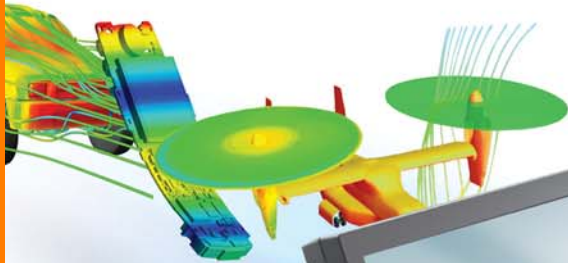


Robust Design Optimization of a Centrifugal Compressor concerning Fluid-Structure Interaction and Manufacturing Tolerances

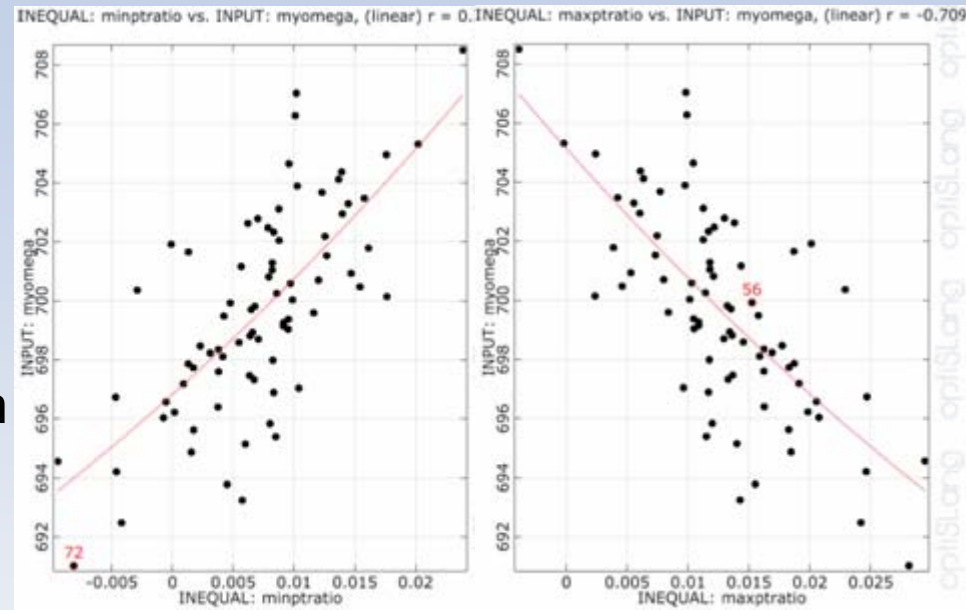


Dirk Roos
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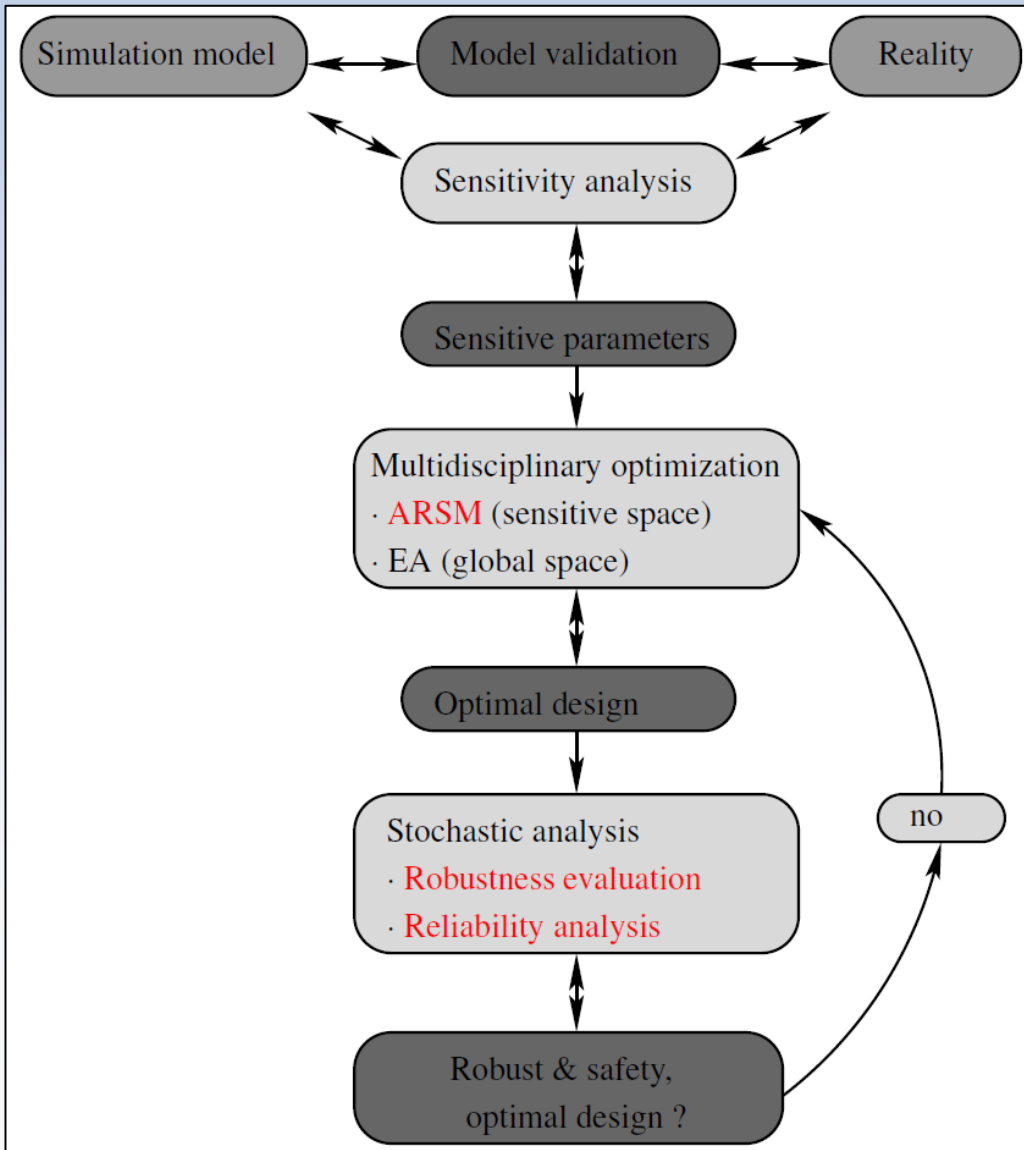
Johannes Einzinger
ANSYS Continental Europe
johannes.einzinger@ansys.com

Conclusion Robustness Analysis

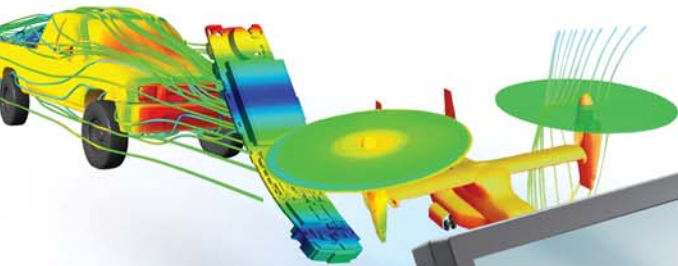
- **Non robust** behavior with respect to
 - **Efficiency**
 - **Total pressure**
- But **acceptable** failure probability level for structural risk
 - Estimation of a Six Sigma Design
- Efficiency: **myeta**
 - RVHubBeta1 as largest as possible
 - RVShdBeta1 as largest as possible
 - RImpeller as smallest as possible
- Total pressure: **ptratio**
 - myomega as largest as possible
 - RImpeller as largest as possible
 - ptratio mean \rightarrow 1.355



Successive Robust Design Optimization



- iterative decoupled loop approach
- in combination with identification of the most significant random and design variables using the multivariate statistic
- first step the robustness evaluation can be used to prove the predictive capability of the simulation model and to
- identify the most important parameters to solve reliability analysis, efficiently
- it is necessary to evaluate robustness and safety of the design



