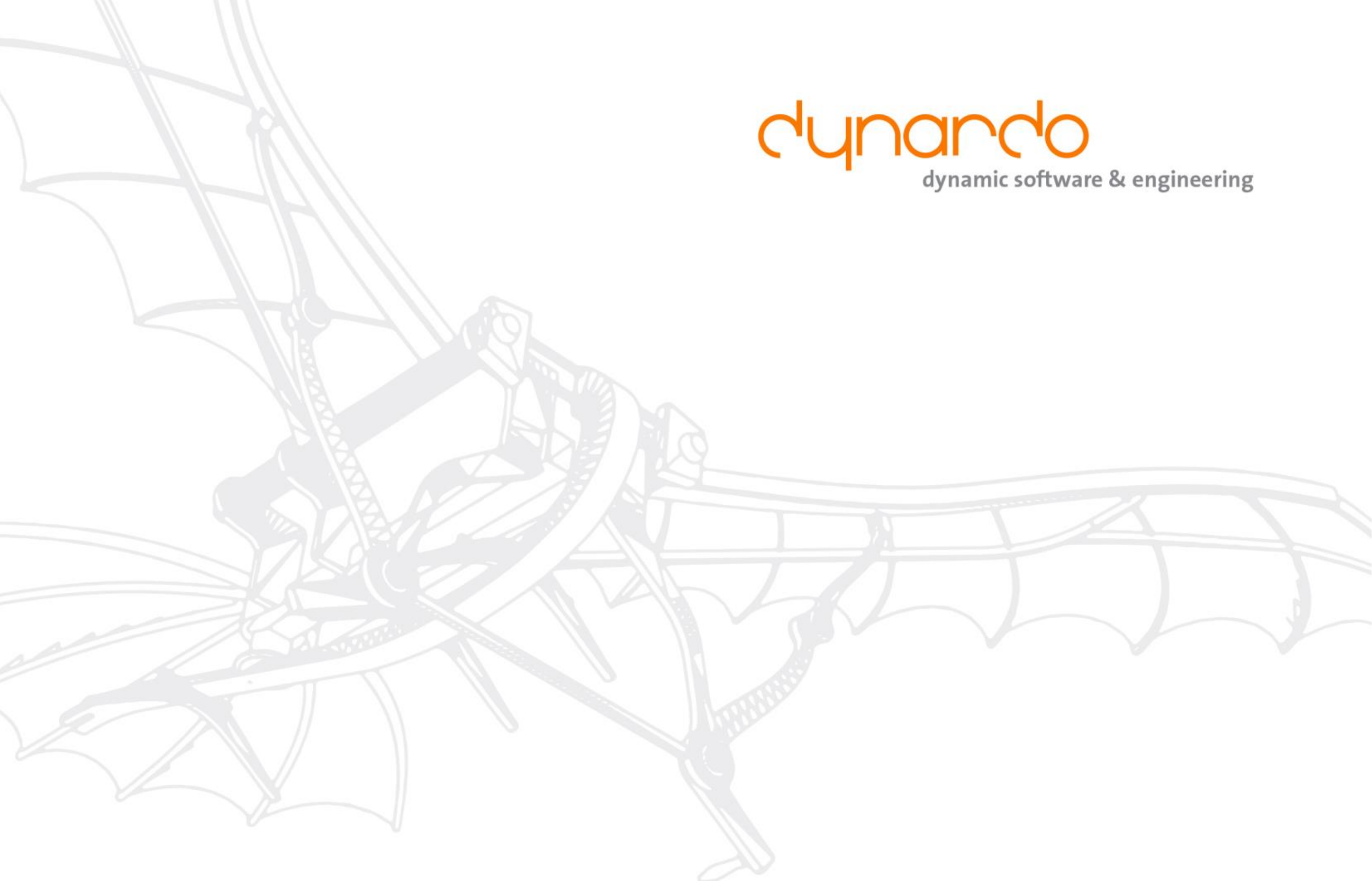

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Lectures

Random Fields in Statistics on Structure

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presented at the Weimar Optimization and Stochastic Days 2009
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Random Fields in Statistics on Structure

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Abstract

Properties of engineering structures or structural parts are usually of random nature, due to manufacturing tolerances, material scatter or random loads. For the numerical robustness assessment within the virtual product development, such randomness has to be taken into account, by applying correct statistical modeling. Processes like manufacturing simulation with random parameters or analyses with random load cases induce random results which are distributed on the examined structure. For the analysis of such spatially random phenomena, as well as for the simulation of imperfect structures, random field methodology provides the correct, moreover effective parametric, by which the most significant information can be filtrated from the data. Statistics on Structure (SoS) offered by dynardo is a software for the analysis of given random data. The present article covers a brief theoretical background of random fields, an overview on SoS and demonstrates the analysis of random data by an example from crash simulation.

