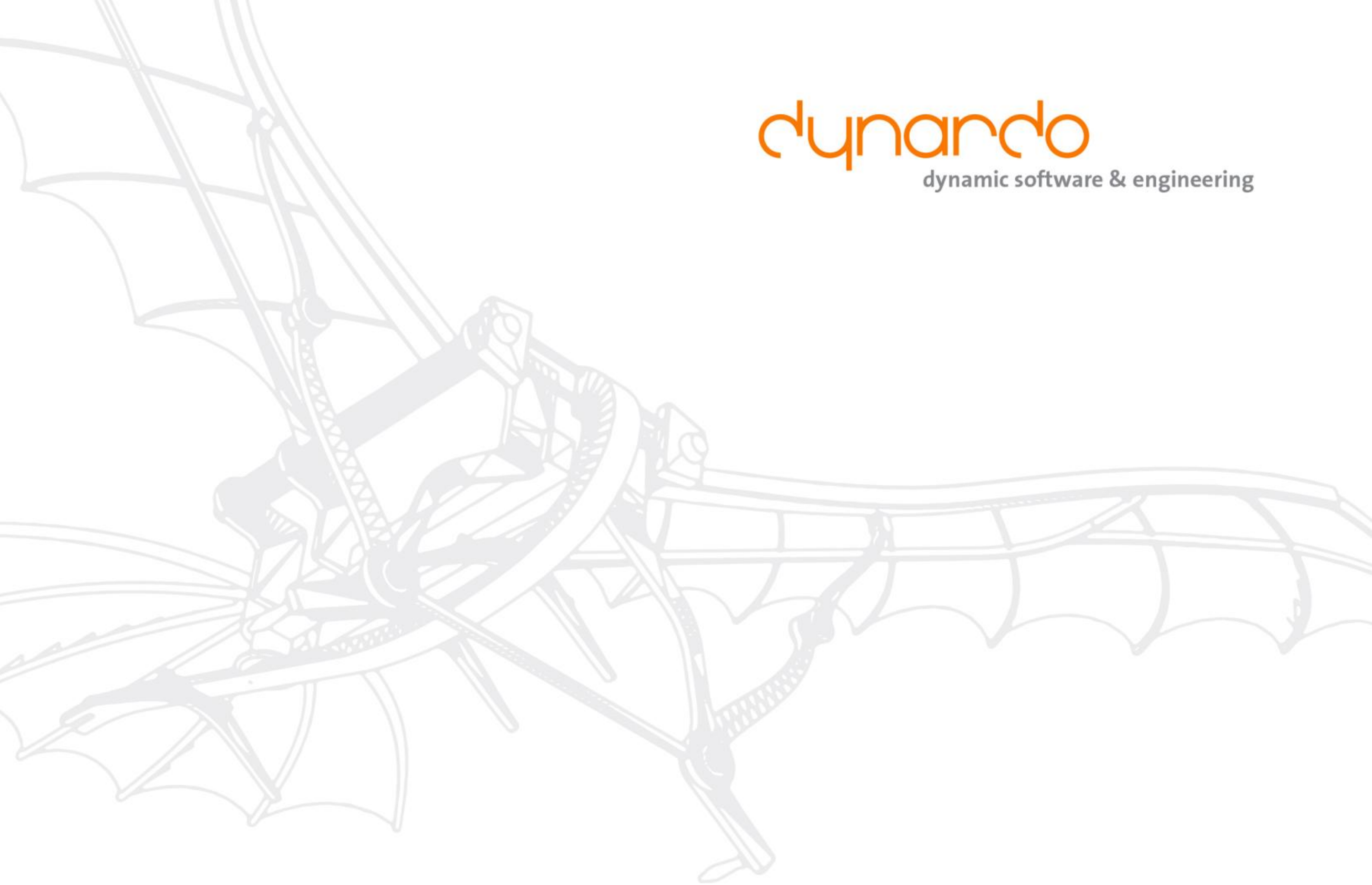

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Lectures

Optimizing of hydraulic fracturing procedure using numerical simulation

Johannes Will

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“Optimizing of hydraulic fracturing procedure using numerical simulation”

Johannes Will

DYNARDO – Dynamic Software and Engineering GmbH, Weimar

Abstract

This paper presents a modeling approach for three-dimensional simulation of hydraulic fracturing of jointed rock. Dynardo developed a 3D-hydraulic fracturing simulator and coupled the simulator with leading edge calibration and optimization algorithms to offers the software base to optimize gas production with computer simulation. With the integrated approach, an effective 3D numerical reservoir simulator is available. The input parameters of the numerical model from the best available well log and reservoir data must be calibrated from the direct diagnostics measurements to assign the correct level of importance to various mechanisms of the hydraulic fracturing. Only with this degree of diagnostic characterization of the hydraulic fracture it is possible to truly understand the controls on the evolution of the fracture network geometry of hydraulic fractures. With this approach, a predictive model for the design of hydraulic fracture can be developed for a reservoir. By use of the predictive, calibrated model, the hydraulic fracturing design can be optimized to provide the required conductivity the hydraulic fracture design to maximize the gas production.

For three dimensional modeling, analysis and post processing FEM simulator **ANSYS®** is used. The second key component is the material library of jointed rock **multiPlas**, which simulates the fracturing process. The third key component is the optimization tool **optiSLang**, which is used for calibration of the reservoir simulator and finally used for optimization of gas production. The simulator was set up and verified in Barnett Shale production area. The reservoir model was characterized with 7 rock layers and up to 4 sets of joints per rock layer. With the help of optiSLang a sensitivity study of the 200 physical and fracturing design parameters was performed to identify and calibrate the important physical parameters. The calibrated mode is then used to predict and optimize the gas production rates.

Keywords: Calibration, Optimization, hydraulic fracturing, jointed rock, Barnett shale, ANSYS, multiPlas, optiSLang

1 Introduction

Natural gas and oil reservoirs are often located in layered rock formations with low permeability like the Barnett Shale in Texas, US. In order to mine the reservoirs, the stimulation of the reservoir by increase the permeability becomes necessary for a profitable oil or gas production. Hydraulic fracturing is used routinely in the gas and oil industry to create a large network of permeable fractures which connects the production well with the greatest possible volume of reservoir rock for profitable gas production rates. Goal of the optimization of the hydraulic fracturing procedure is maximizing the fractured reservoir rock volume which results in the maximization of gas production. With today's available measurement technologies like seismic mapping, the hydraulic fracturing performance of a well can be measured. Even with the availability of diagnostic technologies it is important to understand that simply measuring the dimension of a hydraulic fracture treatment is still a post processing picture. It does not predict how a different design of hydraulic fracturing in the same well would have grown. Neither does it predict how the same design would behave in a different well. So far very often, expensive in situ trial and error approaches are used to identify the most successful hydraulic fracturing design.

Of course design and optimization of hydraulic fracturing using computer simulation is very promising and the only way to mine difficult reservoirs profitably.

