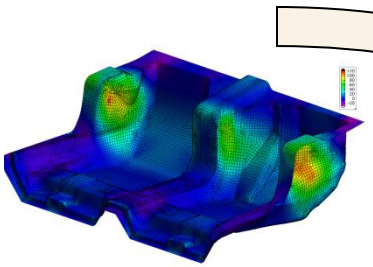


Spatial Correlation = random fields

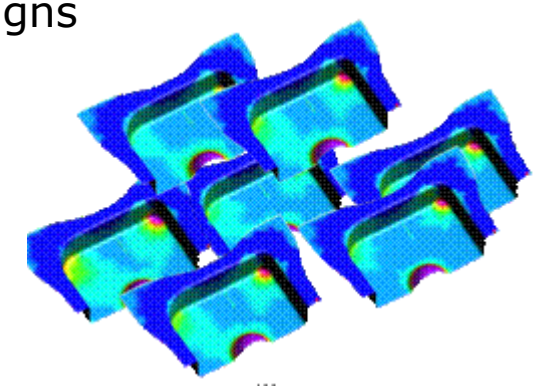
Variance based Robustness Analysis

1) Define the robustness space using scatter range, distribution and correlation

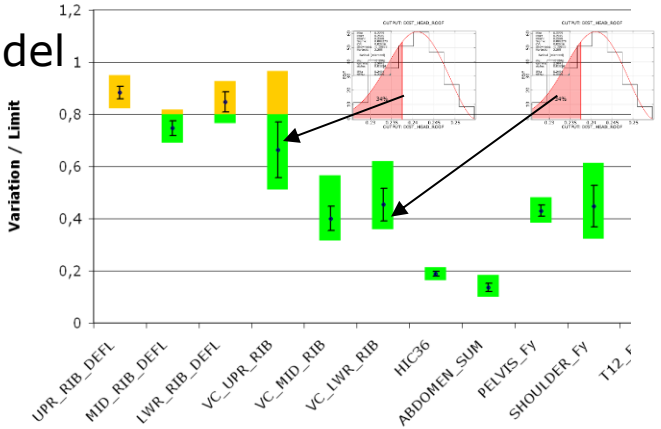




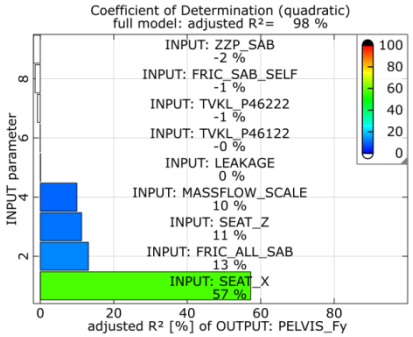
2) Scan the robustness space by producing and evaluating *n* designs



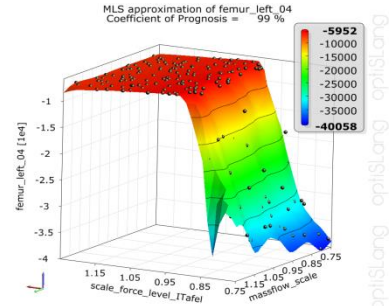
3) Check the variation



4) Check the explainability of the model

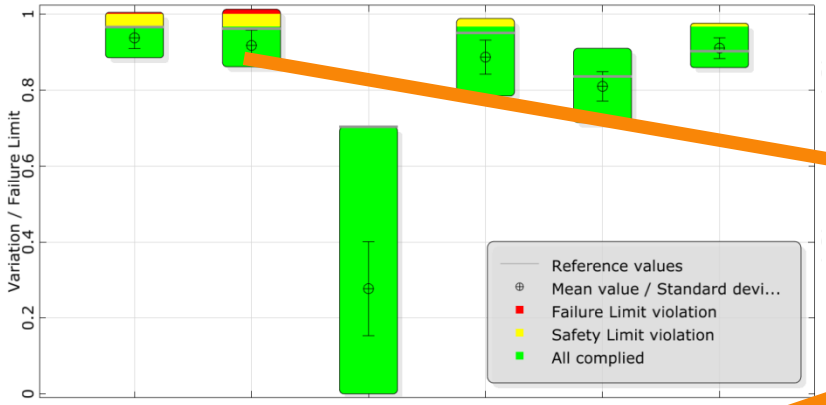


5) Identify the most important scattering variables

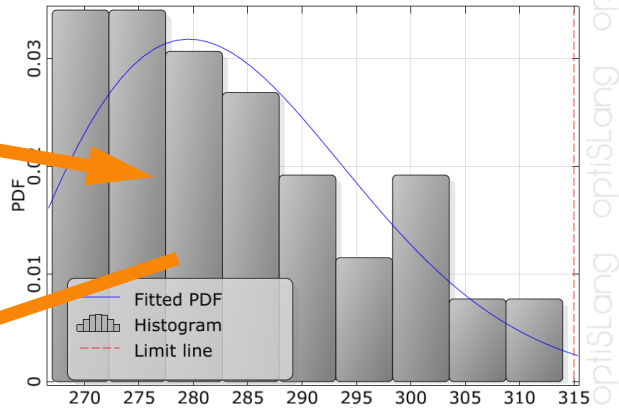


Robustness Measures

Traffic light plot

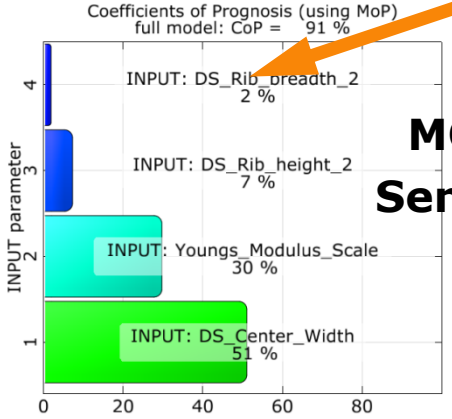


Histogram & Statistical Data



Statistic data			
Min:	267	Max:	314
Mean:	284.4	Sigma:	12.49
CV:	0.04393		
Skewness:	0.5633	Kurtosis:	2.338
Fitted PDF: Rayleigh			
Mean:	284.4	Sigma:	12.49
Limit x = 315			
P_rel:	1	1 - P_rel:	0
P_fit:	0.983168	1 - P_fit:	0.016832
Sigma-Level:	2.4496		

MOP/CoP Sensitivities

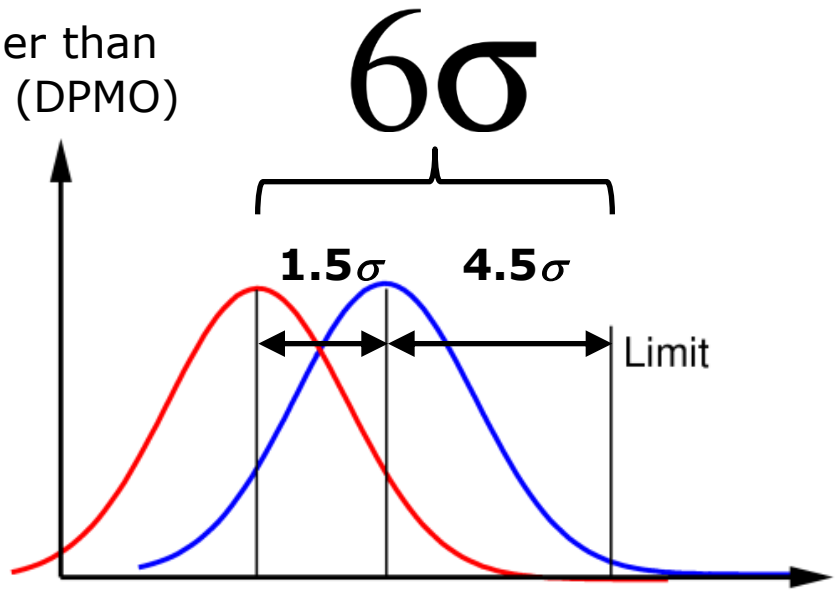


Six Sigma and Design for Six Sigma (DFSS)

- Methodology for **quality management** and **process improvement**
- Most common approaches:
  DMAIC - Define – Measure – Analyze – Improve – Control (existing process)
  DMADV – Define – Measure – Analyze – Design – Verify (new process)

- Six Sigma requires a failure level smaller than **3.4 defects per million** opportunities (DPMO)

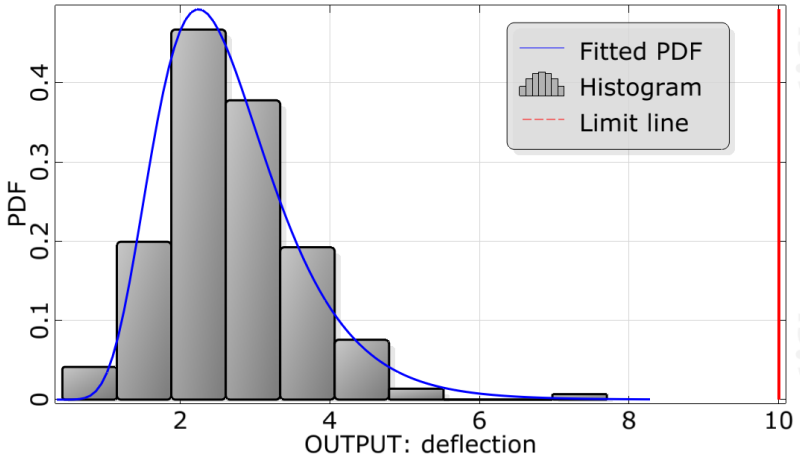
- Assuming a normal distribution a **4.5 sigma safety margin** is required
- An additional empirically based **1.5 sigma shift** was introduced to consider a long term move of the mean value



➢ **Variance-based robustness analysis** is a suitable tool within a Six Sigma quality management process

Variance based Robustness Analysis

- Sufficient estimates of **mean** and **variance** with 50 to 100 samples
- Distribution fit and extrapolation of **small event probabilities** may be very inaccurate
- More precise **reliability methods** should be applied to verify small probabilities



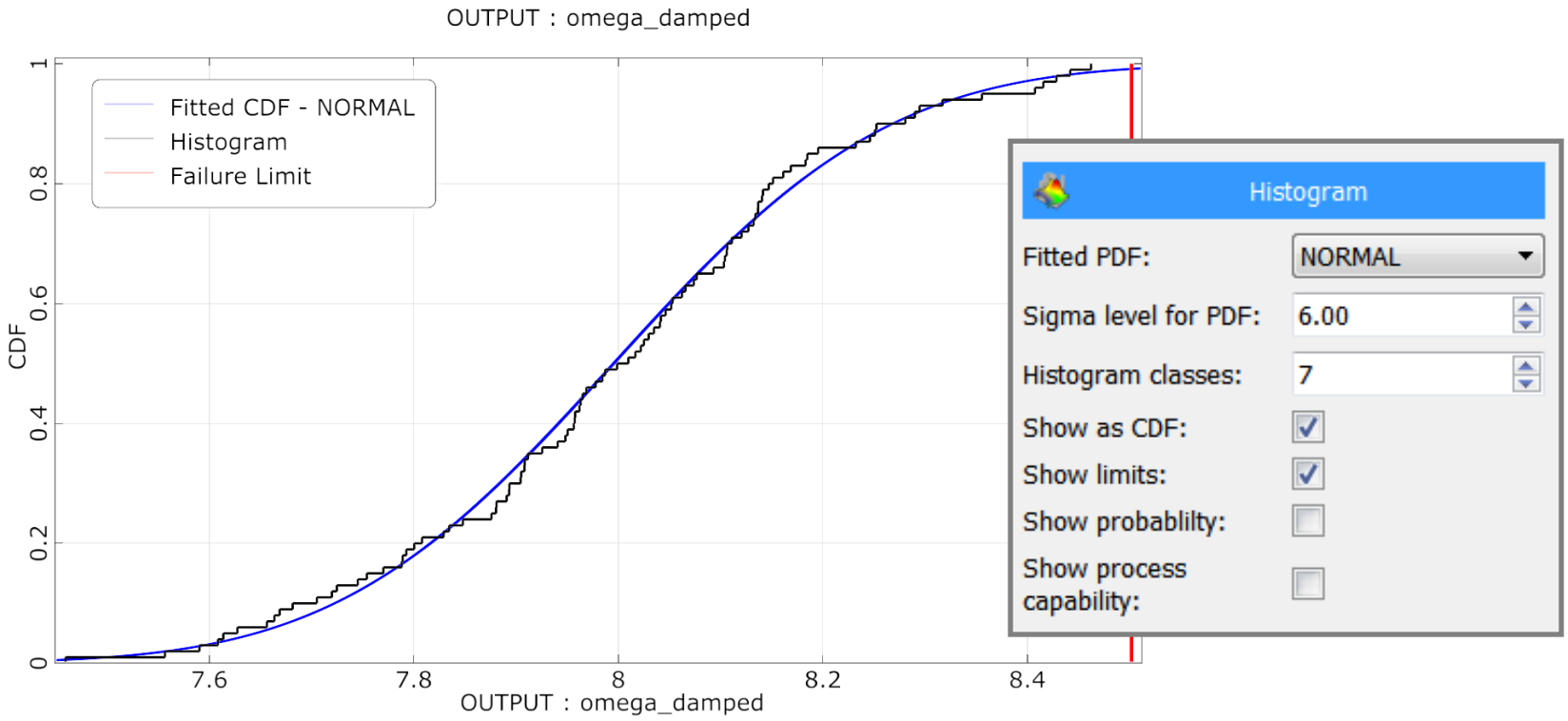
Statistic data			
Min:	0.4254	Max:	7.704
Mean:	2.67	Sigma:	0.9357
CV:	0.3505		
Skewness:	1.017	Kurtosis:	6.465

Fitted PDF: Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	1	1 - P_fit:	2.33147e-015
Sigma-Level:	7.83397		

Fitted PDF: Log-Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	0.999974	1 - P_fit:	2.56303e-005
Sigma-Level:	7.83397		

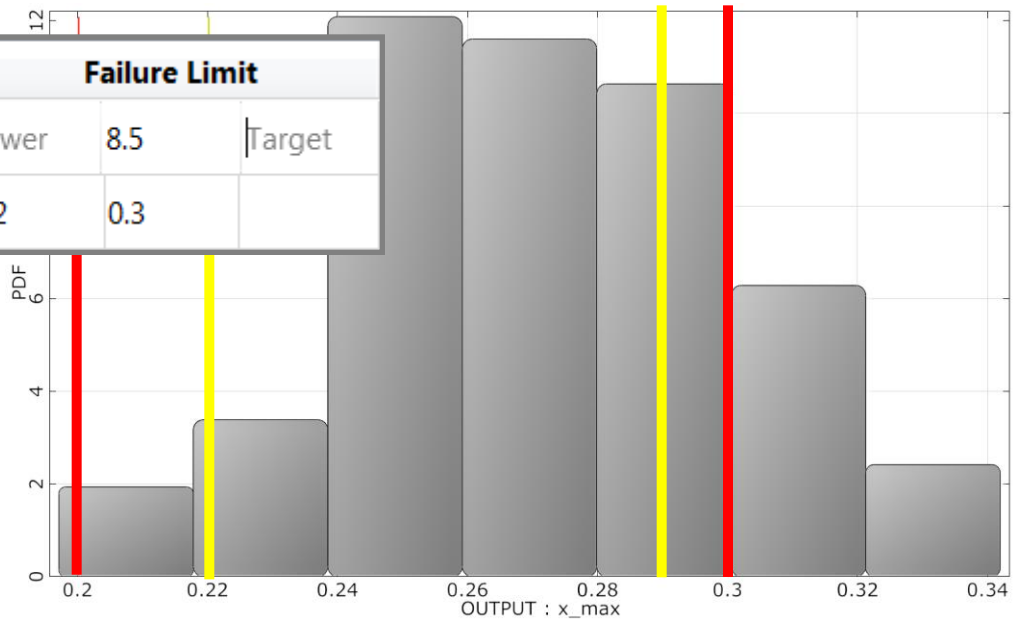
Distribution Fit

- Automatic fit compares deviation of empirical (sample) distribution function with analytical CDF of candidate distribution types
- ➢ Recommended distribution type has minimum sum of squared errors
- Single distribution type is fitted via moments to data points