Design Optimization II

Robustness Evaluation II

Robust Design
Optimization

Reliability Analysis



Design Optimization II



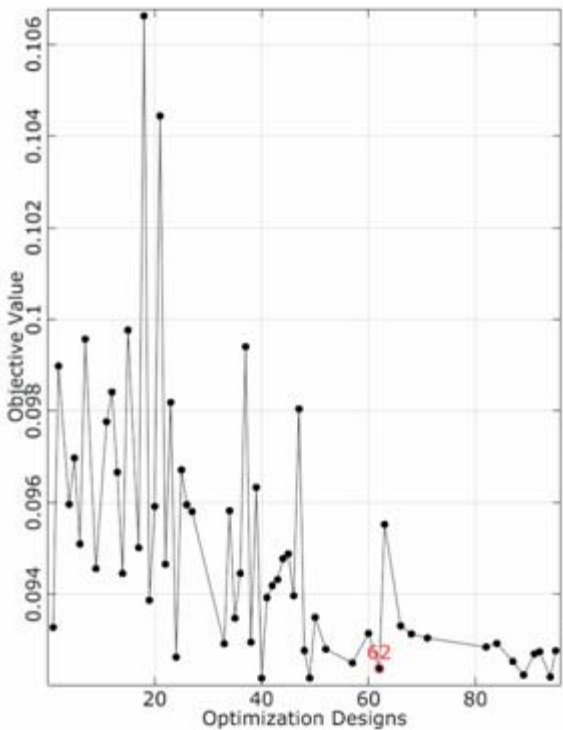
Opti	Robust	Output	Strings	Constraints	Objectives				
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Type	Format	Active	Const...
1	myomega	699.76	699.76	699.0	703.0	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
2	InletWidth	53	53.6136610657...	52.5	57.5	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
3	ExitWidth	26	27.8049298398...	26.5	28.5	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
4	RImpeller	305	292.556879245...	291	300	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
5	HubBeta1	-48	-52.5	-55	-49.5	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
6	HubBeta3	-25	-27.017132519...	-28	-26.5	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
7	ShdBeta1	-55	-60.267623161...	-60.5	-59.5	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
8	RVHubThk1	45	45.0	35	66.0	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
9	RVHubBeta1	60	66.0	62.0	68	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
10	RVShdBeta1	60	62.8548646835...	60.0	64.0	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
11	RVShdThk1	45	45.0	35.0	55.0	continuous	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
12	HubBeta2	-25	-25.0	-27.5	-22.5	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
13	ShdBeta2	-45	-45.0	-49.5	-40.5	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
14	ShdBeta3	-30	-30.0	-33.0	-27.0	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
15	HubThk1	1	1.0	0.8	1.2	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
16	HubThk2	6	5.91963645103...	5.0	7.0	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
17	ShdThk1	1	1.03011230706...	0.8	1.2	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
18	ShdThk2	6	6.0	5.0	7.0	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
19	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
20	RVBlades	24	24	21.6	28.7999999999...	continuous	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>

Cancel OK

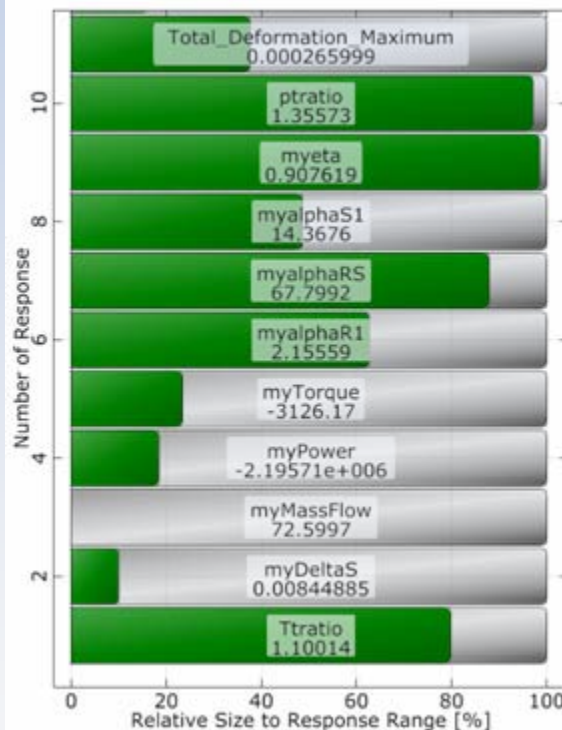
Design Optimization II: ARSM



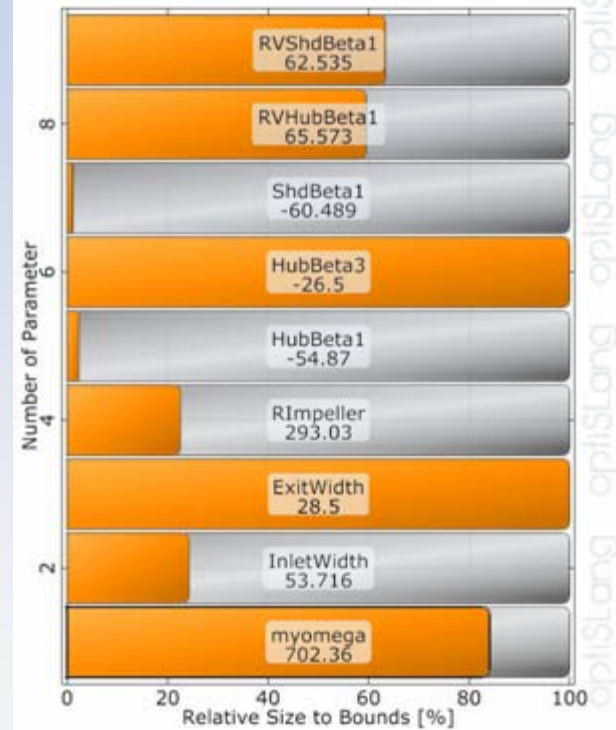
Objective History



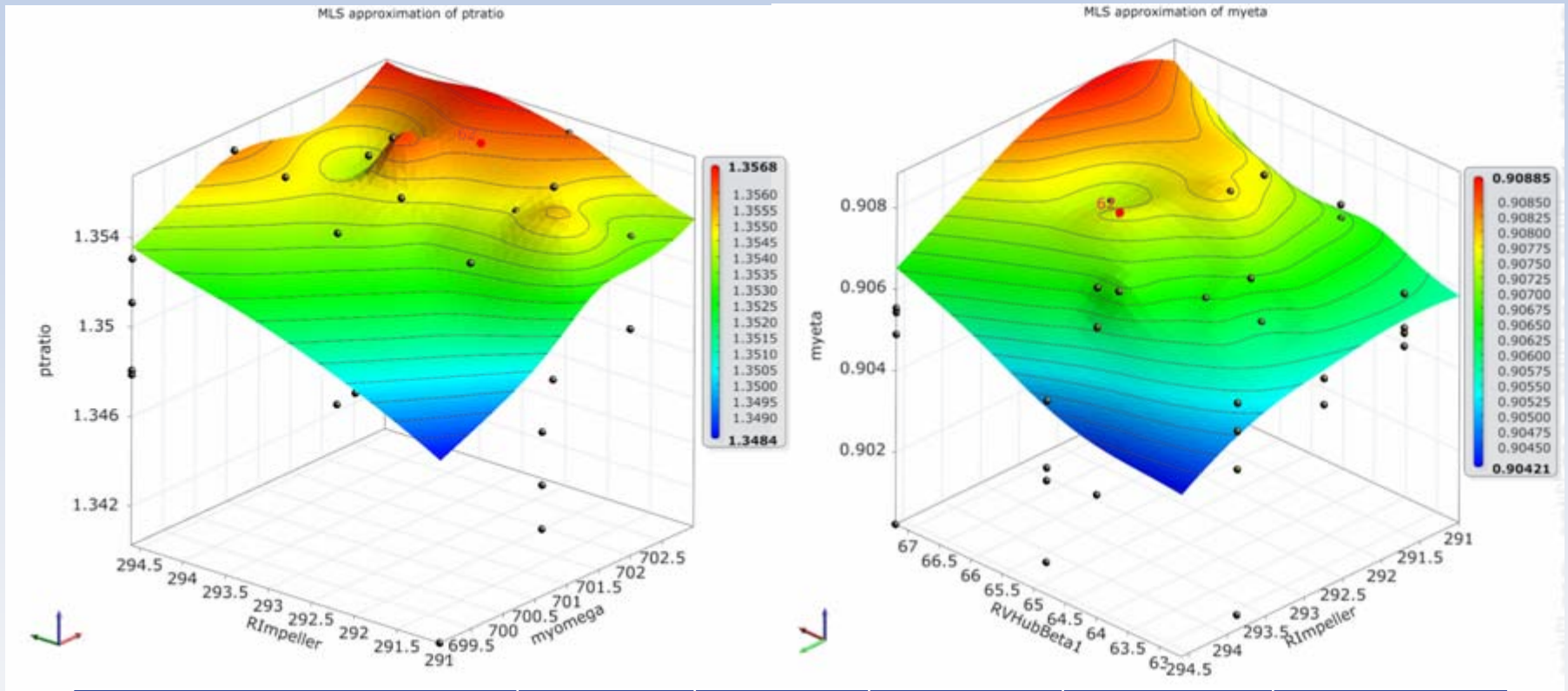
RESPONSE DATA: (Best Design #62)



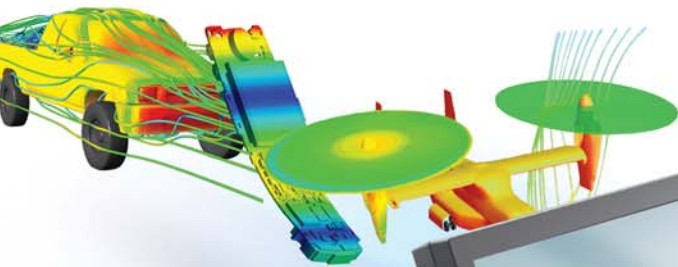
Best Design #62



Design Optimization II: ARSM



	Initial	SA	ARSM I	EA I	ARSM II
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356
Efficiency [%]	86.72	89.15	90.62	90.67	90.76
#Designs	-	100	105	84	62