Hydraulic fracture design models are used today as a prediction tool for the optimization of hydraulic fracturing. However, they all suffer from incomplete understanding of the mechanics of propagation of a hydraulic fracture in a formation. Therefore, the two technologies must be combined such that direct physical measurements of growth of hydraulic fracture can be coupled to a 3D simulator of hydraulic fracturing. The input parameters of the model - from the best available well log and reservoir information - must be calibrated from the direct diagnostics measurements to assign the correct level of importance to various mechanisms of the containment of the hydraulic fracturing. Only with this degree of diagnostic characterization of the hydraulic fracture and coupled modeling is it possible to truly understand the controls on the evolution of the geometry of hydraulic fractures. With this integrated approach, a predictive model for the design of hydraulic fracture can be developed for a reservoir. By use of the predictive, calibrated model, the simulation can be optimized to provide the required conductivity and maximum effective length of the hydraulic fracture to maximize the productive economics.

The base for a successful computer based simulation for productive use is the ability to calculate the fractured volume with sufficient accuracy and efficiency. Especially in cases three dimensional effects have to be taken into account, the common commercial software tools in oil and gas business reaches their bounds. There is a need for an effective modeling and simulation software tool to run three dimensional problems effectively. Effective 3D hydraulic fracturing simulation does not only mean that one hydraulic fracturing process can be modeled and simulated. To optimize the hydraulic fracturing design we need a parametric model and the simulation of one design has to be highly effective because during calibration and optimization the calculation of a couple of hundred different designs becomes necessary. Therefore the balance between accuracy and efficiency is the challenge to use computer based simulation of hydraulic fracturing for productive use.

1.1 Do we really have a three dimensional problem?

When the reservoir show significant anisotropic in situ stress or strength conditions true three dimensional modeling become urgent. In case of layered reservoirs, the layer impact in fracture growth also needs to be included in the simulation. In case of jointed rock reservoirs, like shale's the strength anisotropies of the joint system are often one of the dominant factors for fracture growth and therefore need to be included in the constitutive material equations for the mechanical behavior of jointed rock. Research in the last decades pointed out that there was, so far, no commercial simulator available which can simulate hydraulic fracturing in three dimensional, layered and jointed rock formations. A state of art report [2] pointed out a list to improve commercial simulation tools:

- If fracture growth is dominated by the three-dimensional insitu stress state and the three dimensional joint system then constitutive models for fracture growth orientation and fracture complexity from three dimensional (3D) state-of-stress taking into account 3D rock mechanical characteristics and natural fracture orientation are necessary
- Fracture growth in composite layers, along and through layer interfaces, is not well understood and is not well captured in most current models
- Main physical parameters of fracture growth performance are insitu stress, insitu pore pressure, fracture closure stress and mechanical rock parameters (young modulus, joint system, joint strength parameter).

Therefore, a 3D simulation model has to represent at least three main physical phenomena:

- reopening of the joint system
- fluid flow in the joint system
- permeability increase of the stimulated rock volume

1.2 How to deal with the uncertainties in reservoir parameters?

After having obtained a suitable hydraulic fracture simulator the next challenge is to deal with the uncertainty of the reservoir parameters like material parameter, layer dimensions or in situ conditions. Even from the best available reservoir and well test data a lot of parameters have large uncertainties or have to be taken from

the literature or experience. Here, efficient ways of performing sensitivity analysis to identify the most important input parameters and to calibrate the numerical models to experimental data will become urgent. For that task Dynardo's optimization tool optiSLang [1] is used for calibration of the reservoir simulator and for optimization of gas production. With optiSLang we perform numerical sensitivity studies using optimized stochastic sampling strategies (Latin Hypercube class) which scan the design space followed by statistical measurements of importance of individual model parameters. In that paper, we here restrict our selves here to introducing only that statistical measurement (Coefficient of Importance- CoI) a little bit more in detail which is used to select the most important uncertain model parameters. The calibration and optimization process will then concentrate on that important parameter only.
The coefficient of importance (CoI) measures the amount of response variation which results from the input variation of single uncertain parameter. The base of measurement of importance is different correlation measurements like linear, quadratic or monotonic non linear correlation (Spearman rank order correlation). Finally, this correlation hypothesis is used to predict the importance which shows the highest correlation coefficients.
For more details of algorithms we refer to software documentation [1] or some papers of CAE-based calibration and optimization.

At the documented application, in chapter 3, at Barnett field we will have nearly 200 uncertain parameters to deal with. Here, we face the challenge of identifying the few most important uncertain or unknown parameters out of a large range of parameters. Key inputs to the sensitivity study are lower and upper bounds to every uncertain parameter. Within those bounds, optiSLang creates a sampling of possible design configurations, and an automatic process of evaluation the sample set is used to generate and evaluate every design configuration. optiSLang process automation includes the automatic update of reservoir geometry, automatic brick meshing and automatic calculation of the non linear load history analysis of hydraulic fracturing event.

With that approach, we can automatically identify the key parameters, verify and increase our understanding of the main physical phenomena and set up the base for successful calibration of the simulator.

2 3D hydraulic fracturing simulator

For effective three dimensional parametric modeling, analysis and post processing in chapter 3, the FEM simulator ANSYS® is used. Here, some important background of the numerical simulation is introduced.

First, the reservoir geometry is generated and an automatic brick meshing is performed. In order to be able to introduce the anisotropic in situ stress condition the single reservoir layers are initialized separately and bound together with bonded contact. The non linear load history analysis then starts with the in situ stress and pore pressure initialization, followed by a coupled transient fluid flow mechanical analysis. For the matter of integration the simulation into a calibration process, the complete simulation flow including geometry generation, meshing and the hydraulic fracturing simulation is automated and all uncertainties needs to be parameterized (see flow chart at figure 1). The parameter of the jointed rock contains deformation, strength and flow conditions. The geometric parametric contains uncertainty in layer thickness as well parametric well position. The hydraulic fracturing process is simulated with fluid flow analysis for the pore pressure change in the rock mass as a result of water injection and coupled with mechanical analysis for fracture growth. For consideration of fluid flow in the joints as well fracturing and reopening of the joints a smeared continuum approach is used.