Limits

	Dimension	Safety Limit		Failure Limit		
1	omega_damped		8.3	Lower	8.5	Target
2	x_max	0.22	0.29	0.2	0.3	



- Define lower and/or upper safety and/or failure limits

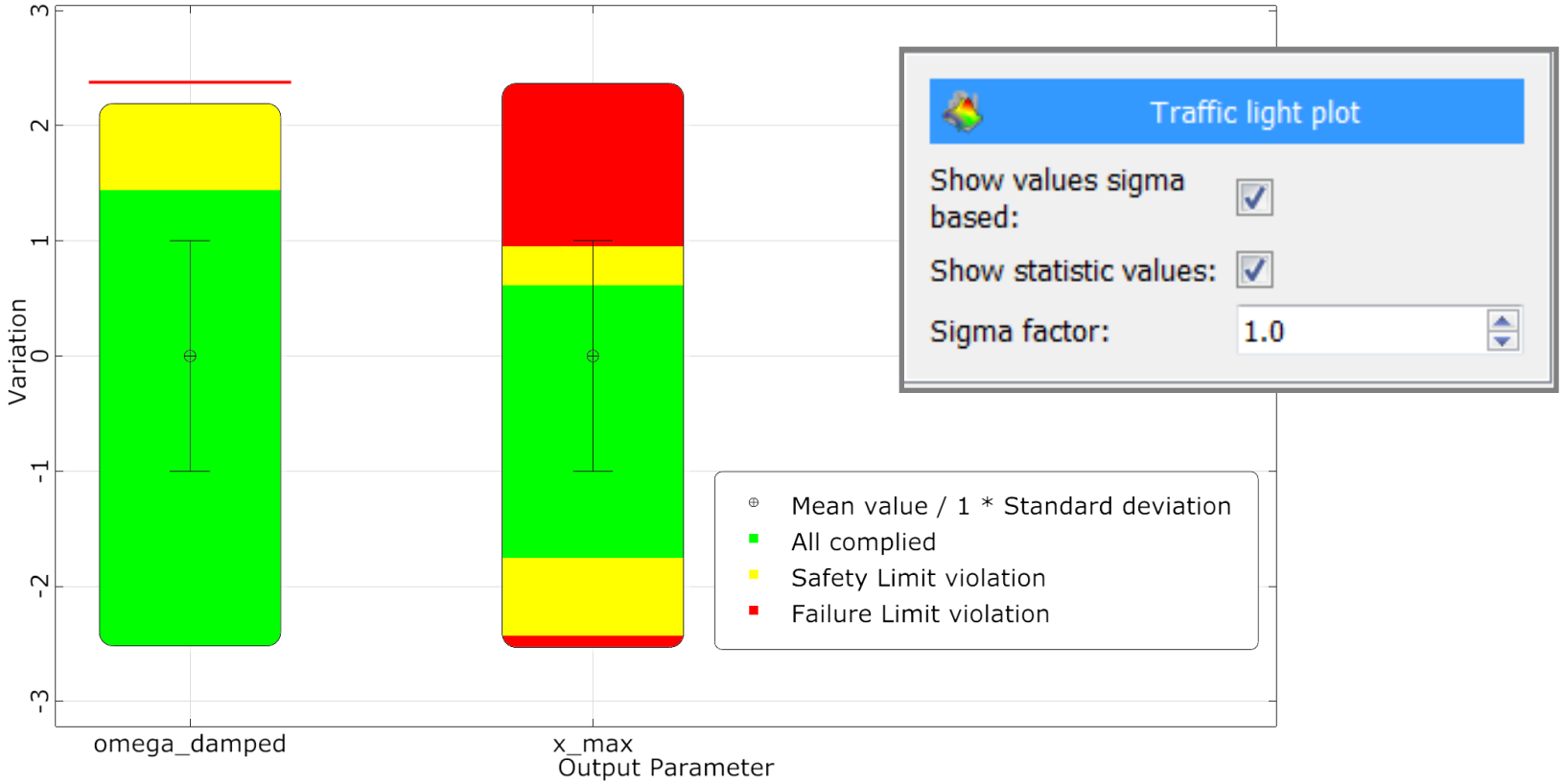
➢ Limits are indicated in the histogram and traffic light plot

➢ Probabilities of violating the limits are calculated

Limit : Safety Limit			
	Lower value = 0.22	Upper value = 0.29	Total
P_rel:	0.05	0.32	0.37
Sigma-Level:	1.74967	0.616846	
Limit : Failure Limit			
	Lower value = 0.2	Upper value = 0.3	Total
P_rel:	0.01	0.18	0.19
Sigma-Level:	2.42582	0.954919	

Traffic Light Plot

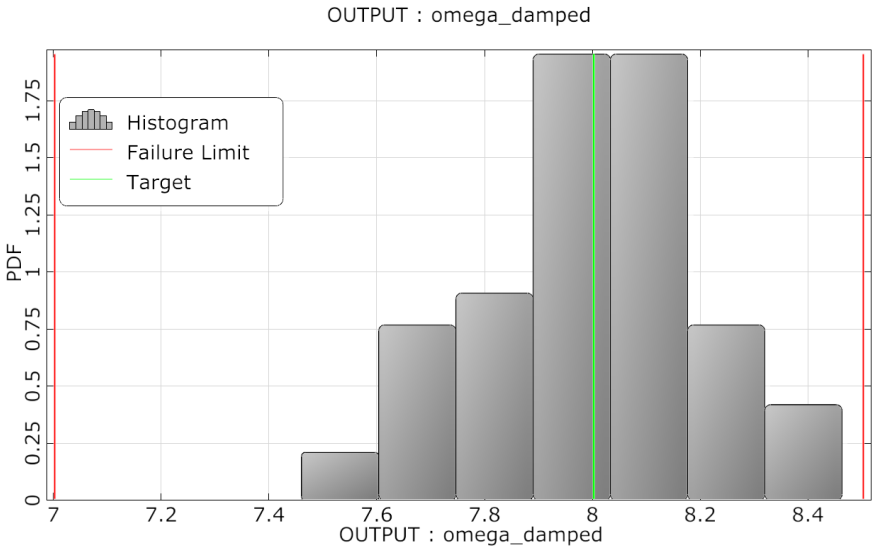
- Shows observed scatter of responses in original scale or standardized w.r.t. mean value and standard deviation
- Failure and safety limits are indicated

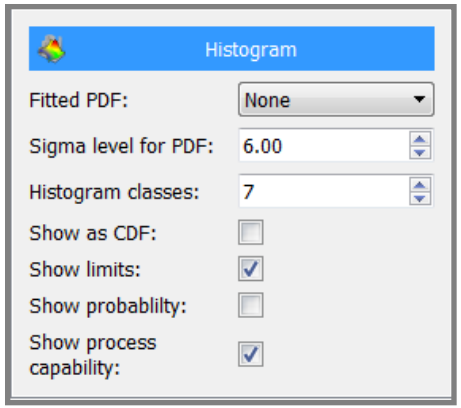


Process Cabability Indices

- Considers lower and upper failure limits and possible target
- Considers estimated sample mean and standard deviation
- Assumes normal distribution
- **Six Sigma** requires **process capability ≥ 2.0**

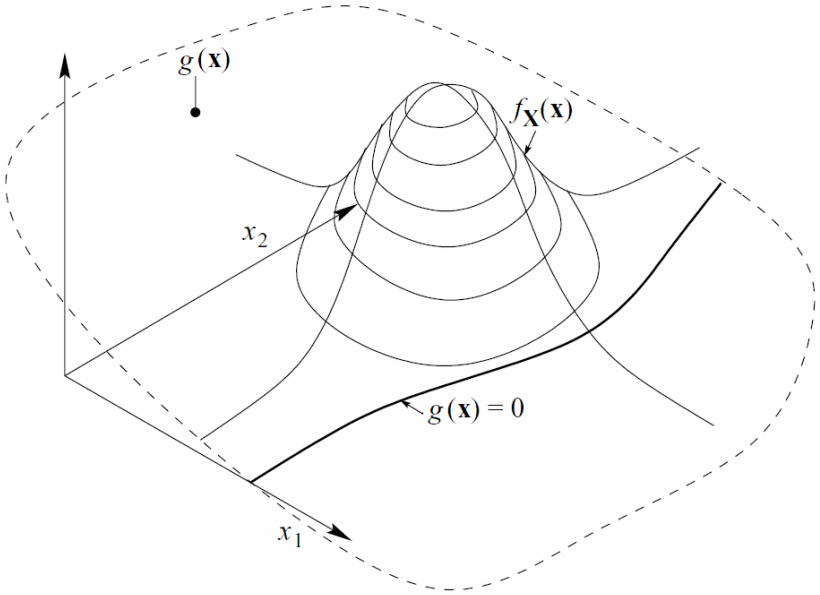
$$\hat{C}_{pk} = \min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right]$$





Process capability index			
LSL = 7 | USL = 8.5 | Target = 8			
C_p,lower:	1.55597	C_p,upper:	0.792532
C_p:	1.17425	C_pk:	0.792532
C_pm:	1.17375	C_pkm:	0.792197

Reliability Analysis



$$P_F = P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right]$$

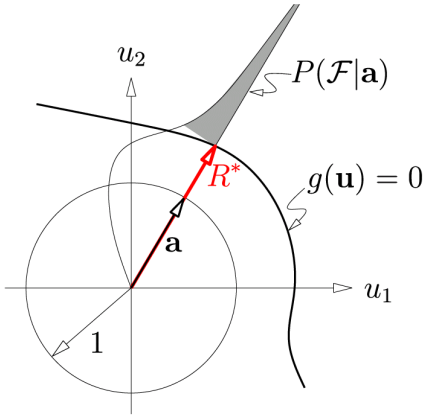
$$= \int_{g(\mathbf{x})\leq 0} \cdots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I\left(g(\mathbf{x})\right) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

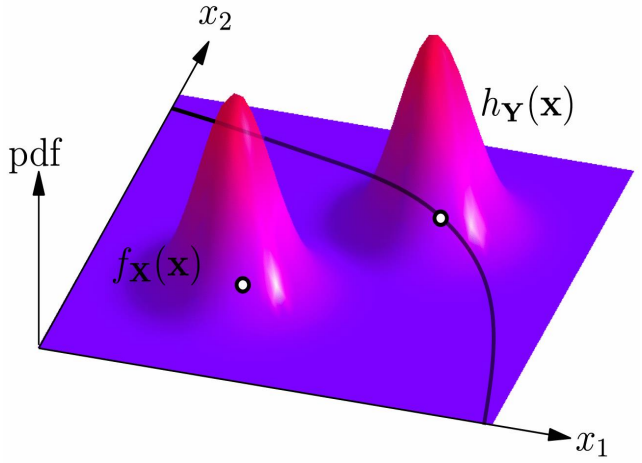
- Limit state function $g(\mathbf{x})$ divides random variable space $\mathbf{X}$ in safe domain $g(\mathbf{x})>0$ and failure domain $g(\mathbf{x}) \leq 0$
- Failure probability is the probability that at least one failure criteria is violated (at least one limit state function is negative)
- Requires accurate uncertainty knowledge

Advanced Methods for Reliability Analysis

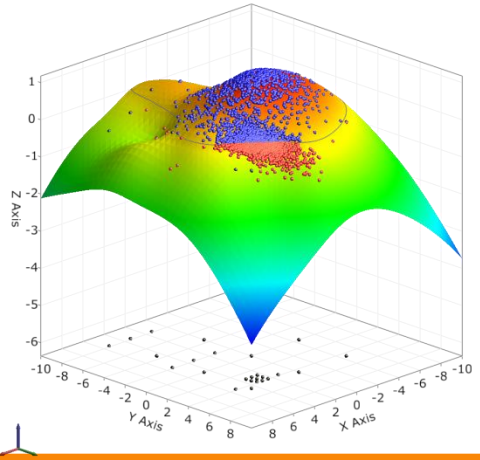
Directional Sampling



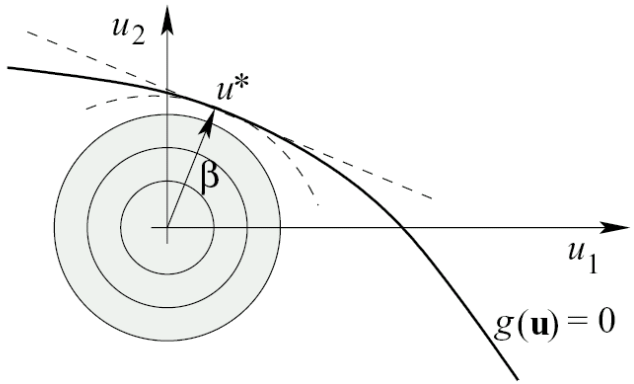
Adaptive Importance Sampling



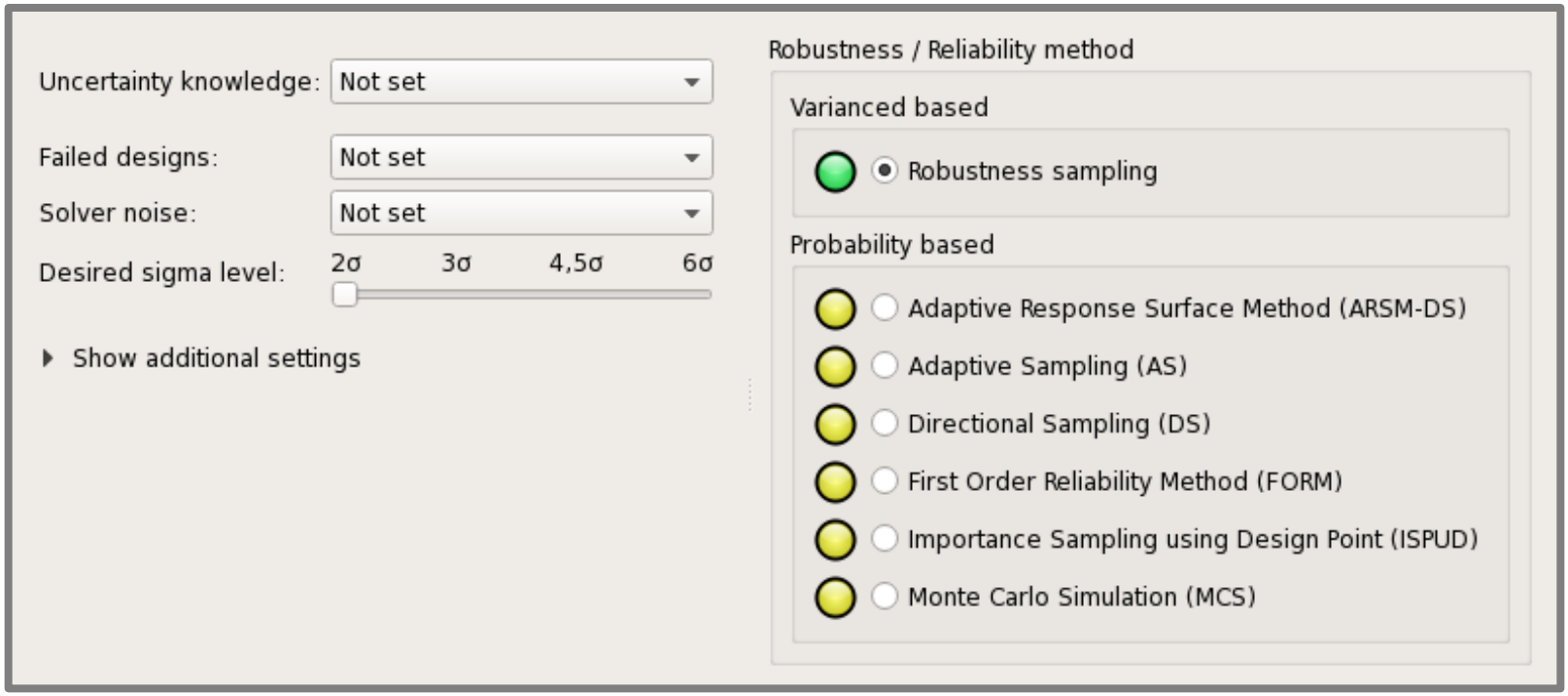
Adaptive Response Surface Method



First Order Reliability Method



Robustness/Reliability Wizzard



- If **no limit state** is defined or
- If the uncertainty knowledge is **not qualified**

