Random Field Developments

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with contributions by
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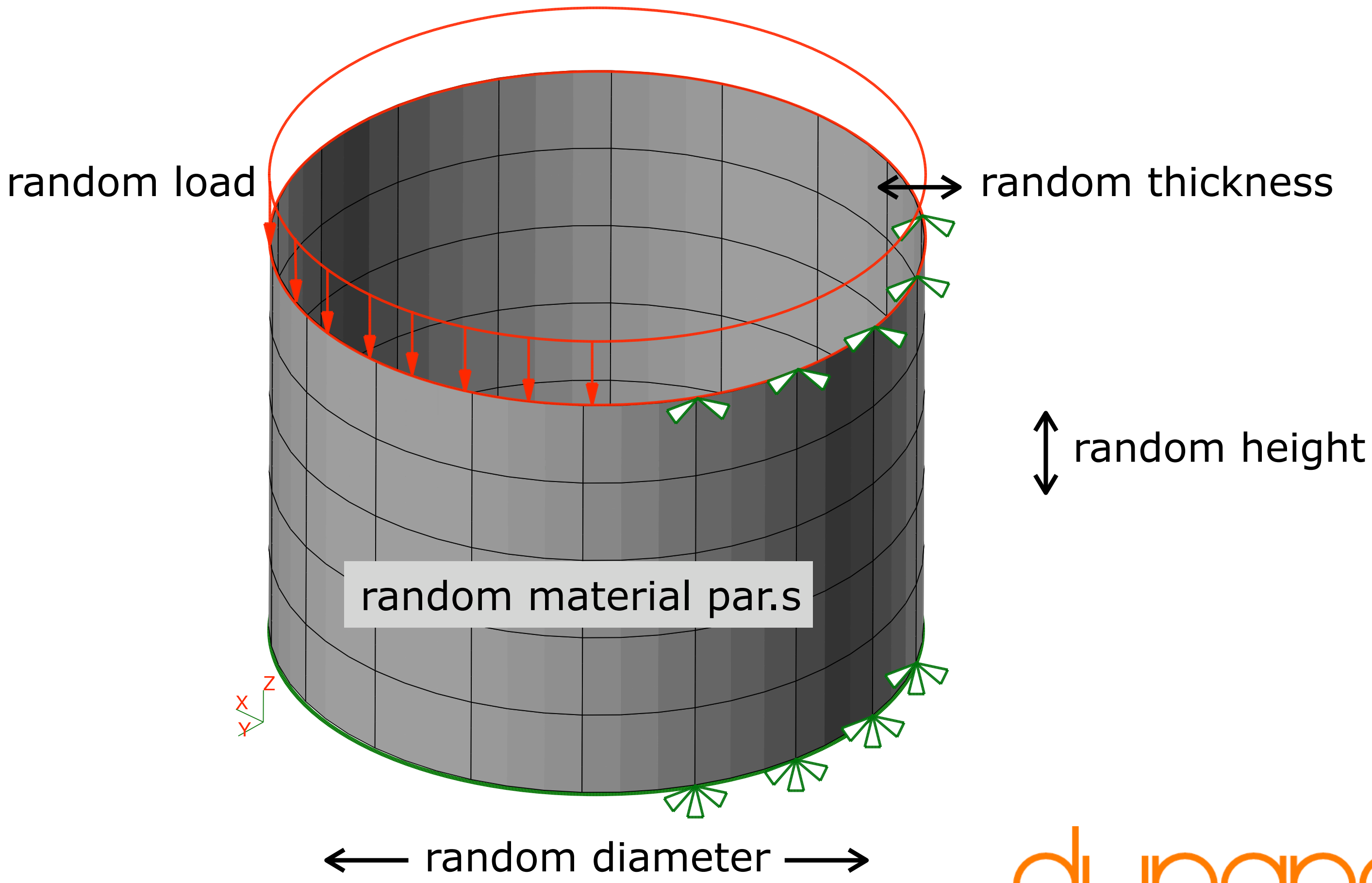
Introduction

What is a Random Field NOT?

Random variation of single parameters (CAD, material, load, ...)



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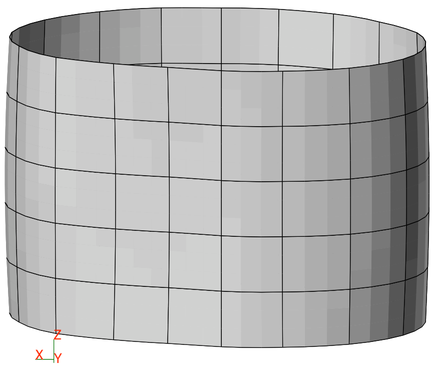
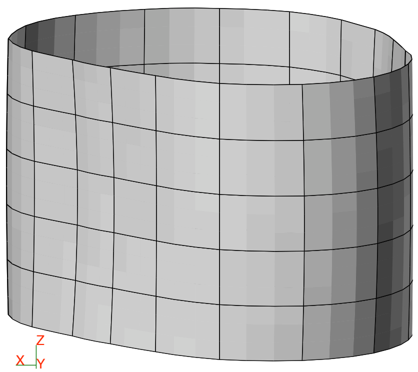
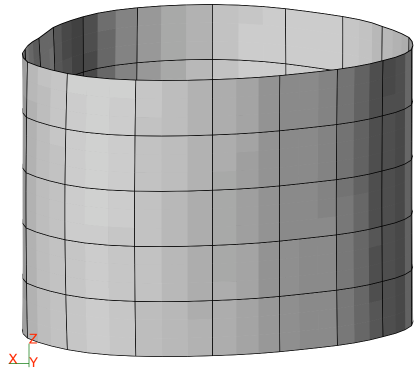
Introduction

