dynardo
dynamic software & engineering

DVM Workshop Zuverlässigkeit und Probabilistik
München, November 2017

Varianzbasierte Robustheits-optimierung unter Pareto Kriterien

Veit Bayer
Thomas Most

Dynardo GmbH
Weimar

Robustness Evaluation

How to Define the Robustness of a Design?

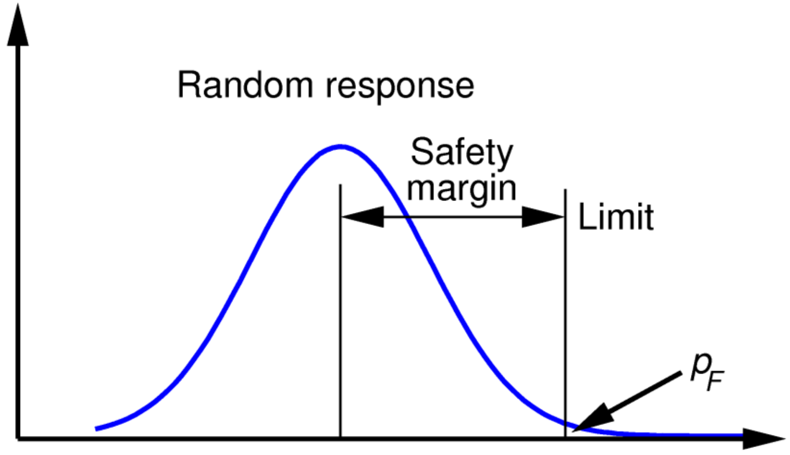
- **Intuitively:** The performance of a robust design is largely unaffected by random perturbations

- **Variance indicator:** The coefficient of variation (CV) of the objective function and/or constraint values is smaller than the CV of the input variables

- **Sigma level:** The interval mean+/- sigma level does not reach an undesired performance
(e.g. design for six-sigma)

- **Probability indicator:** The probability of reaching undesired performance is smaller than an acceptable value

Robustness in terms of constraints



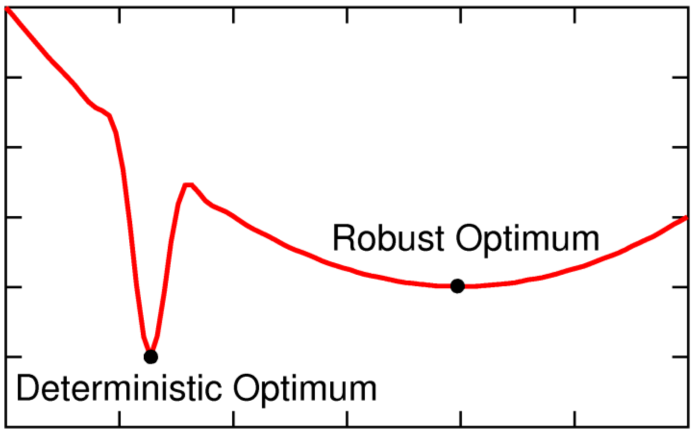
- Safety margin (sigma level) of one or more responses *y*:

$$(y_{limit} - \mu_Y)/\sigma_Y \geq a$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective