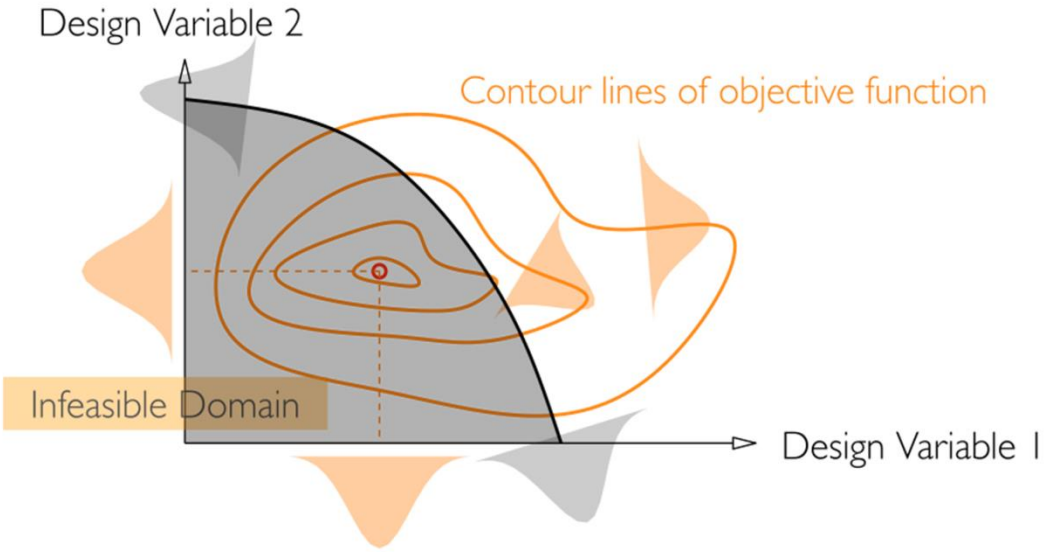
➢ Robustness sampling is recommended

➢ However, an extrapolation for more than 3 sigma is difficult

Robust Design Optimization

Robust Design Optimization

- Robust Design Optimization (RDO) optimizes the design performance while taking into account scatter of design (optimization) variables *and* other tolerances or uncertainties
- As a consequence of input scatter the location of the optima as well as the contour lines of constraints may vary



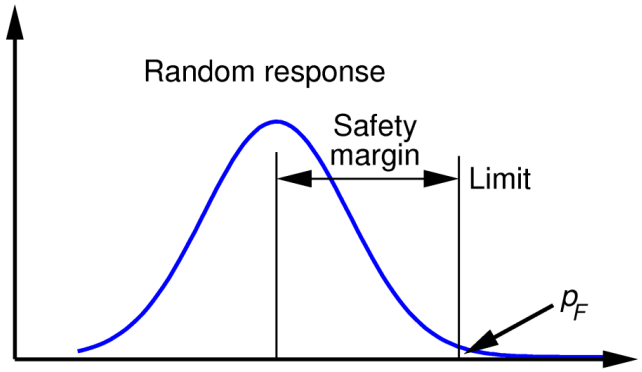
- To proof Robust Designs, safety distances are quantified with variance or probability measures using stochastic analysis

Methods for Robust Design Optimization

Variance-based RDO

- Safety margins of all critical responses are larger than a specified sigma level (e.g. Design for Six Sigma)

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$



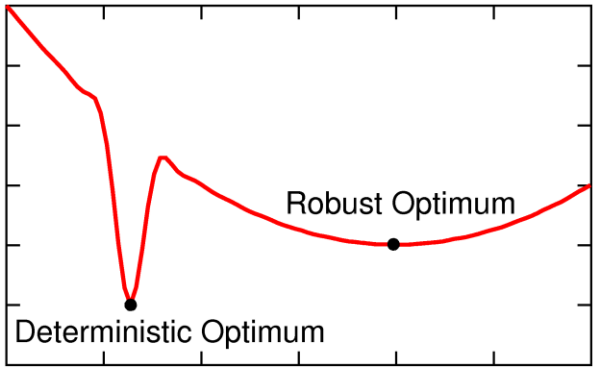
Reliability-based RDO

- Failure probability with respect to given limit states is smaller as required value

$$p_F \leq p_F^{target}$$

Taguchi-based RDO

- Taguchi loss functions
- Modified objective function

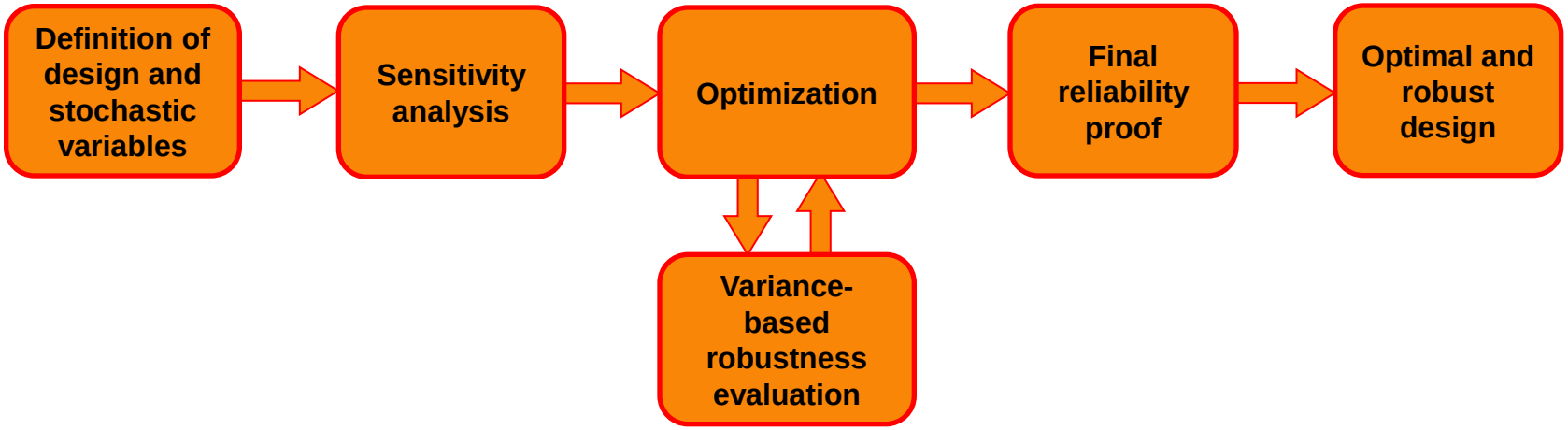


Coupled Robust Design Optimization

- Fully coupled optimization and robustness/reliability analysis
- For each design during the optimization procedure (nominal design), the robustness/reliability analysis is performed
- Applicable to variance-, reliability- and Taguchi-based RDO

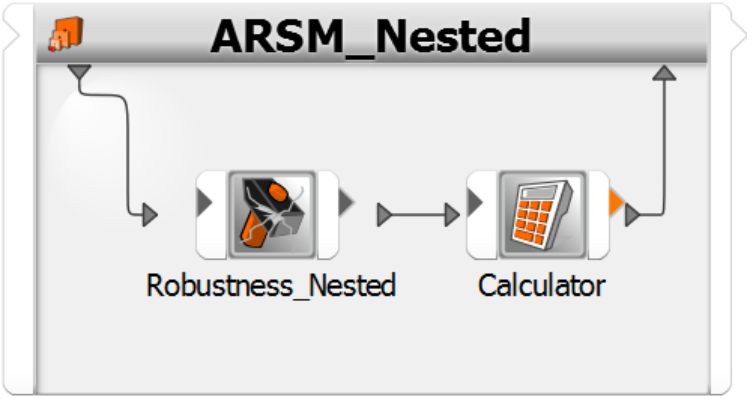
➢ Our efficient implementation uses small sample variance-based robustness measures during the optimization and a final (more accurate) reliability proof

➢ But still the procedure is often not applicable to complex CAE models



Coupled RDO in optiSLang

- Nested loop enables the coupled RDO
- Optimizer has to handle statistical errors of inner robustness analysis
- Sigma level as constraint



	ID	Type	Value	Expression
1	mean_omega	REAL	4.47124	mean(omega_damped)
2	mean_x_max	REAL	0.62342	mean(x_max)
3	stddev_omega	REAL	0	stddev(omega_damped)

Criteria

Name	Type	Expression	Criterion	Limit	Evaluated expression
obj_mean_x_max	Objective	mean_x_max	MIN		0.62342
constr_mean_omega	Constraint	mean_omega+4.5*stddev_omega	≤	8.5	4.47124 ≤ 8.5
new					

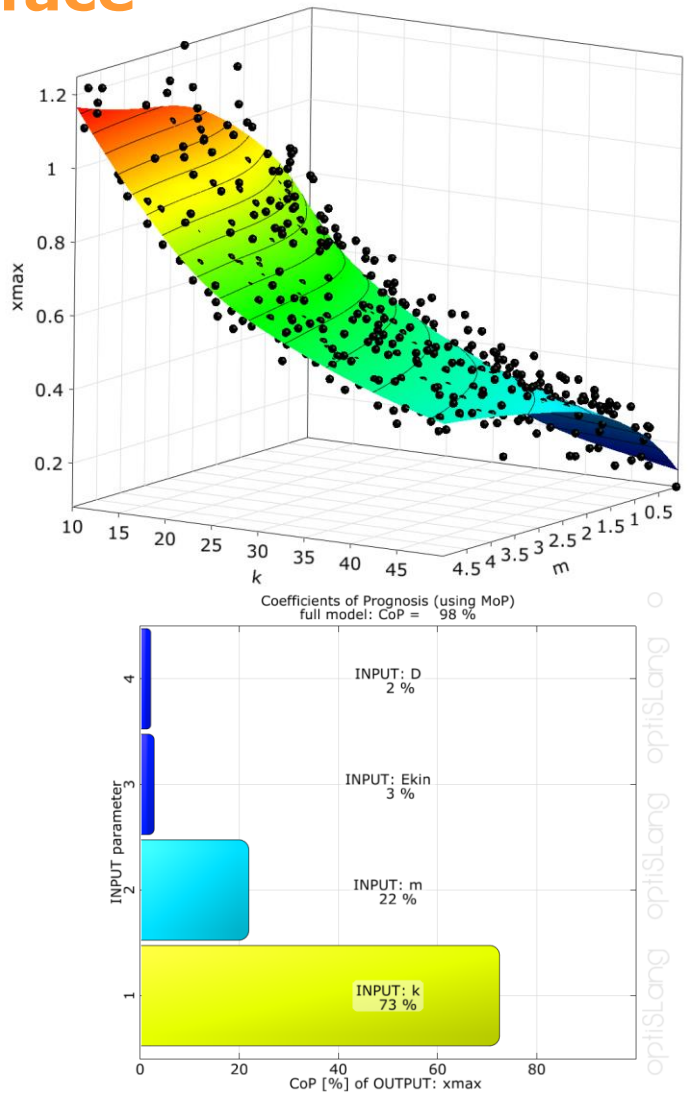
RDO on Global Response Surface

- Approximation of model responses in mixed optimization/stochastic space
- Simultaneous RDO is performed on a global response surface
- Applicable to variance-, reliability- and Taguchi-based RDO
- Approximation quality significantly influences RDO results

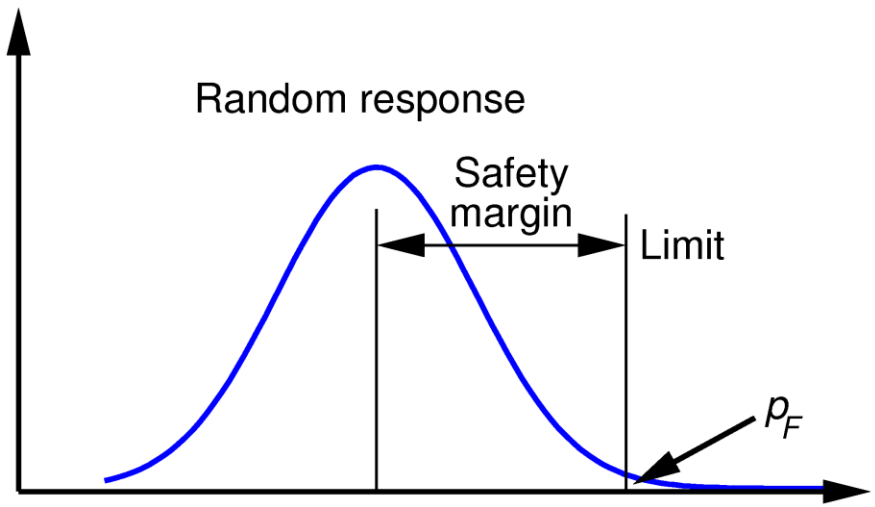
➢ Final robustness/reliability proof is required

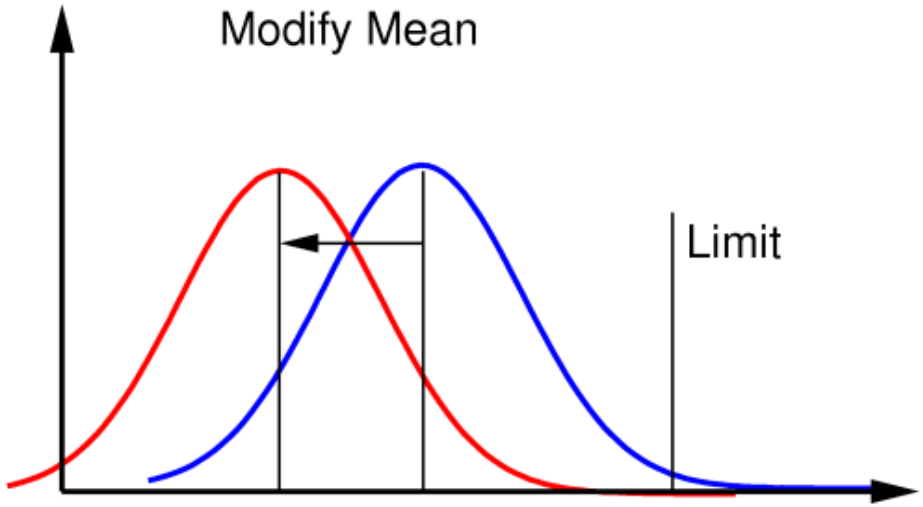
- Pure stochastic variables have small influence compared to design variables