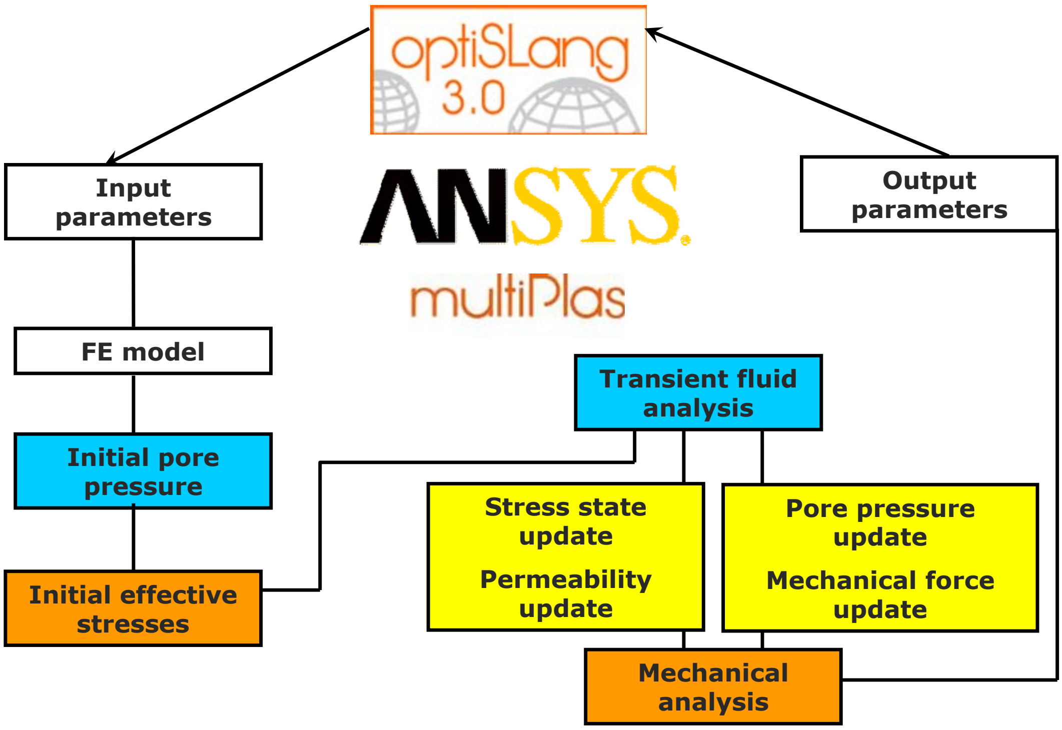

Figure 1 Process flow chart of the calculation of multiple design configurations

It is noted that powerful capabilities in parametric modeling, mapped meshing, contact analysis, the numerical efficiency of non linear mechanical and fluid analysis and the user programmable post processing are essential for industrial application. Effective 3D hydraulic fracturing simulation does not only mean that one hydraulic fracturing process can be modeled and simulated. In order to optimize the hydraulic fracturing design we need a parametric model and the simulation of one design has to be highly effective because during calibration and optimization the calculation of a couple of hundred different designs becomes necessary.

2.1 Mechanical analysis

The smeared continuum approach uses a deformation tensor of the jointed rock mass which is composed from stiffness of rock and multiple joint systems. The stress strain conditions of jointed rock is given by

$$\sigma = D_{RM}\left(\varepsilon^{tot} - \varepsilon^{pl}\right) \tag{1}$$

where

^ D_{RM} jointed rock deformation matrix resulting, resulting from rock and joint stiffness $D_{RM} = D_R + D_J$

The constitutive material models of elastic plastic behavior use effective stress:

$$\sigma_{tot} = \sigma_{eff} + np \tag{2}$$

where

$n = \left(1\ 1\ 1\ 0\ 0\ 0\right)^T$

p pore pressure

2.2 Fluid flow analysis

Because of the very low permeability of the in situ initial jointed rock, fluid flow will occur mainly in the initiated or reopened joint system. The resultant permeability is defined with an anisotropic permeability tensor of the smeared rock mass. Using Darcy's laminar flow approach in a smeared continuum having a joint system the flow velocity in the direction of a joint set result to

$$v_{FT} = \frac{q}{A} = k_{TF}\frac{2a_i}{d} I \tag{3}$$

respectively transformed into global coordinate system $\{v_{FT}\} = K\{I\}$ (4)

where

v_{FT}	flow velocity
K	anisotropic permeability tensor of jointed rock mass
$2a_i$	joint thickness / joint opening
A	cross section
d	joint frequency

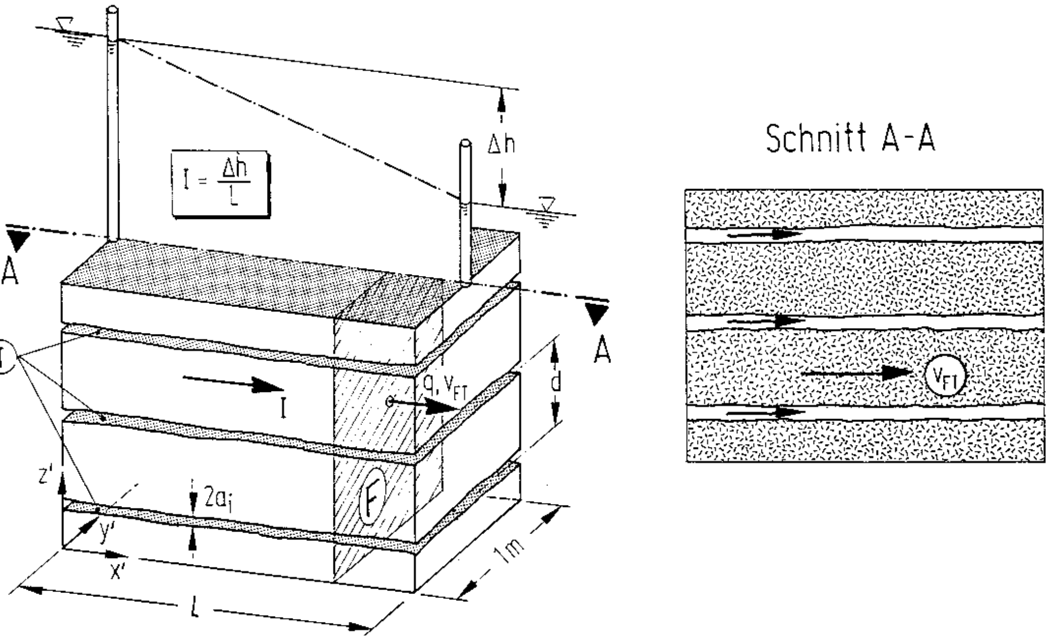

Figure 2: filter velocity in case of one set of joints (picture from Wittke, [6])

2.2.1 Coupling of fluid flow and mechanical analysis

After generating the in situ stress conditions in the mechanical domain and the in situ pore pressure conditions a coupled load history analysis is performed. The hydraulic fracturing event starts with a transient fluid flow step which affects the initial pore pressure field. After every fluid time increments the incremental change in mechanical forces from pore pressure chance will be introduced in the mechanical analysis. The forces on every discretization point of the smeared continuum are computed from the pore pressure gradients

$$\left\{F^{Str}\right\} = \gamma_W \, V_G \, \{I\} \qquad (5)$$

where

F^{Str}	force vector
I	gradients of pore pressure
γ_W	water density
V_G	related volume

2.2.2 Coupling of mechanical and fluid flow analysis

At the mechanical step, a non linear elastic plastic analysis is performed and in case of violating strength limits, fracture initialization and fracture growth occurs. The main physical effect in hydraulic fracturing is the significant increase in permeability in case of fracture growth. In the case of plastic strain increments, the anisotropic permeability tensor of the jointed rock mass is updated with a non linear relation to the anisotropic plastic strain tensor. This relationship is very important to the fracture growth and has to be calibrated to the reservoir conditions.

