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Minimize simulation effort: metamodels vs. reduced order models

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Introduction

- Complex system (many parameters, computationally expensive, slow, ...)
- Needed: Fast and reasonably accurate response (e.g. for real-time applications such as control systems)
- Possible choices:
  - Reduce model complexity based on essential physical features (“reduced order model”)
  - Replace model based on mathematical simplicity (“metamodel”)

Reduced order model

- Purpose: Faster analysis
- Properties
  - Need to understand and represent physics
  - May be applicable for many different load cases
  - Very suitable for time dependent phenomena (structural dynamics, convection-diffusion processes)
  - Can be tricky in the presence of strong nonlinearity
- Typical example: Modal analysis

Metamodel

- Purpose: Faster analysis, simplify and understand complex relations (e.g. in robustness analysis)
- Properties
  - Mathematically formulated black box
  - Suitable for arbitrarily nonlinear I/O relations
  - Requires extensive training data
  - Very difficult to extrapolate
  - Time-dependent problems may be tricky
- Typical example: Linear response surface model

Common properties

- Based on previous experience
  - Knowledge of physical processes
  - Acquired experience through “training”
- Limited range of applicability
  - Nonlinearities
  - Number of input variables
- NOTE: Approaches complement each other

➽ Combination may be better than the sum of the individual parts!

Metamodels by regression

- Adjust a model to experiments

$$Y = f(X, \mathbf{p})$$

- Set of parameters

$$\mathbf{p} = [p_1, p_2, \dots, p_n]^T$$

- Experimental values for input X and output Y

$$(X^{(k)}, Y^{(k)}), k = 1 \dots m$$

- Search for best model by minimizing the residual

$$S(\mathbf{p}) = \sum_{k=1}^{m} \left[Y^{(k)} - f(X^{(k)}, \mathbf{p}) \right]^2 ; \quad \mathbf{p}^* = \text{argmin } S(\mathbf{p})$$

Linear regression

- Linear dependence on parameters (not on variables!)

$$f(X, \mathbf{p}) = \sum_{i=1}^{n} p_i g_i(X)$$

- Necessary condition for a minimum

$$\frac{\partial S}{\partial p_j} = 0; \quad j = 1 \dots n$$

- Solution

$$\sum_{k=1}^{m} \left\{ g_j(X^k)[Y^k - \sum_{i=1}^{n} p_i g_i(X^k)] \right\} = 0; \quad j = 1 \dots n$$

$$\mathbf{Qp} = \mathbf{q}$$

Coefficient of determination

- Defined by correlation between experimental data and model predictions

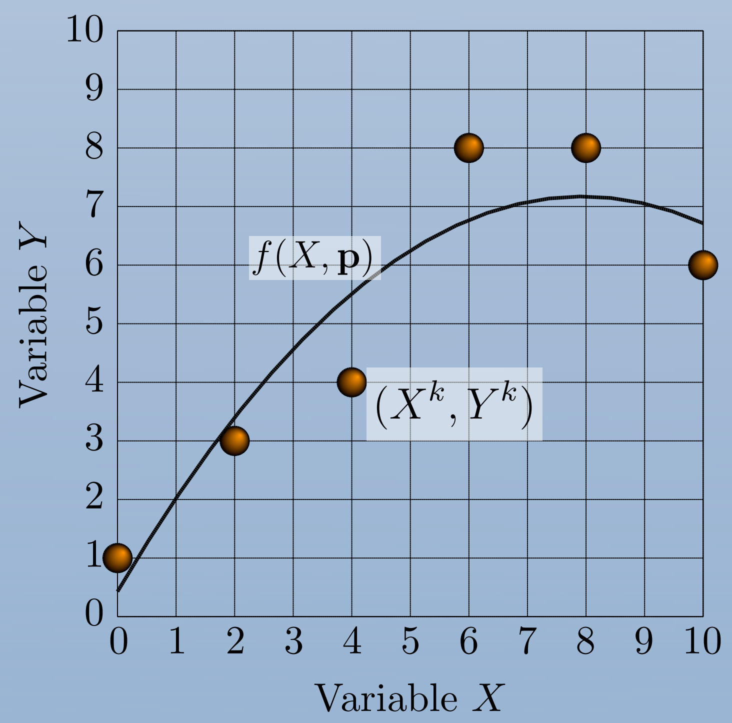
$$R^2 = \left(\frac{\mathsf{E}[Y \cdot Z]}{\sigma_Y \sigma_Z}\right)^2 ; Z = \sum_{i=1}^{n} p_i g_i(X)$$

- Adjusted (reduced) COD for small sample sizes

$$R^2_{adj} = R^2 - \frac{n-1}{m-n}\left(1 - R^2\right)$$

- In the previous example

$$R^2 = 0.86; \quad R^2_{adj} = 0.63$$



Selection of important variables

- Coefficient of determination can be utilized to select important parameters and/or variables
- Starting from a suitable regression model with sufficiently large COD (> 0.80) parameters/variables are eliminated one at a time
- Reduction of COD indicates relevance of parameter/variable
- Coefficient of importance COI between 0 and R^2