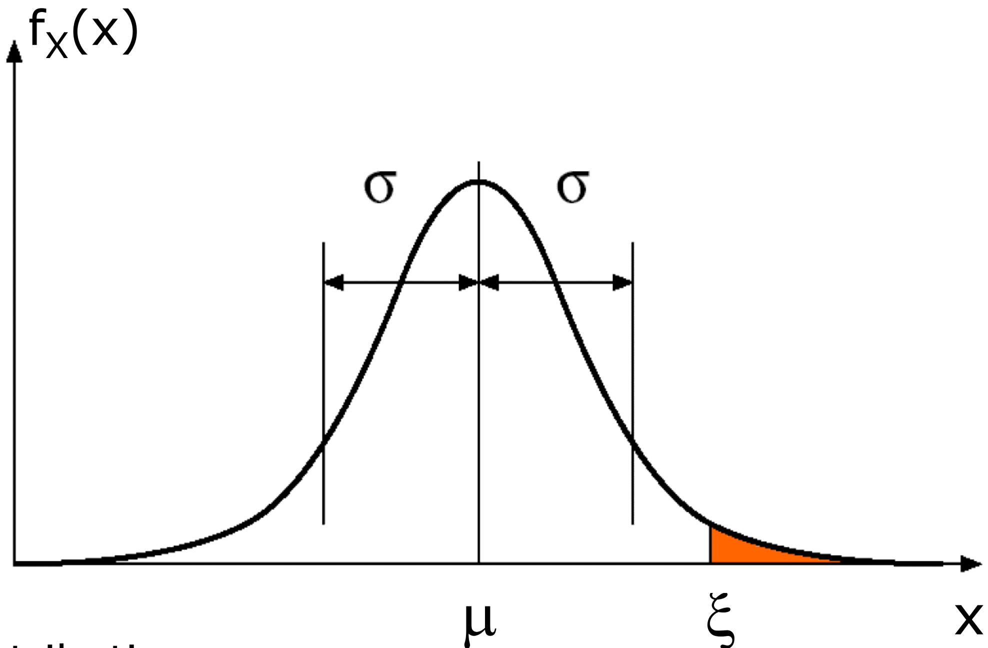


- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Exceedance Probability

- Probability of reaching values above a limit



- Gaussian distribution:

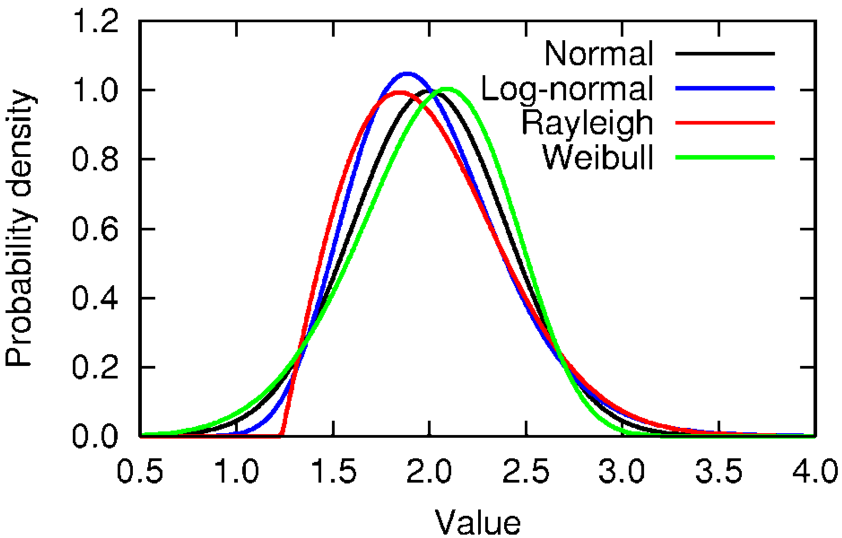
$$P_\xi = P[X \geq \xi]$$

ξ	μ	$\mu + \sigma$	$\mu + 2\sigma$	$\mu + 3\sigma$	$\mu + 4\sigma$	$\mu + 5\sigma$
P_ξ	$5.0 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-2}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-5}$	$2.9 \cdot 10^{-7}$

Sigma Level vs. Exceedance Probability

- The sigma level can be used to estimate the probability of exceeding a certain response limit
- Since the distribution type of the response is generally unknown, this estimate may be very inaccurate for small probabilities (sigma levels larger than 3)
- The sigma level deals with single limit values, whereas the failure probability quantifies the event, that any of several limits is exceeded