➢ Important local effects in the stochastic space may be not represented



RDO Approach 1: Modify the Mean





- Mean values of design variables are modified
- Standard deviation or Coefficient of Variation (CoV) is kept constant

➢ Mean values of responses are modified, scatter may change

- Nominal design is changed until required safety margin is reached

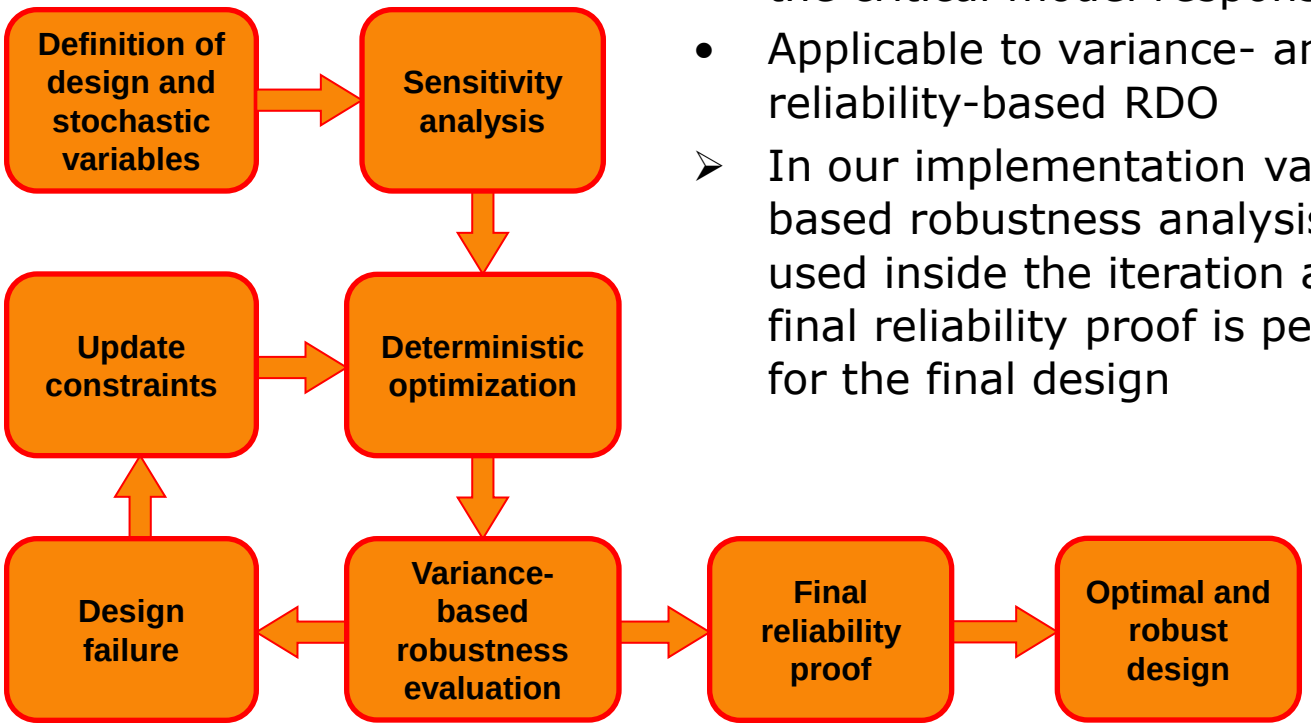
$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Optimization goal may be e.g. minimization of mass

Iterative Robust Design Optimization

- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety margins are adjusted for the critical model responses
- Applicable to variance- and reliability-based RDO

➢ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design



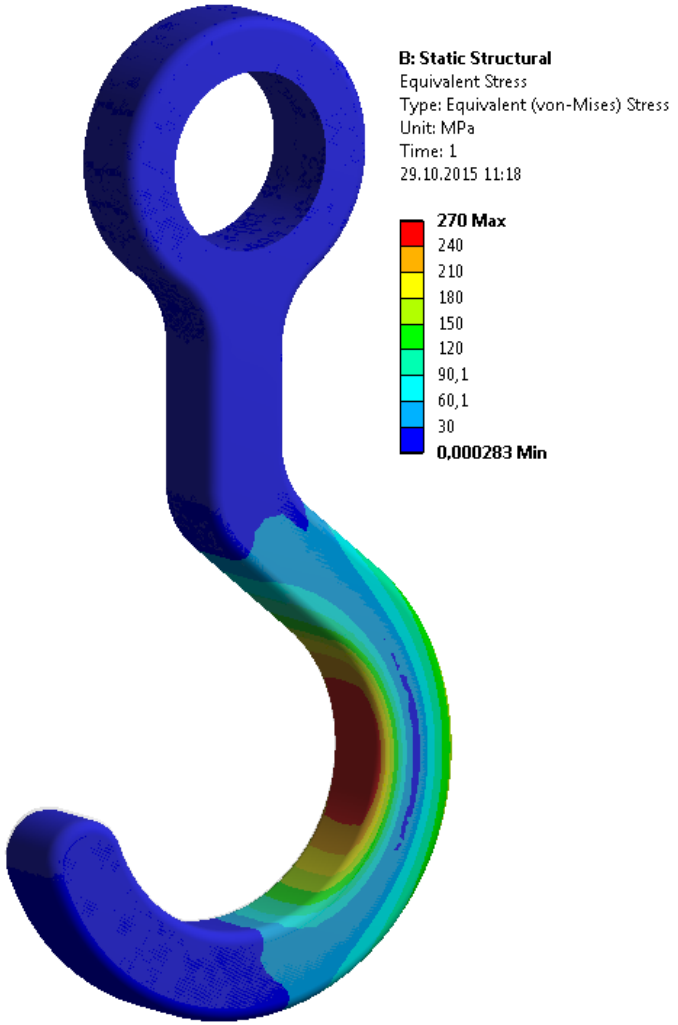
Example: Steel Hook

Deterministic Optimization

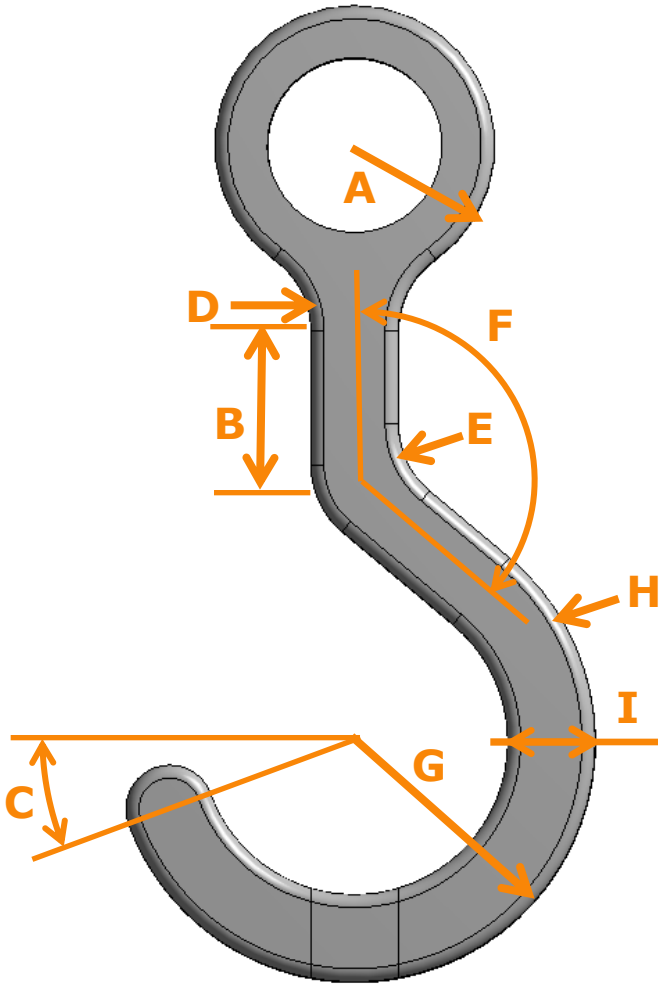
- Minimize the mass
- The maximum stress should not exceed 300 MPa
- 10 geometry parameters are varied for the design variation

Robustness requirement

- Proof for the optimal design that the failure stress limit is not exceeded with a 4.5 sigma safety margin
- 16 scattering parameters are considered (geometry and material properties and the load components)



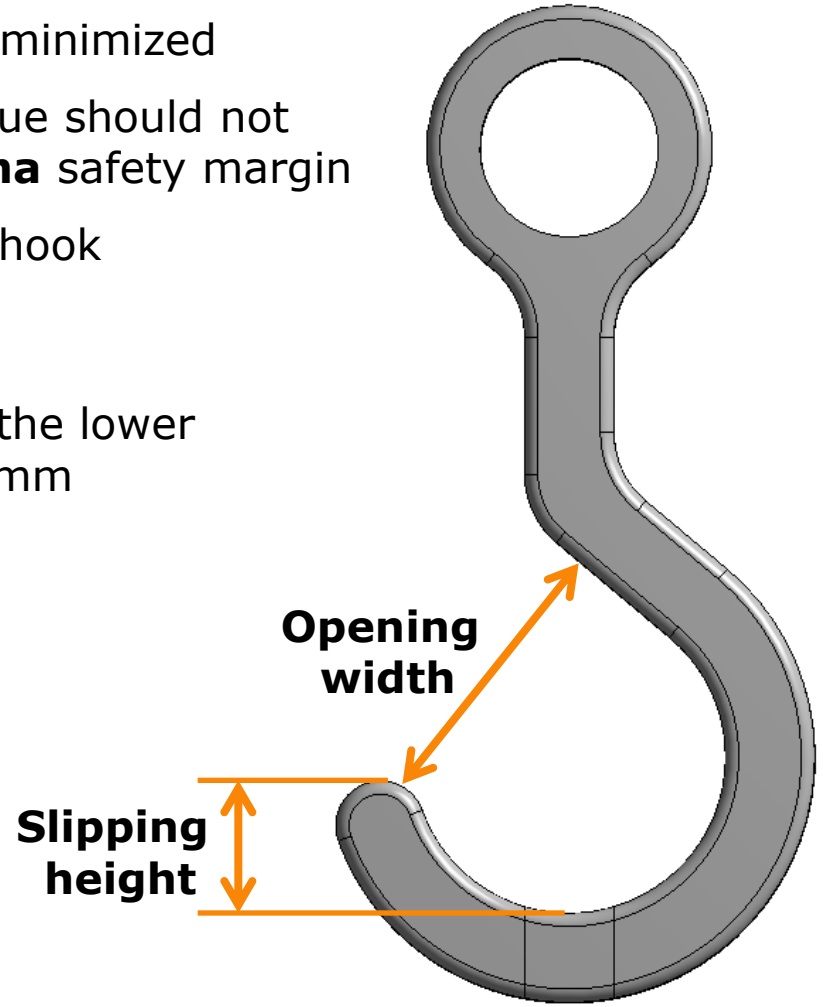
Example: Steel Hook



A	Outer_Diameter	28-35 mm
B	Connection_Length	20-50 mm
C	Opening_Angle	10-30 °
D	Upper_Blend_Radius	18-22 mm
E	Lower_Blend_Radius	18-22 mm
F	Connection_Angle	120-150 °
G	Lower_Radius	45-55 mm
H	Fillet_Radius	2-4 mm
I	Thickness	15-25 mm
	Depth	15-25 mm

Example: Steel Hook

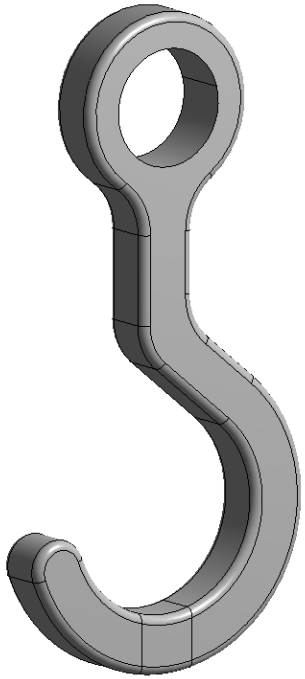
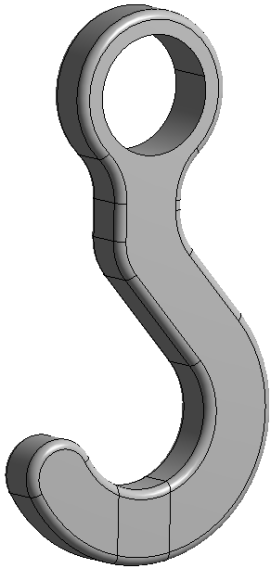
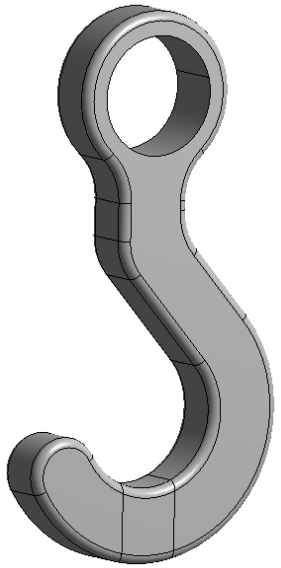
- Total **mass** of the hook should be minimized
- **Maximum equivalent stress** value should not exceed 300 MPa within a **4.5 sigma** safety margin
- **Slipping height** of the deformed hook should be larger than 5 mm within a **4.5 sigma** safety margin
- **Opening width** (undeformed) of the lower half circle should be minimum 50 mm in the nominal design



Initial nominal values

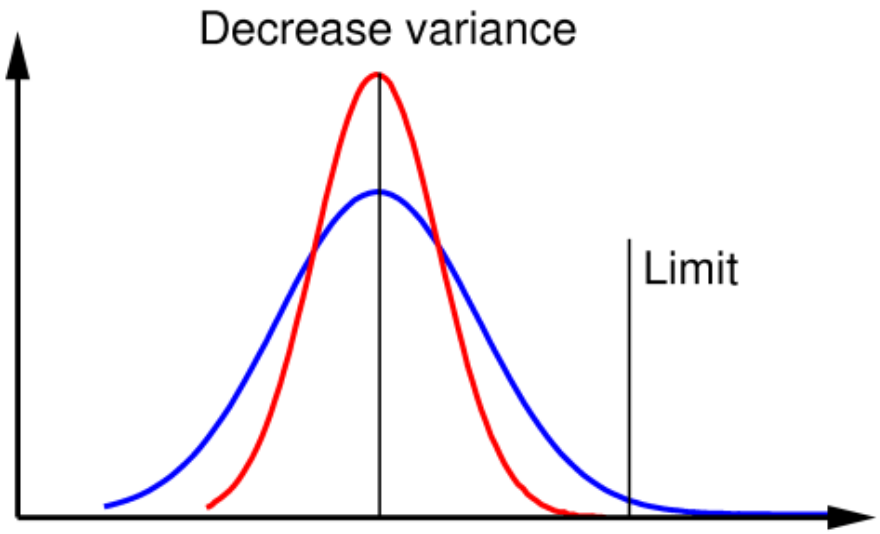
• Mass	1100 g
• Maximum stress	270 MPa
• Slipping height	28 mm
• Opening width	64 mm