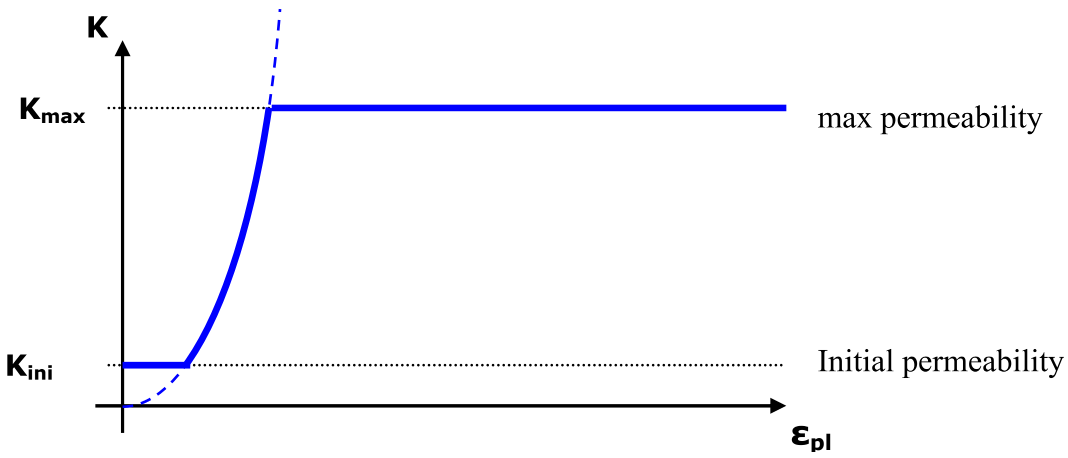

Figure 3: Relationship between plastic strain and permeability

2.3 Constitutive material models for fracture growth

An important key component is Dynardo's library of constitutive models for jointed rock **multiPlas**, which simulates the three dimensional fracturing of the jointed rock mass during the fracture growth. multiPlas uses an elastic plastic smeared volume approach, having constitutive models for the "intact rock" and up to four sets of joints. These mean joints are not modeled discrete and strength conditions of intact rock and rock joints are checked at every discretization points in parallel. The material models in multiPlas, using rate-independent plasticity. The material models are characterized by the irreversible strain that occurs once yield criteria are violated. It is assumed that the total strain vector can be divided into an elastic and a plastic component.

$$\{\varepsilon\}^{tot} = \{\varepsilon\}^{el} + \{\varepsilon\}^{pl} \tag{7}$$

where

$\{\varepsilon\}^{el}$ – elastic strain vector
$\{\varepsilon\}^{pl}$ – plastic strain vector

The yield criterion limits the stress space.

$$F(\{\sigma\},\kappa) \le 0 \tag{8}$$

where

$\{\sigma\}$ - stress vector
κ - hardening or softening parameter

If the stress computed using elastic deformation matrix exceeds the yield criteria (F>0), then plastic straining occur. Plastic strains will be computed by flow rule

$$d\varepsilon^{pl} = \lambda \frac{\partial Q}{\partial \sigma} \tag{9}$$

where

λ - plastic multiplier (which determines the amount of plastic straining)
Q - plastic potential (which determines the direction of plastic straining)

The plastic strains reduce the stress state so that it satisfies the yield criterion (F=0). By using associated flow rules, the plastic potential is equal the yield criterion and the vector of plastic strains is arranged perpendicularly to the yield surface.

$$Q = F \tag{10}$$

By using non-associated flow rules

$$Q \neq F \tag{11}$$

effects that are known from experiments like dilatancy can be controlled more realistically. Introducing the dilatancy effects is particular important in case of fracture shear failure which is one of the most important fractures growth conditions in hydraulic fracturing.

The hardening / softening function $\Omega(\kappa)$ describes the movement of the initial yield surface. For the strain driven hardening/softening equations in multiPlas the scalar value κ serves as a weighting factor for plastic strain.

$$d\kappa = d\kappa(\varepsilon^{pl}) = d\varepsilon_{eq}^{pl} \tag{12}$$

Please note that because of lack of experimental data for softening behavior of jointed rock mass simple assumption of strength degradation from in situ strength to residual strength are often used.

The numerical implementation in multiPlas is carried out by using the return-mapping method. The return mapping procedure is used at the integration point level for the local iterative stress relaxation. It consists of two steps:

1. elastic predictor step:

$$\sigma_{i+1}^{trial} = \sigma_i^* + D\, d\varepsilon_{i+1}^{tot} \tag{13}$$

2. plastic corrector step:

$$\frac{d\sigma}{d\lambda} = -D\,\frac{\partial Q}{\partial \sigma} \tag{14}$$

2.3.1 Dealing with Multi-surface plasticity

Of course the jointed rock mass contain multiple yield criteria, a tension and shear criteria for the intact rock and every set of joints. The consideration of different failure mechanisms is possible by a yield surface built up from several yield criteria. In the stress space, a non-smooth multi-surface yield criterion will then develop. The numerical implementation in multiPlas is carried out using an effective and consistent numerical treatment of multi-surface plasticity [4].

The elastic plastic algorithm has to deal with singularities at intersections from different yield criteria (e.g. F1 to F2 as represented in figure 4).

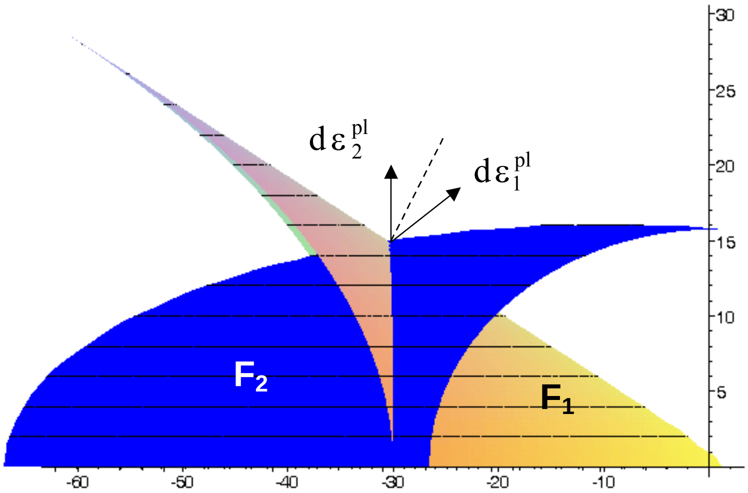

Figure 4 Intersection between the two flow criteria F1 and F2

The consistent numerical treatment of the resulting multi-surface plasticity must deal with the possibility that many yield criteria are active simultaneously. This leads to a system of n=j equations:

$$\left\{\frac{\partial F_n}{\partial \sigma}\right\}^T D\, d\varepsilon = \sum_{j=1}^{Set\ of\ active\ YC} \left[\left\{\frac{\partial F_n}{\partial \sigma}\right\}^T D\,\frac{\partial Q_j}{\partial \sigma} - \frac{\partial F_n}{\partial \kappa_n}\frac{\partial \kappa_n}{\partial \lambda_j}\right] d\lambda_j \tag{15}$$

The solution of this system of equations generates the stress return to flow criteria or within the intersection of flow criteria’s. Contrary to single surface plasticity, exceeding the flow criterion is no longer a sufficient criterion for activity of the plastic multiplier for each active yield criterion. An activity criterion needs to be checked.

$$d\lambda_j \geq 0 \tag{16}$$

This secures that the stress return within the intersection is reasonable from a physical point of view.