$$\mathrm{COI}_i = \mathrm{COD} - \mathrm{COD}_i$$

Importance by ANOVA

- Eliminate e-th parameter p_e from regression

$$f_e(X, \mathbf{p}) = \sum_{i=1, i \neq e}^{n} p_i g_i(X)$$

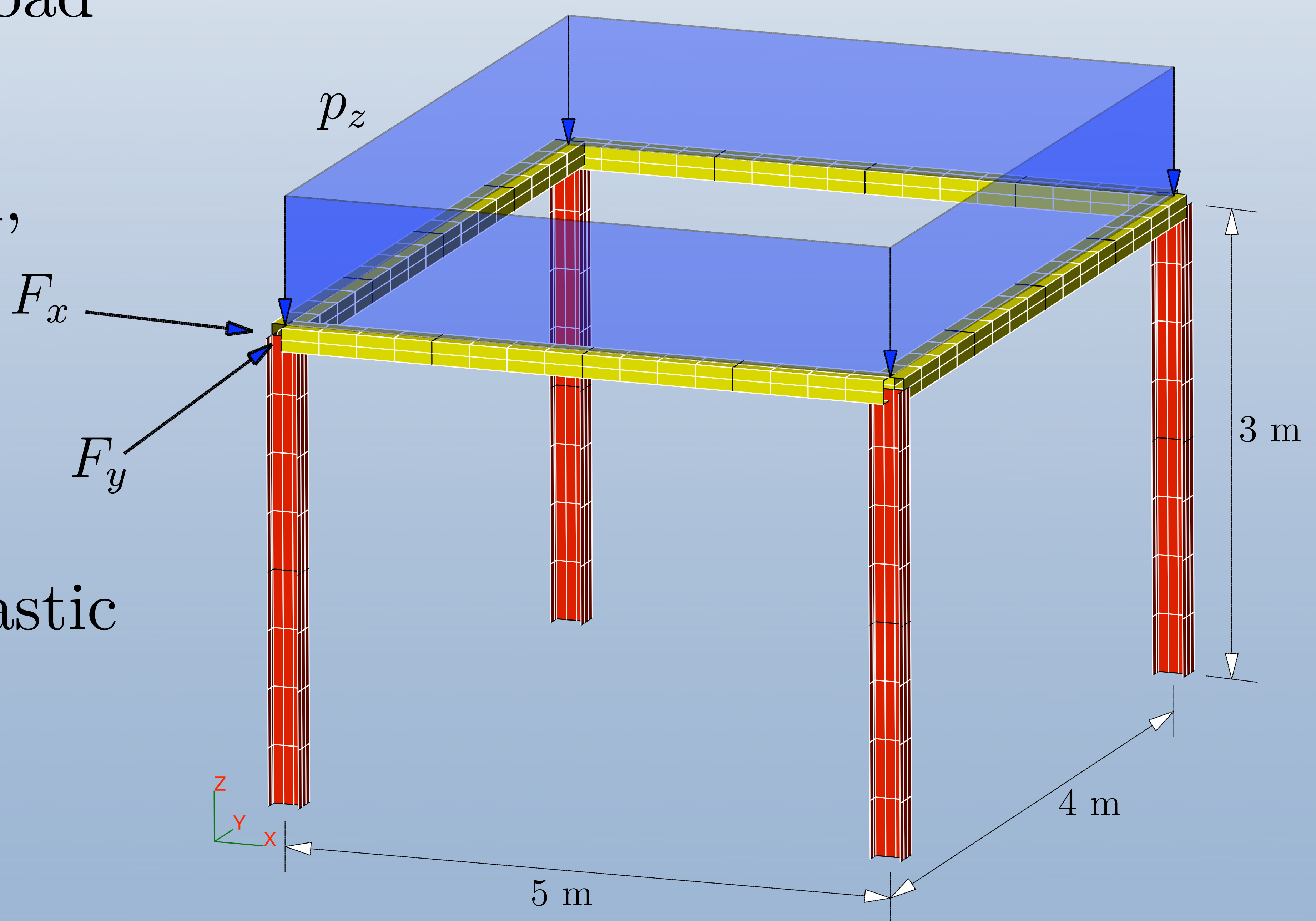
- Compute residual S_e of reduced regression
- Compute F statistic

$$F = \frac{1}{m-n} \frac{S_e - S}{S}$$

- Large values of F indicate higher importance of p_e

Reliability of space frame

- Structure and load configuration
- Plastic material, deterministic
- Random Loads
- Collapse due to formation of plastic zones



Probability of failure

- All variables are Gaussian

RV	Mean	Std. Dev.
p_z [kN/m]	12.0	1.2
F_x [kN]	30.0	3.6
F_y [kN]	40.0	4.8

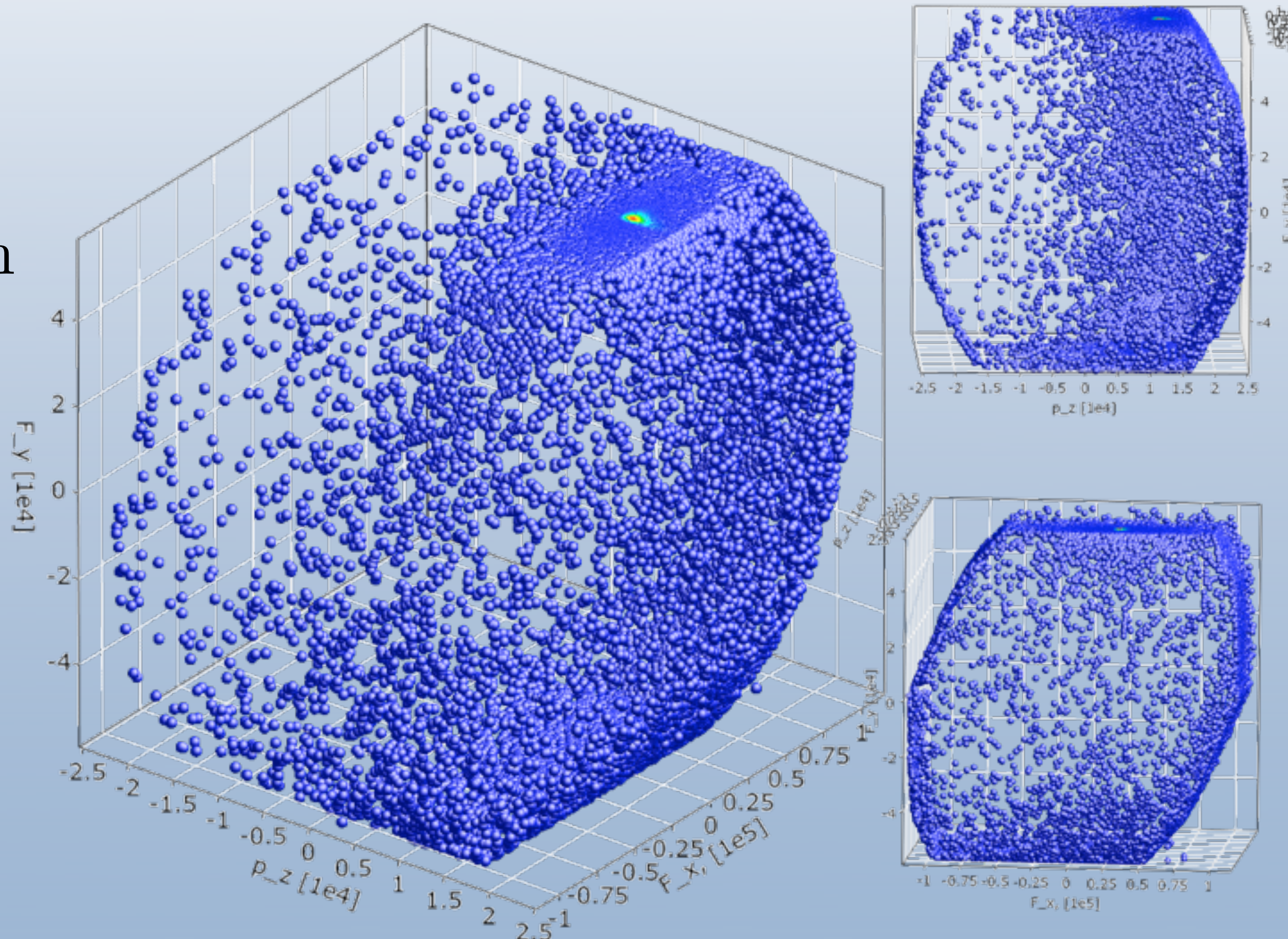
- Directional sampling, 15.000 samples

$$P(\mathcal{F}) = 4.3 \cdot 10^{-5}; \quad \beta = 3.93$$

with a standard error of 3%.