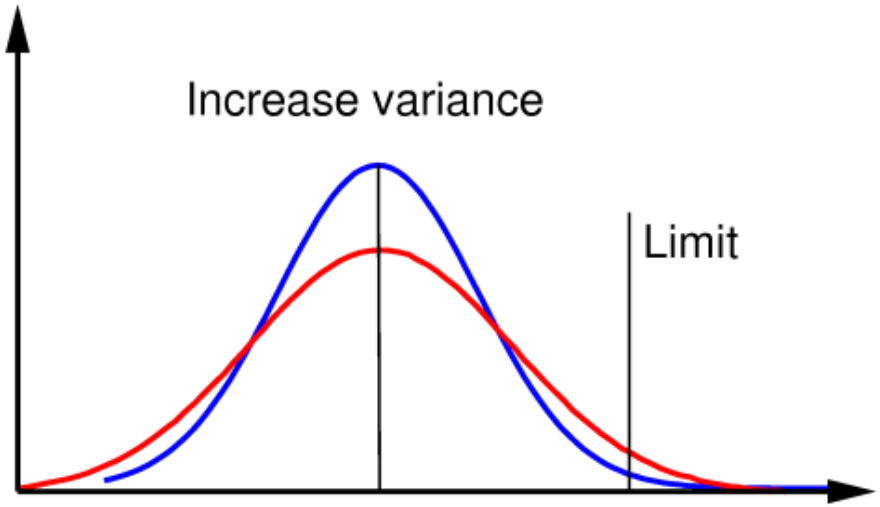
Example: Steel Hook

	Initial Design	1st RDO Step	2nd RDO Step
			
Mass	1100 g	854 g	959 g
Mean stress	270 MPa	180 MPa	160 MPa
Failure probability	23%	10^{-4}	10^{-6}
Reliability index	0.61	3.77	4.79

Optimizing Uncertainties

RDO Approach 2: Modify the Variance





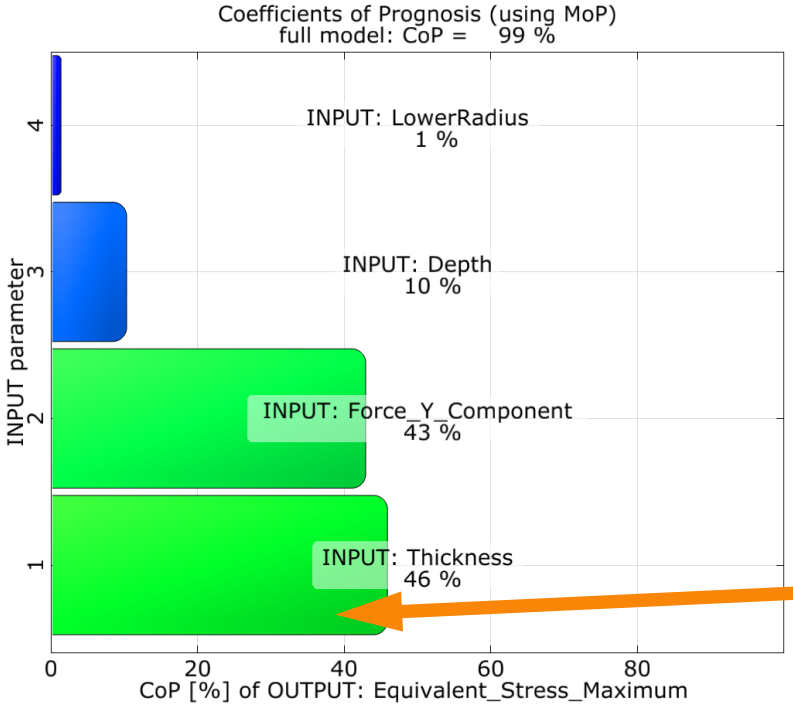
- Nominal design is kept constant
- Mean values of design variables are constant
- Mean of response may change slightly
- Standard deviations or CoVs of input variables are modified until safety margin in model responses is reached

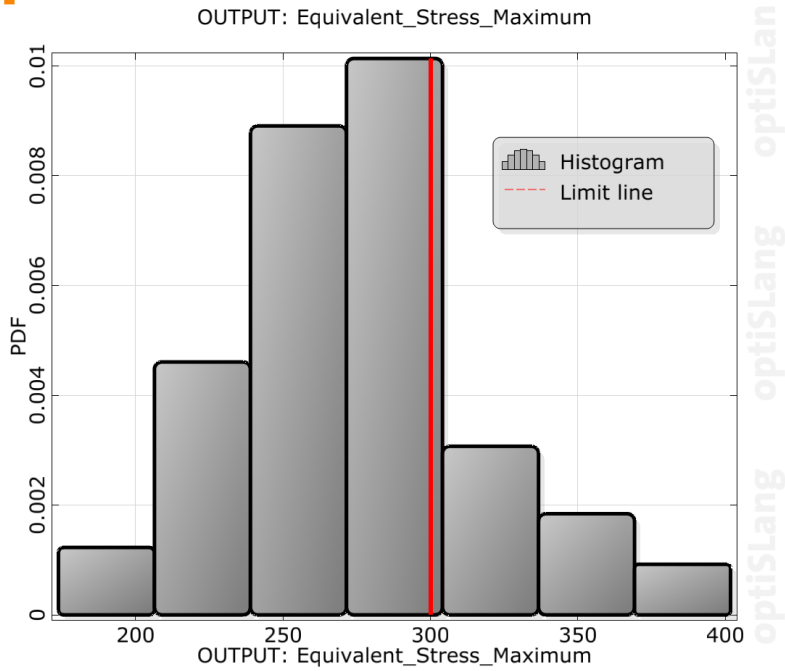
$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Increase of variance of input parameters may also be an objective!

Straight Forward Approach

- Use e.g. varianced-based sensitivity measures to identify most relevant input scatter
- Reduce or increase scatter if possible
- Nominal values are kept fixed





Statistic data			
Min:	173.8	Max:	401.8
Mean:	274.1	Sigma:	42.09
CV:	0.1536		
Skewness:	0.4233	Kurtosis:	3.334
Limit x = 300			
P_rel:	0.77	1 - P_rel:	0.23
Sigma-Level:	0.615076		

Maximum Variance Condition

- Maximizing the variance of inputs as objective
  - Straight forward for only one scattering input variable
  - Scalarization approach for more variables
- Requires lower and upper bounds of input scatter
- Safety margin of model responses as constraint condition

Maximum Volume of n-dimensional hyper-ellipsoid

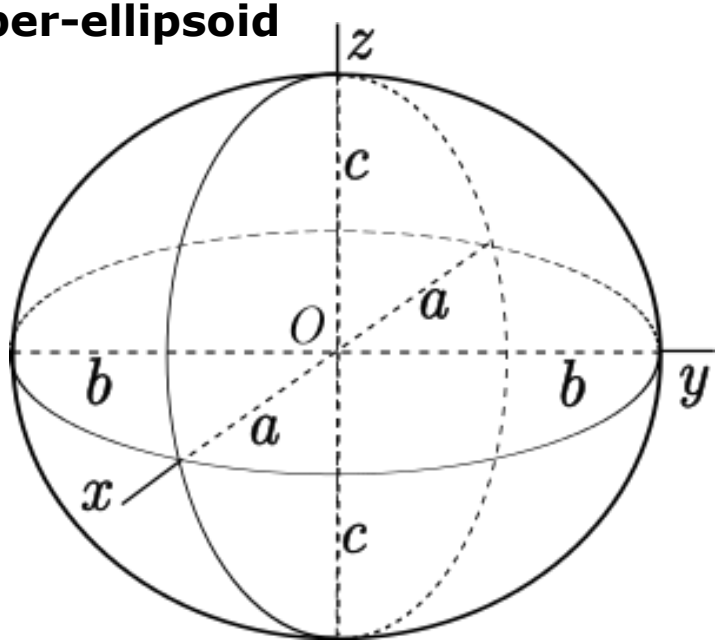
$$V_n = (a_1 a_2 a_3 \cdots a_n)\frac{2}{n}\frac{\pi^{\frac{n}{2}}}{\Gamma(n/2)}$$

- Axes lengths a,b,c are assumed
  e.g. as CoVs of scattering variables

$$\prod CoV(X_i) \rightarrow max$$

- Equivalent additive objective

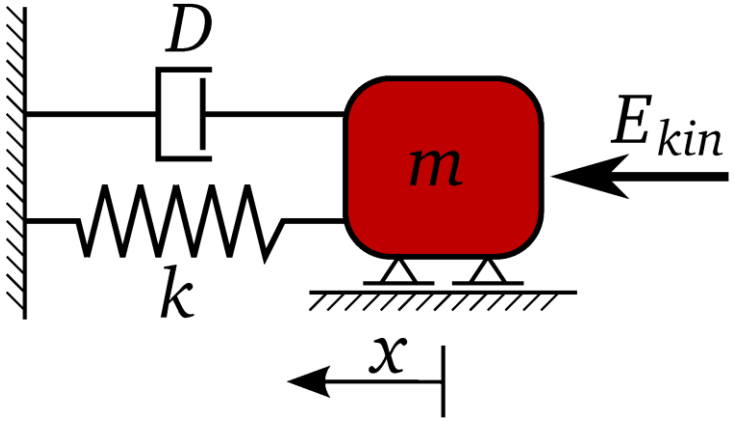
$$\sum \log(CoV(X_i)) \rightarrow max$$



Example: Damped Oscillator

- Single degree-of-freedom system excited with initial kinetic energy

$$E_{kin} = \frac{mv_0^2}{2}, \quad x_0 = 0$$



- Equation of motion of free vibration:

$$\ddot{x} + 2D\omega_0\dot{x} + \omega_0^2 x = 0$$

- Un-damped and damped eigen-frequency

$$\omega_0 = \sqrt{k/m} \qquad w = w_0\sqrt{1 - D^2}$$

- Maximize variance of *k, m, D* and *Ekin* w.r.t. safety constraint that damped eigen-frequency is outside of 4.5σ level

$$\omega_{safetylimit} = 8.5 \text{ rad/s}$$

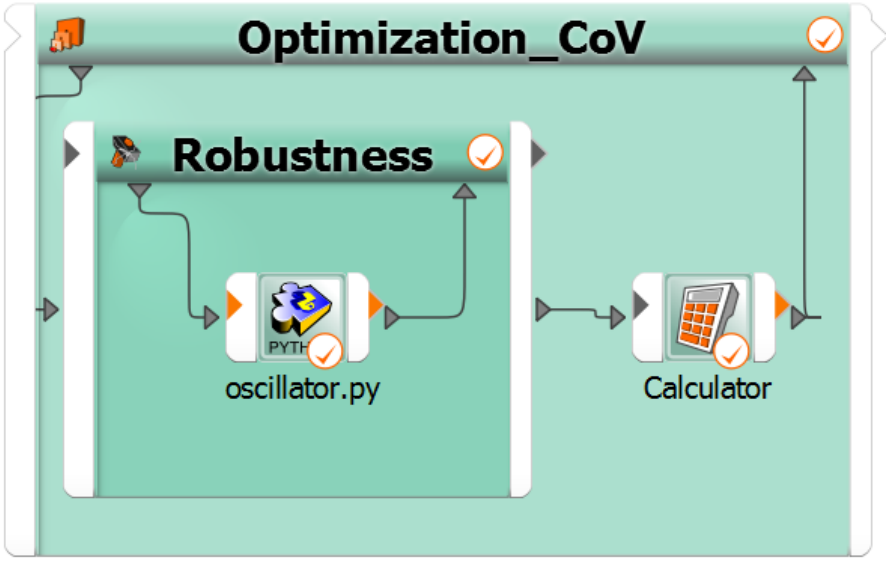
Example: Damped Oscillator

- Definition of standard normal random variables
- Random model inputs are dependent parameters as function of deterministic mean and CoV and one standard random variable each
- CoVs or standard deviations may be considered as optimization parameters
- Definition of normal and uniform type is straight forward

	Name	Parameter type	Reference value	Constant	Operation	Range		PDF	Type	Mean	Std. Dev.
1	D	Dependent	0.02	☐	D_mean*(1+D_unit*D_cov)						
2	D_cov	Optimization	0.1	☐		0.01	0.3				
3	D_mean	Optimization	0.02	☑		0.01	0.03				
4	D_unit	Stochastic	0	☐					NORMAL	0	1
5	Ekin	Dependent	10	☐	Ekin_mean*(1+Ekin_unit*Ekin_cov)						
6	Ekin_cov	Optimization	0.1	☐		0.01	0.2				
7	Ekin_mean	Optimization	10	☑		9	11				
8	Ekin_unit	Stochastic	0	☐					NORMAL	0	1

Example: Damped Oscillator

- Coupled RDO
- Objective is directly given from input CoV values (design variables)
- Constraint is calculated from statistical moments of model responses

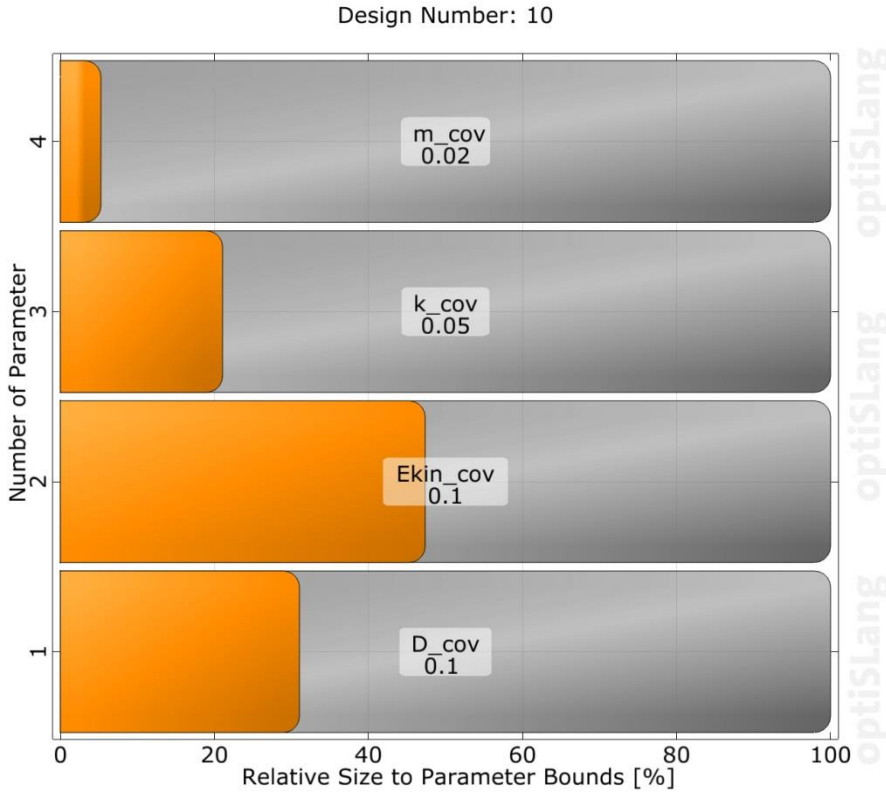


Type	Expression	Criterion	Limit
Objective	log(D_cov)+log(Ekin_cov)+log(m_cov)+log(k_cov)	MAX	
Constraint	4.5*std_omega	≤	8.5-mean_omega