3 Application – hydraulic fracturing simulation in Barnett Shale reservoir

3.1 Modeling

The simulator was set up and verified for gas production wells in Barnett Shale production area. For the here documented well, core measurements of pay zone (fig.8), image logs for rock properties (fig.5 & 8) of layers, step down test and seismic fracture mapping measurements (fig.10) were available and used to build and calibrate the reservoir model. The reservoir model was characterized with 7 rock layers and 4 sets of joints per rock layer.

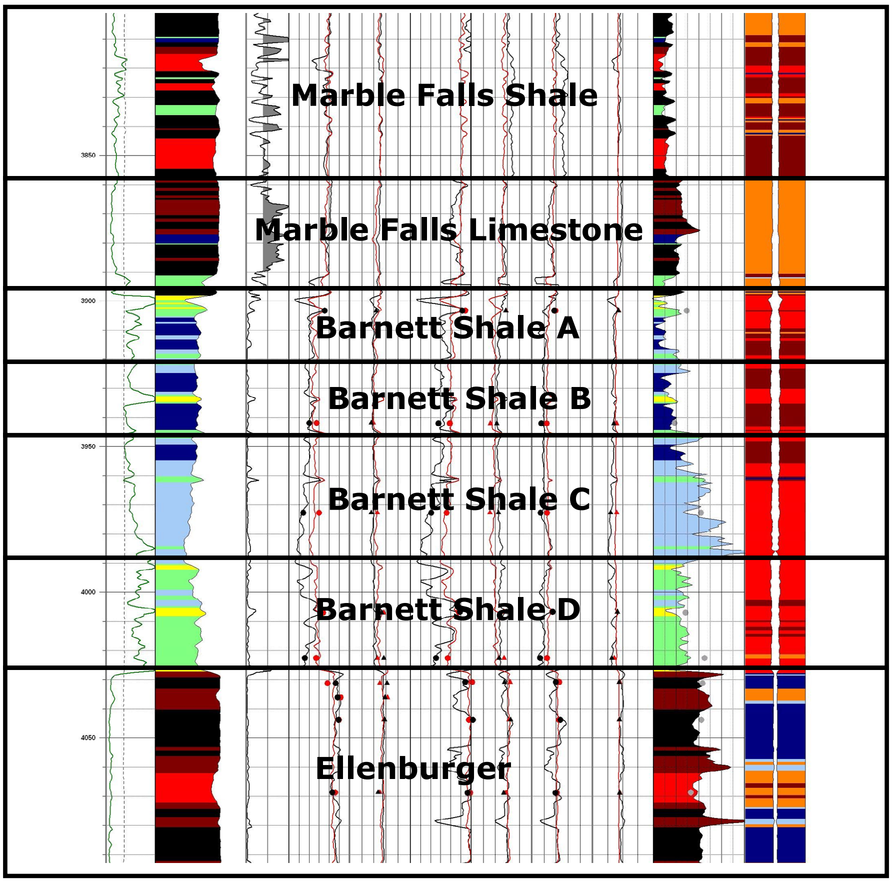

Figure 5 Image log measurement data of the 7 rock layers

The parametric numerical model included geometric parameters like layer thickness and location, material parameters of deformation, strength and hydraulic properties as well as the hydraulic fracturing design parameter.

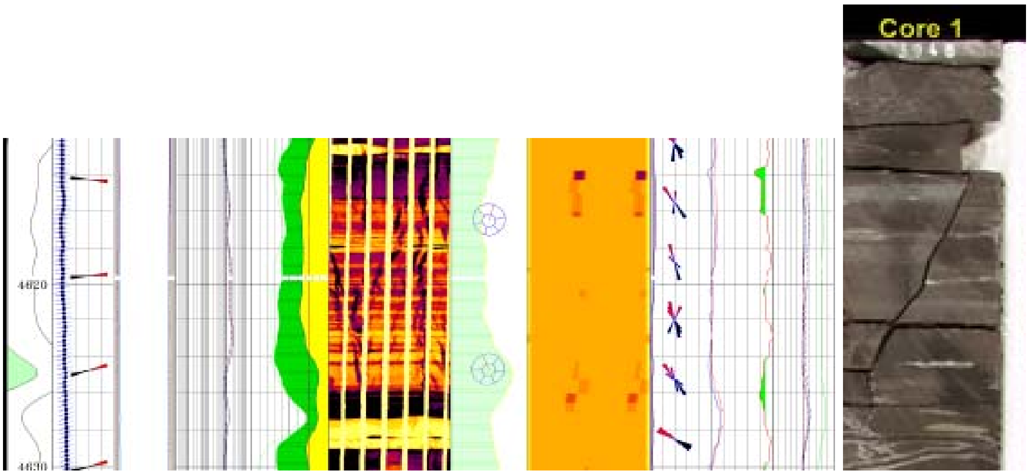

Figure 6 right: image log of well will fracture characteristics
left: core picture with bedding plane and joint

The most significant anisotropy is the bedding plane of the shale. Furthermore, location and frequency of fractures and their characteristics (open, closed, well induced) are identified from well logs and core measurements. From that data in addition to the bedding plane, 3 sets of joints for every rock layer are derived.