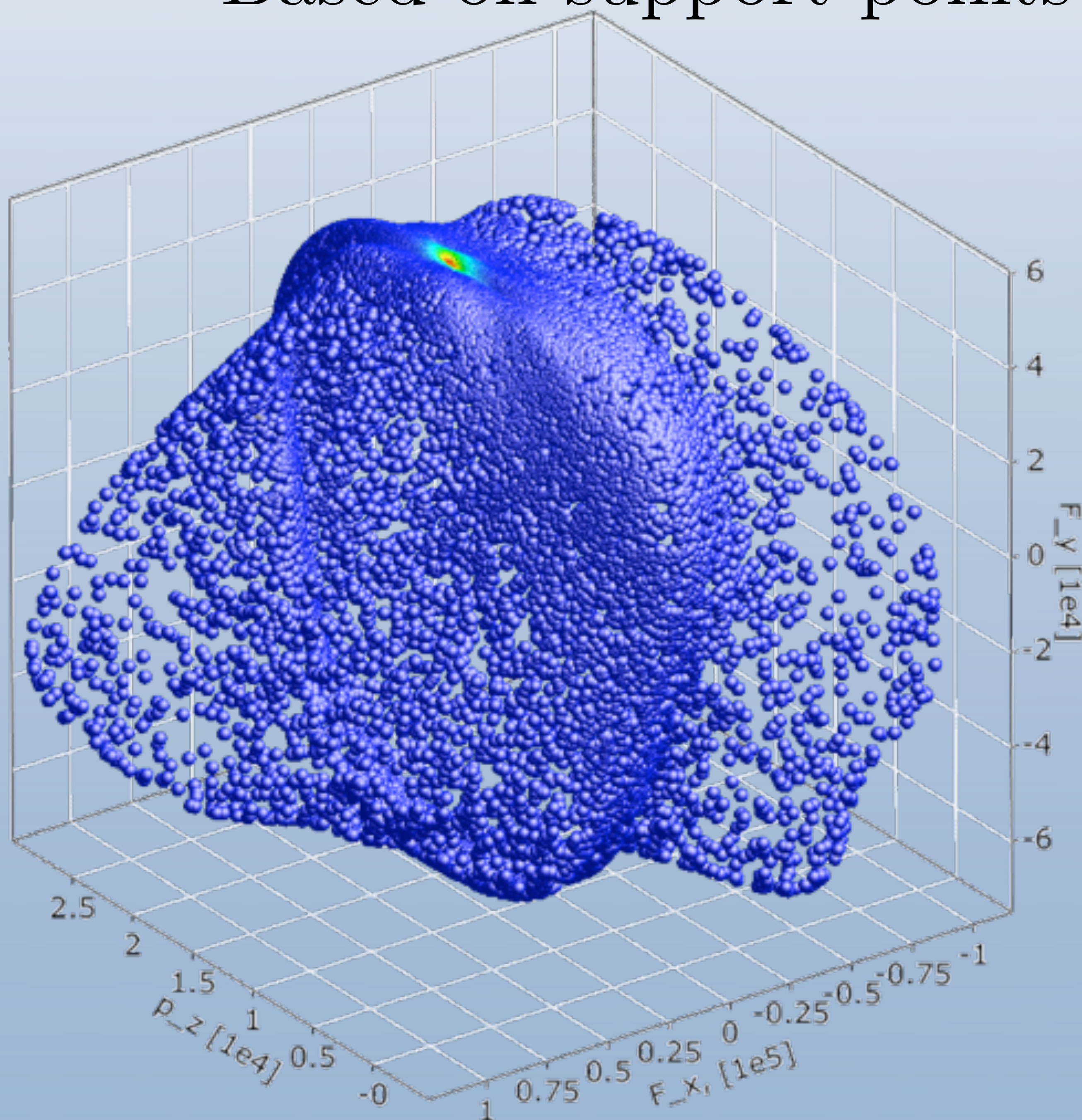
Actual limit state

- From directional sampling with 15.000 points
- Color indicates distance from origin in standard Gaussian space