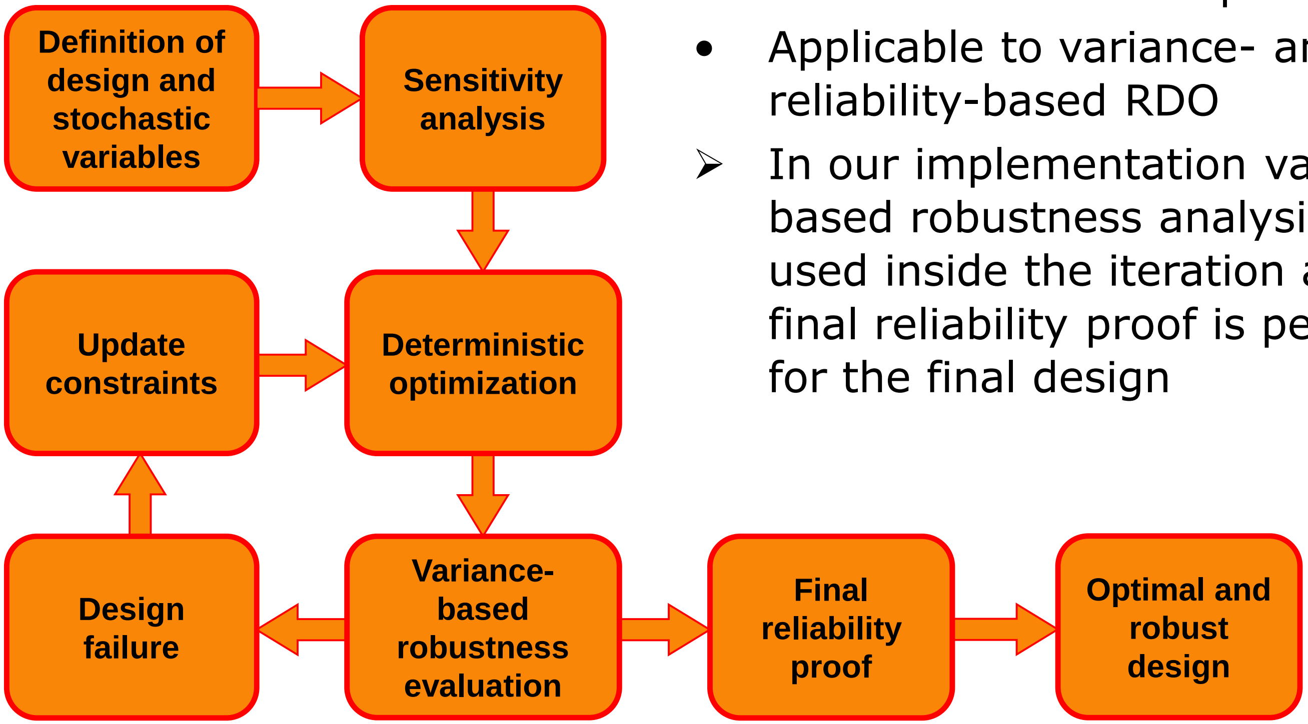
➢ Reliability analysis should be applied to proof the required safety level



Distribution	Required sigma level (CV=20%)		
	p_F = 10^{-2}	p_F = 10^{-3}	p_F = 10^{-6}
Normal	2.32	3.09	4.75
Log-normal	2.77	4.04	7.57
Rayleigh	2.72	3.76	6.11
Weibull	2.03	2.54	3.49

Robust Design Optimization

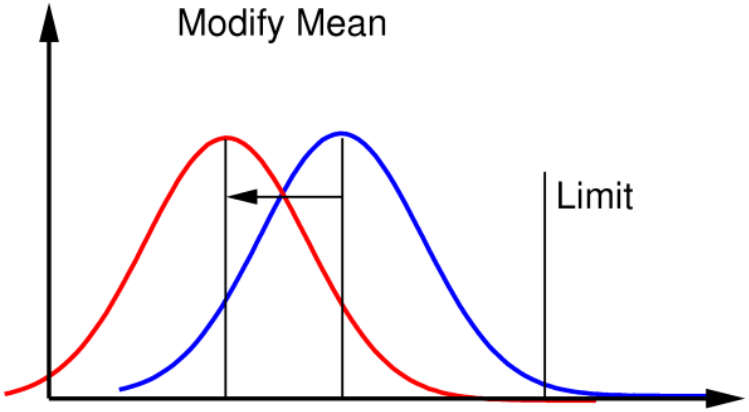
Iterative Robust Design Optimization



- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety factors are adjusted for the critical model responses
- Applicable to variance- and reliability-based RDO
- ➢ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design

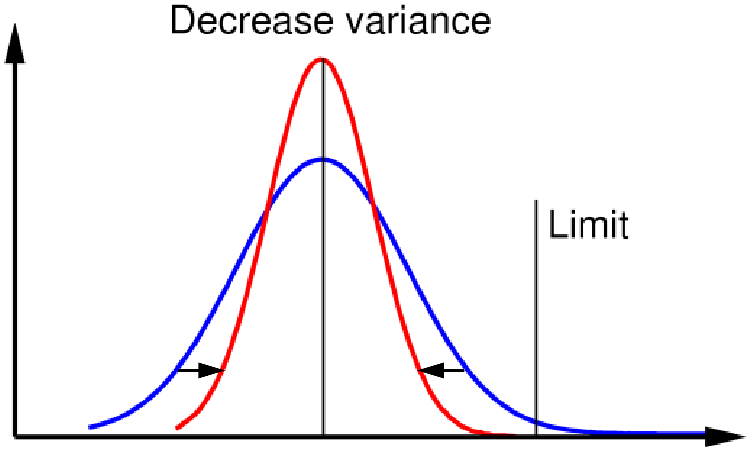
How to Optimize Robustness

Adapt safety margin (as constraint)