Design Optimization II

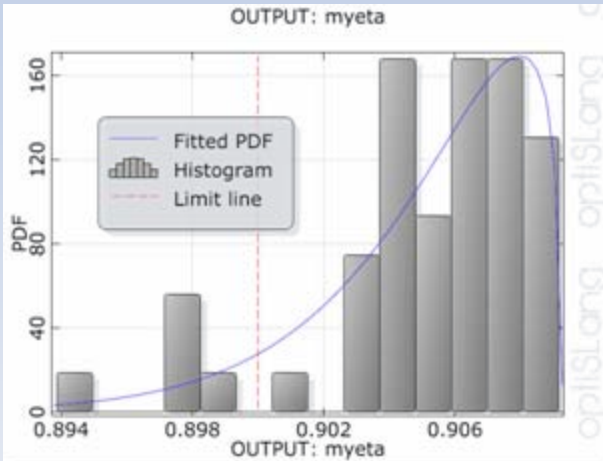
Robustness Evaluation II

Robust Design
Optimization

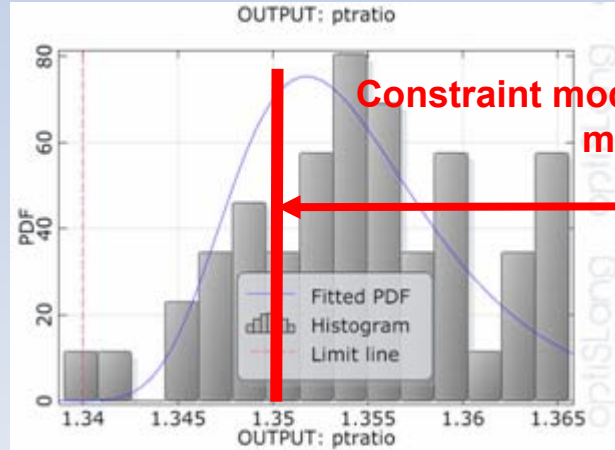
Reliability Analysis



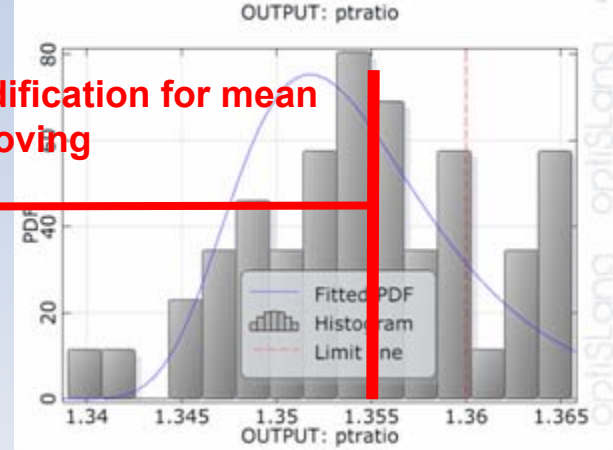
Robust evaluation II: LHS



Statistic data	
Min: 0.8939	Max: 0.9092
Mean: 0.9051	Sigma: 0.00329
CV: 0.003635	
Skewness: -1.487	Kurtosis: 4.997
Fitted PDF: Extreme Typ III (Min) Weibull	
Mean: 0.9051	Sigma: 0.00329
Upper cut: 0.9092	
Limit x = 0.9	
P_rel = 0.102041	P_fit = 0.0808577



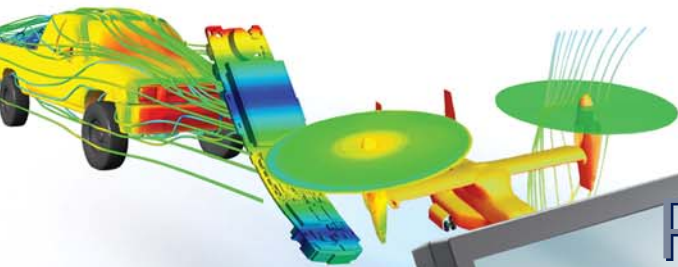
Statistic data	
Min: 1.339	Max: 1.366
Mean: 1.355	Sigma: 0.006258
CV: 0.00462	
Skewness: -0.1853	Kurtosis: 2.623
Fitted PDF: Fisher-Tippett	
Mean: 1.355	Sigma: 0.006258
Limit x = 1.34	
P_rel = 0.0204082	P_fit = 1.4437e-005



Statistic data	
Min: 1.339	Max: 1.366
Mean: 1.355	Sigma: 0.006258
CV: 0.00462	
Skewness: -0.1853	Kurtosis: 2.623
Fitted PDF: Fisher-Tippett	
Mean: 1.355	Sigma: 0.006258
Limit x = 1.36	
P_rel = 0.795918	P_fit = 0.831174

**Tolerance limit $\eta < 90\%$
~8% outside**

**Tolerance limit $\Pi_T > 1.36$
~17% outside**



Design Optimization III

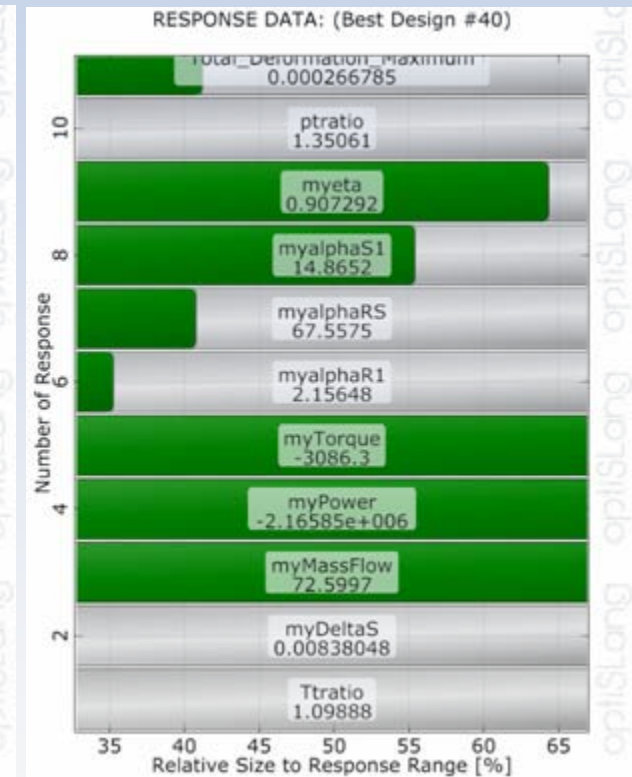
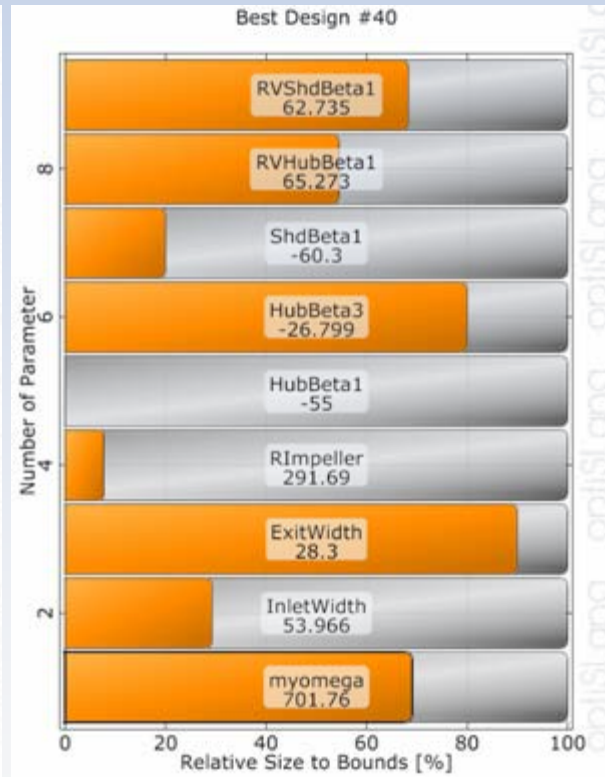
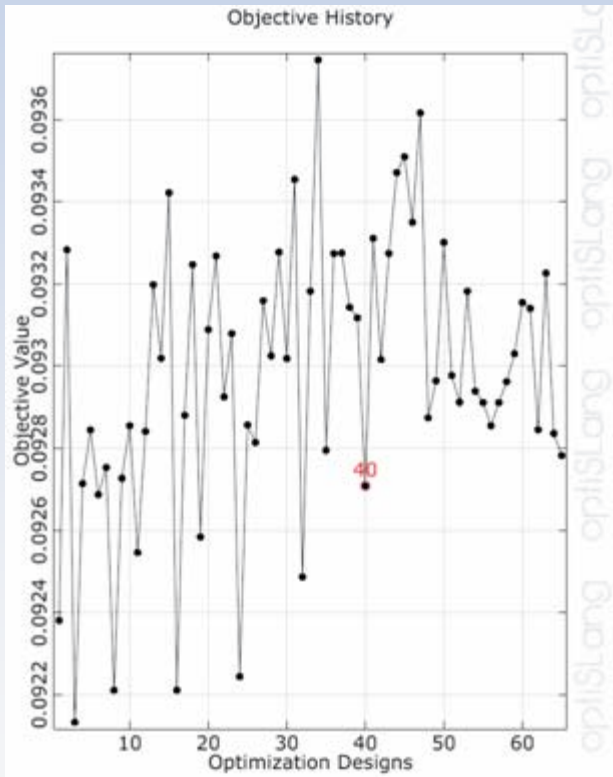
Robustness Evaluation III