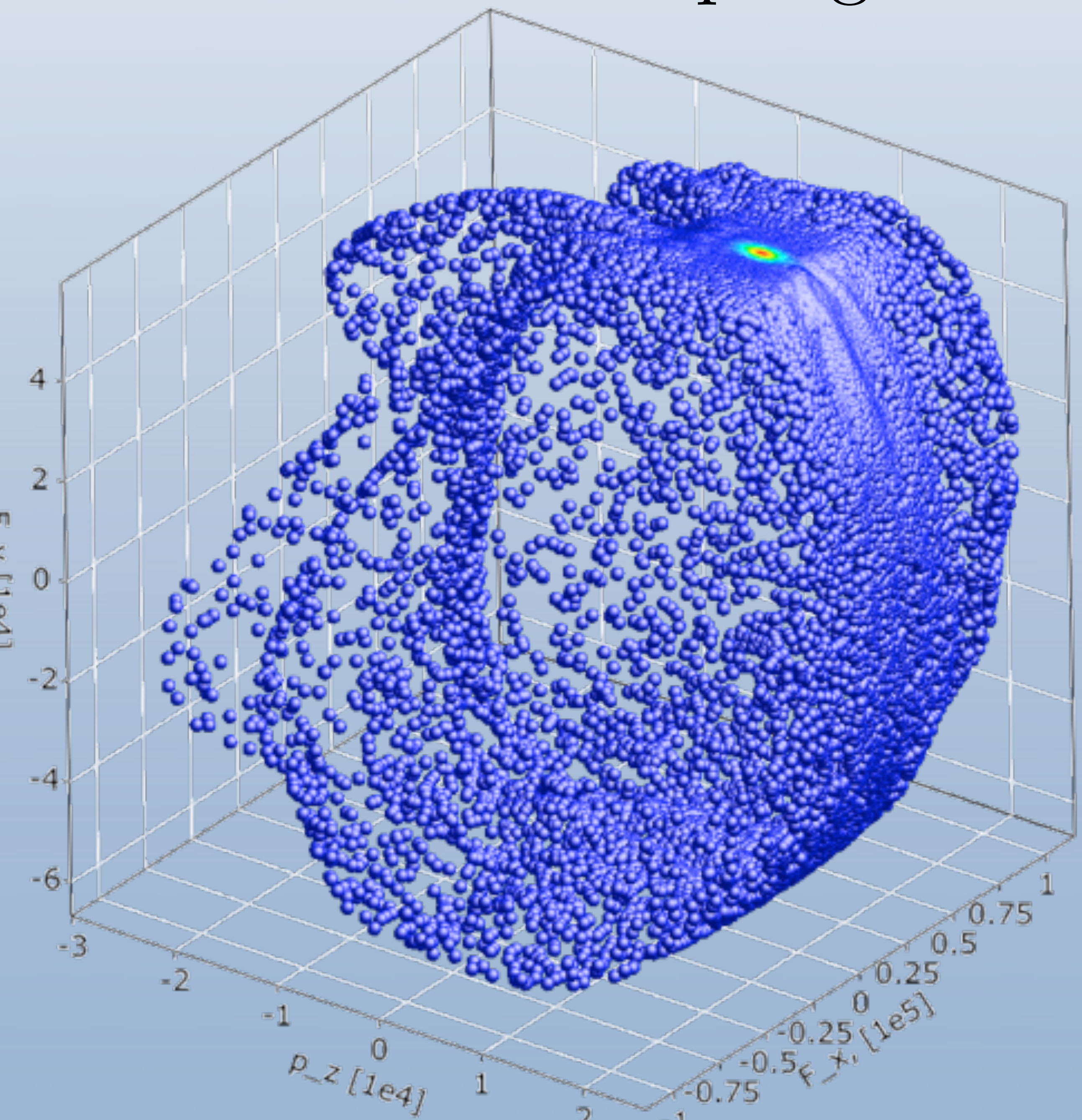


Approximation by MLS

- Based on support points from directional sampling



50 support points



1000 support points

Estimated failure probability

- Approximation results

Method	m	$P(\mathcal{F}) \cdot 10^{-5}$	β
Shepard	50	2.1	4.1
	1000	2.1	4.1
RMLS	50	2.2	4.1
	1000	3.7	4.0
ANN	50	5.9	3.9
	1000	3.3	4.0
Quadratic	10	7.7	3.8

- Reference value (directional sampling, 15.000 samples)

$$P(\mathcal{F}) = 4.3 \cdot 10^{-5}; \quad \beta = 3.9$$

Speedyne: Basic Concept

- Hybrid solution strategy
  - Multi-body approach (Rigid body dynamics)
  - Finite element method (continuum mechanics)
- Explicit time integration
  - Increase critical time step by modal reduction
  - Eliminates high-frequency responses
- Suitable for drop test analysis

History of speedyne development

- 2000-2001: theoretical base of modal projection method, verification with simple examples
- 2002: verification of FE-tire model and comparison with LS-DYNA
- 2003-2006: verification for drop test analysis
- 2006: base of super stable contact algorithms, automatic segment based contact
- 2007-2009: industrial examples are running with super stable contact

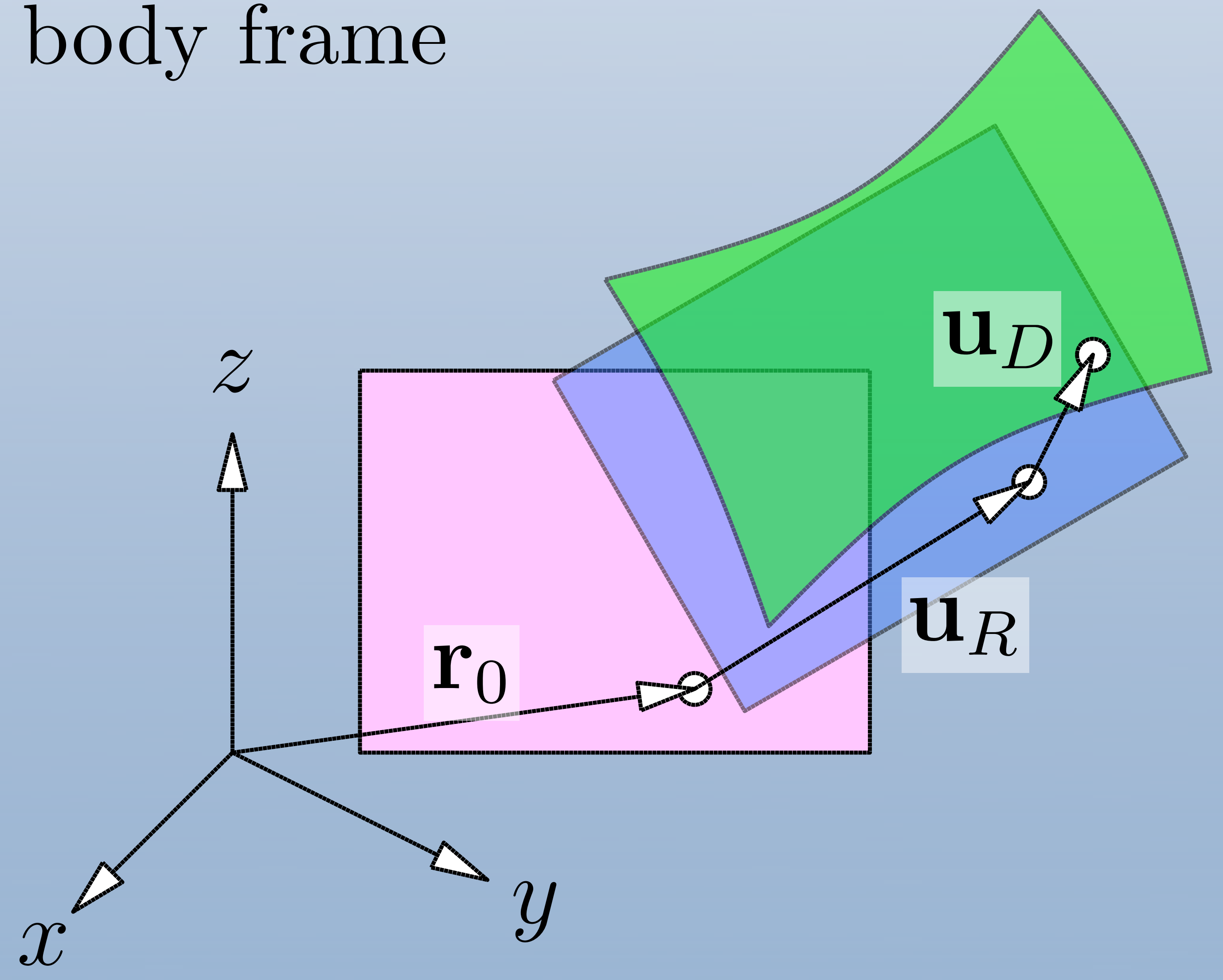
Speedyne for application

- Final product level simulation can based on legacy FE code
- Speedyne can be used for concept level, for optimization and stochastic analysis (reduction of legacy FE model leads to significant speed-up of single simulation)
- Speedyne can be used for long simulation times (e.g. multiple impacts) due to enhanced stability

Basic assumptions

- Additive decomposition of displacement field
  - Rigid body motion in inertial frame
  - Deformation in body frame

$$\mathbf{u} = \mathbf{u}_R + \mathbf{u}_D$$



Range of applicability

- Time step resulting from explicit integration must be significantly smaller than required from physics
- Physics dominated by frequency range covered by relatively large time step
- Contact must not dominate numerical stability - largely not relevant any more due to super-stable contact handling
- Geometrical nonlinearity (tension stiffening) must remain small in order to keep time step large