Keywords: random fields, robustness, reliablity, crash simulation

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1 Introduction

In the development of products, e.g. in the automotive branch or for high-level consumer goods, two important trends can be observed. On one hand, more and more structural parts or even entire structures are optimized by numerical methods. On the other hand, hardware tests for experimental qualification of these parts are reduced and, at least partially, are substituted by methods of virtual prototyping. Since requirements on structures nowadays often reach physical limits, optimized structures tend to react sensitive towards manufacturing tolerances, material scatter or varying load conditions, if these were not considered in the development phase. Hence for the assurance of product quality, avoidance of recalls and fulfillment of safety requirements, an optimization must be accompanied by a robustness and reliability assessment. For this purpose, random influences are generated by Monte Carlo methods and the resulting, then also random, performance is assessed by statistical means (Bucher, 2007; Will, 2007).

Processes like manufacturing simulation with random parameters or analyses with random load cases induce random results which are spatially distributed on the examined structure. Pointwise evaluation of the results, i.e. the search for maximal deformations or stresses, does not make use of the information inherent in the data and may even lead to misinterpretation if the localization is not tracked. Random field methodology provides the correct parametric for obtaining dependable results from the analysis of such spatially random phenomena. Application of random fields for the analysis of given data on a structure, e.g. results of finite element computations with random inputs, provides several levels of insight: First, the distribution of scatter on the structure is observed and hot spots are located. Next, random field data can be decomposed into scatter shapes, which can be ranked by their contribution to the total scatter and reflect the "mechanisms" of the random phenomenon observed. Further statistical analysis, mainly by means of Coefficients of Determination and Coefficients of Prognosis (Most and Will, 2009) allow for a ranking of the influence of random inputs on single scatter shapes. In other words, it can be seen where on the structure appears the highest scatter of results caused by which input parameter.

The following section gives an overview on "Statistics on Structure" (SoS), a software offered by dynardo for statistical assessment and post-processing of random data, which are plotted directly on the structural model. The program applies random field methodology for reducing data and filtrate the most relevant information. Thus section 3 explains the theoretical background, which is helpful for interpretation of the results. The mentioned evaluations are demonstrated by an application example from robustness analysis of a structural car part, section 4.

2 Statistics on Structure

Statistics on Structure (SoS) offered by dynardo is a software for post-processing and analysis of given random data on structures. Typical applications are:

- measurements on the structure, e.g. deviations from the designed geometry, or wear;
- results of structural analyses with random input parameters, e.g. manufacturing process simulation, random loads.