Robust Design
Optimization

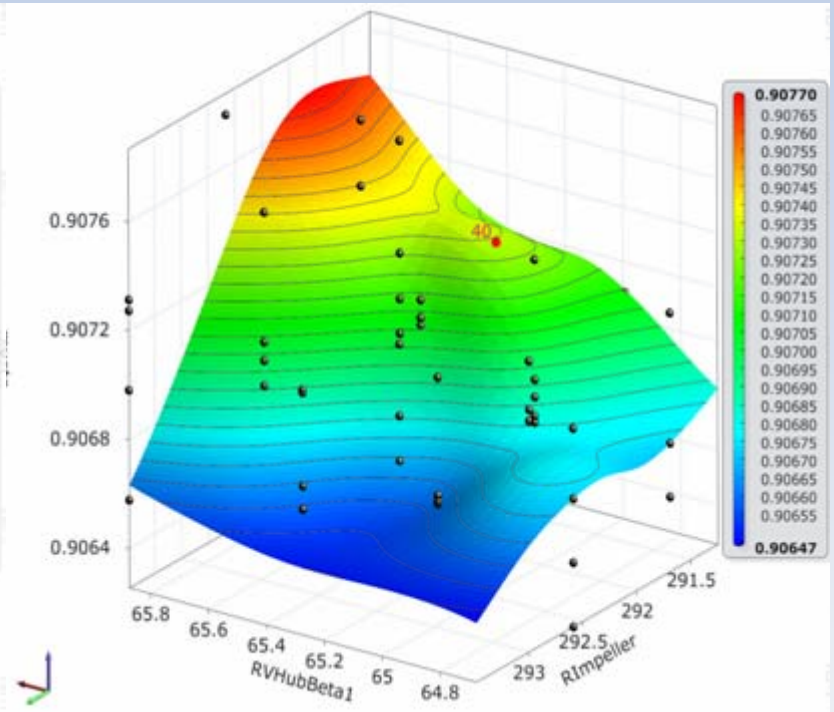
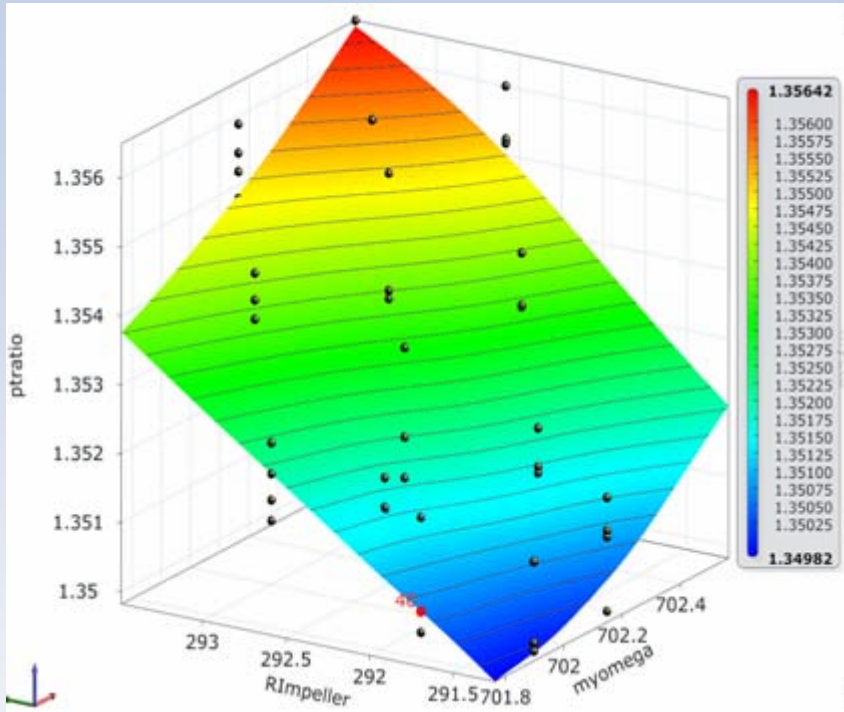
Reliability Analysis



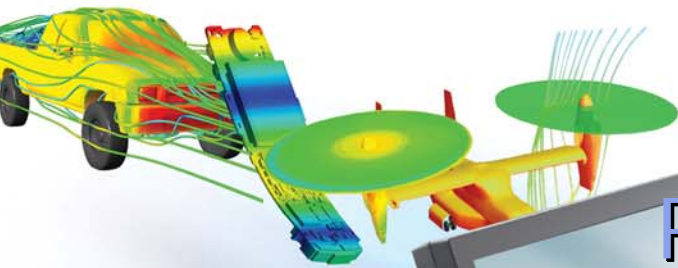
Design Optimization III: ARSM



Design Optimization III: ARSM



	Initial	SA	ARSM I	EA I	ARSM II	ARSM III
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356	1.351
Efficiency [%]	86.72	89.15	90.62	90.67	90.76	90.73
#Designs	-	100	105	84	62	40



Design Optimization III

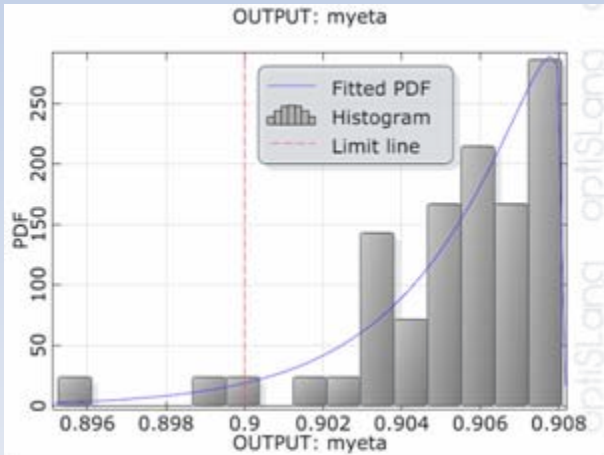
Robustness Evaluation III

Robust Design
Optimization

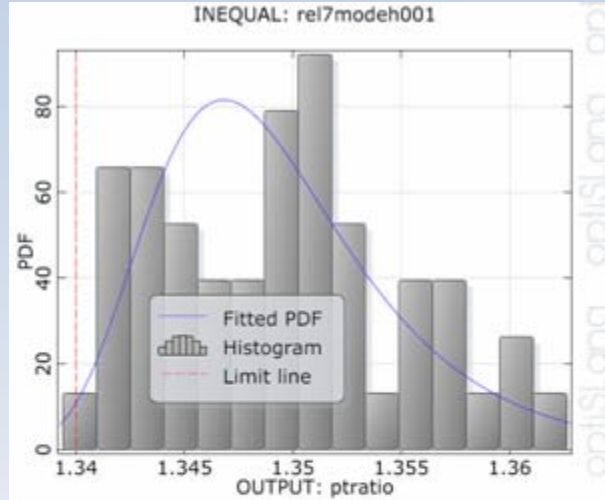
Reliability Analysis



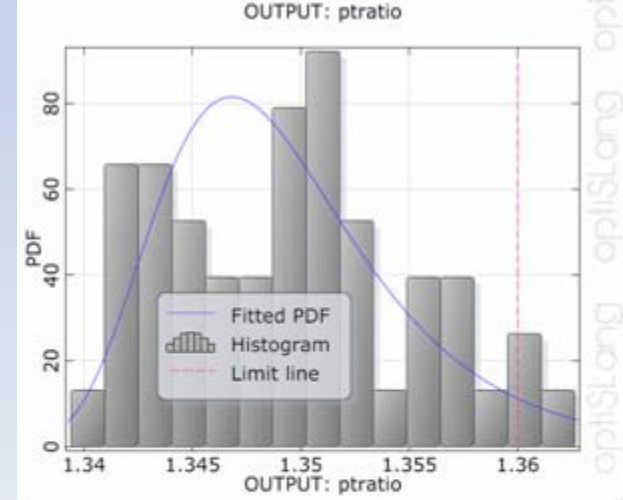
Robust evaluation III: LHS



Statistic data	
Min: 0.8953	Max: 0.9081
Mean: 0.9053	Sigma: 0.002554
CV: 0.002821	
Skewness: -1.868	Kurtosis: 7.173
Fitted PDF: Extreme Typ III (Min) Weibull	
Mean: 0.9053	Sigma: 0.002554
Upper cut: 0.9081	
Limit x = 0.9	
P_rel = 0.0612245	P_fit = 0.0453854



Statistic data	
Min: 1.339	Max: 1.363
Mean: 1.349	Sigma: 0.005782
CV: 0.004285	
Skewness: 0.3347	Kurtosis: 2.38
Fitted PDF: Fisher-Tippett	
Mean: 1.349	Sigma: 0.005782
Limit x = 1.34	
P_rel = 0.0204082	P_fit = 0.01082



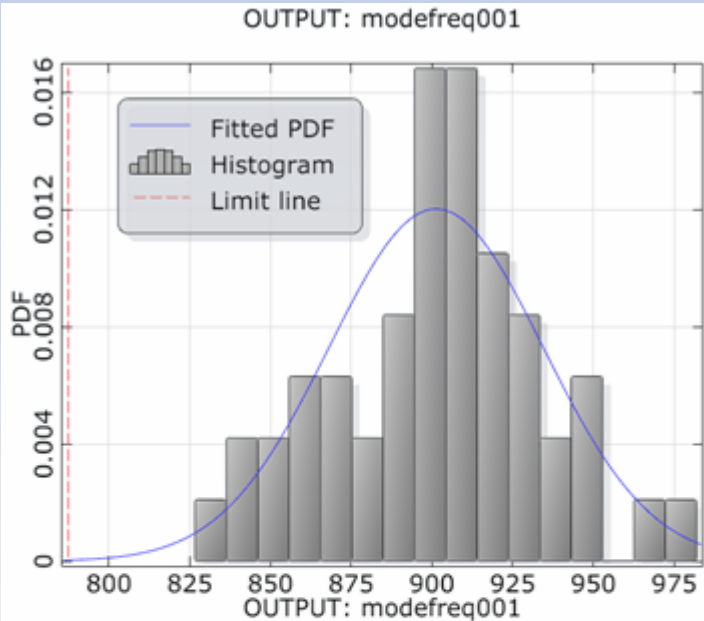
Statistic data	
Min: 1.339	Max: 1.363
Mean: 1.349	Sigma: 0.005782
CV: 0.004285	
Skewness: 0.3347	Kurtosis: 2.38
Fitted PDF: Fisher-Tippett	
Mean: 1.349	Sigma: 0.005782
Limit x = 1.36	
P_rel = 0.938776	P_fit = 0.947814