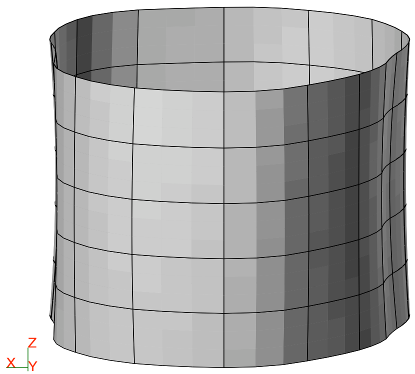
Random Field:

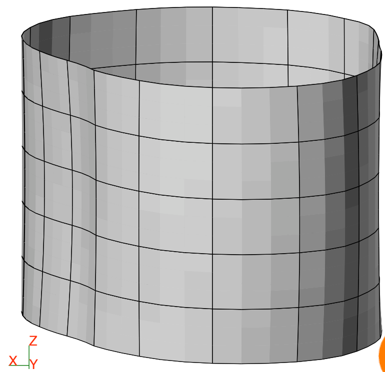
- Random variation of a property (geometry, material, load, ...) over space.
- Spatial domain defined by the observed structure.
- A property takes random values at each point on the structure.
  Values at different locations may be correlated.





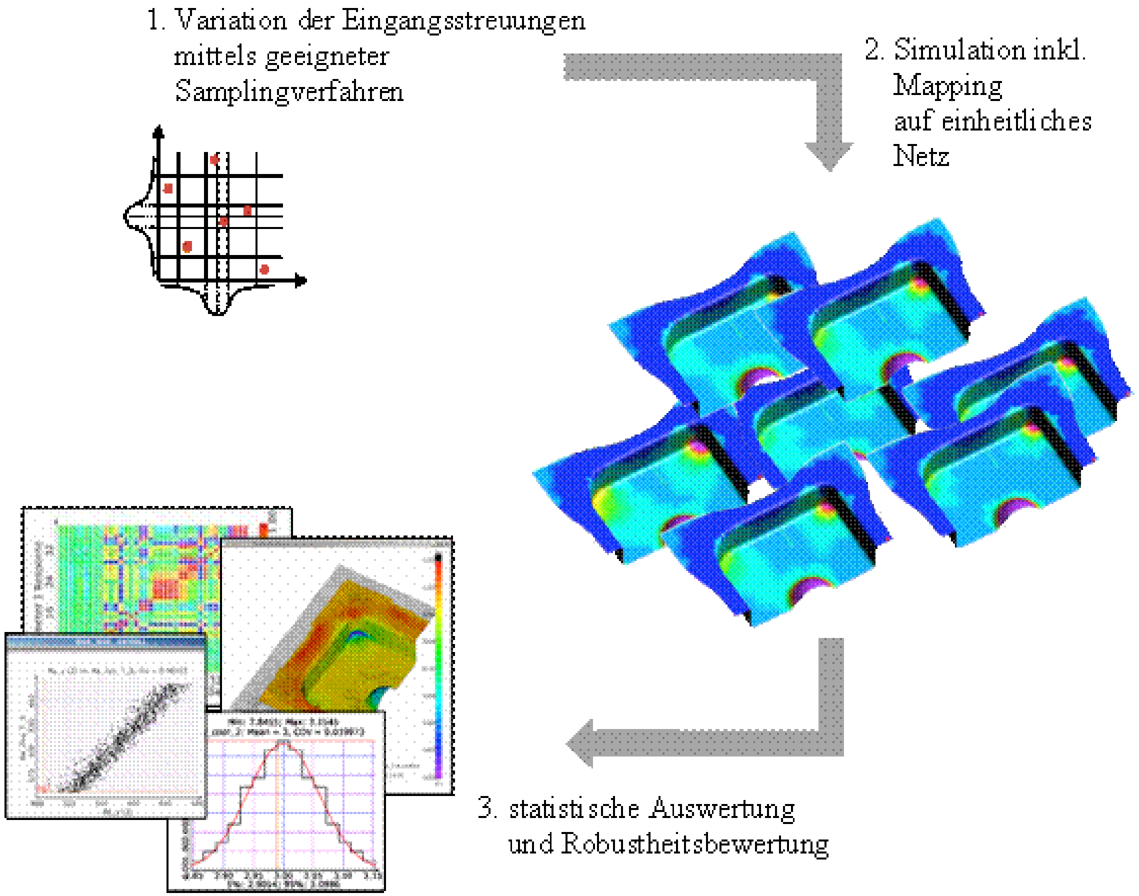






Introduction

Application of Random Fields: e.g., robustness analysis



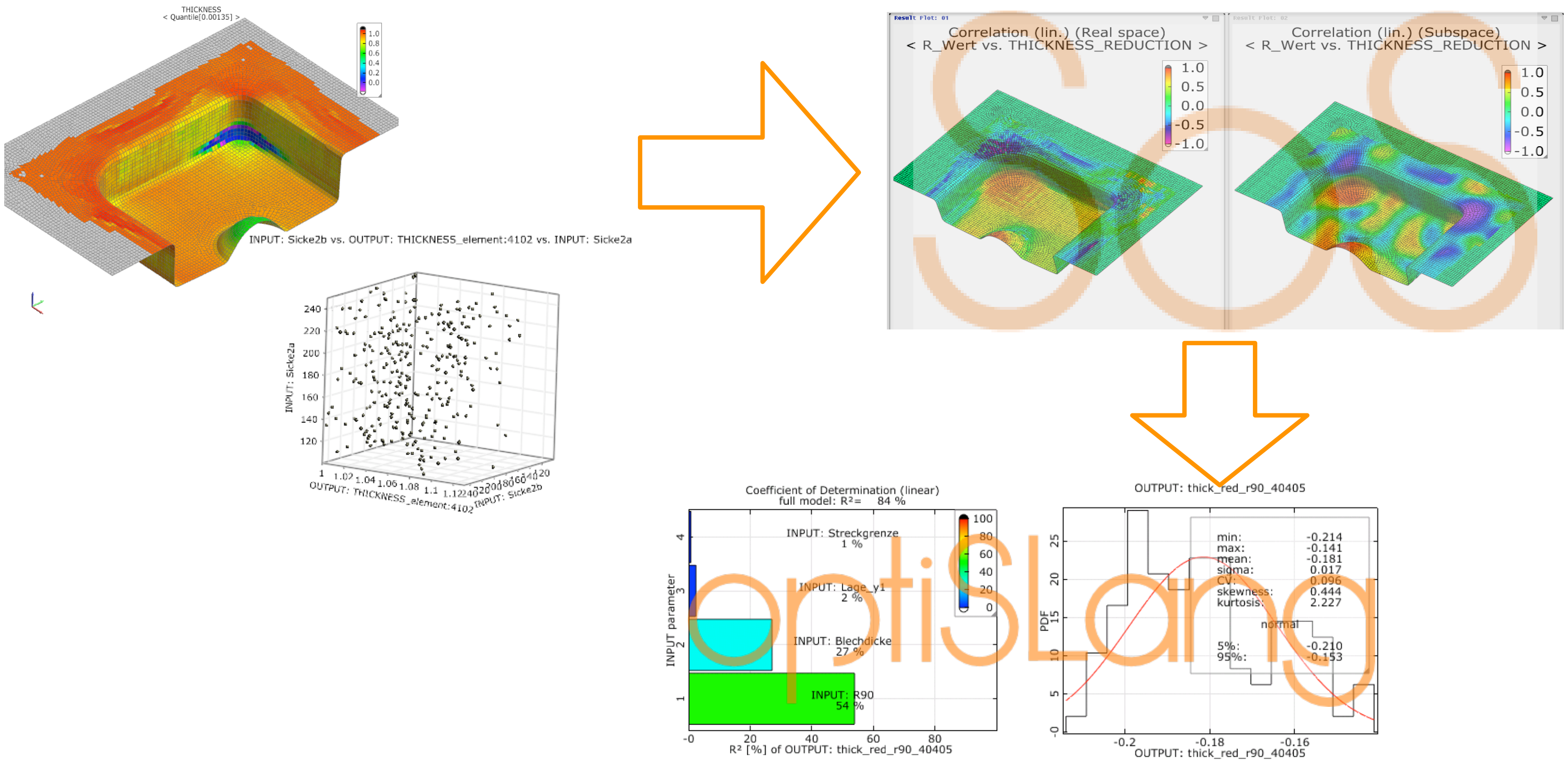
[Will, Bucher, Ganser, Grossenbacher: Computation and visualization of statistical measures on Finite Element structures for forming simulations. WOST 2.0, 2005]