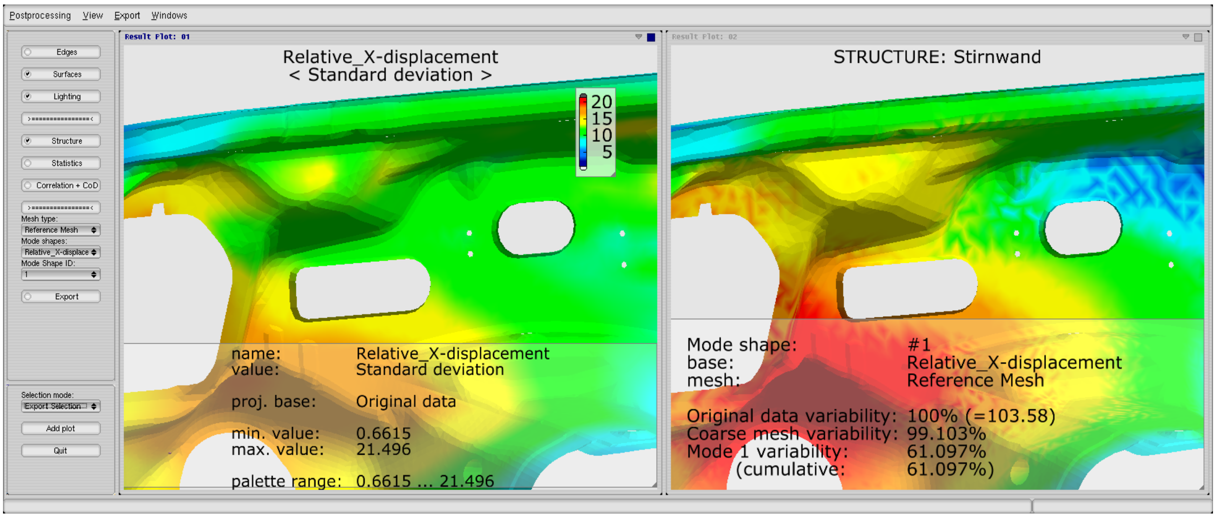

Figure 1: Example of SoS post-processing.

In the latter case, results are available for each node or element of the analysed part and can be related to the random input parameters. Hence, review of results from manufacturing process simulation enables the assessment of manufacturing tolerances at any point on the structure. The statistical relation to the random inputs helps to identify the cause of tolerances. A structural analysis with random inputs – random parameters and loads are as well possible as the tolerances computed beforehand – provides measures of the robustness of the structure. Besides assessment of the global performance, post-processing of analysis results on the strucure with SoS locates the "hot spots", as will be demonstrated in section 4. Such simulations are important steps within robustness assessment of structural parts.

The typical set-up of an SoS application begins with loading a reference finite element structure. Several interfaces for established FE programs are available. Next, a sample result file is read, then results are chosen for post-processing and all inputs and results generated within a robustness analysis are loaded. Typically but not necessarily these data are produced with optiSLang. The next step comprises data reduction and filtration as will be explained in sections 3.2 and 3.3.

Then the post-processing step itself is started. Figure 1 shows the post-processing window. Several sub-windows can be viewed in parallel. The post-processing options comprise:

- Descriptive statistics: single designs and design differences, mean values, standard devations, ranges, maxima and quantiles.
- Correlation and Coefficient of Determination with respect to input parameters.
- Quality capability statistics.
- "Eroded" nodes or elements: location and relative number within the sample of nodes or elements that failed during the previous analyses.
- For the above evaluations, the data are decomposed by help of random field methodology, sect. 3. The resulting scatter shapes of the single random field components can be plotted, which is a good help for interpretation of the scatter.

- It is possible to select regions on the structure or amplitudes of scatter shapes. The respective data are exported for further statistical analysis within optiSLang.

3 Random Fields

3.1 Basics

In simple words, a random field is a random function defined in space. This may be a function of any geometrical or physical property, such as material parameters, deviation from the design geometry, distributed load etc. The domain of the function is given by the structure or structural part under obervation. That is, on any point on the structure, this property takes a random value. The statistical characteristics may differ at different locations, and there may exist a statistical dependency between different points.

Let the random function be denoted as $H(x, \mathbf{r})$, wherein $x \in \mathbb{R}$ is the random property and $\mathbf{r} \in \mathbb{R}^3$ is the local vector pointing to any location on the structure. Figure 2 shows a schematic sketch of a random function, defined on a beam structure. Several *realizations* of the function form the available sample set, the *ensemble*. At any point located by $\mathbf{r}_i$, a random variable X_i can be defined by distribution type and statistical moments (mean value μ_i, standard deviation σ_i) which are evaluated from the given ensemble. Two random variables at different locations, X_1 and X_2, may be statistically dependent. A measure for dependency is Pearson's correlation coeficient, $\rho_{12} = \mathrm{cov}[X_1, X_2]/(\sigma_1\sigma_2)$. Obviously, the correlation between two neighboured points is close to one and diminishes with increasing distance between the two points. For the continuous random function H, the spatial spread of the mean values is characterized by the mean function

$$\mu_H(\mathbf{r}) = \mathrm{E}[H(\mathbf{r})] \tag{1}$$

and the scatter as well as the correlations by the covariance function

$$C_{HH}(\mathbf{r}_1, \mathbf{r}_2) = \mathrm{E}[H(\mathbf{r}_1) \cdot H(\mathbf{r}_2)] - \mu_H(\mathbf{r}_1) \cdot \mu_H(\mathbf{r}_2) \tag{2}$$

Assumed a Gaussian distribution throughout the whole structure, this provides the complete statistical characterization (Vanmarcke, 1983).