**Tolerance limit $\eta < 90\%$
~4.5% outside**

Robust Design

**Tolerance limit
 $1.4 < \Pi_T < 1.36$
~6% outside**

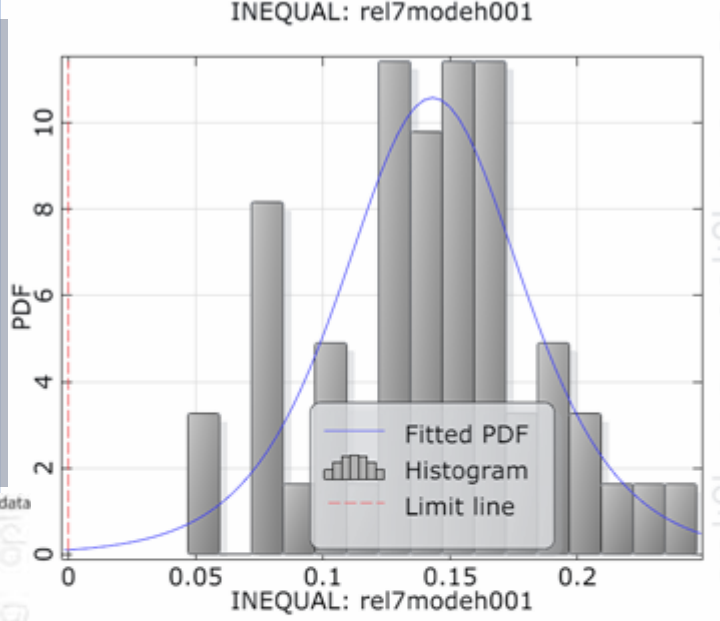
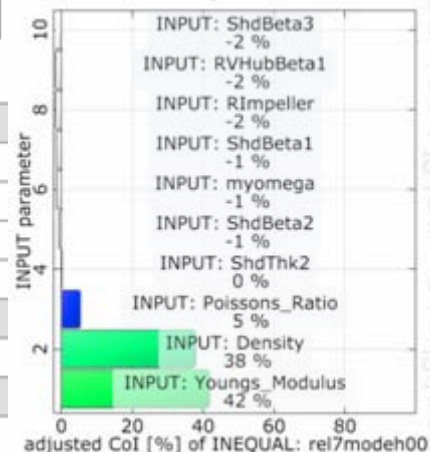
Robust evaluation III: Eigen Frequency Mode 1 Harmonic Index 0



Statistic data	
Min: 826.6	Max: 982
Mean: 901.3	Sigma: 33.18
CV: 0.03682	
Skewness: -0.04593	Kurtosis: 2.862
Fitted PDF: Normal	
Mean: 901.3	Sigma: 33.18
Limit x = 787.4	
P_rel = 0	P_fit = 0.000299089

Tolerance limit:
Resonance point
 ~ 120/240 Hz
 $\mu_{RP001} \sim 937$ Hz
 $\sigma_{RP001} \sim 34$ Hz

Coefficient of Importance (linear) - Spearman ranked data
 full model: adjusted R² = 95 %

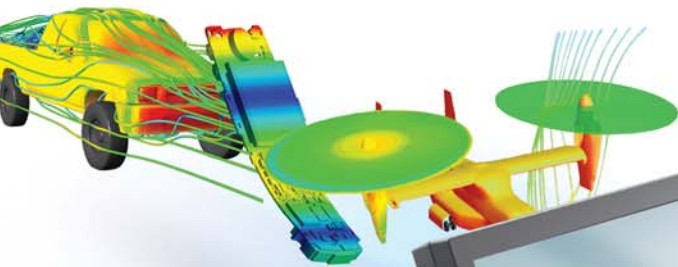


Statistic data	
Min: 0.04693	Max: 0.2468
Mean: 0.1431	Sigma: 0.04281
CV: 0.2991	
Skewness: -0.1285	Kurtosis: 2.936
Fitted PDF: Logistic	
Mean: 0.1431	Sigma: 0.04281
Limit x = 0	
P_rel = 0	P_fit = 0.0023215

$$P(\mathcal{F}) \leq 3.4 \cdot 10^{-6}; (\beta = \sigma_L \geq 4.5)$$

$$P(\mathcal{F}) \approx \Phi(-\beta) \approx \Phi(-0.143 + 0.01/0.0428) = \Phi(-3.11) = 9.4 \cdot 10^{-4}$$

Safety Design?



