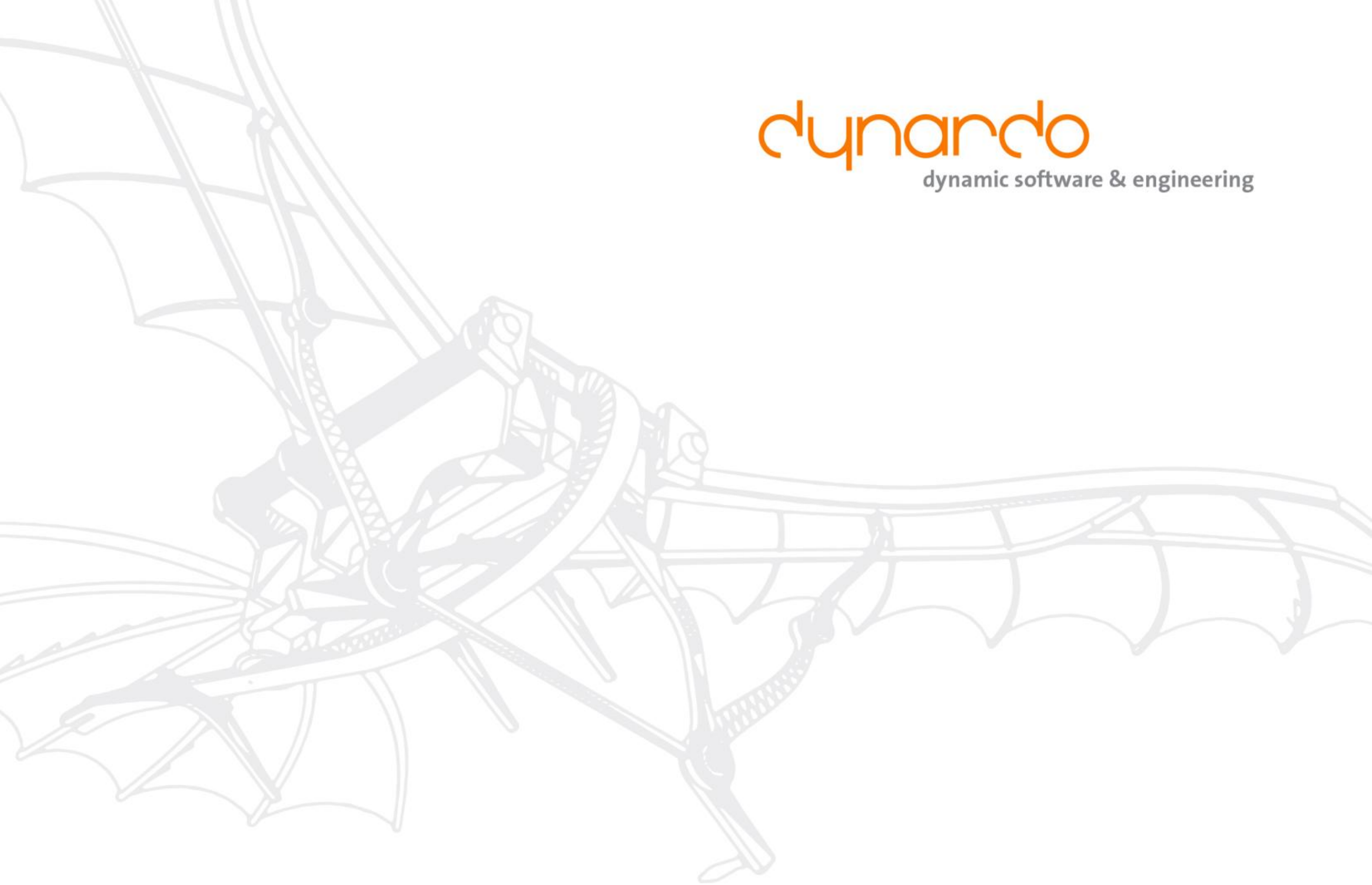

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Lectures

Combination of optimization and robustness evaluation from practical point of view

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Summary:

Optimization as well as robustness evaluation are key technologies of virtual product development. The optimization, i.e. improvement of product characteristics, has been an integral part of this development for several years now. On the other hand, the robustness of a construction, i.e. the reliable function within admissible boundaries, is becoming more and more focused on, recently. In fact, robustness is an additional demand on „optimized“ designs.
The optimization and robustness evaluation are either performed consecutively or simultaneously, and numerous methods are available for this. In the following, existing methods shall shortly be introduced and discussed from a practical point of view regarding their appliance in virtual prototyping processes.
Three optimization method classes are discussed. These are: mathematical optimization methods using gradients, response surface methods, and stochastic search algorithms. Pareto optimization will be shortly mentioned.
Within the robustness analysis, the sensitivity of the unavoidable scatter of environmental conditions and their impact on the most important structural responses is evaluated. Especially for nonlinear structural behavior it is mandatory to analyze the robustness with respect to the most important random variations of the design parameters. Robustness evaluation is restricted to relatively frequently occurring events. To cover rare events, methods of reliability analysis have to be employed.
Additionally, methods for a simultaneous performance of the optimization and the robustness evaluation task are introduced. Two practical applications serve to point out potential application areas of optimization and robustness evaluation.

Keywords:

robustness evaluation, stochastic analysis, optimization

1 Introduction

Optimization and stochastic analysis – meaning robustness evaluation as well as reliability analysis – are key technologies of virtual prototyping. The optimization, i.e. improvement of product characteristics, has been an integral part of this development for several years now. On the other hand, the robustness of a construction, i.e. the reliable function within admissible boundaries, is becoming more and more focused on, recently. In fact, robustness is an additional demand on „optimized" designs.
The optimization and robustness evaluation are either performed consecutively or iteratively, and numerous methods are available for this. Nevertheless, their suitability and cost effectiveness– the latter expressed in the number of CAE-solver runs – can not be guaranteed a priori. Besides considerable necessary computational resources, the integration of these methods into the existing prototyping process is likely to necessitate a considerable effort.
Nevertheless, there is a consensus on the fact that a combination of optimization and stochastic analysis is of vital importance, taking into account that the development cycles are becoming more and more short. A great innovation potential and competitive advantage is seen here.

In the following, methods of optimization and of robustness evaluation will be introduced and discussed from a practical point of view, on the basis of applications in the virtual prototyping process. Here, the focus will lie less on the details of the methods but on demands, restrictions, and application areas. For details of the methods reference to literature is made.
Evidently, an evaluation as concrete as possible of the numerous algorithms that shall be comprehensible not only to the specialists in this field inevitably necessitates simplifications and the concentration of hitherto successful applications.
Because of hybrid approaches, specializations, and enhancements of single methods, the areas of application of the methods are no longer exactly outlined but appear to be movable. Nevertheless, it can not be expected for the near future that one single algorithm effectively, i.e. economically, and satisfyingly solving the majority of the optimization and reliability tasks.

2 Optimization

In optimization tasks, the variation space or design space is defined by optimization variables. These can take continuous values between an upper and lower boundary as well as discrete values. The desired properties of an optimal design are defined by objective functions and constraints. Then, optimization methods search the design space for as good an approximation as possible of both objective functions and constraints. When this process involves more than one calculation discipline, the term "multidisciplinary optimization" is applied. When more than one objective function is used, the term "multicriteria optimization" is used. Generally, at least three method classes are available to perform the optimization task: mathematical optimization by means of gradients, response surface methods, and stochastic search strategies.

2.1 Mathematical optimization methods by means of gradients

Mathematical optimization methods [11,12] determining search direction by means of gradient information possess the best convergence behavior towards the optimum of the aforementioned methods. However, they do pose the strictest demands on the mathematical formulation of the problem regarding continuity, differentiability, smoothness, and scalability. Additionally, a highly accurate determination of the gradients is needed.
The most critical point from a practical point of view is the determination of the gradients. For many tasks, gradients of important response variables can not be determined analytically or semi analytically; and a numerical determination often fails, for example for noisy, or non differentiable tasks, or simply cannot provide sufficient accuracy.
Therefore, a successful practical application is restricted mainly on optimization tasks with continuous optimization variables and well posed mathematical problem formulations permitting to determine appropriate gradients, such as linear or nonlinear implicit finite element analyses. Gradient methods should if possible start in admissible design areas, i.e. those areas fulfilling all constraints. To identify local optima, several optimizations starting from different points are recommended.

2.2 Response Surface Methods

For tasks that are not appropriate for mathematical optimization methods and do not involve more than 5 to 15 optimization variables, response surface methods [6] are an attractive alternative. These methods generate an approximation of the design space based on an appropriate set of samples of

the design space by means of approximation functions. The samples should be determined by means of sample patterns (design of experiments – DOE) that are fitted for the applied approximation functions. Generally, the approximation functions possess good mathematical properties, so mathematical optimization tasks can be used for the optimum search in the substitute space.
However, it is not a trivial task to prove that the approximation is utilizable at the interesting regions of the design space as well as sufficiently accurate for the optimization. Therefore, adaption schemes are used to assure the approximation quality. Adaptive response surface methods, zooming and shifting the approximation space until the optimum converges on the response surface, prove to be most successful [13].
A practical use of these methods is restricted mainly by the number of optimization variables. Nowadays, adaptive response surface methods are successfully applied e.g. for noisy tasks with up to 10, in some applications up to 15 optimization variables. This problem class is frequently found in explicit finite element analyses, multibody simulation, crash simulation etc.