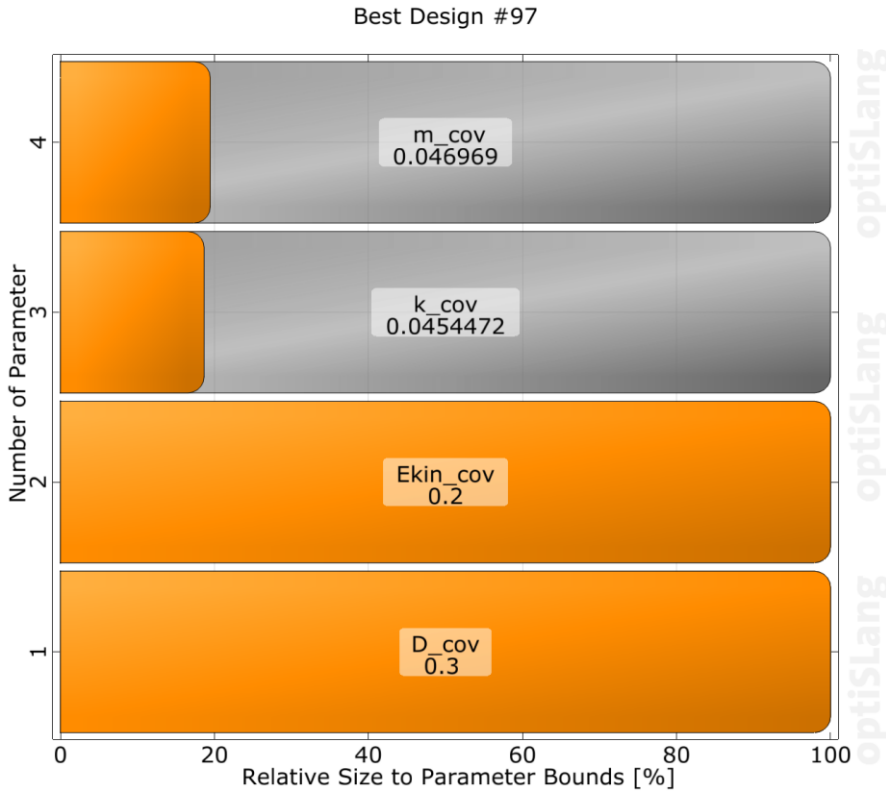
Example: Damped Oscillator

- Variation of input scatter is significantly increased

Initial design: obj = -5



Best design: obj = -3.89



Example: Damped Oscillator

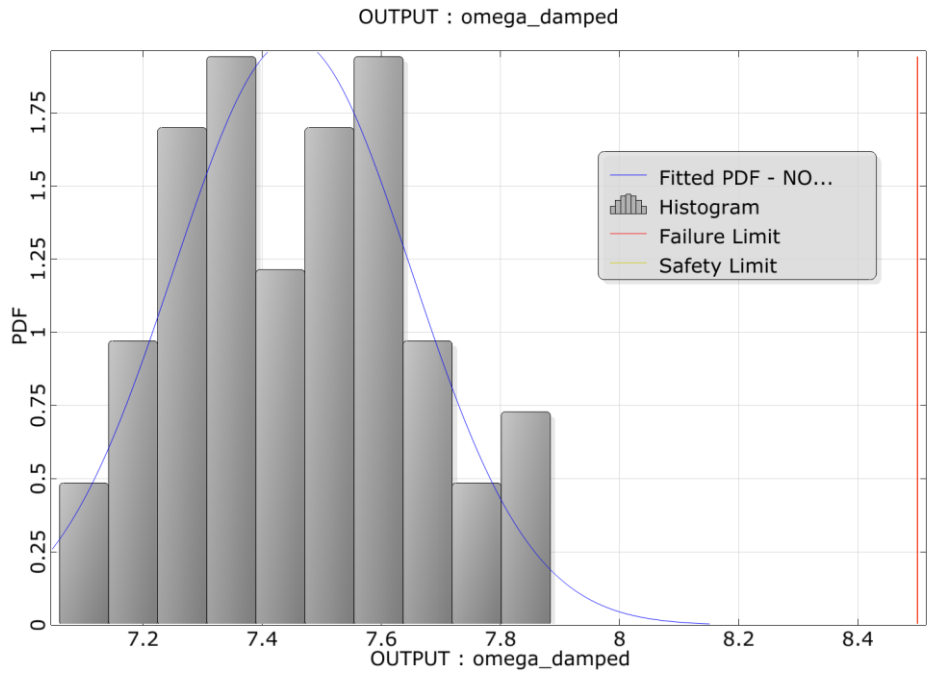
- Variation of response is slightly increased to defined limit

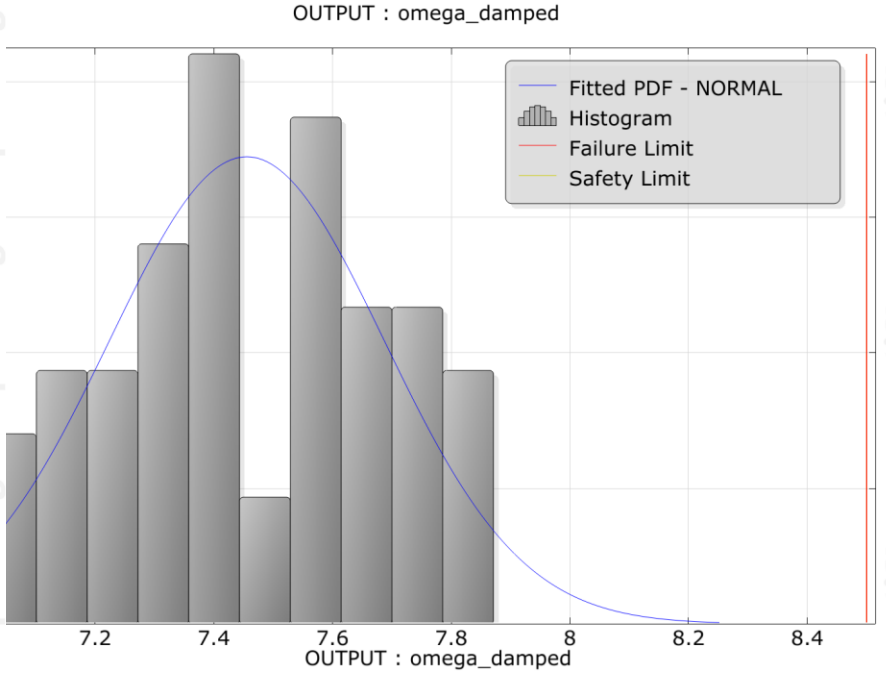
Initial design

Mean:	7.45
Stddev:	0.20
Sigma level:	5.26

Best design

Mean:	7.45
Stddev:	0.23
Sigma level:	4.51

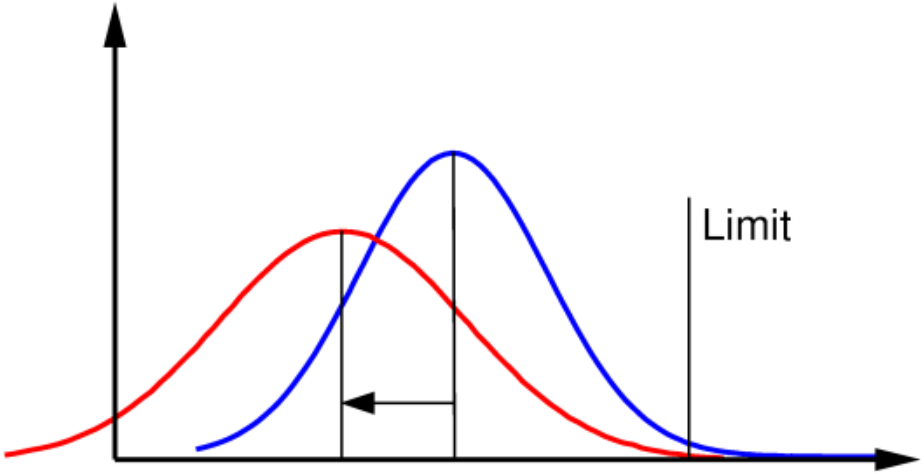


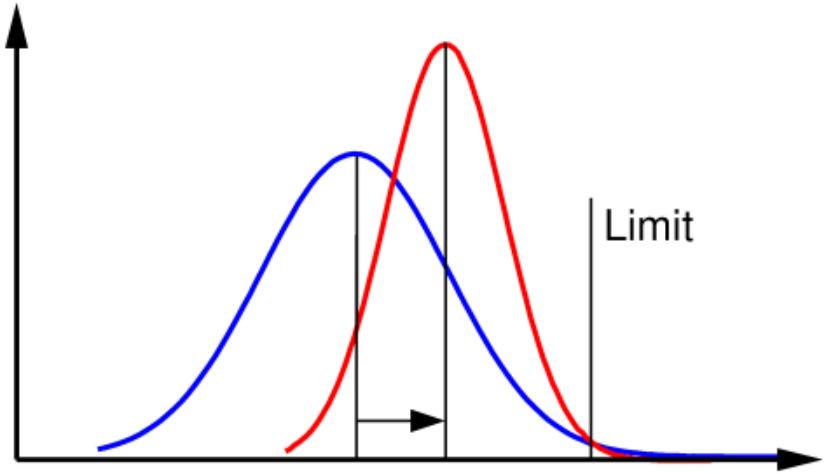


Combining Approach 1 & 2

- Optimize nominal design variables and selected input scatter
- ➢ Results often in conflicting objectives
- Multi-objective Robust Design Optimization to study possible solutions
- Burdensome numerical effort
- ➢ Application of global meta-models in combined RDO space
- ➢ Verification of estimated robustness measures is essential

Increase or decrease variation and/or nominal values ?

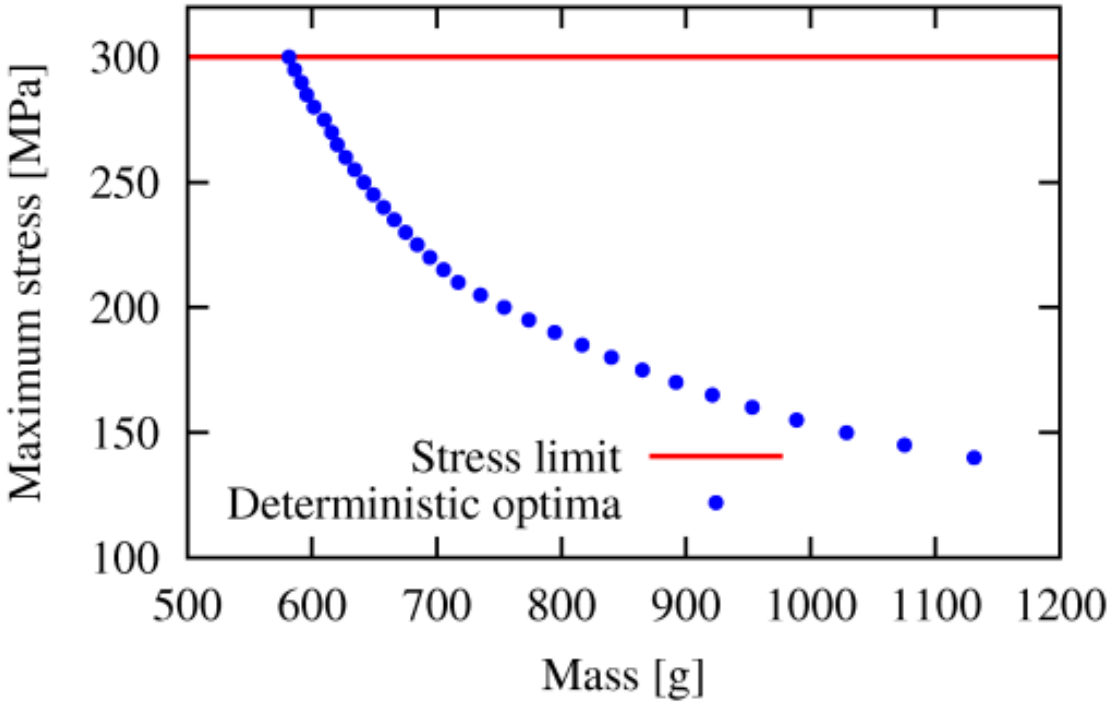


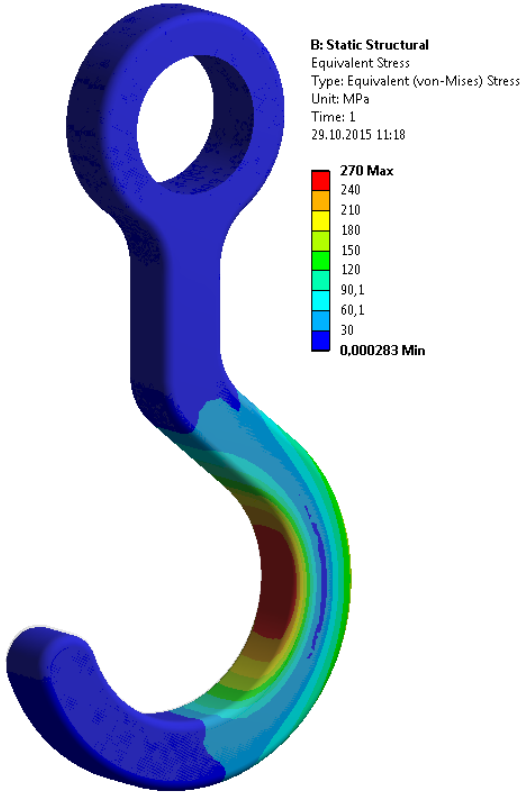


Example: Steel Hook

Deterministic Pareto Optimization

- Minimize the nominal mass
- Minimize the maximum equivalent stress by considering a stress limit of 300 MPa

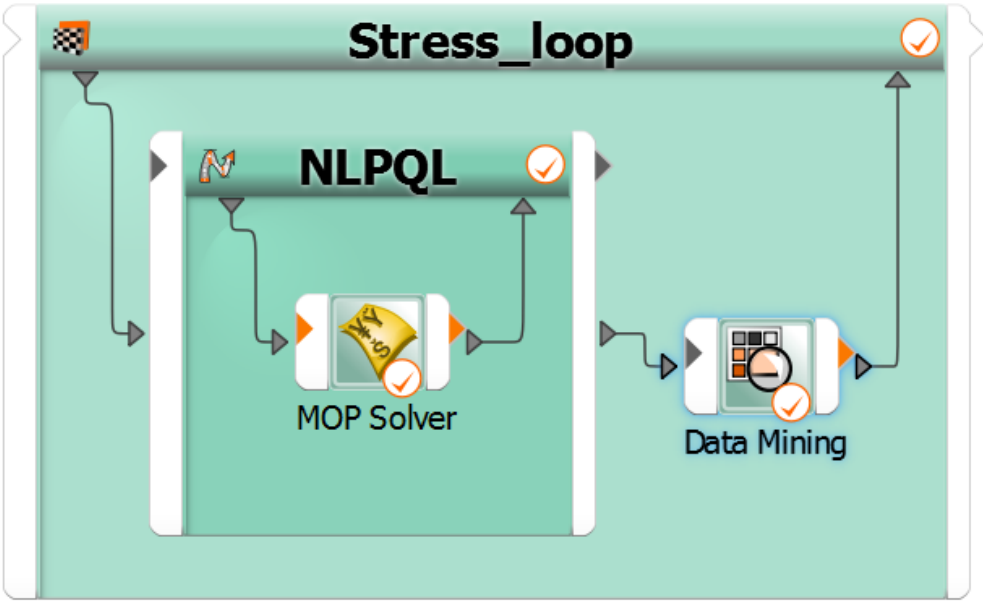




Example: Steel Hook

Nested Pareto Optimization

- Outer system to define the stress limit
- Minimize the nominal mass for each stress limit