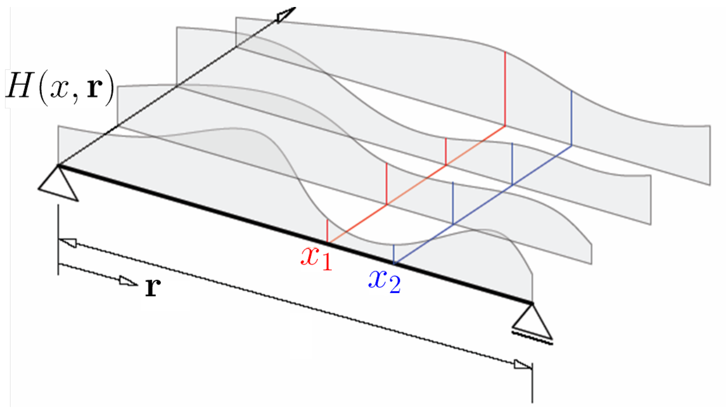

Figure 2: Schematic sketch of realizations of a random field, defined on a one-dimensional structure.

For numerical computations, the random function has to be discretized in order to obtain a finite number of random variables. The discretization is oriented at the observed finite element structure and the type of data, i.e. the discrete support points are either nodes (e.g. for geometry or displacement data) or element mid points (for element data such as stresses, strains, etc). As consequence of the discretization, the random function becomes a random vector $\mathbf{X}(\mathbf{r}) = \{X_i(\mathbf{r}_i)\}$, characterized by the mean vector $\boldsymbol{\mu}_{X_i}$ and covariance matrix $\mathbf{C}_{X_iX_j}$. Each member of random vector is associated to a point on the structure, accordingly for the mean vector and covariance matrix.

3.2 Decomposition

By the following derivation, a parametrization of the random field is found, which consists of a set of independent random variables. This is required for the simulation, i.e. artificial generation of samples, but also provides an effective parametric which is helpful in the analysis of given data in order to filtrate the most significant phenomena. The latter application will be demonstrated in section 4.

Let the mean values be constantly zero. They can as well be subtracted from the data and added later when necessary. The mean-free data are denoted $\mathbf{X}_0$. Then the covariance matrix contains all information of the random field, assumed a Gaussian distribution. With help of an eigenvalue decomposition of the covariance matrix

$$\boldsymbol{\Psi}^T \mathbf{C}_{XX} \boldsymbol{\Psi} = \mathrm{diag}\{\lambda_i\} \tag{3}$$

a set of independent random variables $\mathbf{Y}$ with Gaussian distribution and standard deviations obtained from the eigenvalues of the covariance matrix are defined as

$$Y_i := \mathcal{N}(0\,; \sqrt{\lambda_i}\,; \rho_{i \neq j} = 0) \tag{4}$$

It can be shown that the following transformation between the basic variables $\mathbf{Y}$ and the "real world" variables $\mathbf{X}_0$ holds:

$$\mathbf{X}_0 = \boldsymbol{\Psi}\mathbf{Y} \tag{5}$$

The above is called *Karhunen - Loève series expansion* (Ghanem and Spanos, 1991). The random field of possibly dependent variables is expanded by a series of deterministic shape functions (the eigenvectors of $\mathbf{C}_{XX}$), each scaled by independent random amplitudes. This is the way to simulate a random field. The transformation is reversed in order to decompose a given data set and compute the respective parametric:

$$\mathbf{Y} = \boldsymbol{\Psi}^T \mathbf{X}_0 \tag{6}$$

3.3 Data Reduction

If the data are located at every node or element of a finite element structure, regarding the fact that a covariance matrix is fully occupied, the eigenvalue decomposition is not tractable. In SoS (cf. section 2) there is the option of mesh coarsening, by which the number of random field supports and therefore the dimension of the covariance matrix can be reduced.