2.3 Evolutionary Search Strategies

When neither mathematical method alone nor in combination with response surface methods succeeds, stochastic search strategies remain to solve the optimization task. Of all methods belonging to this class, evolutionary methods in its two forms, genetic algorithms [5] and evolutionary strategies [9], are the most successful ones. The term "stochastic search strategies" is applied here because random events lead to design modification.
Frequently, the application of stochastic search strategies is called design improvement rather than optimization. That is because these methods have a much worse convergence towards the optimum than mathematical methods and necessitate a very large amount of design evaluations to converge.
The main difference between genetic algorithms and evolutionary strategies lies in how the optimization variables evolve. For genetic algorithms, the most important evolutionary process is the random interchange of genes, i.e. optimization variables, between two parent designs in order to create descendents. On the other hand, for evolutionary search strategies, mutation, i.e. random modification, of single genes of a parent design in order to create one descendent is the most important process.
This leads to different advantages and recommendable application areas of both methods. Genetic algorithms are especially suitable for a relatively wide search of the design space. That is why they are often employed for the search of different design areas of comparably good performance (island search) [17] or for a design improvement without any previous knowledge entered into the evolution. In contrast, evolutionary strategies are most appropriate for a design improvement of „pre-optimized" design islands or for construction states for which previous knowledge can be integrated into the start generation or into the evolution operators.
The advantages of both genetic and evolutionary strategies can be combined by using hybrid algorithms, self-adjusting or adaptive evolutionary methods, and thus the speed of design improvement can be increased.

2.4 Sensitivity Studies

As has been pointed out hitherto, profound knowledge of the characteristics of the variation space are is vital for the choice of an appropriate optimization method. It is also a prerequisite for the definition of constraints and objective functions. If this knowledge is not available, sensitivity studies are recommended.
Parameter studies varying single parameters have long since been a common task for the engineer. Similarly, in small parameter spaces, design of experiment methods which systematically calculate single parameters and parameter combinations can be applied. With increasing dimension or nonlinearity of the parameter space, stochastic sampling strategies are preferred to generate the sample set.
An additional advantage of the stochastic sampling strategies compared to the design of experiments is, that they permit a statistical evaluation in the form of correlation and variation analysis of the sensitivities in the variation space. So, instable areas in the design space, hints on the variation potentials of the response variables, or global correlation structures showing which optimization variable has which influence on which response variable, etc. can be identified by means of sensitivity analysis. In this way, sensitivity studies may permit a reduction of the parameter space for subsequent optimization tasks. Moreover, the previous knowledge drawn from the sensitivity studies regarding the properties of the design space often is most helpful for an appropriate formulation of the constraints and the objective functions.

2.5 Pareto Optimization Strategies

It has been pointed out earlier that optimization problems may possess more than one objective criterion (multicriteria optimization).
When these criteria are not conflicting weight strategies can be used to combine several criteria to one objective function. Mathematically, assigning different weights to the single criteria should only have an influence on the convergence speed. In reality, even in cases where the objective criteria are not in conflict, different weights can indeed lead to different optimization results. The reasons for this may be that a local optimum has been found, the optimal area has a very small gradient, or the solution is not converged.
In case the criteria are in conflict, even from a mathematical point of view, there is no longer one single optimum. Instead of that, a number of possible compromise solutions exists. In this case, the weights of the different objective criteria is influencing the optimal compromise in a much stronger way.
As an alternative of calculation single compromise solution, Pareto optimization methods can be applied to determine the compromise set. By means of a posteriori weighting, they permit to choose the optimal compromise solution not before but after the optimization. Successful methods of Pareto optimization generally apply evolutionary methods such as Strength Evolutionary Pareto Algorithms [18].
Practically, a successful application is restricted to two- and three-dimensional multicriteria tasks. This is not so much due to a restriction of the underlying algorithms but to the failure of representation possibilities of the compromise solution. However, a proper representation is essential for a good a posteriori choice.
Taking into account that, instead of one optimal point, a set of optimal points is to be determined, it is hardly surprising that generally the computational expense is considerably increased. That leads to the recommendation that Pareto optimization strategies not be used to start the work on an optimization task. They should be applied at a work stage when the structure of the optimization problem (important input and response variables) is well known, and it can be taken for sure that two or three important objective criteria are in conflict.