Super-stable contact

- Apply correction to the velocity field such that momentum and energy are maintained
- Remove/reduce penetrations by appropriately modifying velocities
- Use symplectic integration algorithm to preserve a Hamiltonian close to the total energy (exact for Hamiltonian systems)
- Time step can be kept rather large (depending on relative velocity of colliding parts)

Speedyne - procedure

Initialize kinematics

Compute loads and
restoring forces at $\mathbf{R}_0$, $\mathbf{t}_0$, $\mathbf{u}_0$

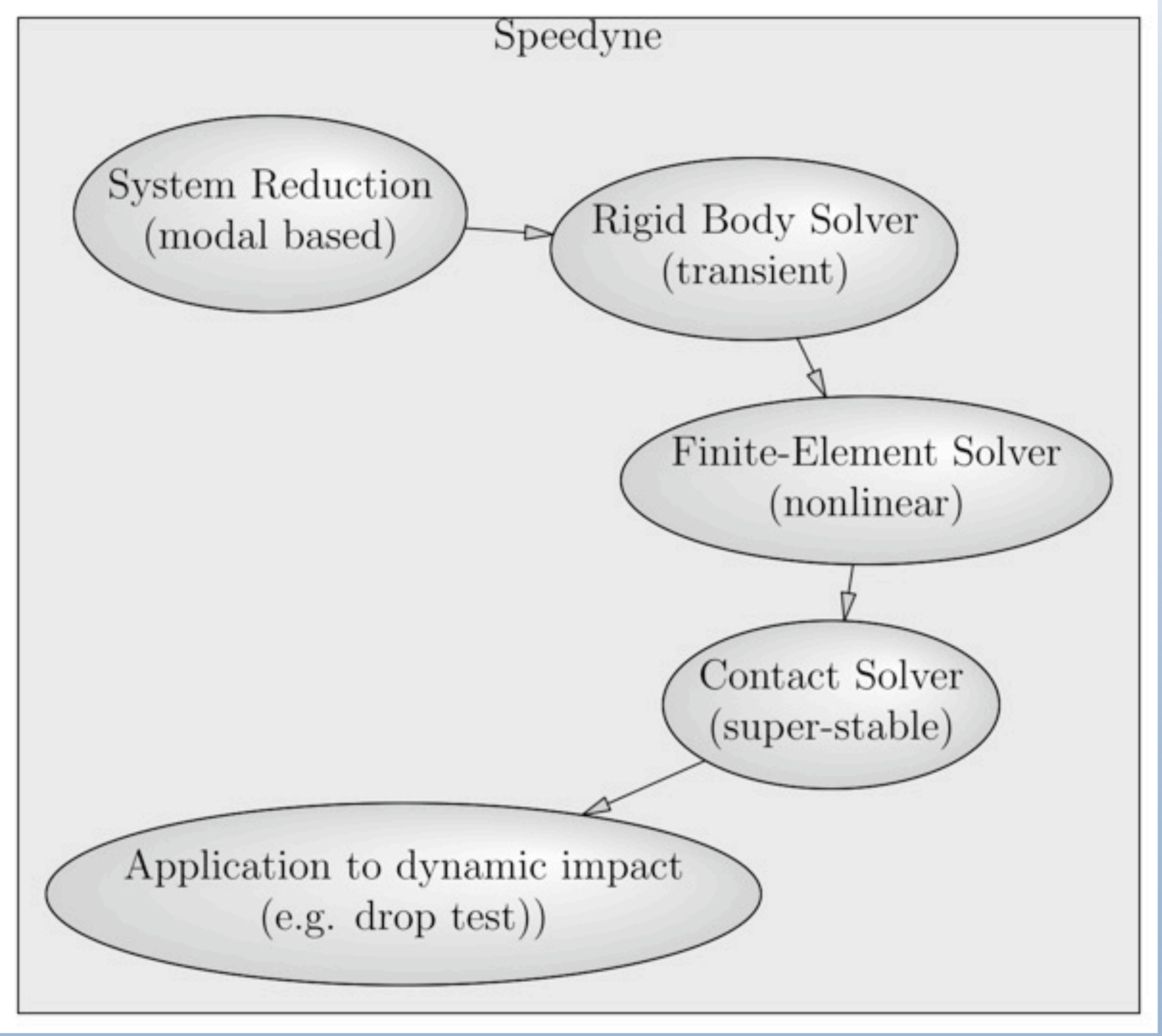
Integrate rigid body
$\mathbf{R}_1, \mathbf{t}_1$