Mesh coarsening is performed by an effective polygon reduction algorithm which keeps the mesh topology, i.e. the relative refinement of the discretization. Data are transferred

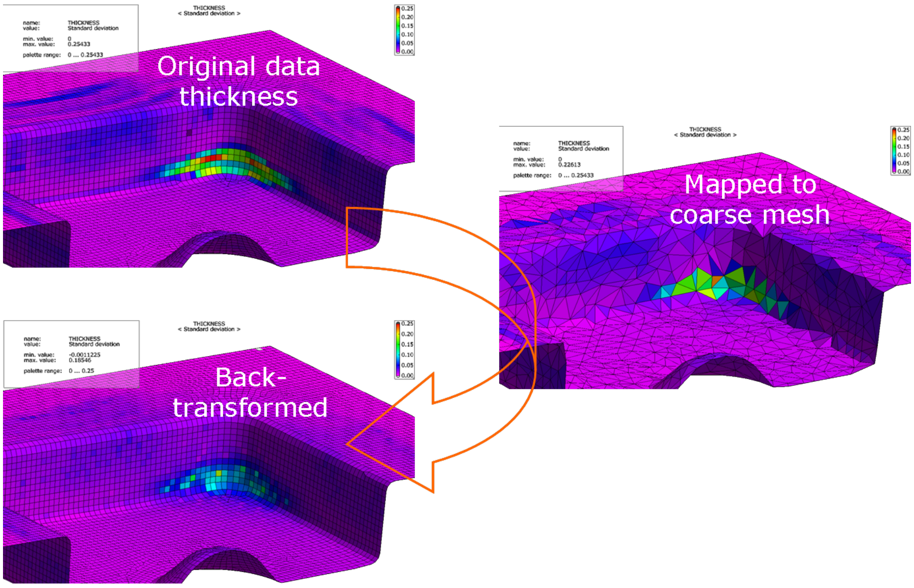

Figure 3: Mesh coarsening and data interpolation for sheet thickness in a metal forming example.

from the original supports to the coarse mesh by distance-weighted local averaging. The number of random variables, i.e. the dimension of the random vector $\mathbf{X}$, equals the number of supports of the coarse mesh. After the decomposition of the coarsened random field and further analyses, results are mapped back for post-processing. Here, a Moving Least Squares Regression (Most and Bucher, 2005) is used.

The effect of mesh coarsening is studied at a fictitious example of a sheet metal forming simulation. Figure 3 shows in the upper left plot the sheet thickness after forming simulation, given at each finite element. Using the same color palette, the data mapped onto the extremely coarsened mesh already lacks the peaks (right plot). After mapping back the data (lower left), the smoothing effect can be observed by comparison to the original data set. One has to keep in mind that this kind of data reduction is a loss of information, however, the smoothing can be desirable for noisy data.

The Karhunen - Loève expansion, eq. (5) ff., can be truncated for further reduction of the dimension. The eigenvalues of the covariance matrix are sorted in descending order. Since the eigenvalues define the variances of the basic variables Y_i, only the variables with highest variances are incorporated in the truncated series. Variables with only a small contribution to the total scatter of the random field are neglected. The truncation error is measured as the variability fraction (Brenner, 1995)

$$Q = \frac{\sum\limits_{i=1}^{N_\lambda} \lambda_i \Big/ \dim(\mathbf{C}_{XX})}{\sum\limits_{j=1}^{N_\text{supports}} \sigma_j^2 \Big/ N_\text{supports}} \tag{7}$$

The numerator is the sum of variances of the considered basic variables after truncation, the denominator is the sum of all variances in the data set, which may have been reduced before by mesh coarsening. Thus both numerator and denominator are normalized to the dimension of the underlying mesh. Typically only a small number of basic variables suffices to represent over 90% of the total variability, as will be shown in the example, sec. 4.

In a similar way, the contribution of a single variable to the total scatter can be calculated as

$$Q = \frac{\lambda_i \Big/ \dim(\mathbf{C}_{XX})}{\sum\limits_{j=1}^{N_{\text{supports}}} \sigma_j^2 \Big/ N_{\text{supports}}} \tag{8}$$

Because each variable is the random scale of one shape function, this single contribution is an important measure for analysis of the decomposed random field. An important shape, that is one with high variance, often reveals the "mechanism" that causes the random scatter on the structure. The statistical relation of the sample of amplitudes computed by (6) to the random input parameters gives further insight into the causes of scatter.

4 Example from crash analysis

The application presented here is a re-analysis of a project published in Will and Frank (2008), which exploits new features of the current SoS release. The performance of a load bearing part of a car body subjected to a crash load case is studied within a robustness evaluation. Hardware tests of an early stage of development showed plastic buckling, which could not be reproduced by deterministic methods of virtual product development.