3 Robustness Evaluation

As an introduction of the term „robustness evaluation“, the concept of stochastic computation methods shall very shortly be described.

3.1 Stochastic Computation Methods

The previous remarks were based on a purely deterministic concept. This concept does not take into account any uncertainties, i.e. scatters, and thus describes and analyzes but one possible state of the design, the boundary conditions and the loads, which is taken as the base for the evaluation. Speaking in terms of stochastic methods, this is equivalent to a mean value analysis (expected values). In case the scatter around the mean values of the input variables is small and the resulting scatter of important response variables are small, a deterministic analysis describes the task with sufficient accuracy. This does not hold for cases, when knowledge of resistance or loads is uncertain or highly scattering, or when a nonlinear scatter transmission behavior leads to response variables outside of tolerance areas. Here, the influence of scattering input variables has to be investigated.
Obviously, engineers have taken this into account even before the introduction of stochastic computation concepts, e.g. by means of variant studies or worst case scenarios. However, considering the increasingly complex computation models and the increasing demands on quality and reliability, and last but not least for economical reasons, a quantitative evaluation of the scatter by means of stochastic computation methods is required.
Such a quantitative evaluation necessitates basic knowledge of the in reality existing scatters of input variables, which is than described statistically by distribution functions. Then, the scatter of the result variables or the probability of events in the result spaced is assessed by means of stochastic and statistical computation methods.

Basically, the achievable level of accuracy, expressed here as the achievable level of probability in the result space, depends on the accuracy of the knowledge of the scattering input variables, in this case the knowledge of the distribution functions. That means, if events shall be excluded with a probability of 5%, relatively coarse knowledge of the uncertainties and input scatter may suffice. If a rare event (e.g. one of a million) shall be covered a much more detailed knowledge of the input scatter is needed.
Additionally, the chosen stochastic computation method must be capable of guaranteeing a desired accuracy of the estimation of the statistical variables. While the computation engineer is able to check the result of an optimization using his previous assessment standards, the statistical measures of

stochastic computations often cannot be checked in a conventional way. For these reasons it is recommendable to introduce stochastic analyses into virtual prototyping processes step by step beginning with robustness evaluations [16]. The results of robustness evaluations should be brought into agreement with experiences, and the underlying scatter transmission mechanisms should be understood. With robustness evaluation relatively coarse knowledge of assumed input scatters already permits the determination of their sensitivity towards the scatter of the result variables. At the same time, relatively frequent events, i.e. with a probability of occurrence in the order of magnitude of 5% or more, can be covered.

If reliability of less frequent events is desired methods of reliability analysis are necessary. These methods estimate small probabilities by means of approximation of the limit state function (FORM/SORM [8]), or using special sampling strategies (adaptive sampling [2], directional sampling [4]), or combining both (ISPUD [1]). As an economical use of reliability analysis methods often is restricted to small parameter spaces, robustness evaluations are often an essential preliminary stage to reduce this space. Note that, generally, for small probabilities of occurrence, a much higher computational expense will be necessary and the distribution functions of the scattering input variables have to known much more accurately.

3.2 Robustness Evaluations

Robustness evaluations evaluate scatters around mean values of input variables and their influence on the scatter of the result variables. Additionally to the estimation of the response scatter, robustness evaluations can serve to identify the scattering input variables that most contribute to the response scatter, and, as has already been mentioned, to cover relatively frequent events.
In contrast to sensitivity analyses as presented in section 2.4, the scatter of input variables is not described by an upper and lower boundary. In robustness analyses, this is done by distribution functions representing the assumed existing scatters around the mean values. A number of possible designs are generated by means of sampling methods. After determination of the sample set with CAE-solver runs, mean values, variation coefficients and distribution functions are estimated from the histograms of the response variables. These serve to quantitatively evaluate the scatter of the response variables.
The next step is the determination of the correlation structures. To this end, correlation coefficients are computed, and a principal component analysis (PCA) is performed. The linear correlation coefficients serve to evaluate the linear interconnection between the variations of two variables, i.e. of one input variable to one response variable. Contrary, a principal component analysis of the linear correlation matrix investigate correlations of higher dimensions, i.e. significant correlations between one group of input variables and a group of response variables. Additionally, the principal component analysis extracts the single scatter transmission mechanisms from the linear correlation structure. Hence, it permits to decompose the problem into several subspaces, if necessary.