Case I: Given Random Field

Random field data are given

- from simulation of a production process (e.g. sheet metal forming)
- from measurements

and post-processed on the structure or re-imported to optiSLang



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Data Reduction

Modal decomposition of covariance matrix:

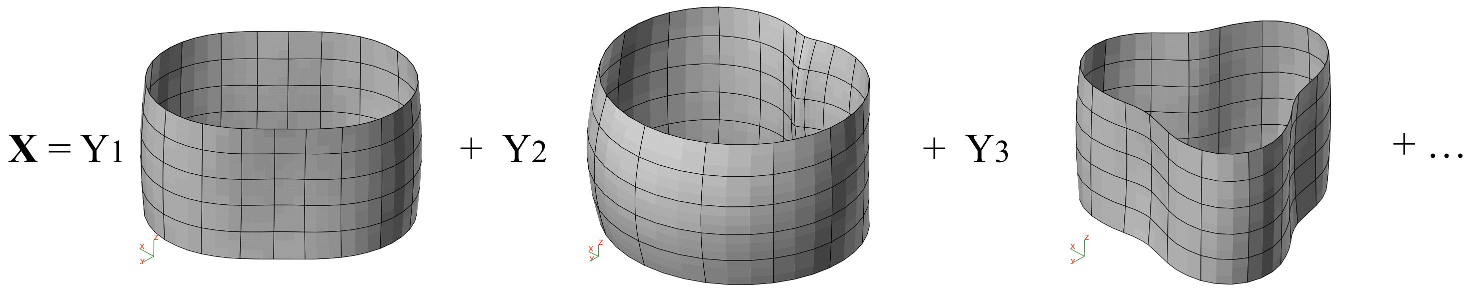
$$\mathbf{\Psi}^T \mathbf{C}_{XX} \mathbf{\Psi} = \mathrm{diag}\{\lambda_i\}$$

Transformation of data into sub-space:

$$\mathbf{Y} = \mathbf{\Psi}^T \mathbf{X} \quad \Leftrightarrow \quad \mathbf{X} = \mathbf{\Psi} \mathbf{Y}$$

… where statistical evaluations are performed.