Integrate deformable body
$\mathbf{u}_1$

Check contact, if yes
modify $\mathbf{R}_1, \mathbf{t}_1, \mathbf{u}_1$

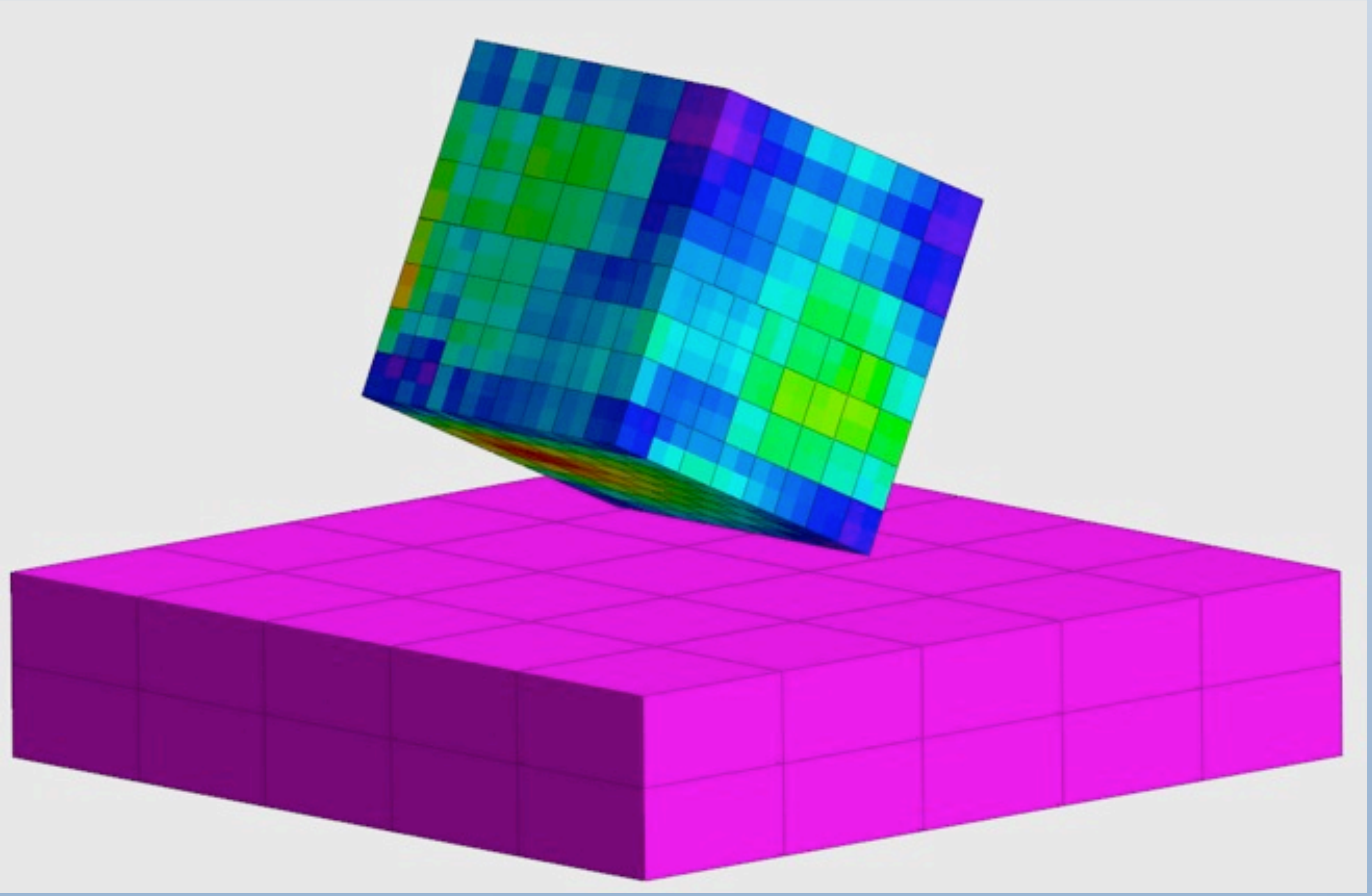
Copy new to old
merge new displacements $\mathbf{u}_1$