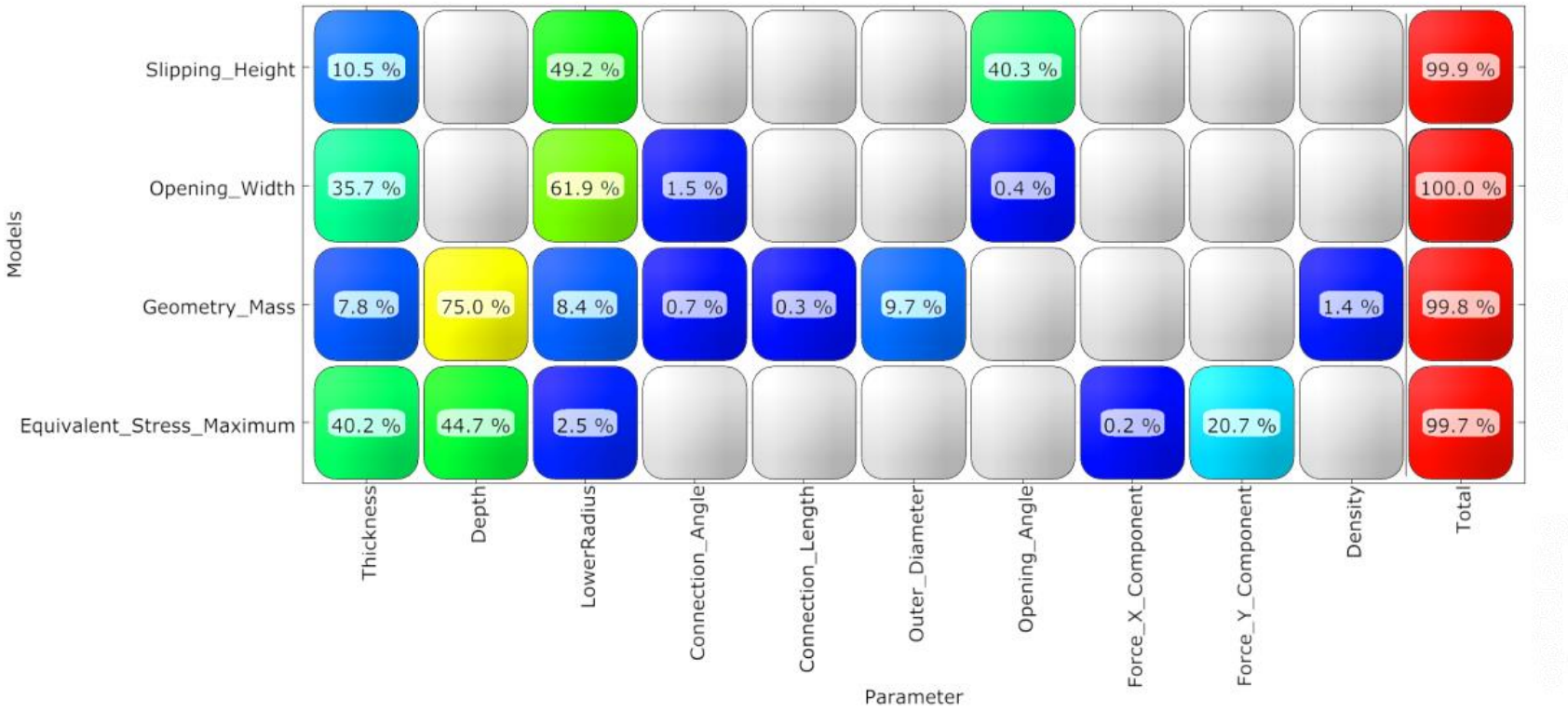


Name	Type	Expression	Criterion	Limit
obj_Geometry_Mass	Objective	Geometry_Mass	MIN	
constr_Slipping_Height	Constraint	Slipping_Height	≥	10
constr_Opening_Width	Constraint	Opening_Width	≥	50
constr_Equivalent_Stress_Maximum	Constraint	Equivalent_Stress_Maximum	≤	Stress_Limit

Example: Steel Hook

Global meta-model in RDO space

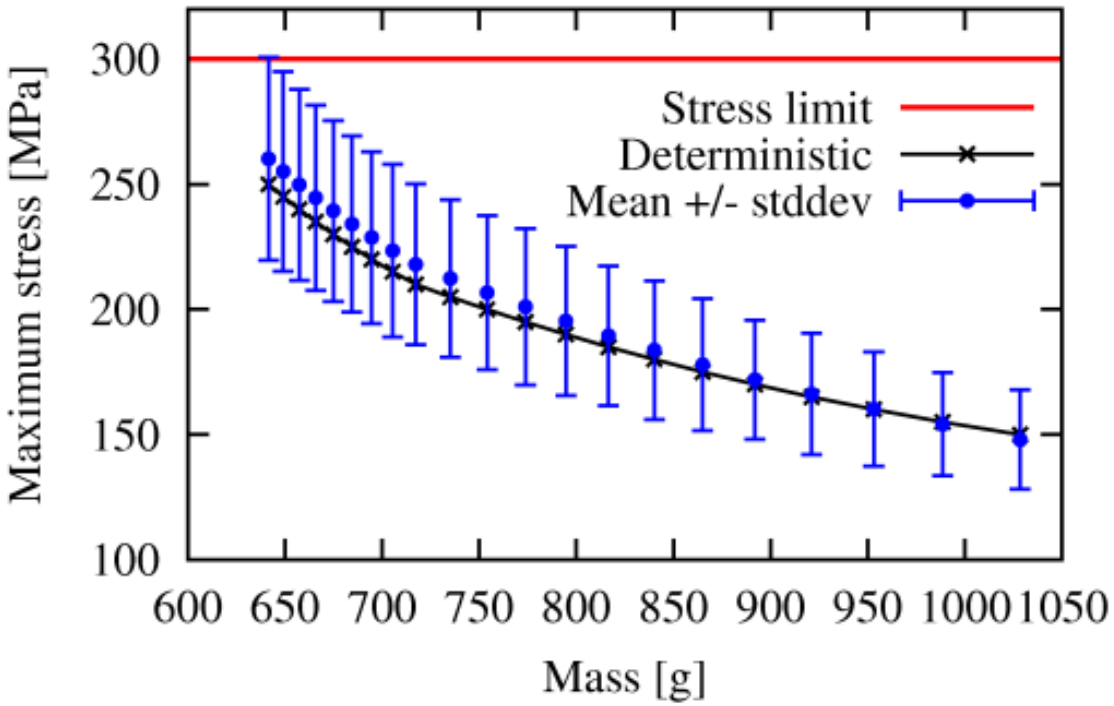
- Generation of support points within optimization bounds +/- 5 sigma
- Optimized space-filling LHS with 500 samples (16 dimensions)



Example: Steel Hook

Robustness Evaluation of the Pareto Frontier

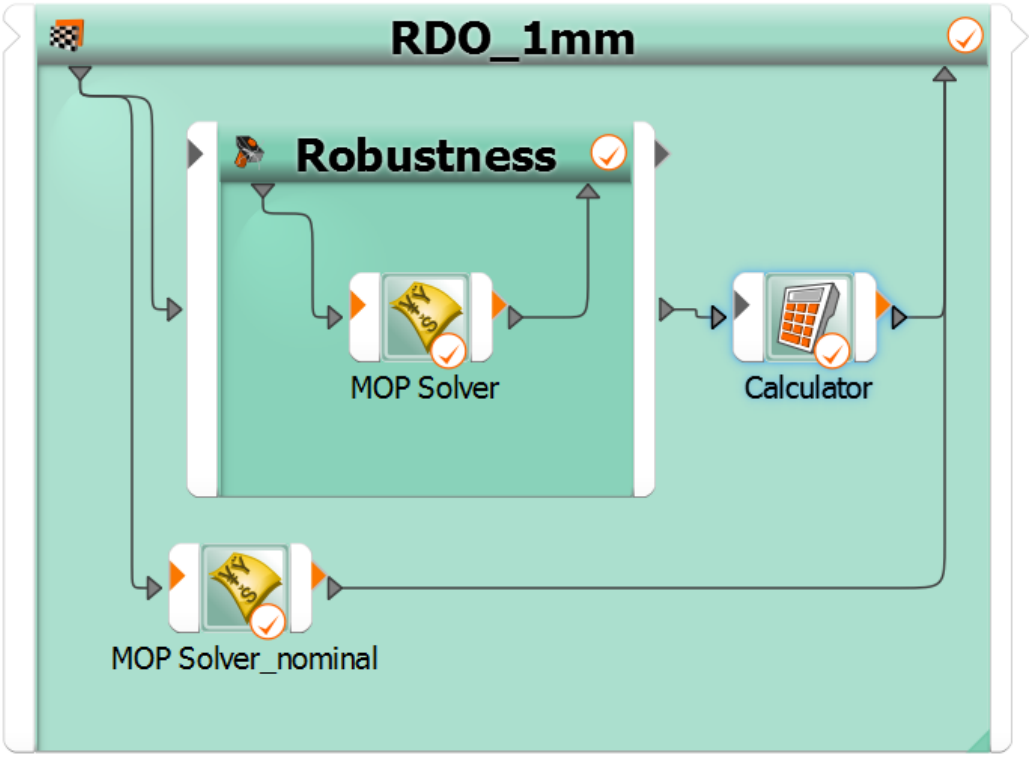
- Evaluation of the stress scatter for each nominal design
- Consideration of well known stochastic material and loading properties and roughly estimated geometry scatter



Example: Steel Hook

Robustness Evaluation of the Pareto Frontier

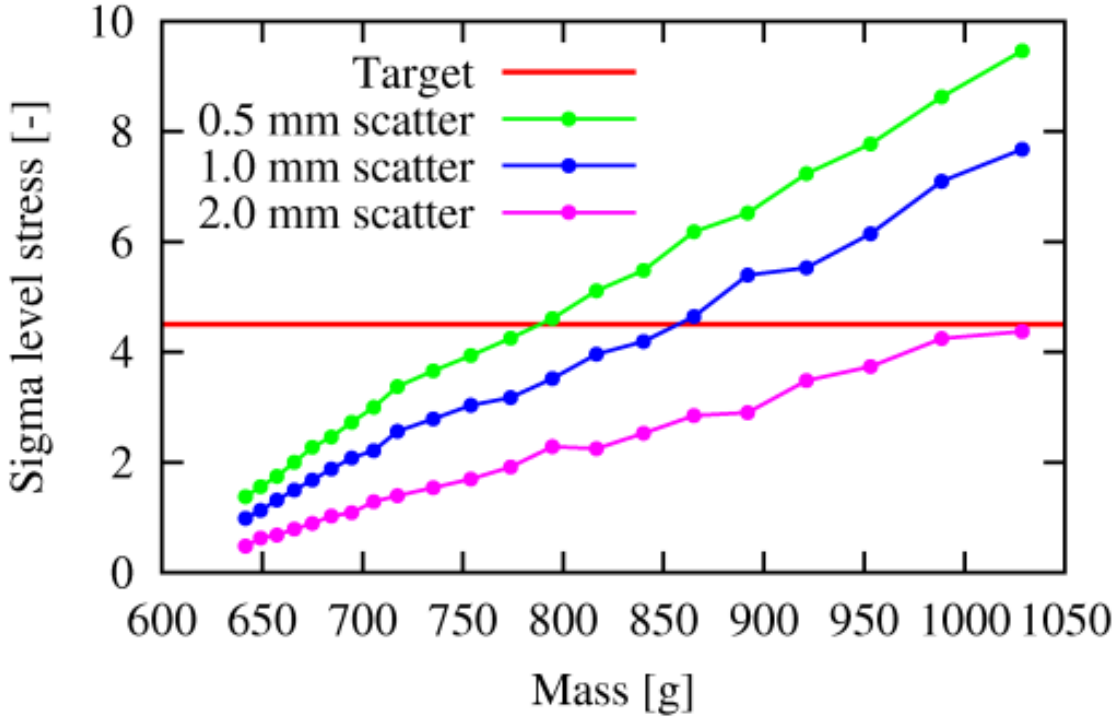
- Standard nested RDO system with given stochastic parameters
- Outer system is a sensitivity with Pareto designs



Example: Steel Hook

Influence of input scatter to safety level

- For each pareto-optimal nominal design the safety level is estimated
- Geometry scatter is varied

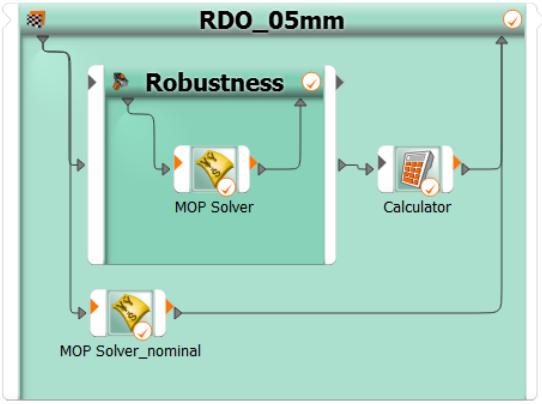


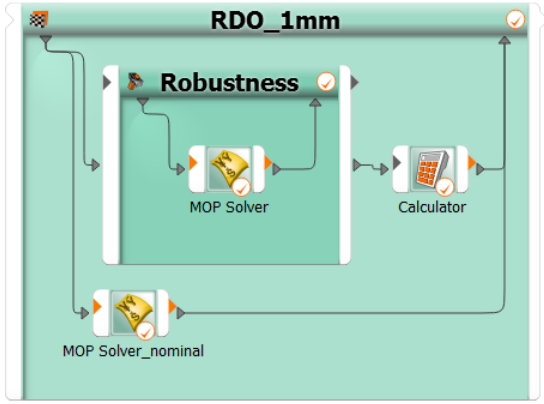
Example: Steel Hook

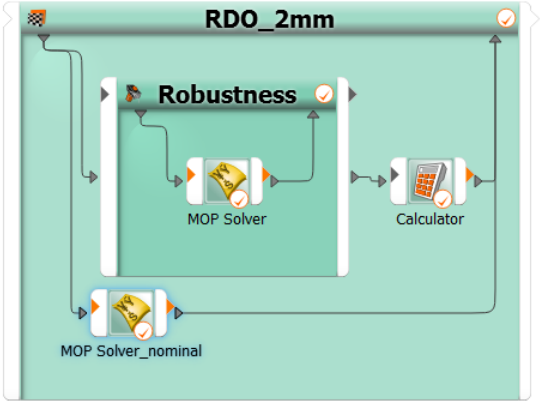
Influence of input scatter to safety level