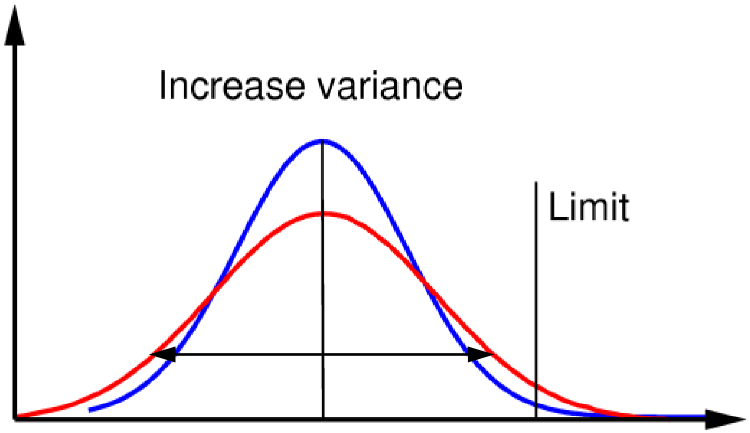


- Shift of mean response from deterministic optimum is possible
- Scatter may vary with different location of the mean

Adapt scatter (in input space)

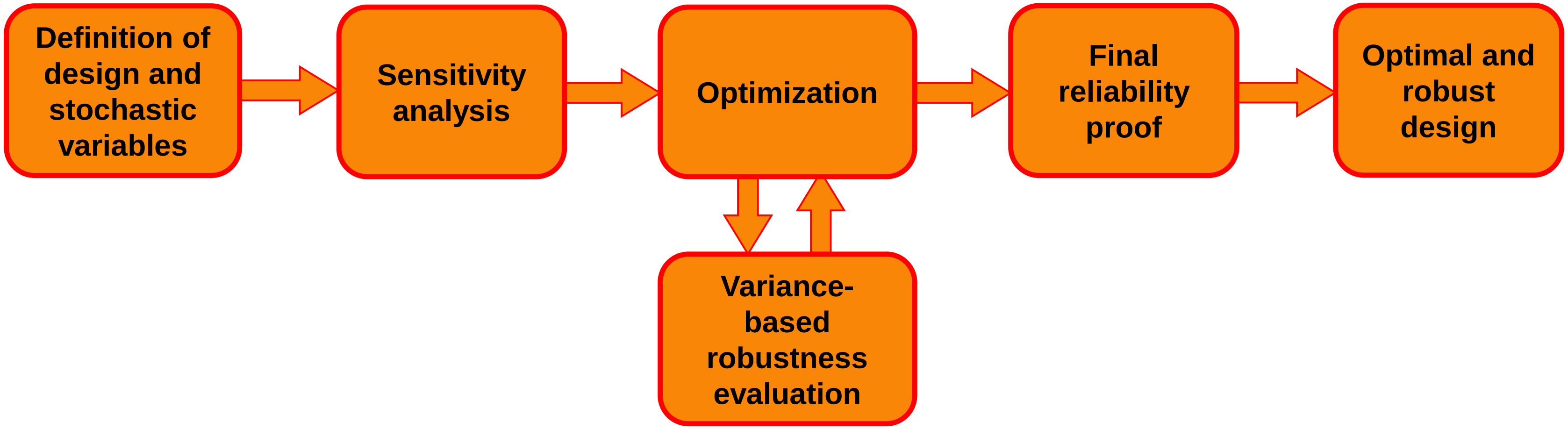


… but avoid "overdesign"



Coupled Robust Design Optimization

- Fully coupled optimization and robustness/reliability analysis
- For each optimization (nominal) design the robustness/reliability analysis is performed
- Applicable to variance-, reliability- and Taguchi-based RDO
- ➢ Our efficient implementation uses small sample variance-based robustness measures during the optimization and a final (more accurate) reliability proof
- ➢ But still the procedure is often not applicable to complex CAE models