Design Optimization

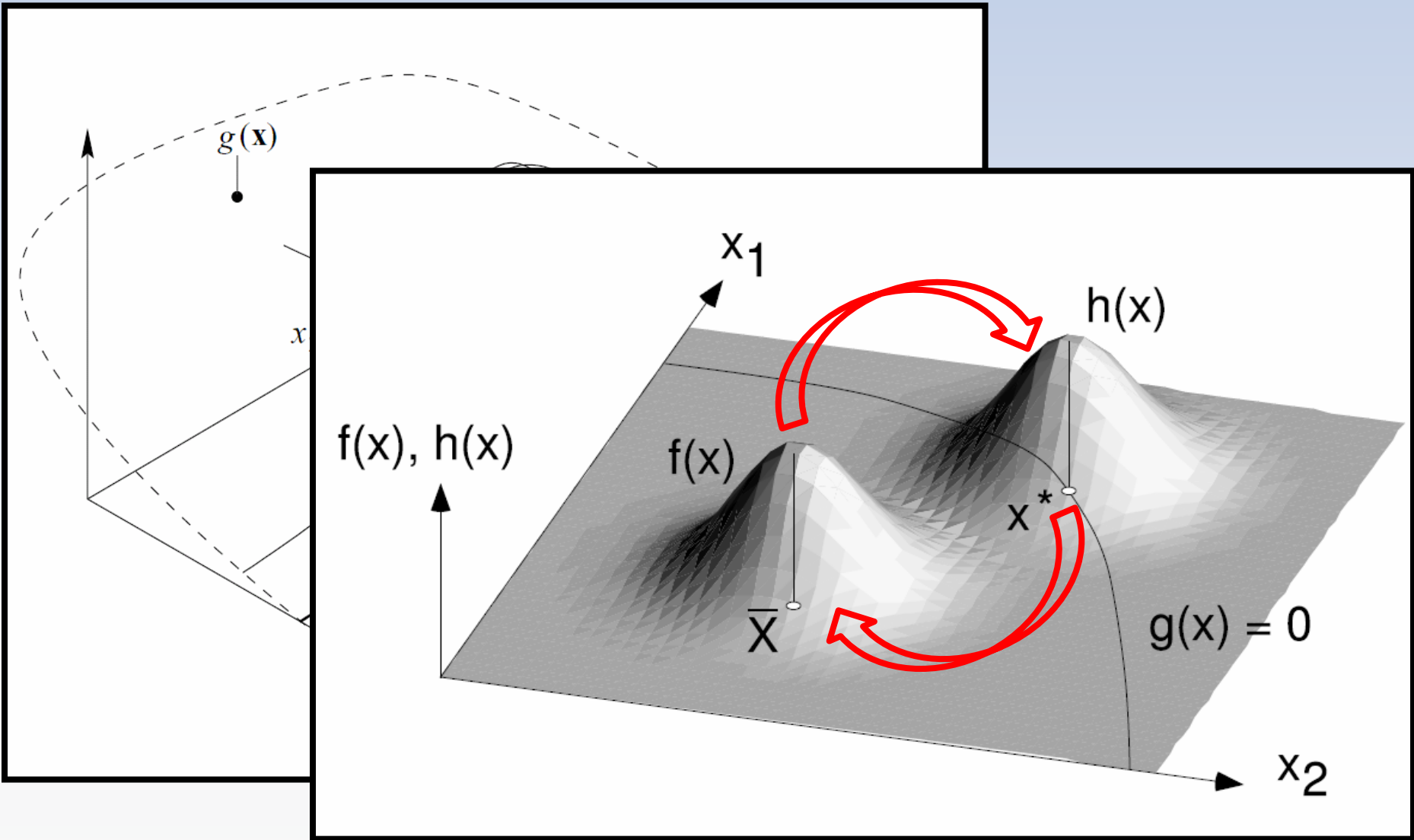
Robustness Evaluation

Robust Design
Optimization

Reliability Analysis

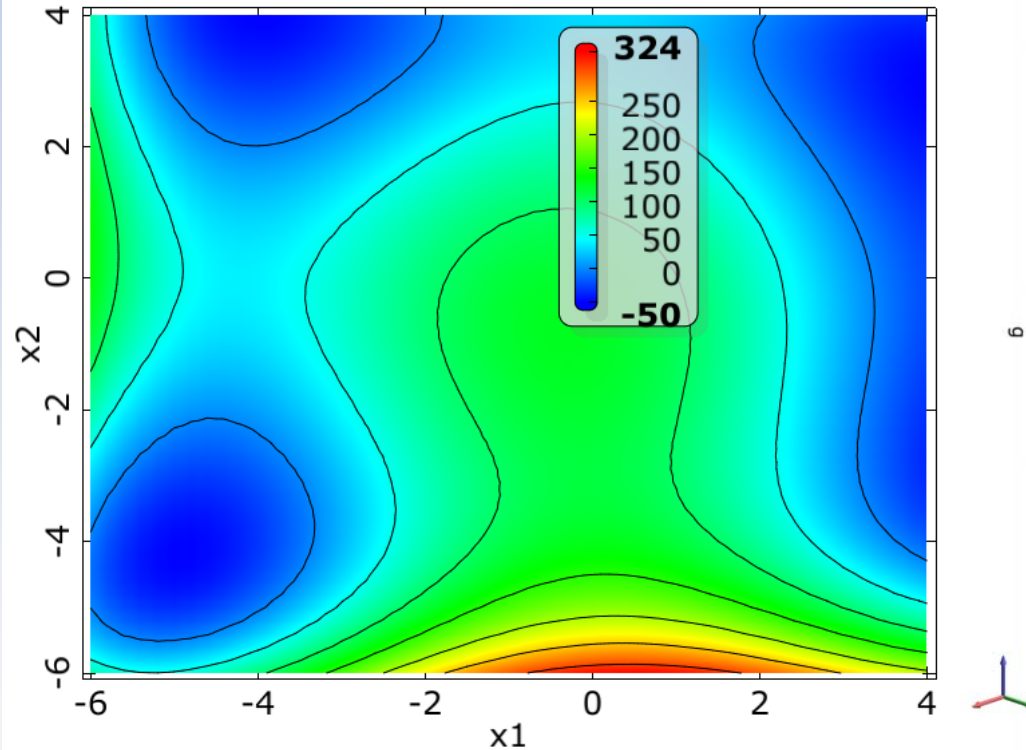


Reliability Analysis

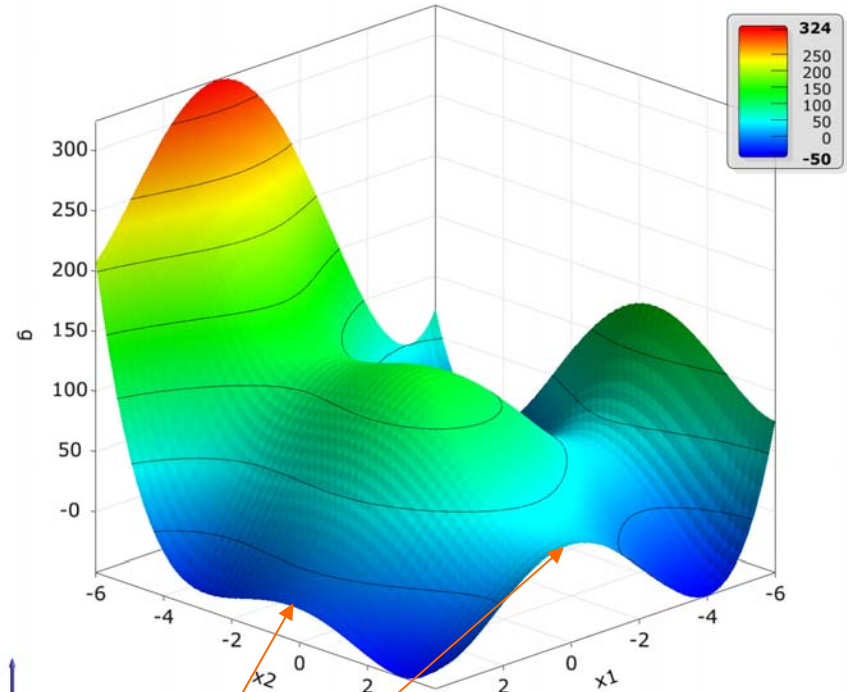


Reliability Analysis

MLS approximation of g



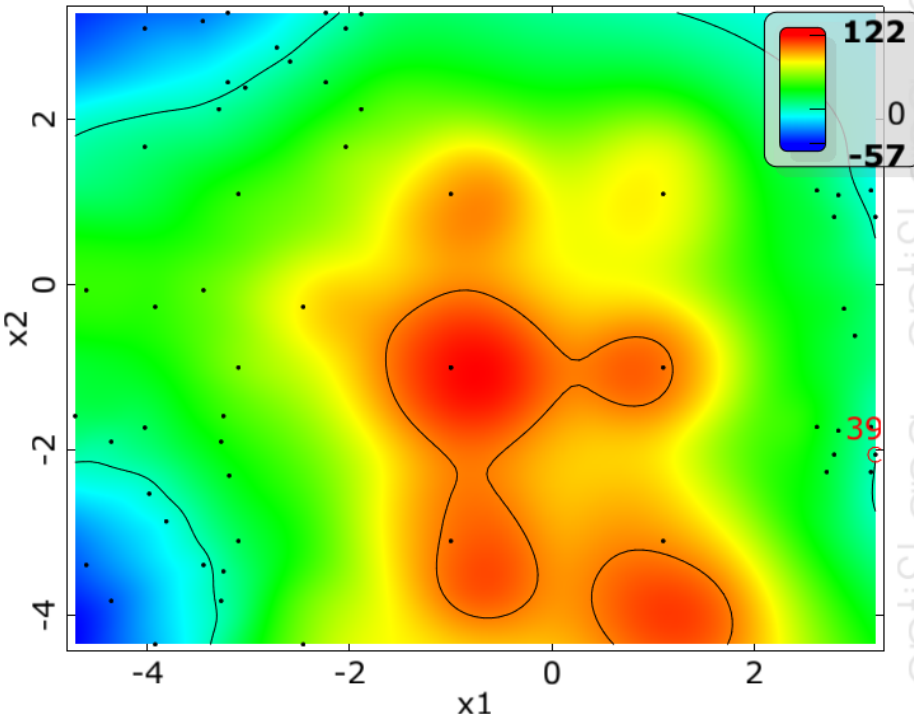
MLS approximation of g



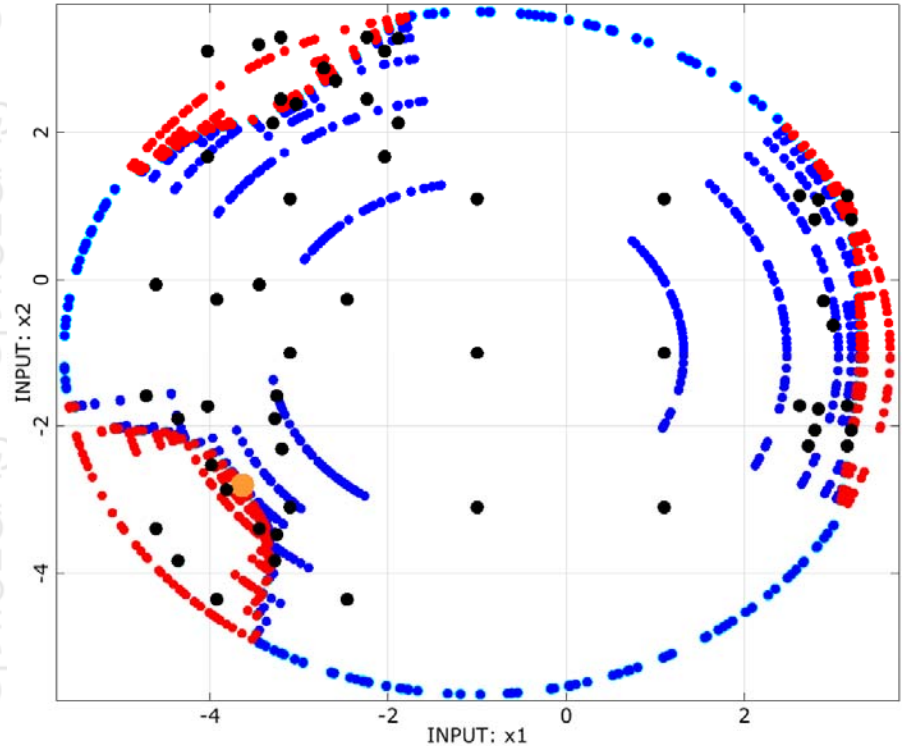
- Himmelblau function
- Nonlinear two dimensional state function $g(x_1, x_2)$
- Nonlinear limit state function $g(x_1, x_2) = 0$
- Three separated domains with high failure probability density