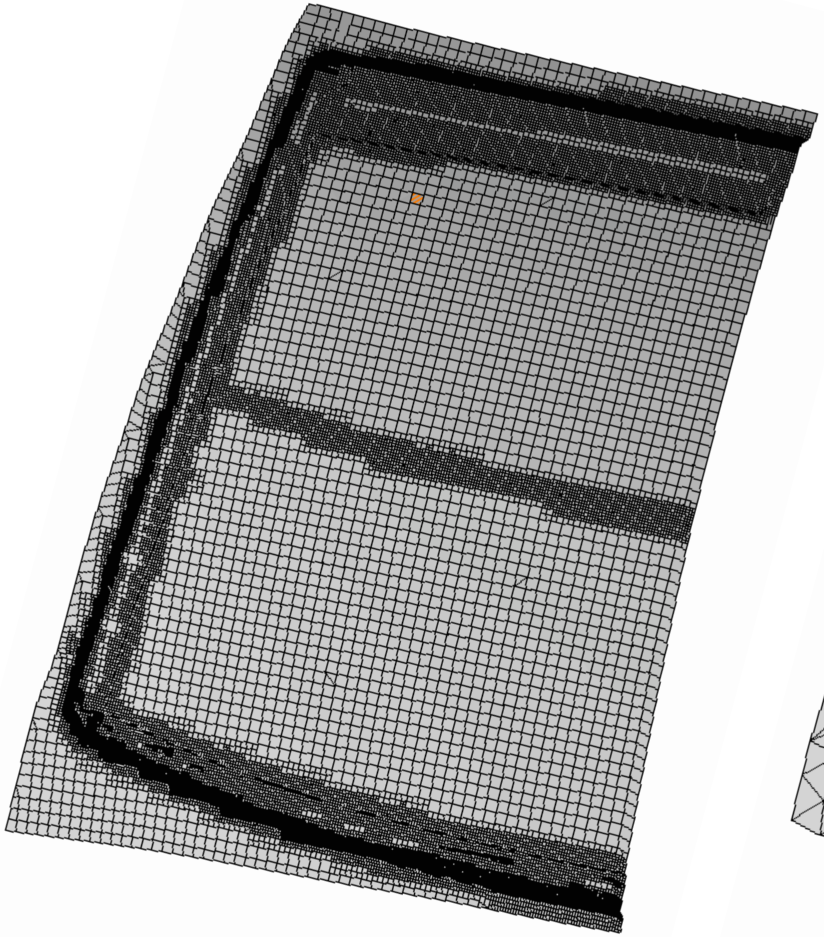
Plan: use covariance matrix of data to identify important imperfection shapes.

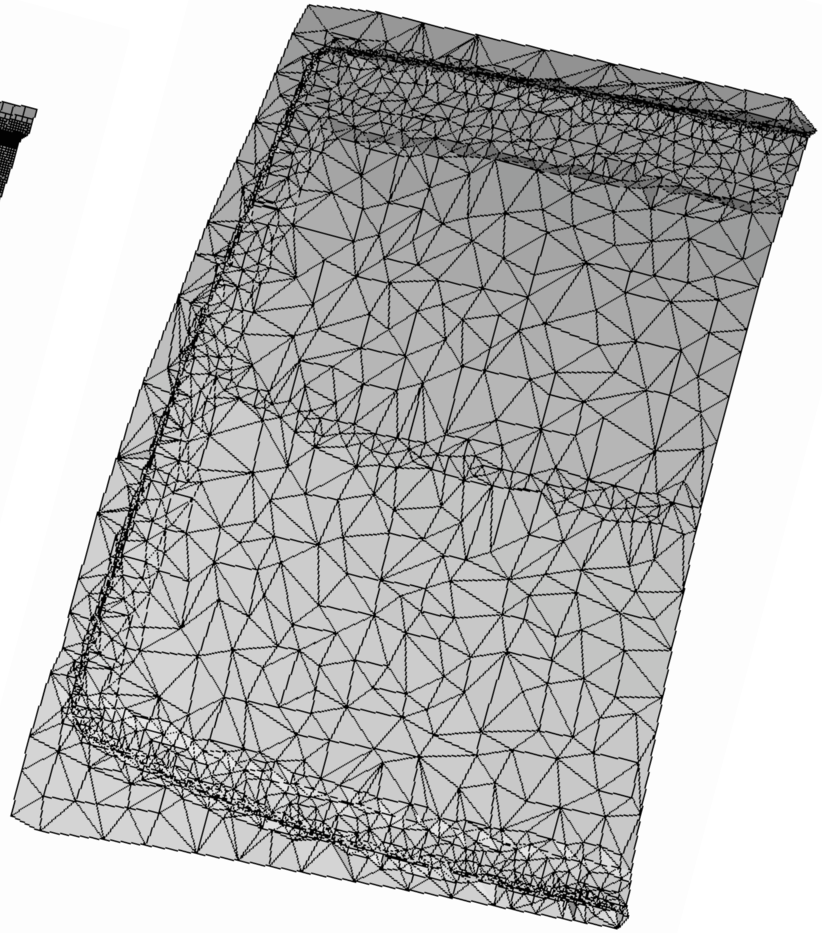


Expansion of random field by random amplitudes and mode shapes.