Analysis chain

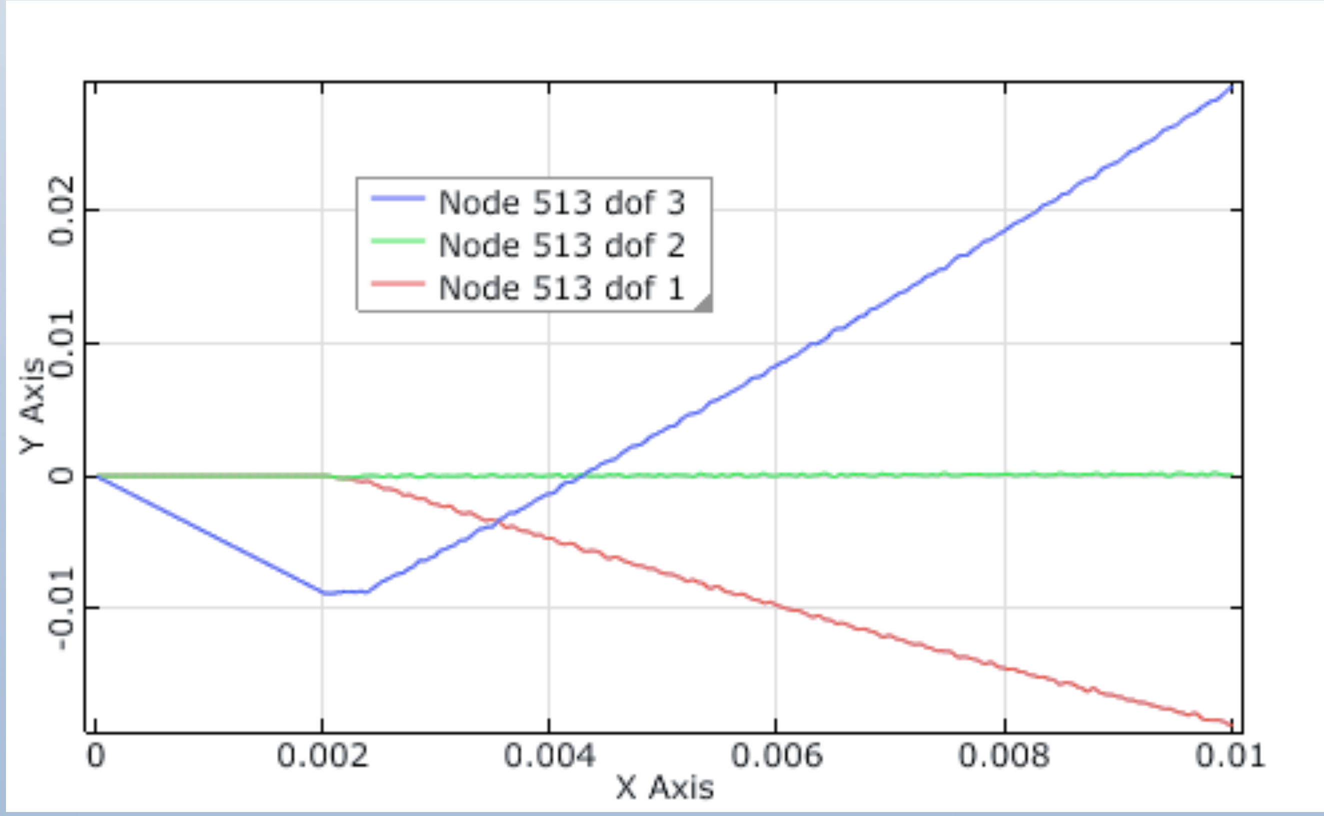


Test example

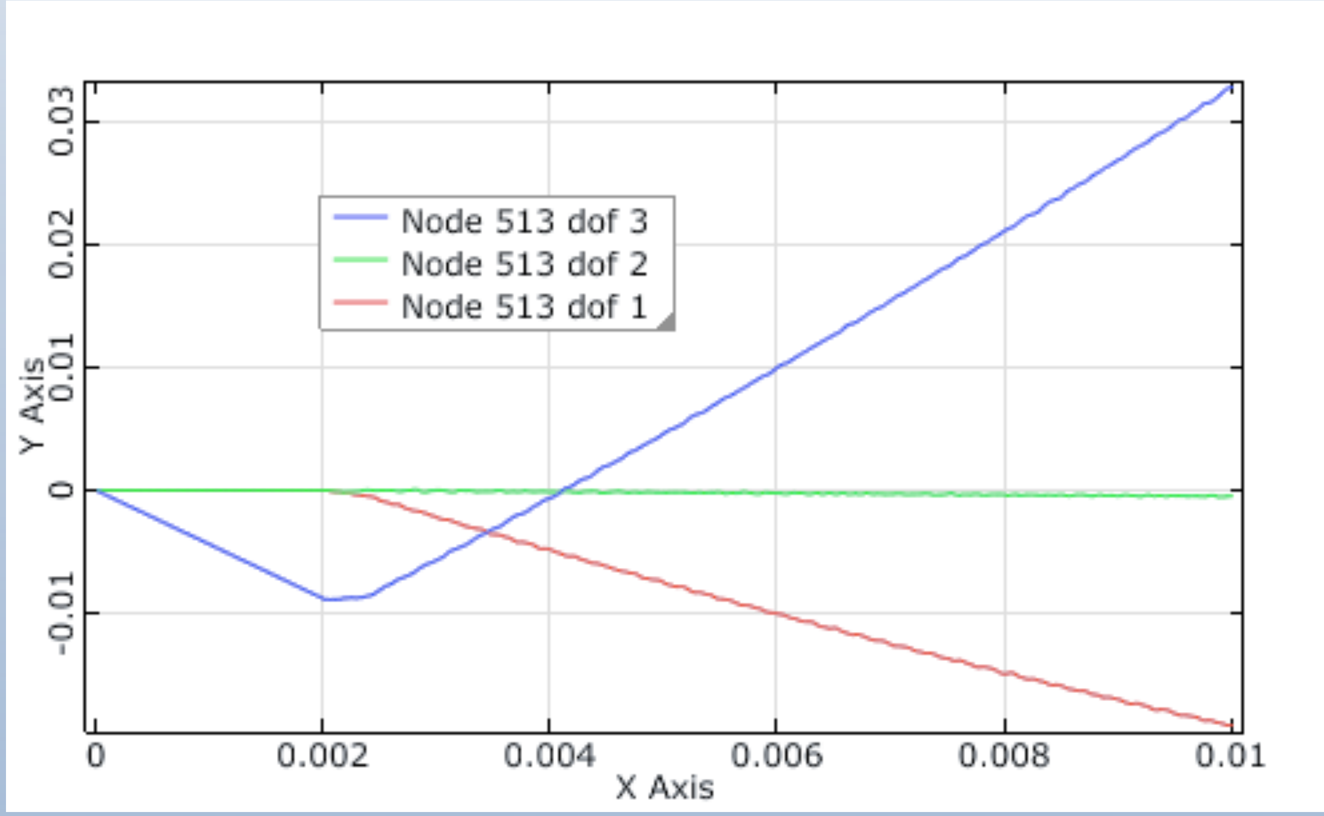
- Simple cube, hexahedral elements, one tie