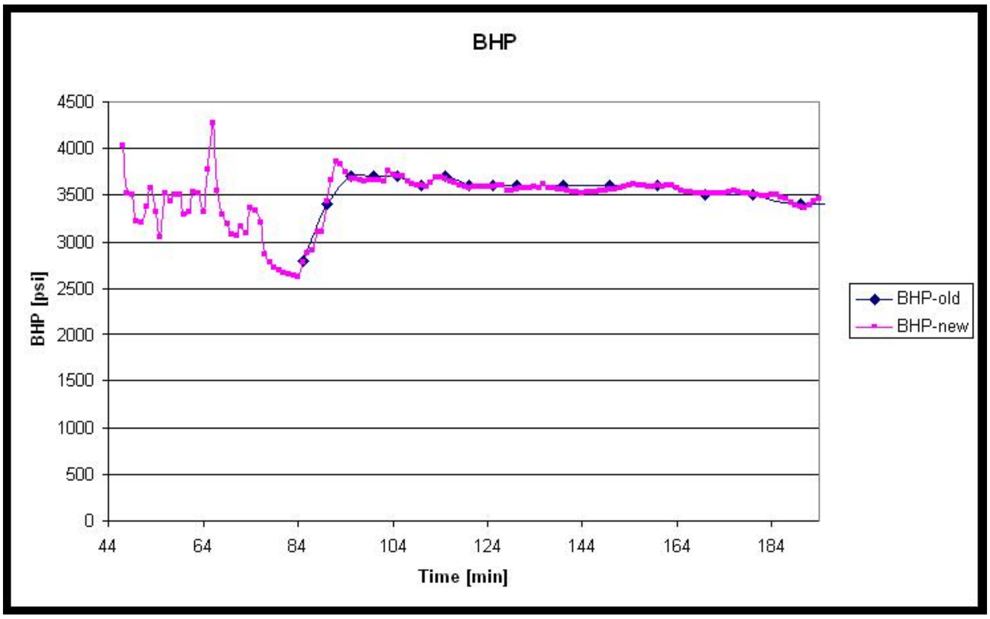

Figure 7 bottom hole pressure at perforation from hydraulic fracturing of stage 1

The hydraulic fracturing design contains 5 stages of pressuring ranging from 100 to 200 minutes of pressure design (fig.7). In the numerical fluid flow simulation, the bottom hole pressure at well perforation is the initial loading for every time step increment.

At first, an analysis using mean values of all parameters is performed. The non linear load history analysis contains an initialization of a mechanical and hydraulic model and the coupled fluid flow and mechanical analysis of the hydraulic fracturing event. The main result of the simulation is the three dimensional body of fractured rock represented by the volume which shows plastic strain resulting from fracture growth (fig.8).

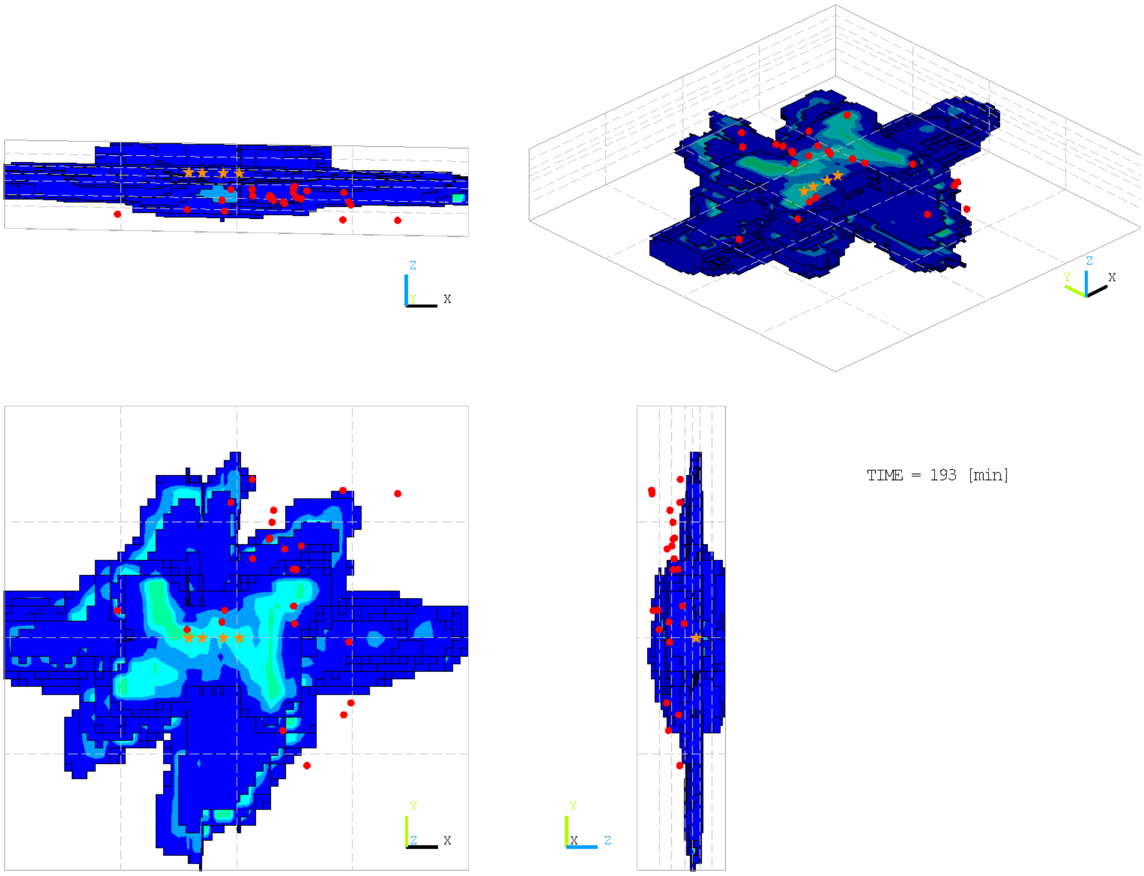

Figure 8 Stimulated rock body after 193 minutes of pressuring (blue: stimulated rock mass from simulation, red: seismic mapping measurements)

An analysis of the history of activities of failure criteria (fig.9) clearly shows that fracture growth is dominated by shear failure in the joint system and the amount of other failure (intact rock failure or tension failure) is negligible. Because of the high strength of intact rock compared to the relatively low strength of joints, this phenomenon is expected. In reality, some cracking of intact rock will occur to connect the joint system but that phenomenon will not occur in the smeared volume approach. Note that in the smeared volume approach, plastic failure and as a result plastic activity occur only at the fracture frontier. Inside the fractured rock, body pore pressure differences disappear and stresses relax (dark blue).

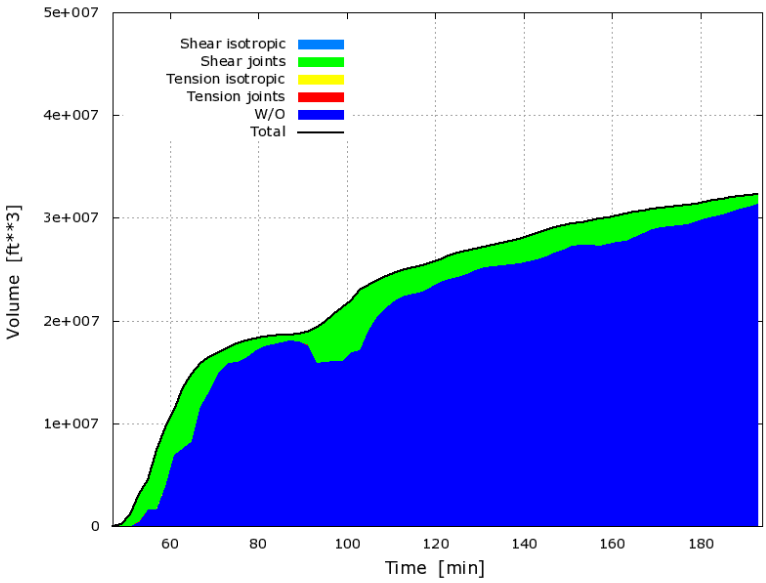

Figure 9 history of activity

3.2 Calibration and Optimization

In the calibration process, first key parameters of the modeling approach like appropriate mesh size, hydraulic time steps, maximum permeability of jointed rock mass or energy dissipation at pore pressure frontier are calibrated to measurement results of step down test and seismic fracture measurements (fig.10). The step down test gives the fracture initiation pressure level and the seismic fracture measurements the cracking dimensions (height, wide and length) in time.