Data Reduction

Data reduction and smoothing via mesh coarsening





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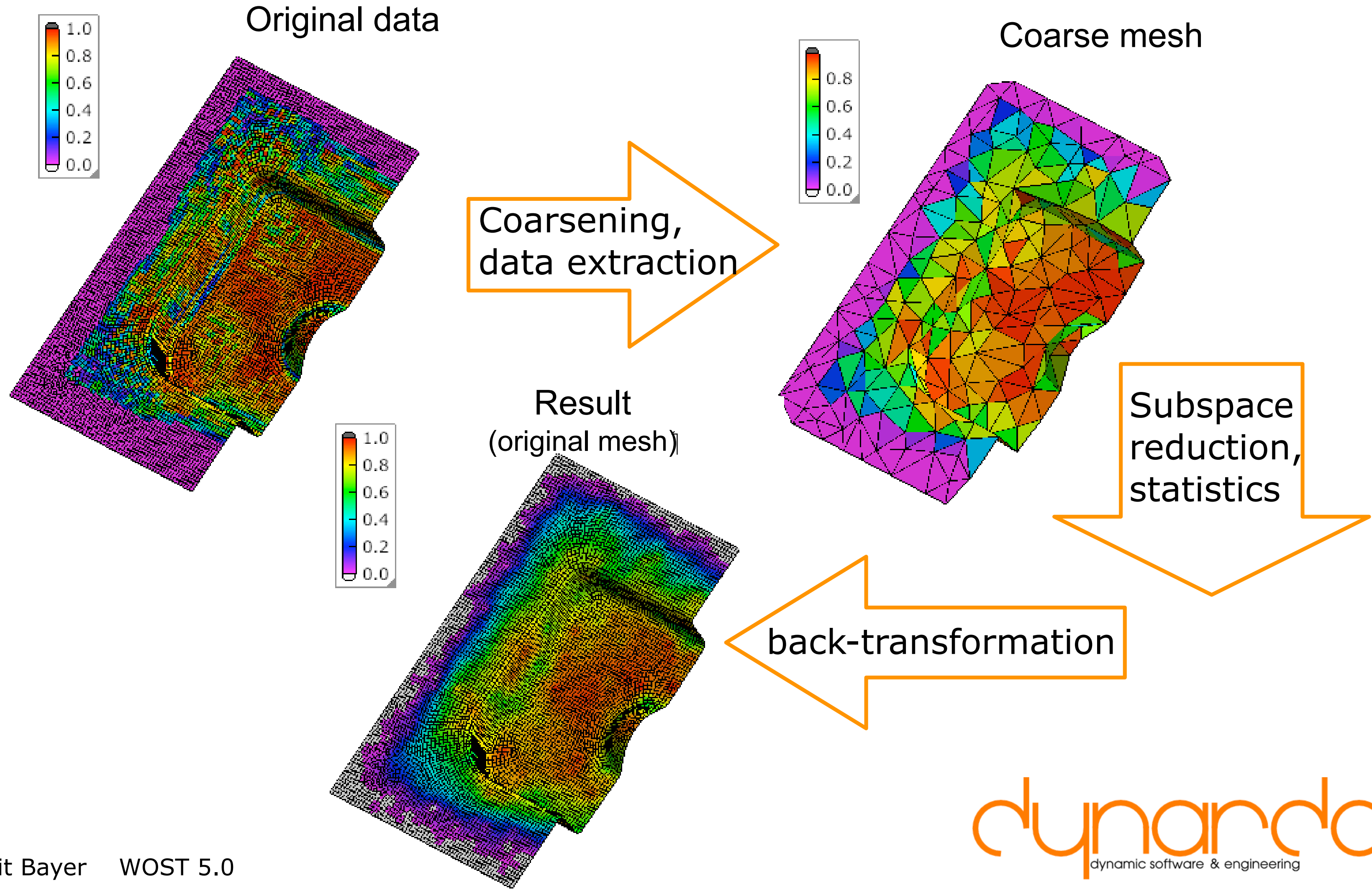
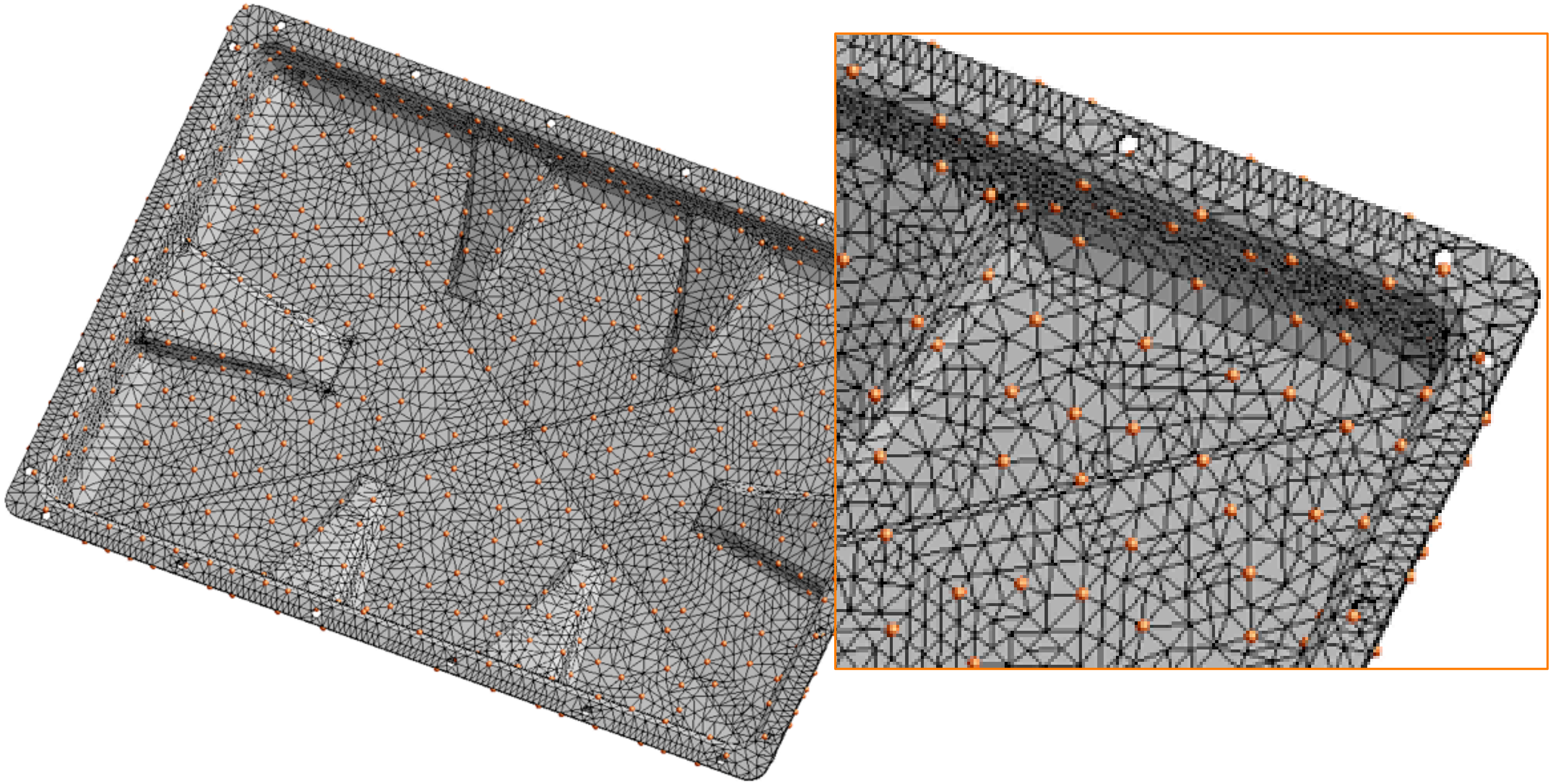
Data Reduction

Smoothing effect by mesh coarsening



Case II: Measured Random Data

Random imperfection measured at discrete points:
grid of data is coarser than structure model.