For the stochastic analysis, the forming processes of several structural parts within the load path were simulated with random production parameters. The yield strength was assumed to vary within the bounds of purchase tolerances. The resulting sheet thicknesses and plastic deformations were modeled by random field parametric and introduced to the crash simulation. Also load parameters such as velocity, barrier position and angle, as well as friction between barrier and car and within the vehicle itself were considered as random. For further details of modeling the inputs, see Will and Frank (2008). SoS helped to analyse the causes of scattering plastic strains and made possible a redesign to remedy this unwanted behaviour. The present study shall identify the most relevant scatter shape of the random field and find the responsible input parameters.

The finite element model comprises 4914 nodes and 4826 shell elements. 150 samples with the mentioned random parameters were generated and put into crash simulation. The result which is studied in more detail is the plastic strain after crash. Figure 4 shows the standard deviations evaluated from all samples. Gray shaded areas did not vary, thus no plastic deformation occured. At about on third from the left, a characteristic area of high scatter resembles the buckling phenomenon.

The sample of plastic strains is decomposed into shape functions and random amplitudes by the methods explained in section 3. Figure 5 shows the three shapes with highest variances. The first shape already covers a large part of the plastic strain characteristics seen in fig. 4, the following shapes add "side effects". Note that the mode shapes of the covariance matrix are unit vectors, so palette colors are not comparable. The truncated series made up of these shapes (cf. eq. 5) and respective amplitudes represents 98% of the total scatter of the plastic strain. This demonstrates the effectiveness of the random field parametric proposed here: The original data set which is 100% of variability consists of one value per finite element, thus almost 5000 random variables in total.

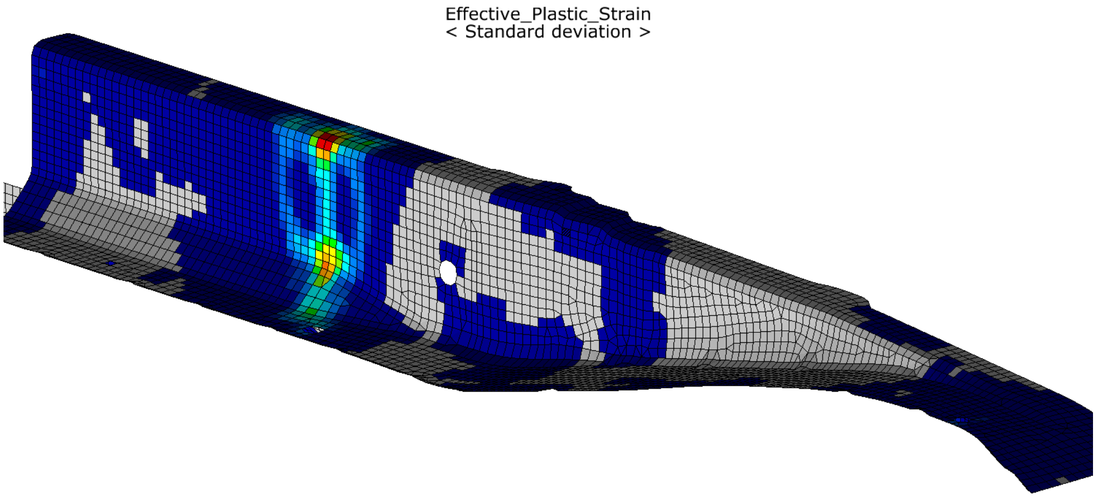

Figure 4: Load-bearing car part: Standard deviaions of plastic strain after crash.

The sample of the first amplitude is computed by eq. (6) and exported as optiSLang result file. In optiSLang, the *Metamodel of optimal Prognosis* (Most and Will, 2009) was determined. This procedure systematically tests several regression models and subsets of the input variables in order to find the best regression for the amplitude. The criterion is the *Coefficient of Prognosis*: while a larger part of the available data is used for determining the regression model, the squared correlation between the remaining data and the regression model is computed to form the CoP. As side effect, the input variables chosen for the optimal model are ranked by the CoP value. 13 out of 55 input variables were chosen by the algorithm to have significant effect on the first amplitude of the plastic strain field. As seen in fig. 6, the shell thickness and plastic deformation after forming of the examined part and the barrier angle have the largest influence. This is the information needed to improve the design of the part.

5 Concluding Remarks

It is demonstrated in the present paper, how the methodology of random fields can be utilized within virtual product development. Variance based robustness assessment, by simulating manufacturing processess or load cases with random parameters, results in spatially distributed random properties of the observed structure. One possible application is to study manufacturing tolerances and formulate quality requirements. Another is the proof of robustness of a strucural part in regard of spatially random properties.

