Sampling based sensitivity analysis: a case study in aerospace engineering

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Introduction

- **Issue:** Understanding the behavior of engineering structures near the limit state, especially buckling of light weight shell structures
- **Point of view:** Absolute values of single indicators (e.g. knock down factors) or failure probabilities give little insight in the mechanisms of collapse
- **Remedy:** Better understanding by means of sensitivity analysis
- **Situation:** Numerical model is given by an Input-Output map

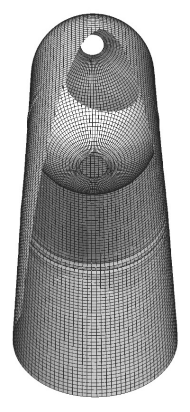
$$Y = g(X_1, \ldots, X_d)$$

where the indicator Y is a function of the design parameters $X_1, \ldots, X_d$
- **Goal:** Sensitivity analysis of output with respect to input

DATA, INPUT – OUTPUT

- **Buckling load:** depending on loads, material, geometry (currently 17 – 130 input variables)
- **Flight critical behavior:** investigation of (currently) $\geq$ 136 output variables
- **Challenges:** excessive computational cost
- IO-map continuous but non-differentiable
- Statistical distribution of input parameters unknown (nonparametric methods required)
- Scarce data: flight scenarios supplied by the *architect*
- **Method of choice:** Monte Carlo simulation, statistical indicators of dependence

Example: original 17 variables



	Description of input parameters	
i	Parameter X_i	Mean μ_i
1	Initial temperature	293 K
2	Step1 thermal loading cylinder1	450 K
3	Step1 thermal loading cylinder2	350 K
4	Step1 thermal loading cylinder3	150 K
5	Step1 thermal loading sphere1	150 K
6	Step1 thermal loading sphere2	110 K
7	Step2 hydrostatic pressure cylinder3	0.4 MPa
8	Step2 hydrostatic pressure sphere1	0.4 MPa
9	Step2 hydrostatic pressure sphere2	0.4 MPa
10	Step3 aerodynamic pressure	−0.05 MPa
11	Step4 booster loads y-direction node1	40000 N
12	Step4 booster loads y-direction node2	20000 N
13	Step4 booster loads z-direction node1	3.e6 N
14	Step4 booster loads z-direction node2	1.e6 N
15	Step4 mechanical loads x-direction	100 N
16	Step4 mechanical loads y-direction	50 N
17	Step4 mechanical loads z-direction	300 N

Input Data – Simplified Model

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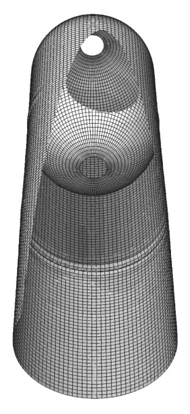

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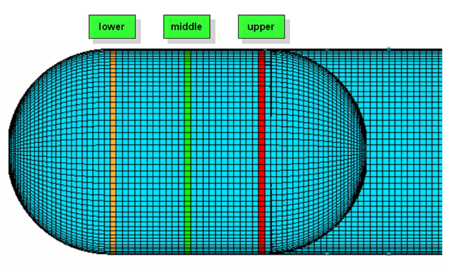
Input:

Uniform distributions with spread $\pm 15\%$ of nominal value

Description of Output Variables

Output measured at 3 rings (100 finite elements each) and aggregated:



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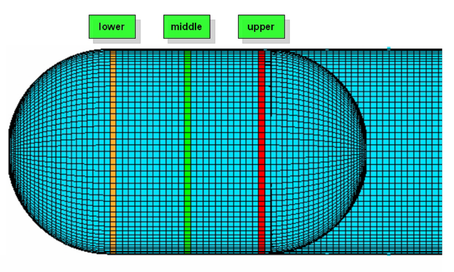


Typical output variables:

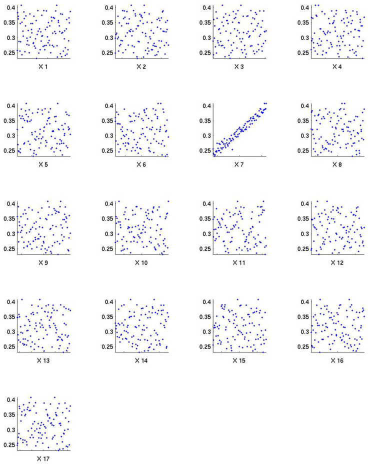
Local variables:

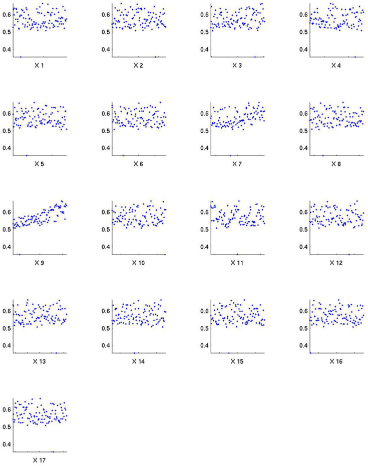
Translations, rotations and stresses, plastic/logarithmic strains, elastic/plastic strain energy densities ...

Global variables:

Summarized elastic/plastic strain energy densities, eigenvalues of the stiffness matrix ...

Input vs. elastic strain energy density: ring 2 min (left) – max (right)





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Remedy: Statistical indicators that remove these effects.

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Indicators in use (X_i vs. Y):

- CC – Pearson correlation coefficient;
- PCC – partial correlation coefficient;
- SRC – standardized regression coefficient;
- RCC – Spearman rank correlation coefficient;
- PRCC – partial rank correlation coefficient;
- SRRC – standardized rank regression coefficient.

EXAMPLE – PCCs AND PRCCs:

1. For each input variable X_i, construct linear regression models

$$\widehat{X}_i = \alpha_0 + \sum_{j \neq i} \alpha_j X_j, \quad \widehat{Y} = \beta_0 + \sum_{j \neq i} \beta_j X_j.$$

2. Compute the residuals

$$e_{X_i \cdot X_{\smallsetminus i}} = X_i - \widehat{X}_i, \quad e_{Y \cdot X_{\smallsetminus i}} = Y - \widehat{Y}.$$

3. Compute the correlation coefficient of the residuals

$$\textbf{PCC}: \qquad \rho_{X_i, Y \cdot X_{\smallsetminus i}} = \rho(e_{X_i \cdot X_{\smallsetminus i}}, e_{Y \cdot X_{\smallsetminus i}}).$$

4. Apply procedure to ranks in place of raw data – **PRCC**.

5. Assessment of significance: Resampling of computed data gives bootstrap confidence intervals at no additional cost.

CCs, RCCs, PCCs, PRCCs, SRCs, SRRCs

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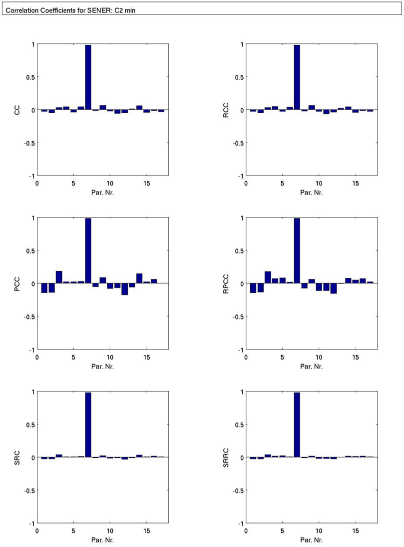