Reliability Analysis

MLS approximation of g



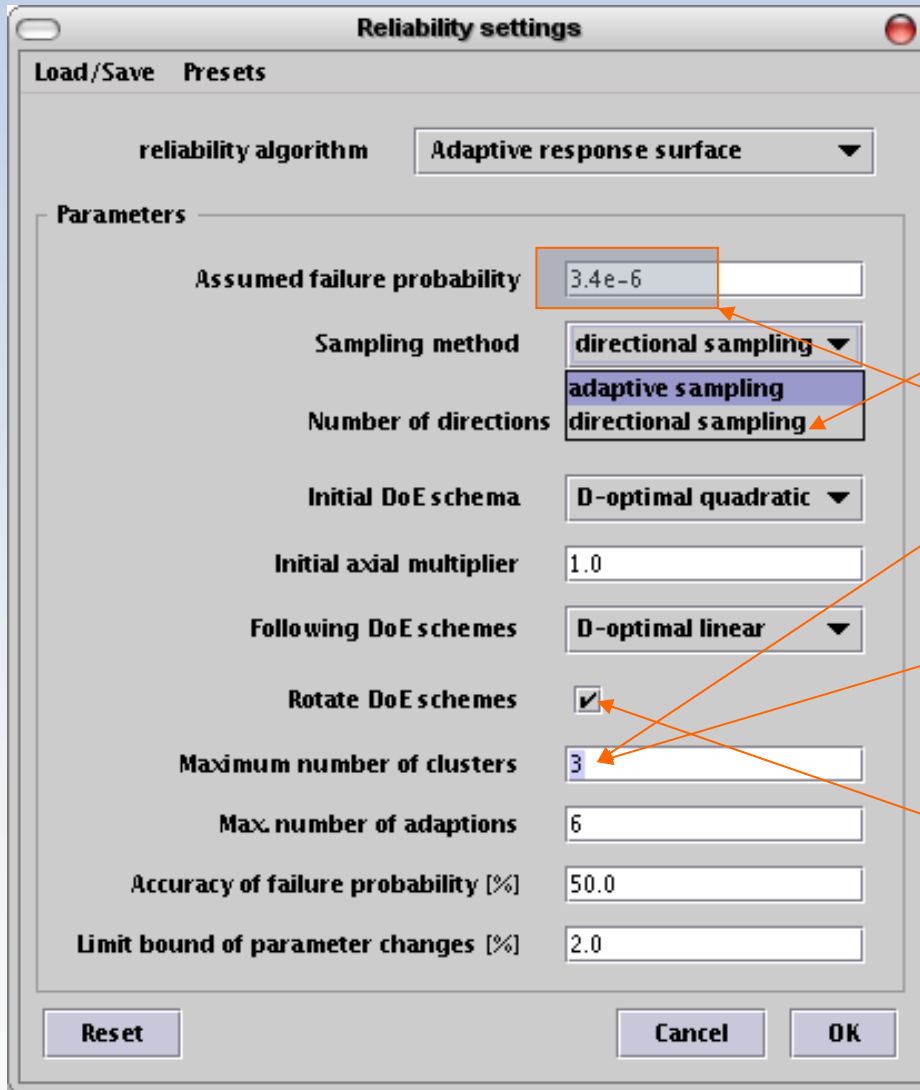
INPUT: x1 vs. INPUT: x2
(5. Approximation)



- Adaptive response surface method
- Directional sampling on MLS
- Design evaluations: 58
- PF = 1.67E-06 (1.99E-06)

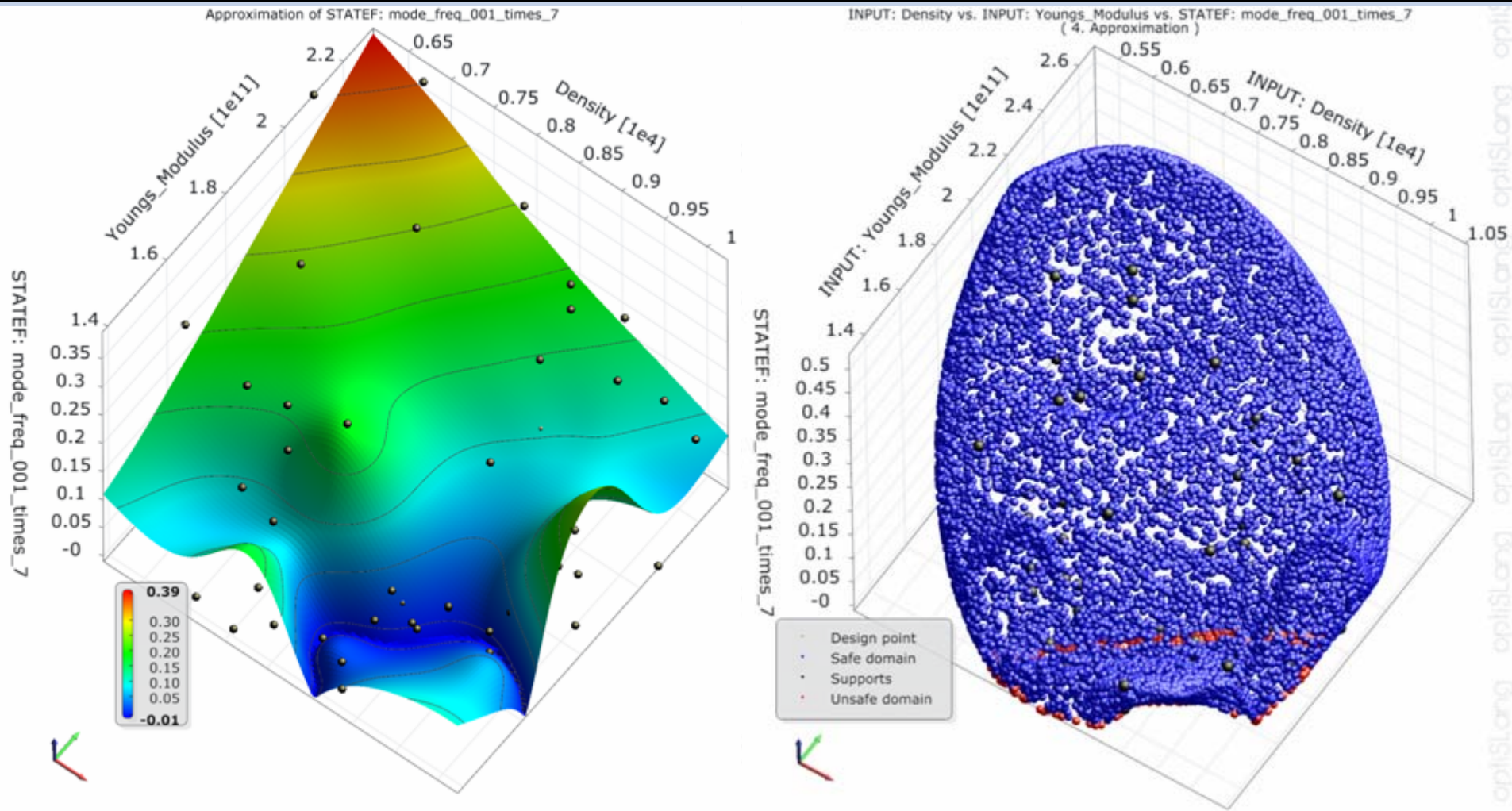
- Sigma level independent
- $n \leq 20$
- Multiple adaptive DOE
- Supports multi-domain limit states

Adaptive response surface approximation



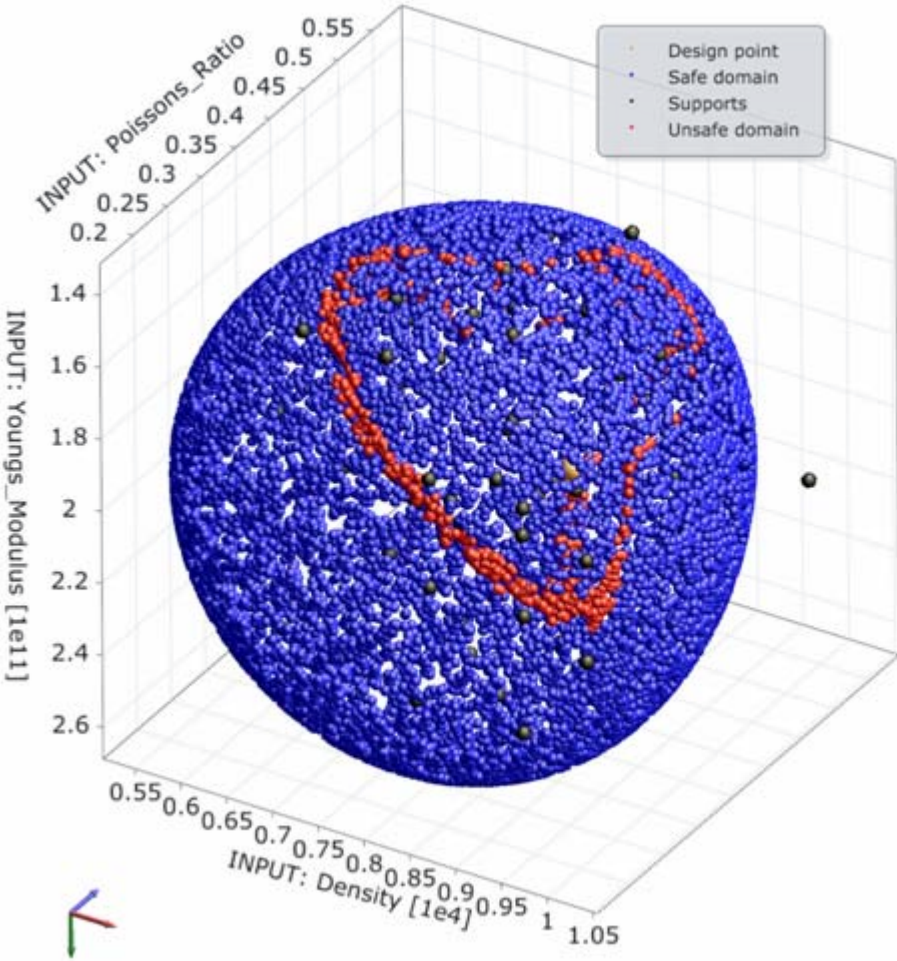
- Sampling methods on the MLS approximation:
 - Adaptive Sampling
 - **Directional Sampling**
 - supports more than two failure domains
 - and sigma level independent
- **Cluster analysis** to detect number of failure domains with high failure probability
- **Rotatable** adaptive designs of experiments to improve the approximation accuracy