- Each input scatter is analyzed in a nested system
- Stress sigma level is evaluated for each nominal design in calculator

	ID	Expression
1	mean_stress	mean(Equivalent_Stress_Maximum)
2	stddev_stress	stddev(Equivalent_Stress_Maximum)
3	sigma_level_stress	(300-mean_stress)/stddev_stress



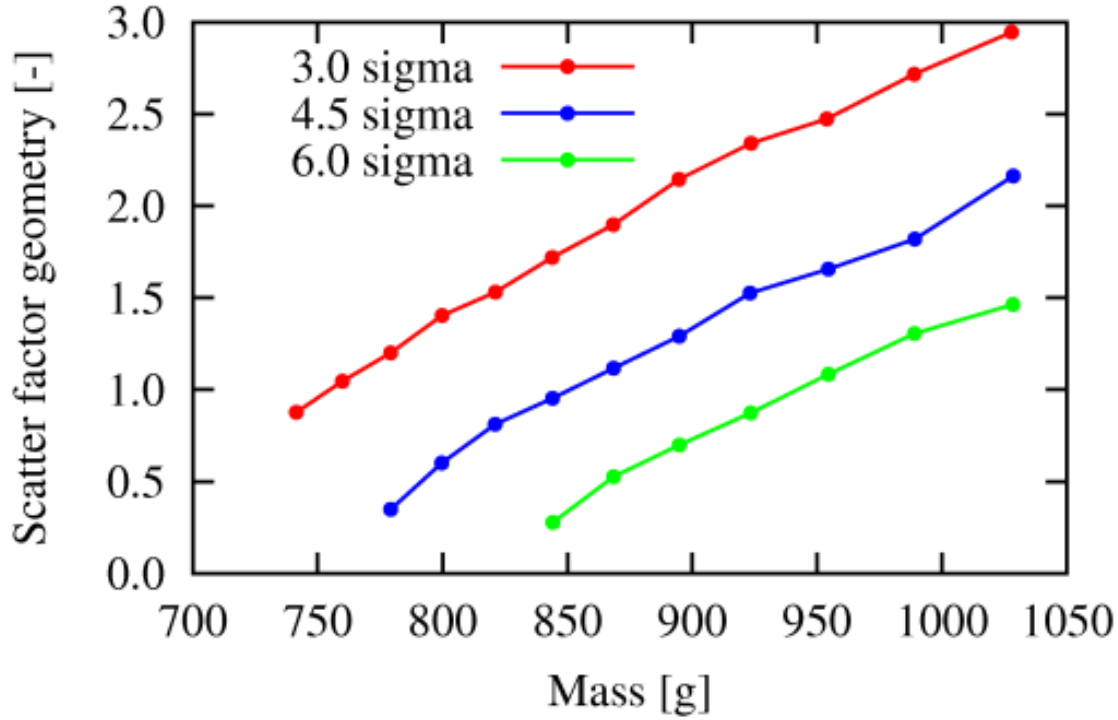




Example: Steel Hook

Maximum input scatter w.r.t. the safety level

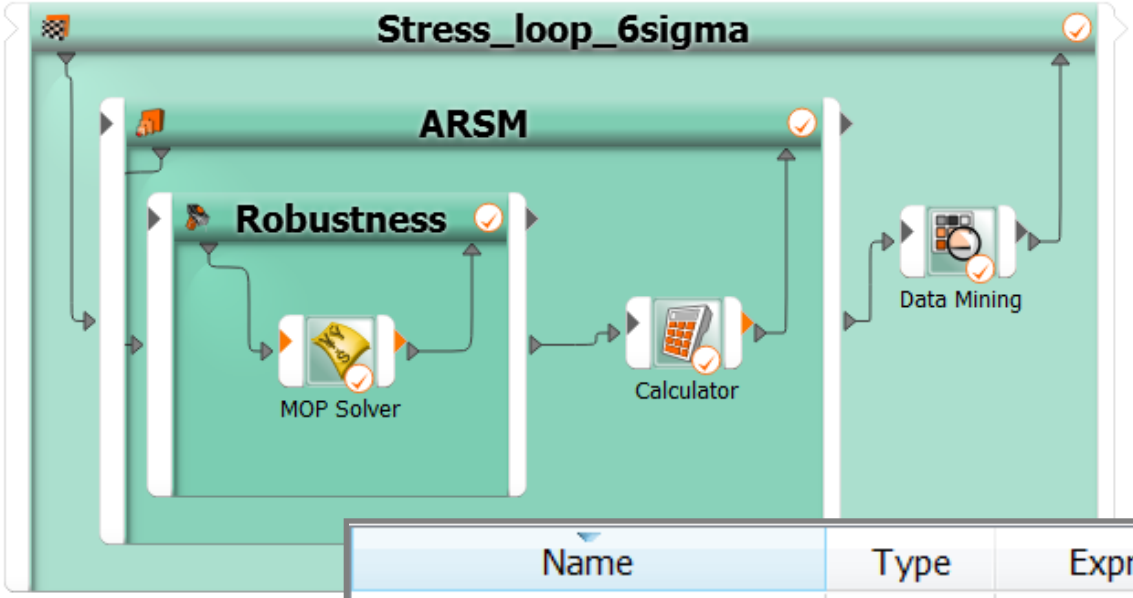
- For each nominal design the maximum possible input scatter is obtained via optimization (3-level nested Robust Design Optimization)
- Investigation for different safety requirements



Example: Steel Hook

Maximum input scatter w.r.t. the safety level

- Outer DOE is a loop over all deterministic Pareto designs
- Optimization maximizes the scatter factor with sigma level constraint

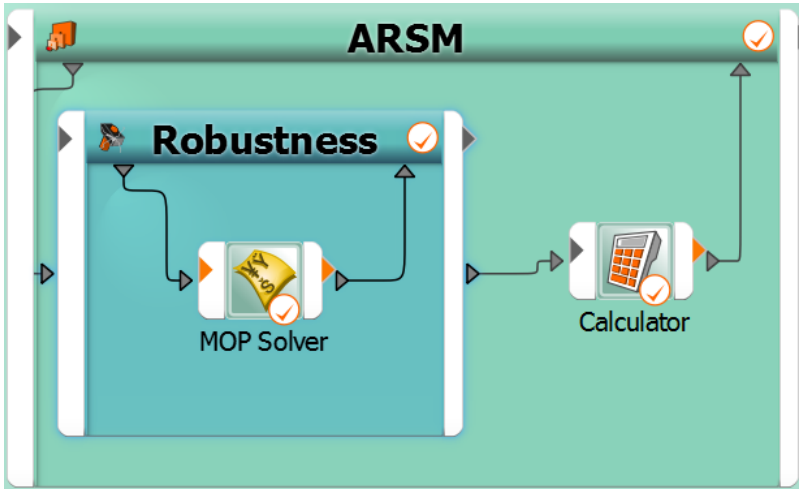


Name	Type	Expression	Criterion	Limit
obj_scatter_factor	Objective	scatter_factor	MAX	
constr_sigma_level_stress	Constraint	300-mean_stress	≥	6*stddev_stress

Example: Steel Hook

Maximum input scatter w.r.t. the safety level

- Scatter factor and mean values are optimization variables
- Geometry parameters are dependent parameters combining the deterministic mean and scatter factor with a standard normal distribution
- For other distribution types this formulation may be more sophisticated



	Name	Parameter type	Operation	Range		PDF	Type	Mean	Std. Dev.
26	scatter_factor	Optimization		0.2	2				
27	Thickness	Dependent	Thickness_mean+scatter_factor*Thickness_std						
28	Thickness_mean	Optimization		15	25				
29	Thickness_std	Stochastic					NORMAL	0	1

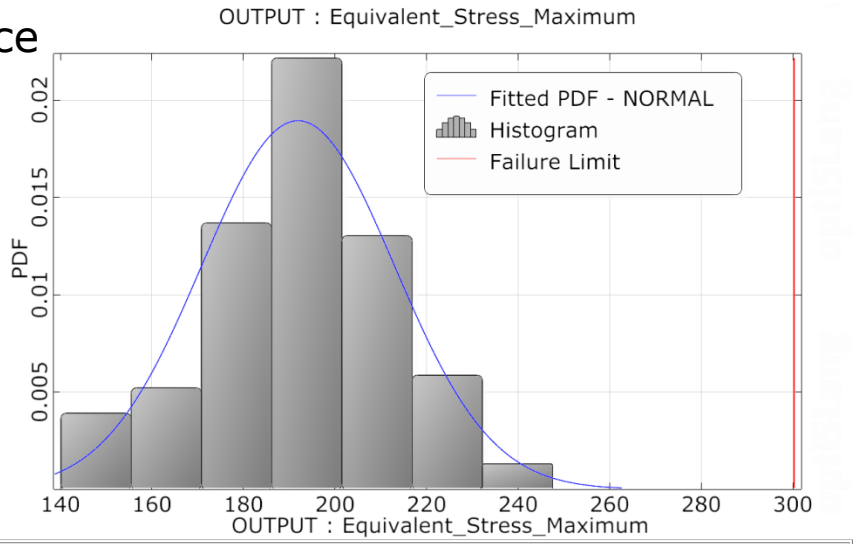
Example: Steel Hook

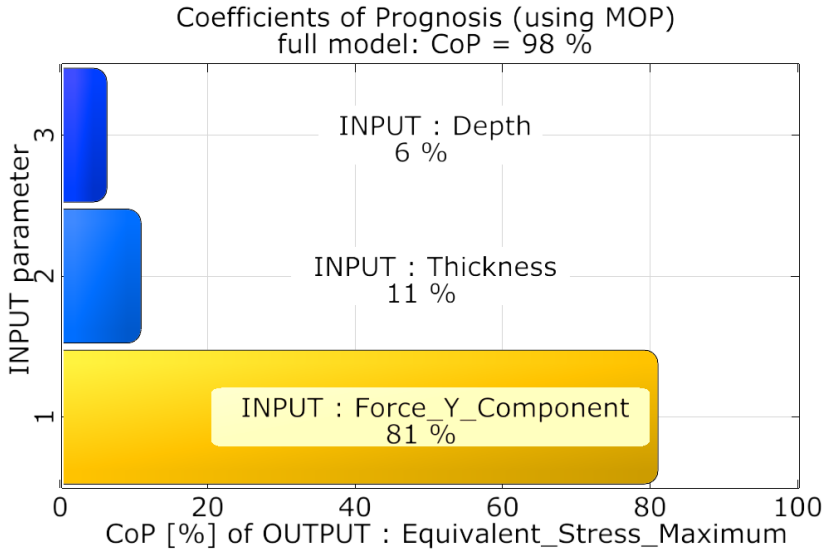
Verification of finally selected nominal design

- 100 LHS samples in stochastic space
- Obtained sigma level is 5.1 > 4.5

➢ Further reliability proof

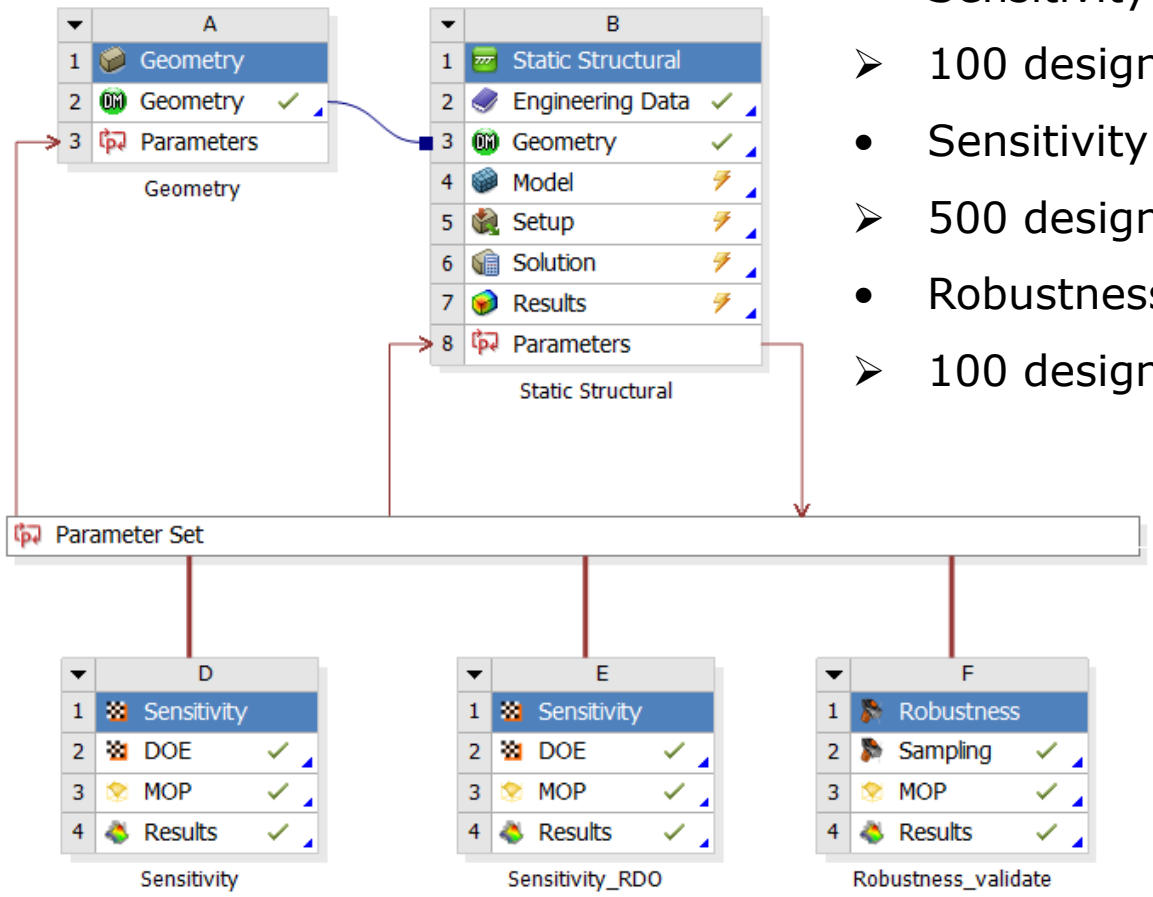
➢ Sensitivity of geometrical scatter is significantly reduced





Statistical data			
Min:	139.843	Max:	247.228
Mean value:	191.645	Standard deviation:	21.0612
CoV:	0.109897		
Limit : Failure Limit			
	Lower value = not set	Upper value = 300	Total
P_rel:		0	0
P_fit:		1.33916e-07	1.33916e-07
Sigma-Level:		5.14478	

Example: Steel Hook



Solver call within ANSYS Workbench

- Sensitivity & MOP in optimization space
  - ➢ 100 designs
- Sensitivity & MOP in RDO space
  - ➢ 500 designs
- Robustness validation
  - ➢ 100 designs

Summary

- Robust Design Optimization is suitable to optimize nominal design parameters as well as the variation of the input scatter
- Production cost may be reduced due to less amount of material but also due to weaker quality requirements to the product itself and to the deliverers
- For an industrial application the investigation of possible compromises between nominal values and scatter limitations (quality requirements) is very promising
- Robust design can be included in the quality management process even for situations with incomplete or rough knowledge of uncertainties