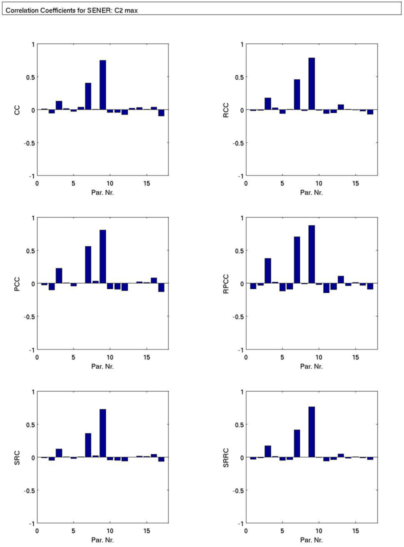

95%-Bootstrap Confidence Intervals

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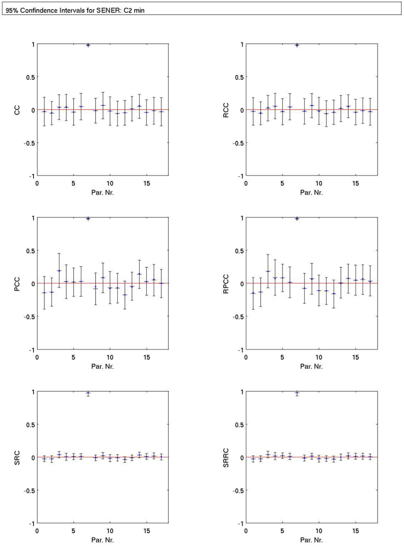

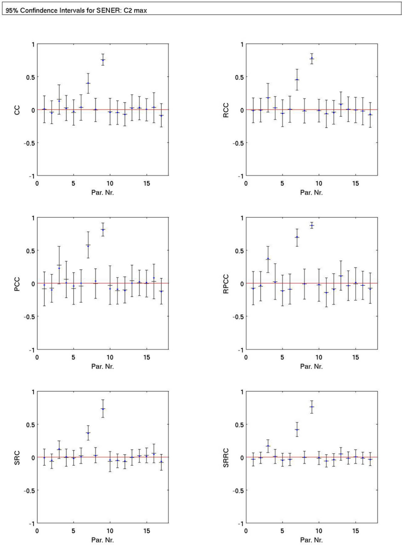

Evaluation

- Selection of variables according to significance in at least one criterion;
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Extension (work in progress):

Modelling of material parameters by stochastic field:
Widely used autocovariance function

$$C(\rho) = \sigma^2 \exp(-|\rho|/\ell) \qquad \text{at spatial lag} \quad \rho.$$

Questions:
– Change of sensitivities when material is stochastic;
– dependence of sensitivities on field parameters σ and ℓ;
– sensitivity of output with respect to field parameters σ and ℓ.

Idea: Perform Monte Carlo parameter variation not with initial values, but at a later stage in the iterations.
Linear equations: Solutions for neighboring systems obtained by a splitting of the stiffness-matrix.

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– do p % of required iteration with average values of input parameters;
– perform Monte Carlo variation of input parameters;
– obtain equilibrium state for each MC realization of parameters;
– perform remaining (1-p) % of iterations.

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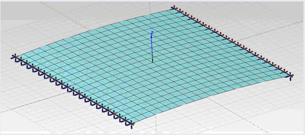
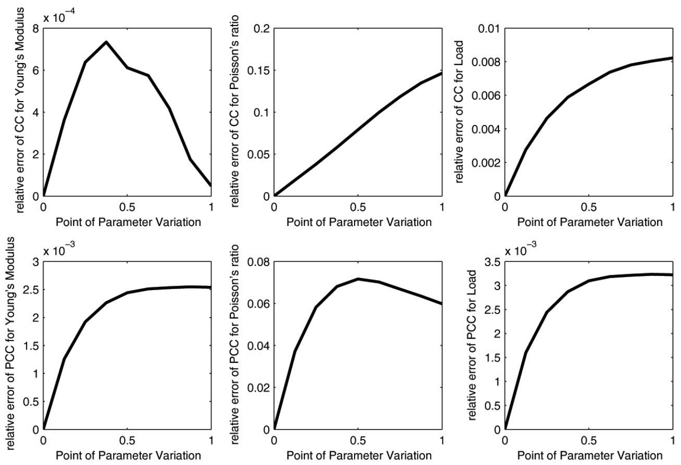
Example: Thin cylindrical roof, loaded with a central point-force.
Output: central displacement;
Input: Young's modulus, Poisson's ratio, load.



Results

Relative error in CCs and PCCs vs. point of parameter variation; 0 ...variation at initial step, 1 ...variation at final step.



Outlook

- Understanding the reliability of structures by means of sensitivity analysis
- Powerful tool: Monte Carlo simulation plus statistical indicators
- Fairly low computational cost: relatively small sample size suffices
- Assessment of statistical significance by resampling at no additional cost
- Non-parametric method
- Wide range of applicability