Data are mapped onto the entire structure by suitable interpolation (<u>Shepard Interpolation</u>, Moving Least Squares, Kriging, …).

Case III: Simulation of a Random Field

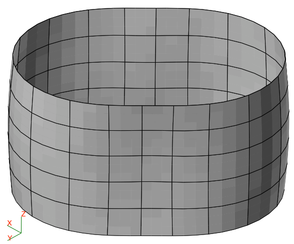
Stochastic properties given either by measured or computed data,
or assumed.

Random field is modelled by mean vector and covariance matrix
and simulated by series expansion: random amplitudes times mode shapes.

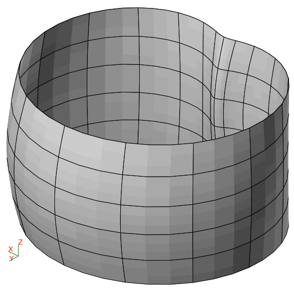
$$\mathbf{\Psi}^T \mathbf{C}_{XX} \mathbf{\Psi} = \mathrm{diag}\{\lambda_i\}$$

$$\mathbf{Y} = \mathbf{\Psi}^T \mathbf{X} \quad \Leftrightarrow \quad \mathbf{X} = \mathbf{\Psi} \mathbf{Y}$$

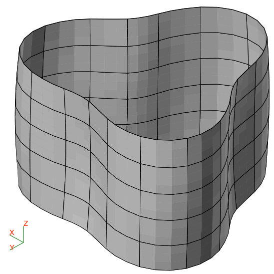
$\mathbf{X} = Y_1$



$+ \; Y_2$



$+ \; Y_3$



$+ \ldots$

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Shrinkage Estimator of Covariance Matrix

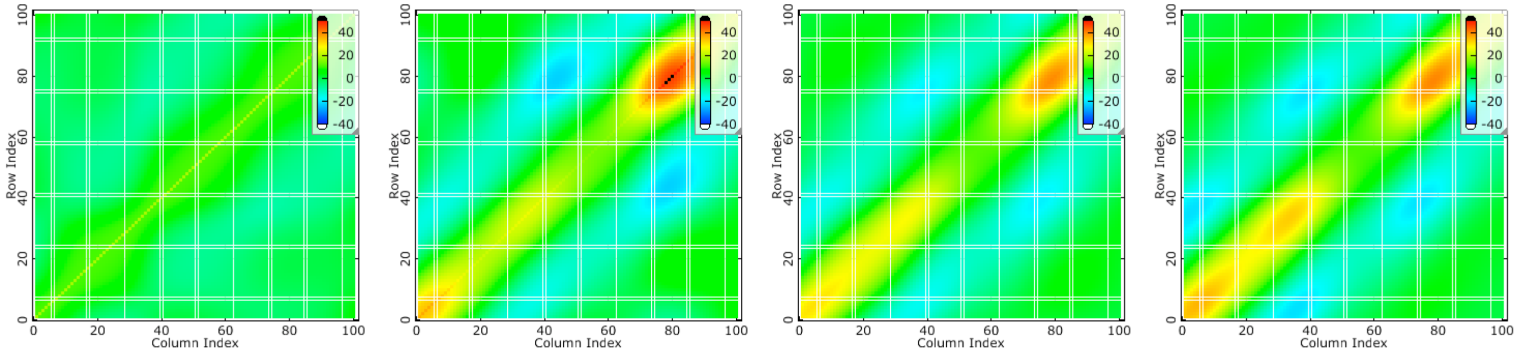
The covariance matrix has to be estimated from given data

- maintaining positive definiteness
- obtaining high confidence.

One way to get this is the Shrinkage estimator:

$$\widetilde{\mathbf{C}} = \lambda \bar{\mathbf{C}} + (1-\lambda)\mathbf{\Gamma}\,, \quad \lambda \in [0;1]$$