By interpretation of the results as random fields and decomposition by the Karhunen - Loève series, sect. 3.2, an effective parametric for the spatial scatter is gained. The shape functions of the Karhunen - Loève series reveal “mechanisms” of the spatial distribution and ease therefore the interpretation of it. Analysis of the statistical relation between random inputs and the simulation results allows to identify and isolate the most significant effects, therefore the detection of the cause of scatter. This is shown in the example of section 4.

It is also possible to model a random field, based on measurements, simulation results

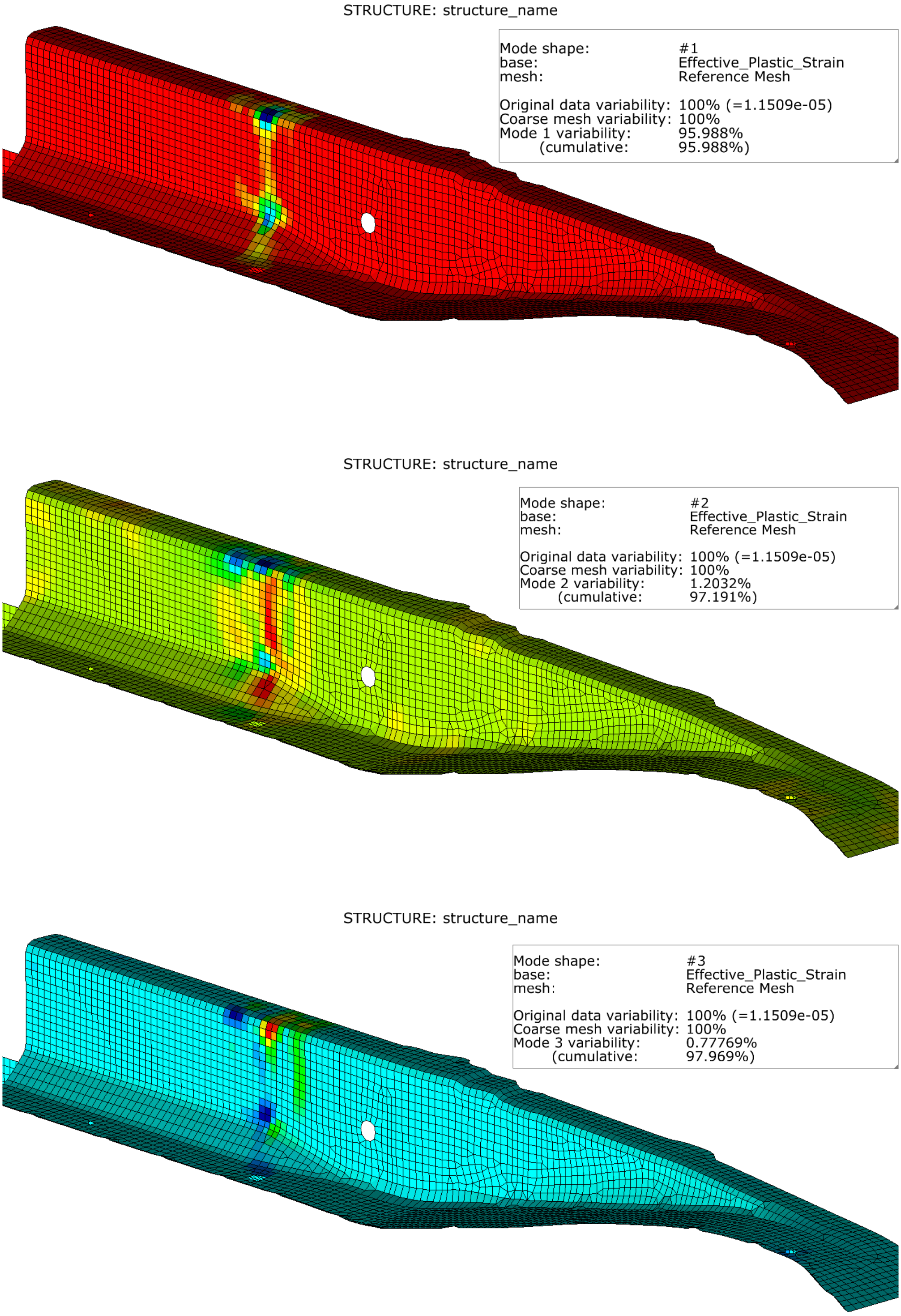

Figure 5: Load-bearing car part: First three shape functions of plastic strain.