Full vs. reduced integration

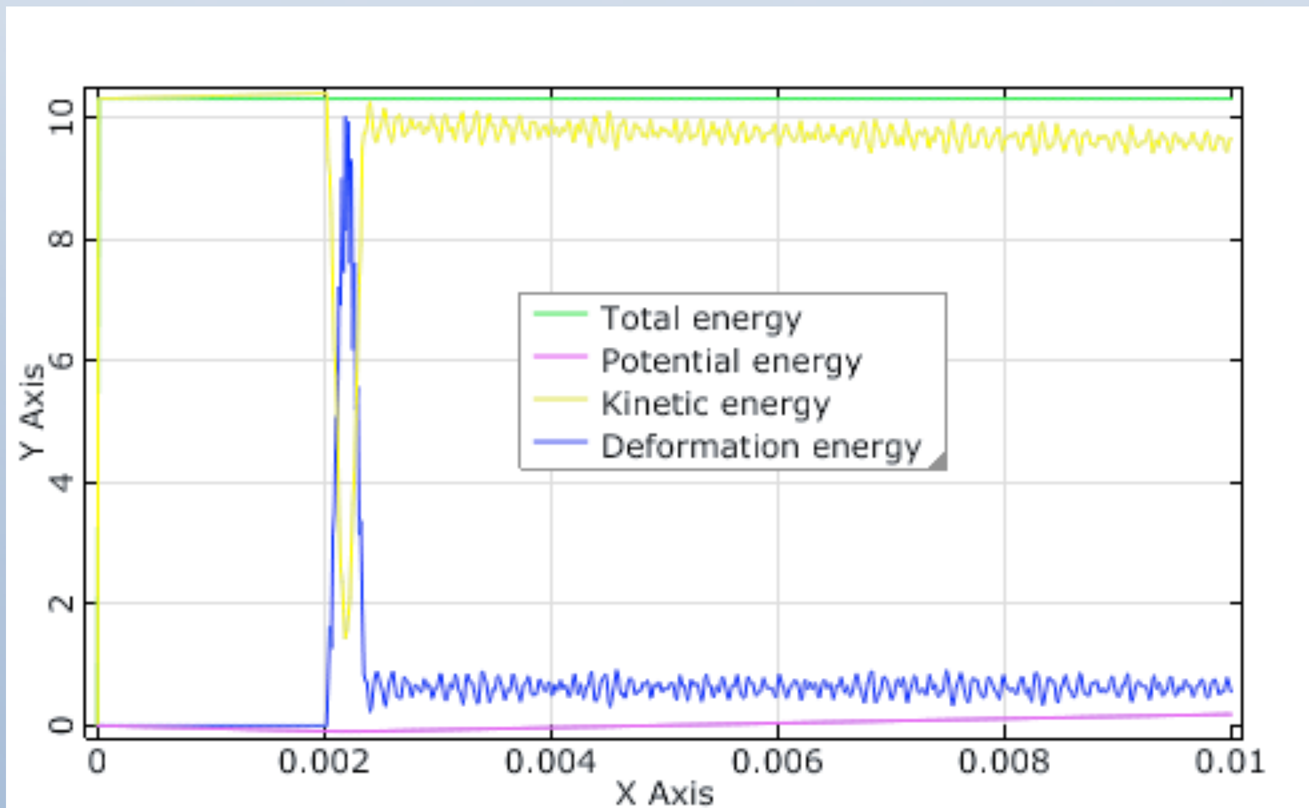


Full

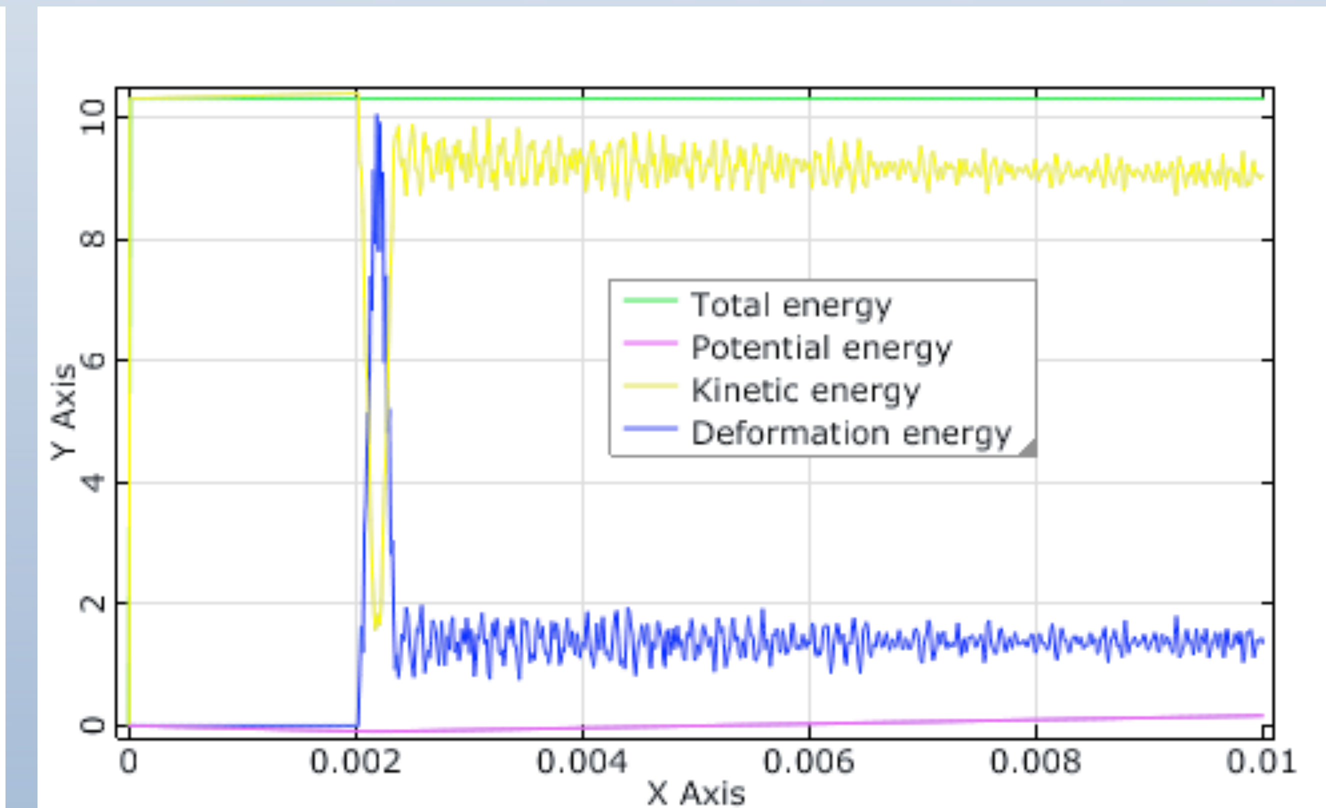


reduced

Full vs. reduced integration



Full



reduced

Export to legacy FE-Solver

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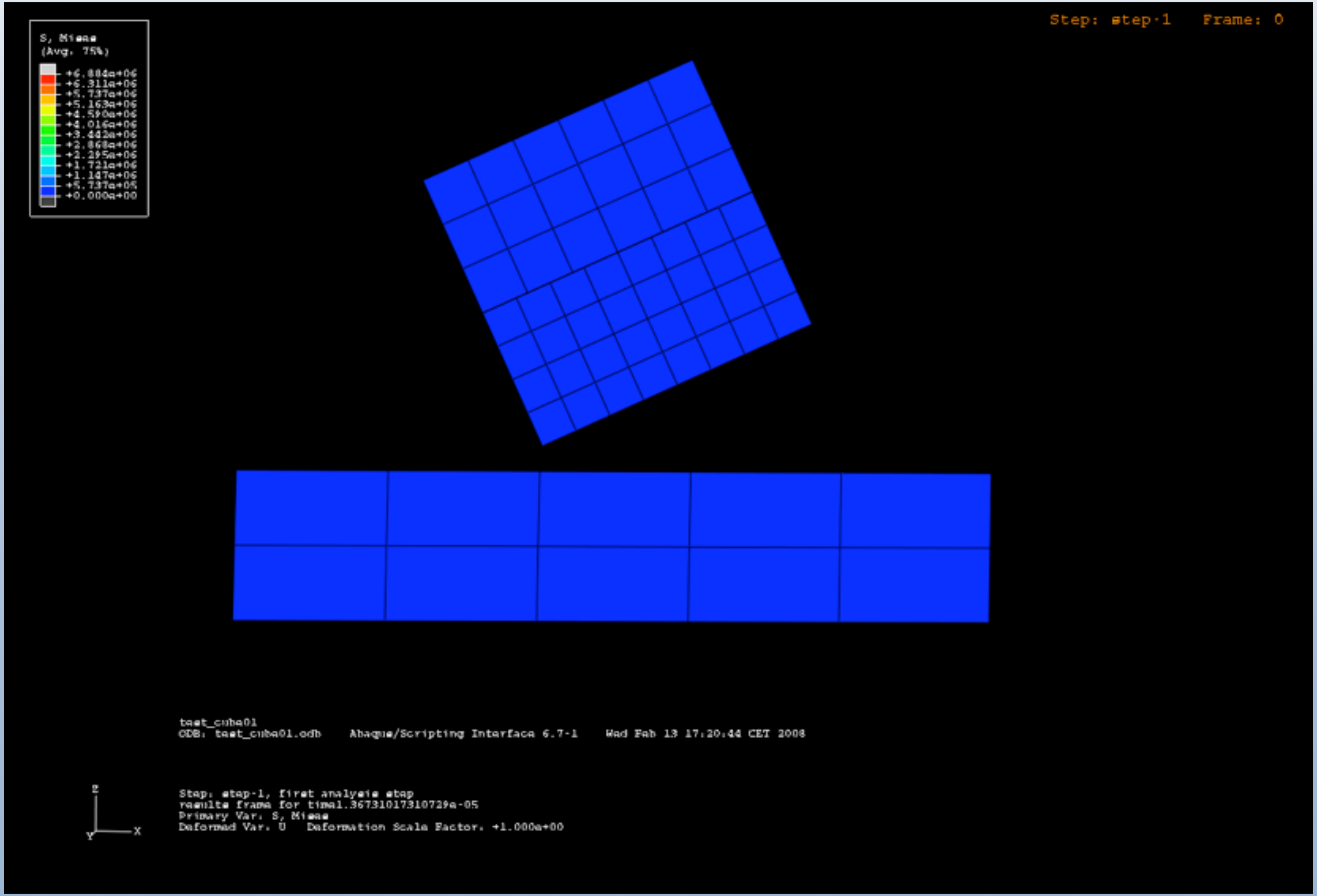

Concluding remarks

- Time-consuming simulation tasks prevent application of optimization and stochastic analysis
- Simulation time can be substantially reduced by
  - reduced order models (based on understanding of physics)
  - Metamodels (based on black-box I/O relations)
- Both approaches have different advantages/ disadvantages
- Combination approach appears promising in order to obtain best results under time constraints
- New development under way: optiSpeed