The sample number should be chosen depending on the statistical measures to be interpreted. There are two main classes of sampling strategies: firstly Monte Carlo sampling, and secondly variants of Latin Hypercube sampling. These last are generally preferred because much less samples are needed to achieve comparable confidence interval of the statistical measures compared to Monte Carlo sampling. If Latin Hypercube sampling is used, the following approximate sample numbers may serve as a guideline: A minimum sample number of two times the number of random responses should be generated for a statistical evaluation of the individual variables, such as mean values, histograms, or variation coefficients. If a statistical coverage of the linear correlation structure is desired a minimum sample number of two times the sum of input and response variables is recommended. This is to be regarded as a starting value because the necessary sample numbers for a determination of the linear correlation coefficients can in some cases be considerably higher. Such cases may be problems with small dimensions (i.e. less than 20 input and response variables), and highly nonlinear or noisy problems. Therefore, convergence studies for important correlation coefficients are recommended. A more precise estimation of the sample number is possible if the order of magnitude of the correlation coefficients, the desired confidence level, and the desired tolerance interval is provided.

3.3 Combination of Optimization and Robustness Evaluation

When optimization and robustness evaluation are first introduced into the virtual prototyping process, an iterative execution of both is probably a good starting point. That means robustness evaluations are performed for optimized designs. It is possible that these have retroactive effects on the optimization

task in case robustness improvement become necessary. In the simplest case input scatter must be decreased or pre-optimized variables are shifted into more robust areas.
A combination of optimization and stochastic analysis may be preferable in order to solve the optimization task as completely as possible. That means that the optimization task additionally includes the scatter minimization of important response variables. Depending on the probability level, two classes of stochastic optimization [10] can be distinguished.

Firstly, variance minimization tasks can be solved using methods of the Robust Design Optimization. Generally, these combine response surface approximations of the deterministic optimization variables, of the scattering optimization variables, and of further scattering design variables, e.g. boundary conditions or loads.
Note that the demands made on the response surface approximation for the variance estimation are not the same as made on those possibly used for the optimization task. If the response space is too strongly smoothed, in case of doubt this leads to much smaller variance estimations on the response surface compared to the real design space. Hence, response surface approximations conserving local information are preferred, e.g. Krigin models, weighted radii, and moving least square approaches. Again, the practical application of response surface methods is limited by the relatively small possible variable number, aggravated by the fact that this number includes both optimization and scattering variables.
Thus, it is again obvious that conflicts between optimization and robustness should be known, and that often a reduction of the parameter spaces by means of sensitivity analyses and robustness evaluation may prove necessary. It is recommendable to perform robustness evaluations for final designs in the original design space. Thus, it can be verified, whether the variance estimation on the response surface corresponds to the variance in the original design space.

Secondly, if an optimization tasks includes probabilities of occurrence, this is called reliability-based optimization. Besides responses surface methods, gradient based methods, i.e. FORM/SORM, are available [3]. In some cases, both response surface methods and gradient based methods fail, because it is neither possible to determine appropriate gradients in both the optimization variable and the scattering variable space, nor is the number of optimization and scattering variables sufficiently small for a response surface use. For these cases combinations of stochastic methods and optimization methods can be applied, e.g. genetic optimization for the optimization variables and FORM for the stochastic variables. However, these methods lead to a further increase of the computational expense and frequently, they are only applicable to problems with small parameter numbers or tend to an exorbitant increase the number of required external CAE solver runs.

4 Applications

In the following, potential application areas of optimization methods and robustness evaluations shall be pointed out on the basis of two application examples. In booth examples the software OptiSLang [7] was used to perform optimization and robustness evaluation.

4.1 Optimization and Robustness Evaluation of an Occupant Restraint System

In the framework of a verification project of optimization methods and robustness evaluations, two parameters seat belt reaction forces and the vent hole size of an occupant restraint system have been optimized. The aim was to increase the number of stars that could be achieved in a Euro NCAP as well as in an US NCAP evaluation. The response space of the multi body simulations was noisy, so genetic optimization strategies as well adaptive response surface strategies were used. Both strategies yielded very similar “optima”, the adaptive response surface strategies showing better convergence, as was expected for a problem with an as little dimension.