$$l(\lambda) = \|\widetilde{\mathbf{C}} - \mathbf{C}\|^2 \;\rightarrow \min$$



N=10 N=50 N=500 true covariance

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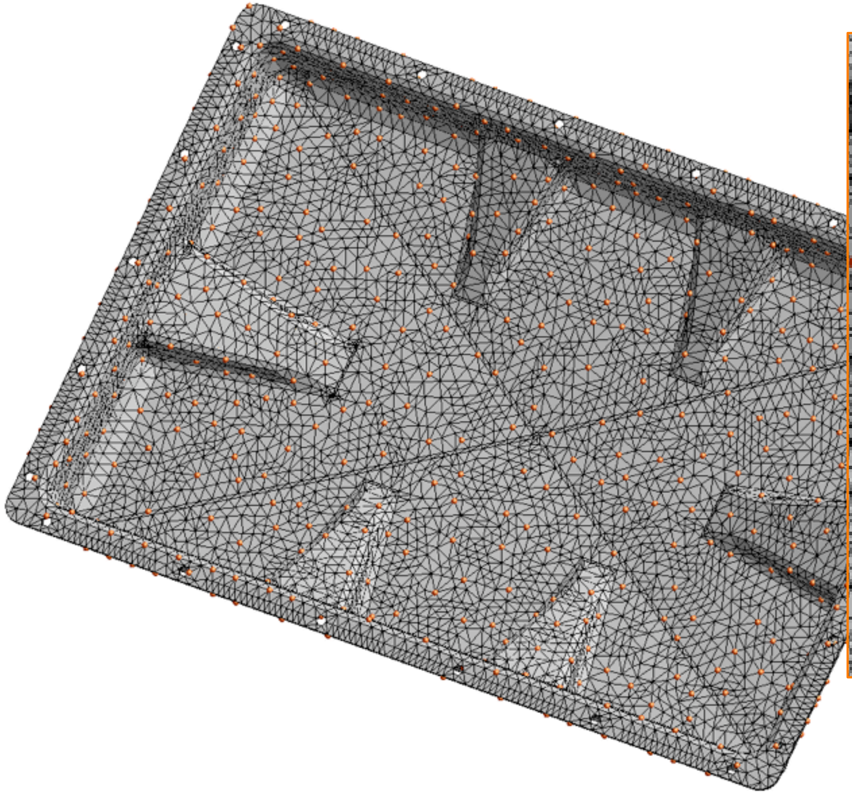
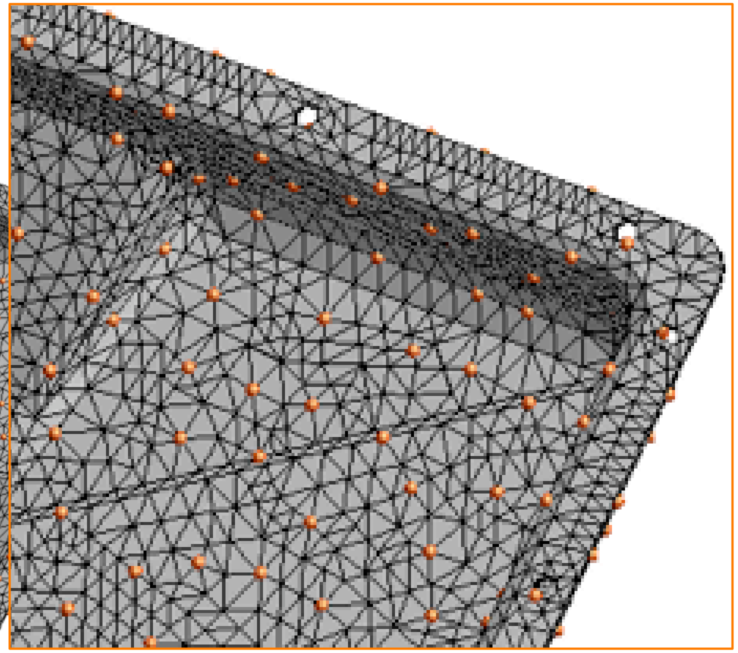
Example 2

Harmonic analysis of an oil pan with geometric imperfections





Model dimensions: 300x450x56 mm, thickness 1.5mm, ~76000 nodes, 10-node tetraeder elements, harmonic analysis in ANSYS.

Example 2

Random geometric deviations
at nodes selected by mesh coarsening.

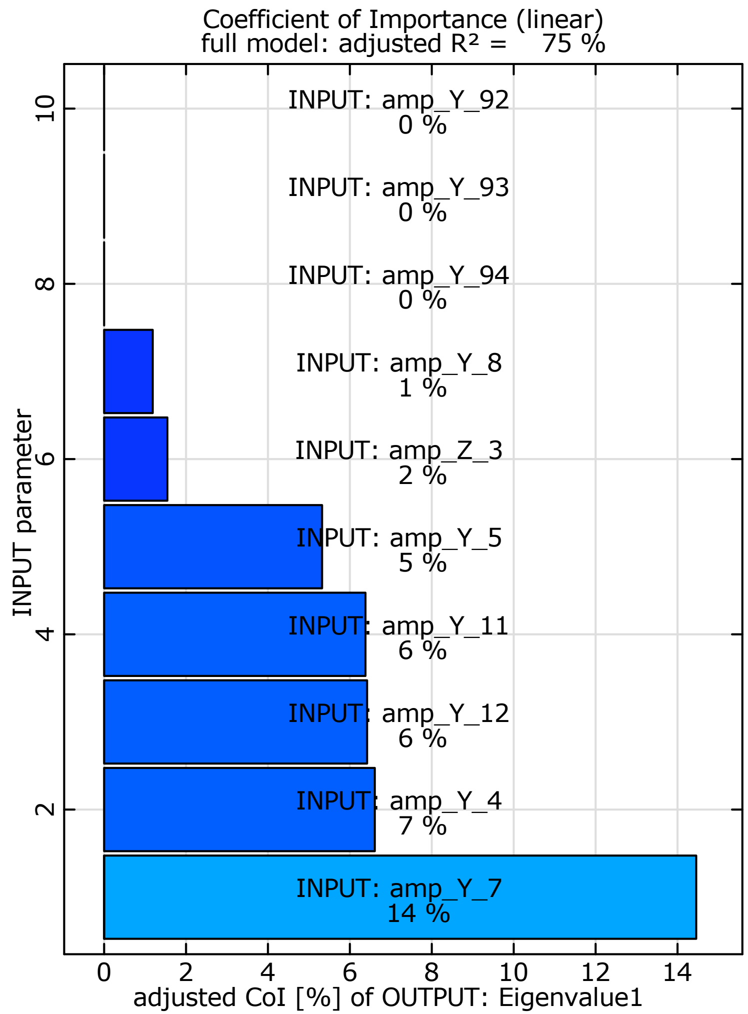
Mapping of random field realisations
by Shepard 3D-interpolation.

Performance criterion for lowest
eigenfrequency $P[f_0 \leq 400\, Hz] = 10^{-4}$

After a pilot simulation by LHS,
7 relevant mode shapes were selected
which influence most the lowest
frequency.

Then, reliability is computed by ARSM
using just 94 structural evaluations.

Random field modelling in SLang,
Simulation, statistical evaluation in
optiSLang.