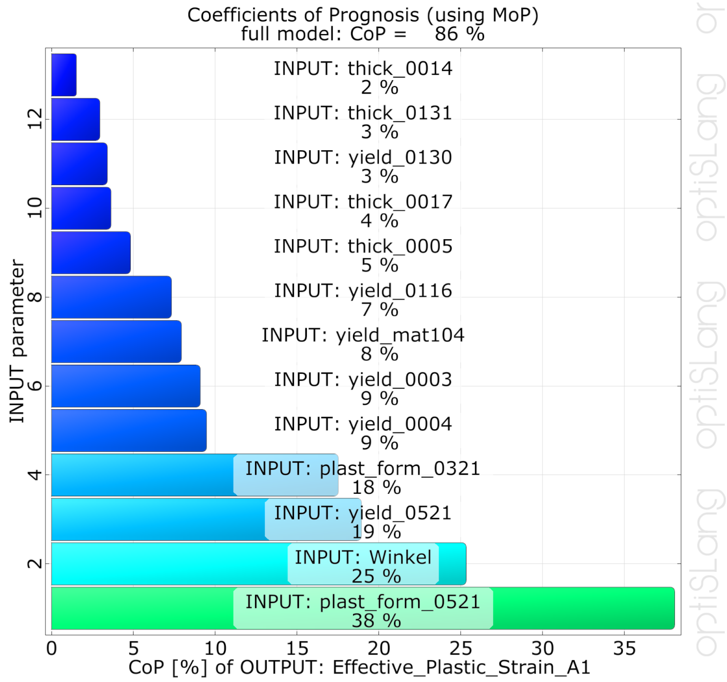

Figure 6: Load-bearing car part: Coefficients of Prognosis of random inputs w.r.t. first random field amplitude.

or pure assumption and generate imperfect structures for a study of their performance.

Statistics on Structure (SoS) is a software yet for post-processing of such spatially scattering data, sect. 2. Structural models and simulation results can be read in several formats. Results of the statistical survey are visualized directly on the structure. Further developments of SoS aim at:

- even more effective parametric in regard of data reduction and filtration of the most relevant information,
- handling of different meshes (e.g. FE-model and measurement grid; meshes for sheet metal forming and crash analysis),
- modeling and simulation of random fields,
- closing the automatic process chain from manufacturing simulation to the performance analysis.

Manifold applications of random fields will become possible that way and will be integrated in user-friendly front-end.

References

C. E. Brenner. *Ein Beitrag zur Zuverlässigkeitsanalyse von Strukturen unter Berücksichtigung von Systemuntersuchungen mit Hilfe der Methode der Stochastischen Finite Elemente*. PhD thesis, Leopold Franzens Universität Innsbruck, 1995.

C. Bucher. Basic concepts for robustness evaluation using stochastic analysis. In *EUROMECH Colloq. Efficient Methods of Robust Design and Optimization*, London, 2007.

SoS – Statistics on Structure, User's Manual, Version 2.3.0. Dynardo GmbH, Weimar, 2010.

optiSLang - the optimizing Structural Language, Users' Manual, Version 3.1.0. Dynardo GmbH, Weimar, Germany, 2009.

R. Ghanem and P.D. Spanos. *Stochastic Finite Elements – a Spectral Approach*. Springer, New york, Berlin, 1991.

T. Most and C Bucher. A moving least square weighting function for the elementfree galerkin method which almost fulfills essential boundary conditions. *Structural Engineering and Mechanics*, 21, 2005.

T. Most and J. Will. Metamodel of optimized prognosis (mop) - an automatic approach for user friendly parameter optimization. In *Weimar Optimization and Stochastic Days 6.0*, Weimar, 2009.

E. Vanmarcke. *Random Fields: Analysis and Synthesis*. MIT Press, Cambridge (MA) USA, 1983.

J. Will. State of the art - robustness in cae-based virtual prototyping processes of automotive applications. In *Weimar Optimization and Stochastic Days 4.0*, Weimar, 2007.

J. Will and T. Frank. Rechnerische Robustheitsbewertung von Strukturcrashlastfällen bei der Daimler AG. In *Weimar Optimization and Stochastic Days 5.0*, Weimar, 2008.