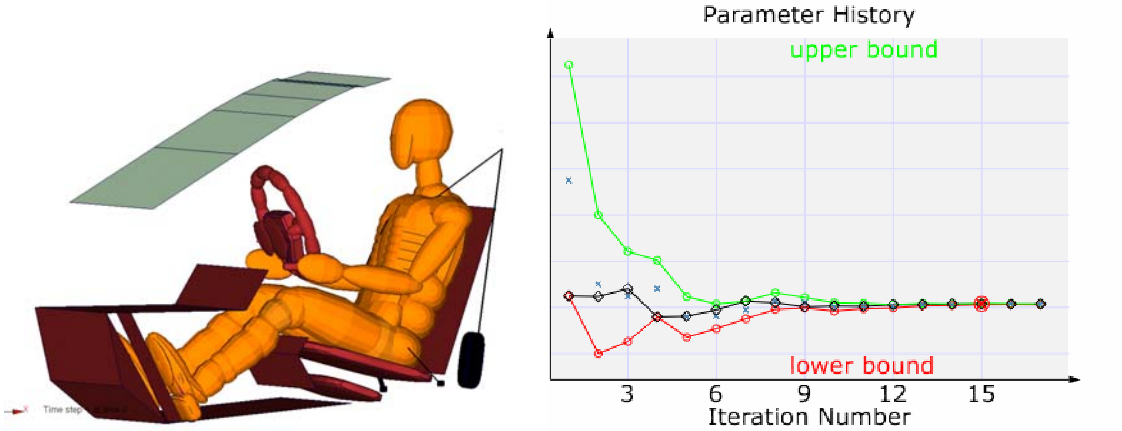

Fig. 1 Occupant restraint system and convergence of the adaptive response surface. After ten approximations, all parameters had converged.

A subsequent robustness evaluation of the optimized design took into account scattering of the optimized variables as well as of dummy position and of airbag characteristics. Robustness problems regarding the Euro NCAP evaluation were found. The input variables responsible for this could be identified from the correlation structures. Additionally, a conflict between the optimization of the seat belt force and the criteria following Euro NCAP and US NCAP was detected. By shifting the mean values of the optimized seat belt forces a design with a somewhat lower performance but with a better mean value and less scatter could be found.
Considering the small dimension of the optimization space with only three variables, a Pareto optimization would have been applicable to determine the compromise set of seat belt force adjustment regarding the achievable number of stars Euro NCAP and US NCAP. It proved more advantageous, though, to include additional variables of the occupant restraint system into the optimization. The conflict regarding the adjustment of the seat belt force could be almost completely dissolved in a parameter space including eleven variables. Moreover, the performance of the occupant restraint system could be increased further.

Fig. 2 Anthill plot seat belt force versus EURO NCAP evaluation. The two decreasing clusters illustrate two mechanisms of performance loss.

4.2 Robustness Evaluations of the Driving Comfort Behaviour

In the second example robustness evaluations have been performed for the driving comfort behaviour of car models [16]. The sensitivity of a great number of scattering variables has been investigated for different load cases. To describe the input scatter Gaussian distribution functions were estimated based on existing knowledge of possible scatter percentages around the mean values.
During the computation, the convergence of the correlation structures, i.e. the linear correlation matrix as well as the principal component structure, have been observed. The correlation structures could be regarded as reliably determined when an increasing number of computations did not yield significant changes. So, the necessary sample number was reached, and the statistical measures are reliable. Fortunately, very stable correlation structures could be observed and the robustness of all load cases could be proofed. In the different load cases only a few variables dominate the correlation and variation structures, and some few dominating nonlinearities of the transmission behaviour can be identified in the anthill plots. Then, robustness evaluations can reliably identify the most important scattering input variables. At the same time, robustness evaluations can give precious hints on the transmission ways of the scatters as well as on their optimization potential.