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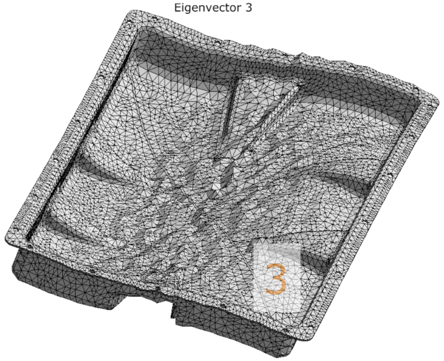
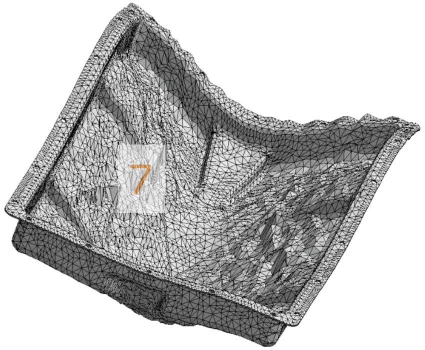
Example 2

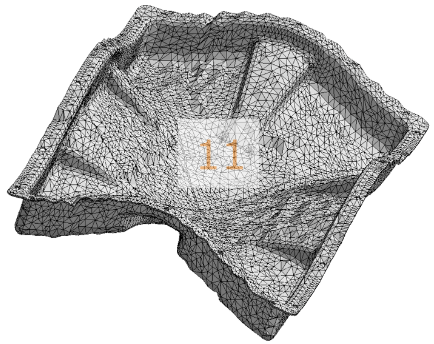
Eigenvector 3



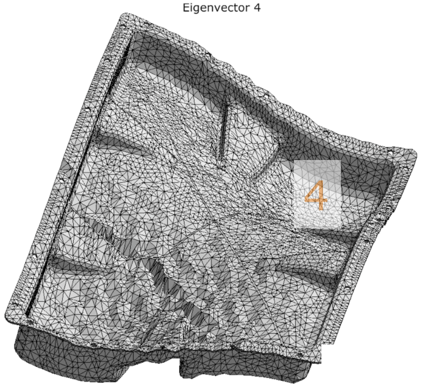
Eigenvector 7



Eigenvector 11

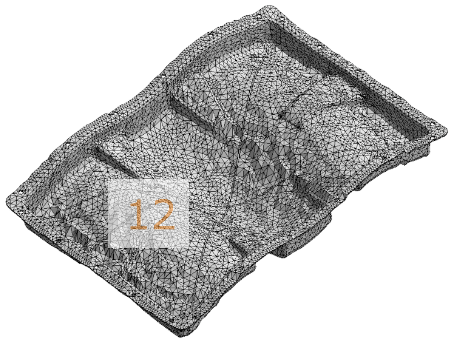


Eigenvector 4



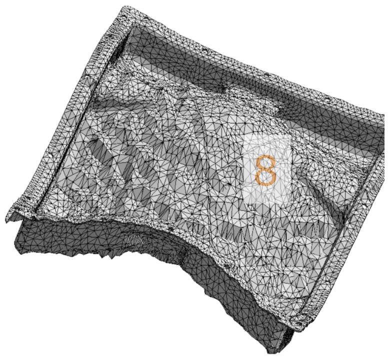
Random field shapes
used for
reliability analysis

Eigenvector 12



Eigenvector 5

Eigenvector 8



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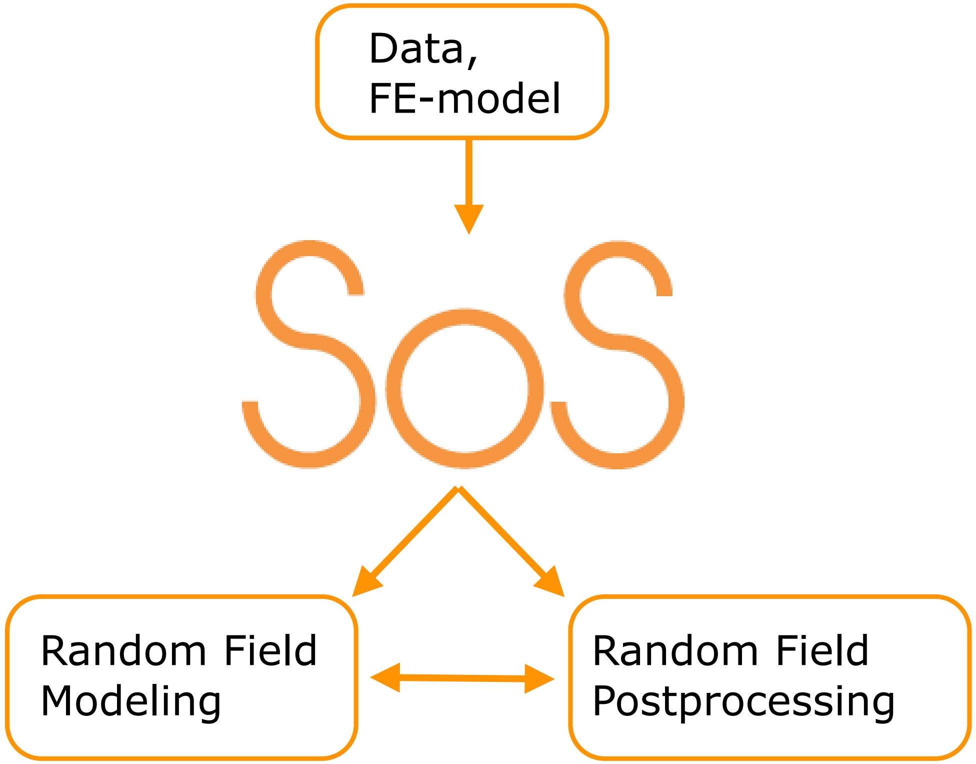
Conclusions

SoS will develop from a random field analyzer
to be a random field modeler and simulator as well.



Conclusions

Means of data reduction maintaining meaningful and interpretable results is a major topic in future developments:

- mesh coarsening,
- efficient modal decomposition,
- choice of relevant mode shapes,
- data interpolation,
- error measures.

For simulation, random field modelling is a key issue:

- estimation of correlations / covariance matrix,
- random field modelling between coarsely spread supports

Interfacing to optiSLang is planned
in order to make use of parametrisation and post-processing.

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