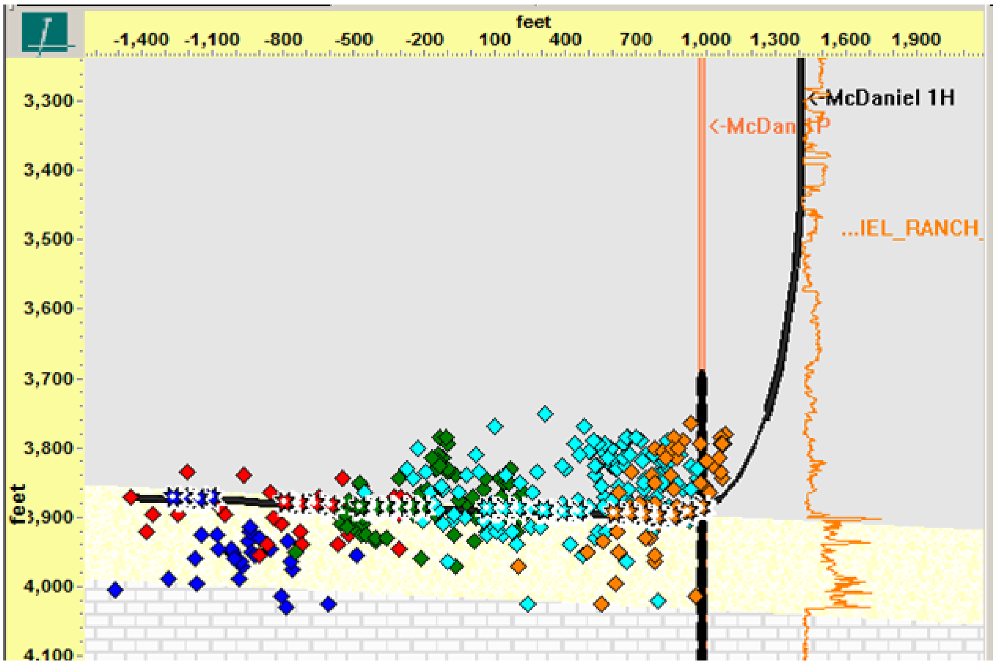

Figure 10 Seismic fracture mapping measurement

Then with the help of optiSLang, a sensitivity study of the 200 geometry, material and fracturing design parameters is performed to identify the important parameters. The most important input to the sensitivity analysis are the lower and upper bounds of the uncertain parameters. Of course, the bounds are not known in detail. The bounds are derived from all available test data from the reservoir, from experience and from the literature. The goal of the sensitivity study is to identify the important model parameter but also to verify the estimated range of uncertain parameter. A first scan of the parameter space did show that some estimated lower parameter bounds of strength values of the joint system are unrealistic because the in situ stress condition could not be initialized with such low joint strength parameters. After adjusting some parameter bounds, we designed a Latin Hypercube sampling to scan the design space of the sensitivity study. The importance of the variables is calculated with statistical measurements of correlation coefficients and coefficient of importance (CoI). After 162 design evaluations we stopped the sensitivity study because the correlation coefficients of the most important model parameter converged above a significance limit (fig.11).

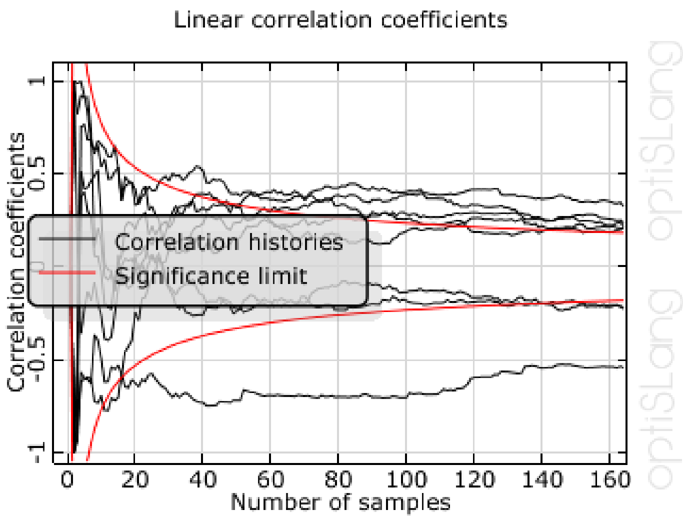

Figure 11 Convergence plot of the most important model parameter

The identified correlations and mechanisms of the important input parameters due to the reservoir response are validated and finally the model is calibrated to the fracture mapping measurements in the sub domain of important model parameter using optiSLang optimization algorithms.

Measurements of importance show that for the total volume of stimulated rock, the most important parameters are the in situ stress conditions (k0-values), the variation of bottom hole pressure height and total pressuring time (fig12). This is expected because the in situ stress conditions define the fracture closure pressure and the bottom hole pressure regime defines the energy to crack the rock mass. The strength parameter of the joint system, especially the friction angle and dilatancy angle, and some hydraulic fracturing design parameter become important for the variation of fracture height, wide or length. This simply means that the total volume is driven by the total energy loading of the hydraulic fracturing and the direction of fracture growth is driven by the joint system but can be influenced by the hydraulic fracturing design.

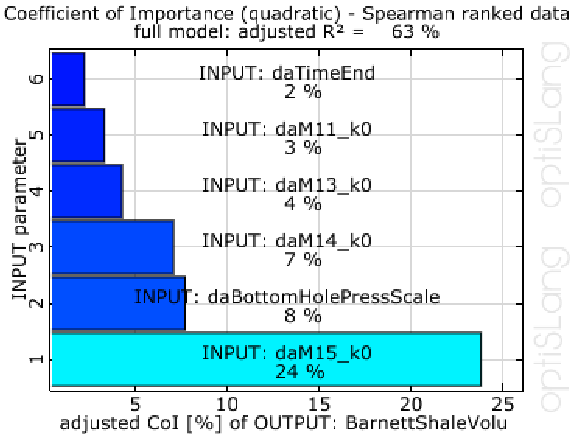

Figure 12 Coefficients of importance shows the influence of input parameters to the variation of total volume of stimulated rock mass