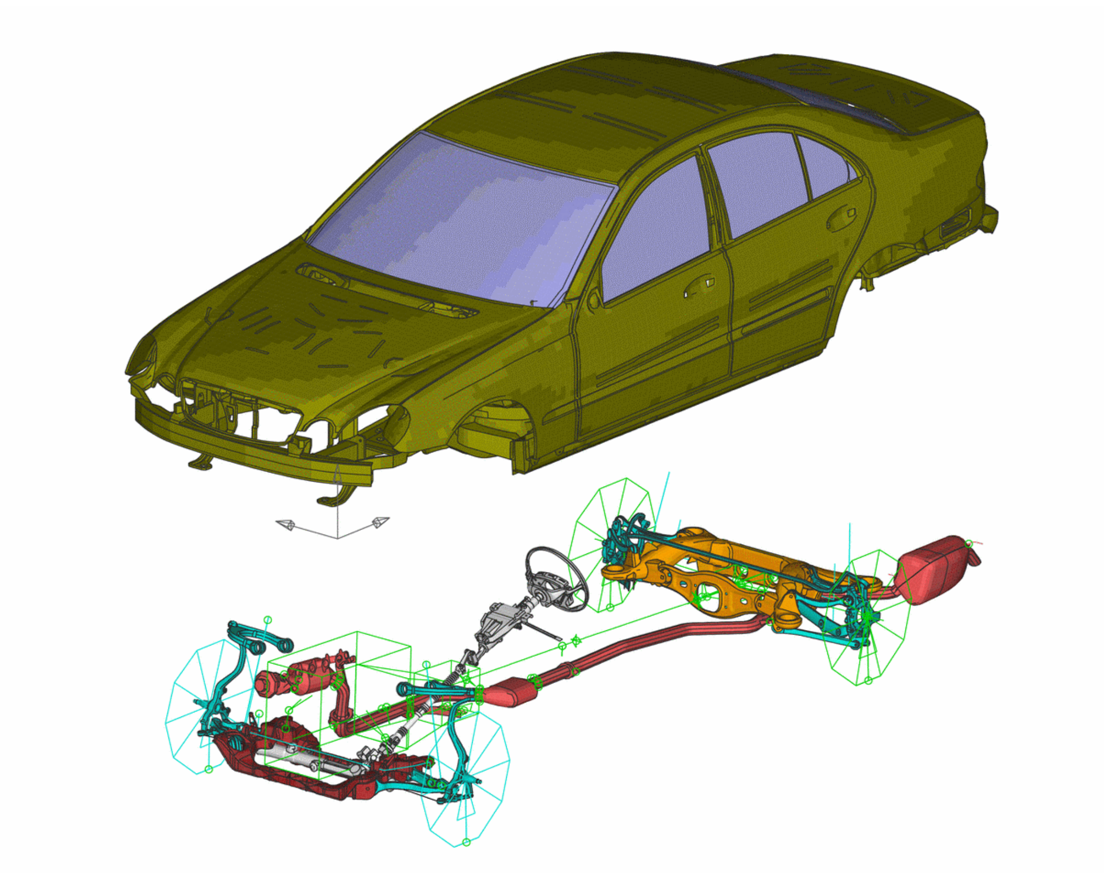

Fig. 3: Finite element vehicle model (explosion view)

In the here presented case, robustness evaluations have been executed for the sound pressure levels at four different positions in the passenger compartment considering 76 scattering stiffness of the suspension system. Two engine excitations were investigated. A total of 199 samples have been computed. In the first load case, all response variables show high correlation coefficients to only one scattering input variable (stiffness of transmission system bearing). That means the scattering of that variable dominates the whole scatter of all response variables.