Reliability Analysis

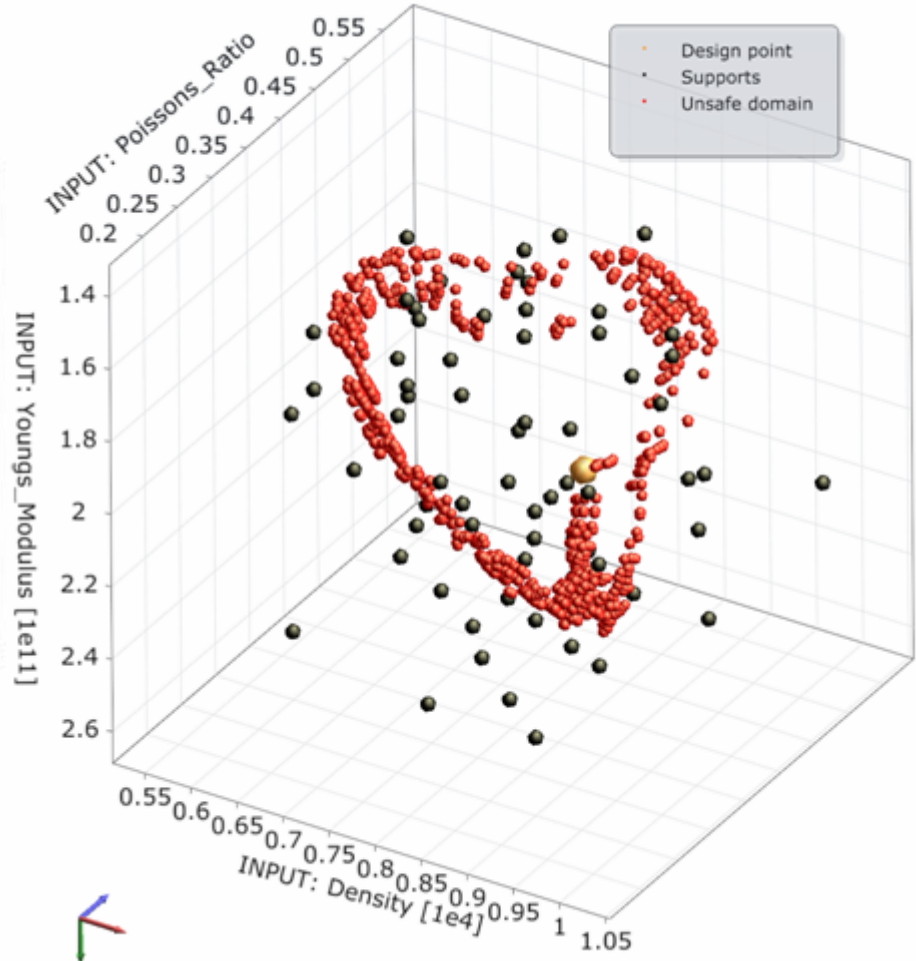


Reliability Analysis

INPUT: Density vs. INPUT: Youngs_Modulus vs. INPUT: Poissons_Ratio
(4. Approximation)

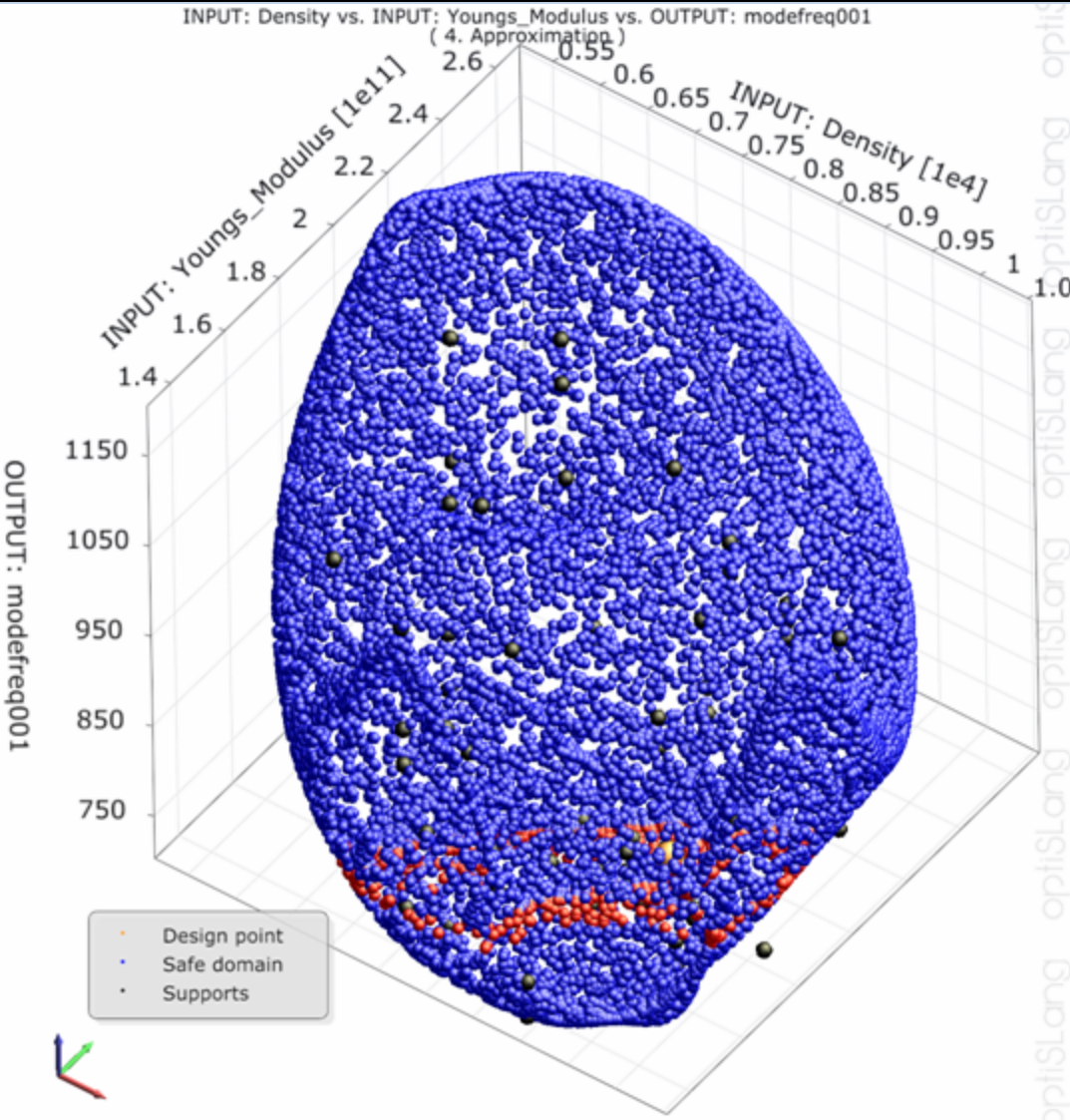


INPUT: Density vs. INPUT: Youngs_Modulus vs. INPUT: Poissons_Ratio
(4. Approximation)



optiSlang optiSlang optiSlang optiSlang optiSlang

Reliability Analysis



Method : Directional Sampling on Adaptive Response Surfaces (ARSM-DS)

Selected data : 4. Approximation

Number of designs : 68 (4 failed)

Complete directions : 10000 / 10000

Number of samples :

Total : 13640

Safe domain : 11891

Unsafe domain : 1749

Failure strings : 0

Unsuccessful : 0

Probability of failure : 2.75e-007 (2.75e-007)

Standard deviation error : 6.667e-008 (6.667e-008)

Most probable failure point:

Density : 9279.86887124

Youngs_Modulus : 177034428177

Poissons_Ratio : 0.285958415968

Distance median - design point (beta) : 4.328

Probability of failure (FORM): 7.526e-006

- $n = 3$ random parameters
- $N = 68$ design evaluations
- $P(\mathcal{F}) \approx 3 \cdot 10^{-7} < 3.4 \cdot 10^{-6}$
- Six Sigma Design

Summary

- Parametric Workflow management
- Automatic and embedded solution
- Parallel and distributed solver runs
- Process integration within optiSLang
- Efficient Robust Design Optimization with
- Quadratic convergence rate and
- **18** design parameters and
- **26** random geometry parameters,
- **including** the **manufacturing tolerances** based random field modelling
- Optimized robust design:
 - **5%** improvement of the efficiency ($\eta < 90\%$, failure rate $\sim 4.5\%$)
 - Tolerance limit ($1.4 < \Pi_T < 1.36$, failure rate $\sim 6\%$)
- **Optimized Six Sigma design** $P(\mathcal{F}) \approx 3 \cdot 10^{-7}$
- $N = 100 + 105 + 84 + 100 + 62 + 50 + 40 + 50 + 68 = \mathbf{659}$ design evaluations
(SA)(EA)(ARSM)(RE)(ARSM)(RE)(ARSM)(RE)(RA)
- Calculation time: 10 days (8 CPUs)