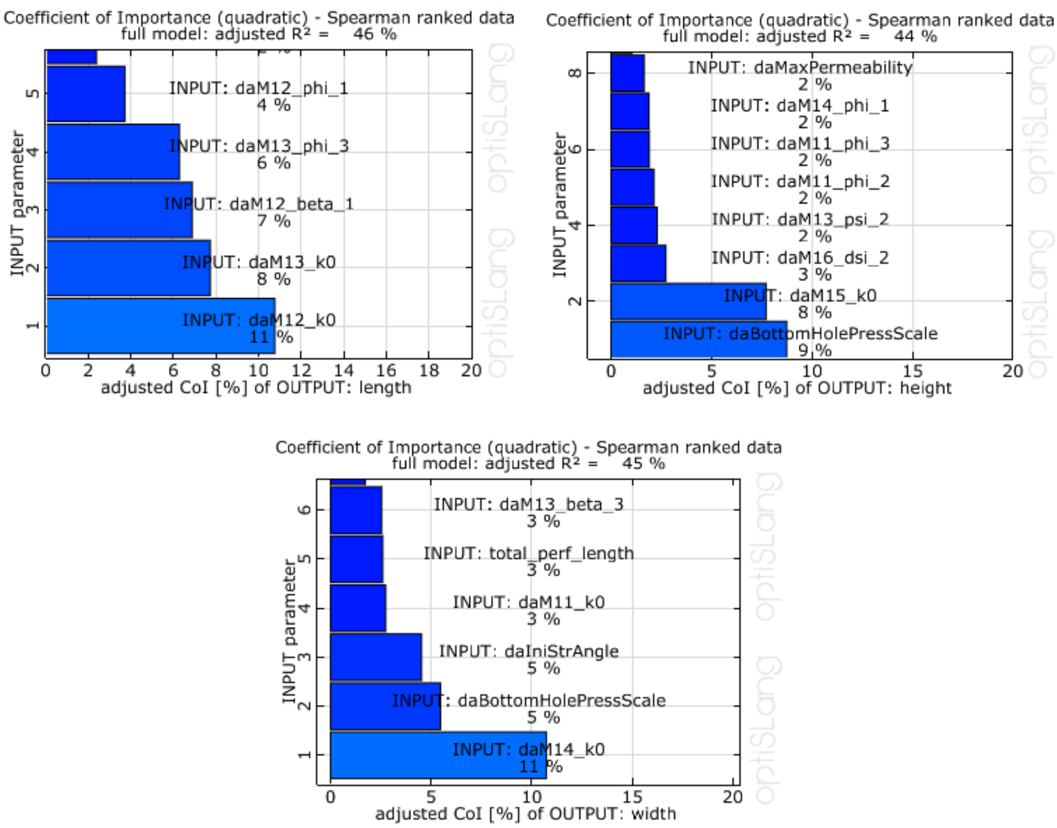

Figure 13 Coefficient of Importance of maximum length and height of stimulated rock volume

In the calibration process, the in situ stress conditions and important joint strength parameter are updated until time and location of seismic fracture measurements show reasonable agreement with the simulation results. The calibrated model is then used to predict the gas production rate of the well. Here, correlation between stimulated volume and 6 month gas production at Barnett field was used. The positive surprise was that the predicted gas production rate from the calibrated model showed very good agreement to the real production rate and much better agreement than the estimated production rates with the help of seismic fracture measurements only. The reason for the better agreement was the better calculation of the volume of the complex 3D body of stimulated rock in the layered reservoir which was not possible using the seismic fracture mapping measurement only.

After understanding of correlations between hydraulic fracturing design parameters and resulting stimulated rock volume, the optimization potential was investigated and it could be shown that an increase of gas production of 25% was possible with just an optimized well position in the reservoir.

4 Summary and Outlook

Using ANSYS and multiPlas, a 3-dimensional, parametric hydraulic fracturing simulator was set up which could be calibrated to seismic fracture measurements and to real production rates of gas wells. High numerical efficiency of the simulator is an absolute must for the calibration process of hydraulic fracturing simulation using 3D models. The simulator did calculate one stage of hydraulic fracturing running for 200 minutes at 2008 up-to-date dual core workstation in 12 to 16 hours. To reach that numerical efficiency, in numerical modeling we concentrated only on first order physical effects, namely the interaction between fracture growth and permeability growth using smeared volume approach and multi-surface plasticity in combination with simplified flow conditions in the smeared rock mass. Of course, very nonlinear flow effects will occur near well borne and discrete and local crack propagation will effect the fracture growth. Yet for the final extension of the stimulated reservoir volume it seems to be that the overall anisotropic stress and strength conditions of the rock mass play the dominant role.

Regardless of which simulator will be used for the hydraulic fracturing simulation, the optiSLang functionality for sensitivity analysis and calibration with dealing of a large amount of uncertainties can be linked to any simulator and will be a key functionality to understand and calibrate complex reservoir models. optiSLang correlation analysis identifies the main reservoir parameter to additionally calibrate the mechanism of how the fracturing design parameter effects the fracture growth. This understanding is urgent for optimizing the gas production rate, especially in layered reservoirs.

The advanced functionality of ANSYS software suite for mechanical and fluid analysis supports the introduction of further physical effects. Enhancements for thermal hydraulic mechanical coupling introducing the effect of elastic, thermal and creep strains to the permeability tensors are in preparation. This physical effects will be important for (hydraulic) fracturing analysis when thermal effects have to be taken into account, when fracturing of nuclear waste disposals needs to be investigated or in geothermal projects using hydraulic fracturing to stimulate permeability in jointed rock reservoirs.

References

[1] optiSLang – the optimizing Structural Language version 3.0.1, DYNARDO GmbH, Weimar, 2009, www.dynardo.de

[2] Weijers, L.: Evaluation of Oil Industry Stimulation Practices for Engineered Geothermal Systems, Report DOE-PS36-04GO94001 from Pinnacle Technologies ,October 2007

[3] Will, J.: The Calibration of Measurement and Simulation as Optimization Problem, Proceeding NAFEMS Seminar Virtual Testing – Simulationsverfahren als integrierter Baustein einer effizienten Produktentwicklung“” April 2006, Wiesbaden, Germany, www.dynardo.de

[4] Will, J.: Variation Analysis as Contribution to the Assurance of Reliable Prognoses in Virtual Product Development, Proceeding NAFEMS Seminar “Reliable Use of Numerical Methods in Upfront Simulations” March 2007, Wiesbaden, Germany, www.dynardo.de

[5] Will, J.: Dissertation Beitrag zur Standsicherheitsberechnung im geklüfteten Fels in der Kontinuums- und Diskontinuumsmechanik unter Verwendung impliziter und expliziter Berechnungsstrategien, Berichte des Institutes für Strukturmechanik 2/99, Bauhaus Universität Weimar, 1999

[6] Wittke, W.: Rock Mechanics, Theory and Application with Case Histories, ISBN/EAN: 3540527192

[7] multiPlas – elastoplastic material models for ANSYS, version 6.0, DYNARDO GmbH, Weimar, 2009, www.dynardo.de