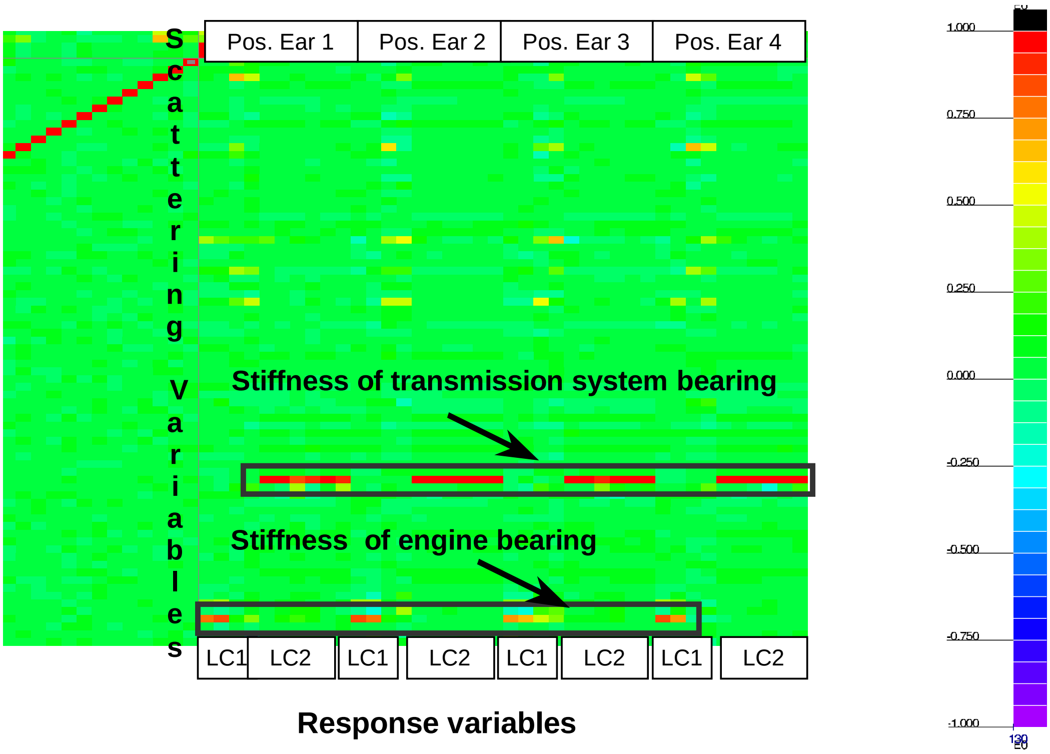

Fig. 4: Matrix of linear correlation

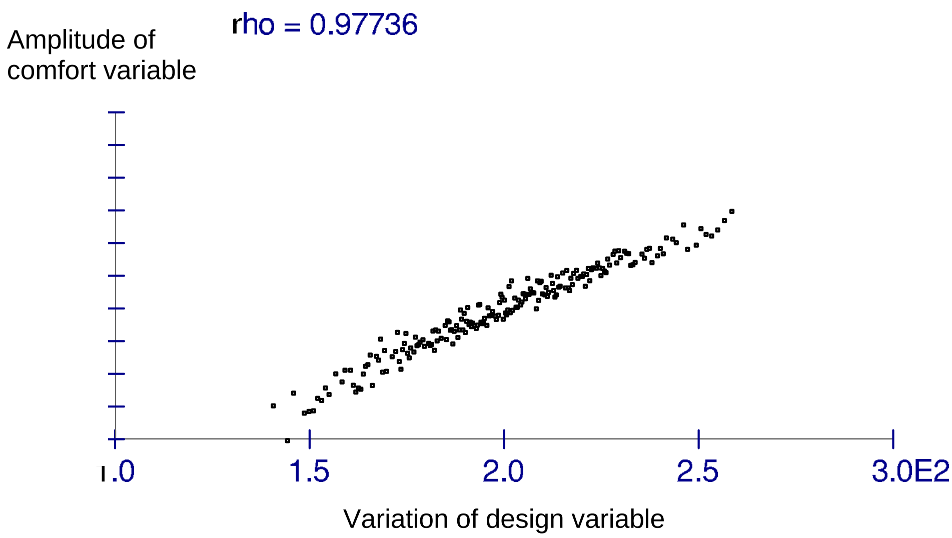

Fig. 5: Anthill plot stiffness of transmission system bearing versus response

In the second load case, also significant correlation coefficients of the response variables to the engine bearing stiffness are found. All in all, the scatters of the sound pressure levels are moderate and below undesired amplitudes. The two dominating input variables could be reliably determined by analyzing the correlation and variation structures. So, these two load cases can be influenced significantly by a variation of these few characteristics.

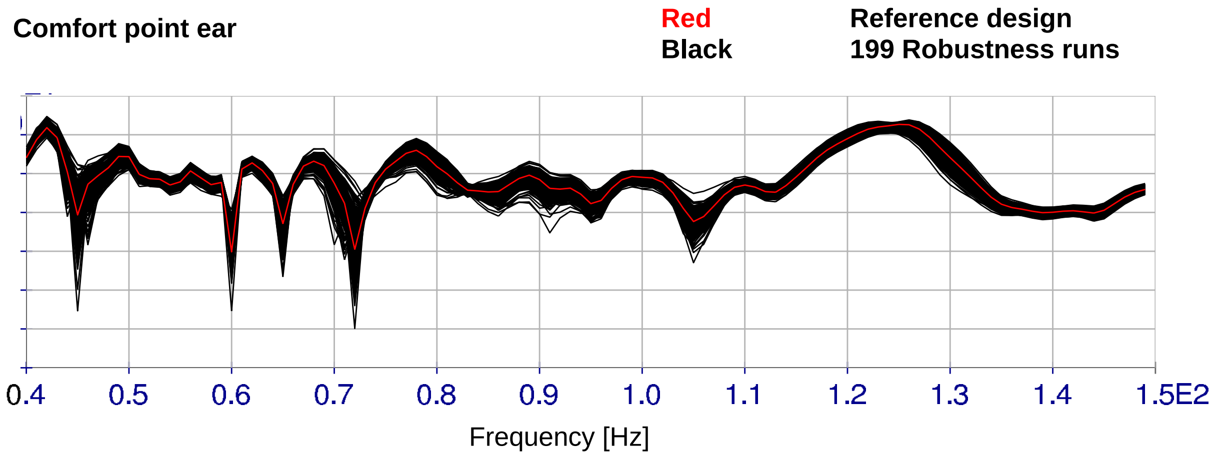

Fig. 6: Scatter visualisation of sound pressure level

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