Example: Steel Hook

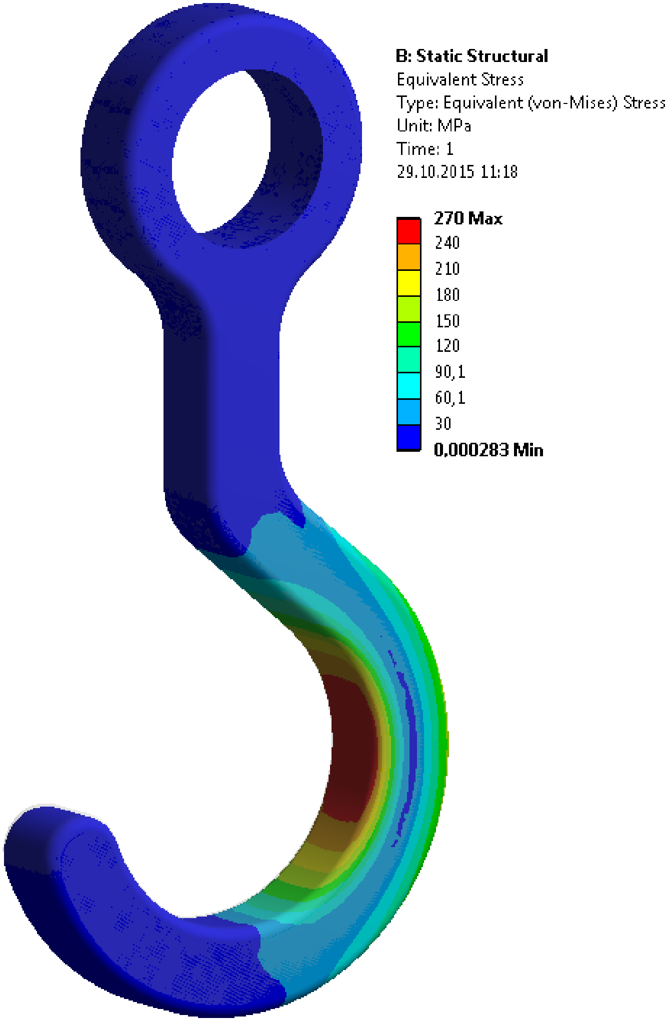
The Robust Design Optimization Task

Deterministic Optimization

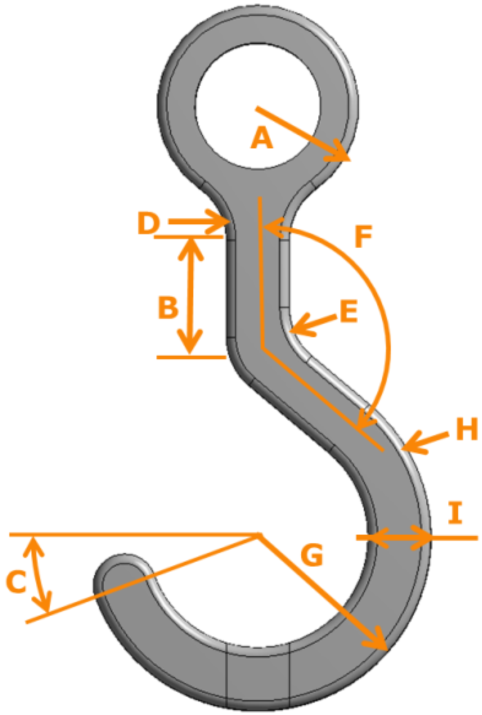
- Minimize the mass
- The maximum stress should not exceed 300 MPa
- 10 geometry parameters are varied for the design variation

Robustness requirement

- Proof for the optimal design that the failure stress limit is not exceeded with a 4.5 sigma safety margin
- 15 scattering parameters are considered (geometry and material properties and the load components)



Optimization Parameters; Initial Design



A	Outer_Diameter	28-35 mm
B	Connection_Length	20-50 mm
C	Opening_Angle	10-30 °
D	Upper_Blend_Radius	18-22 mm
E	Lower_Blend_Radius	18-22 mm
F	Connection_Angle	120-150 °
G	Lower_Radius	45-55 mm
H	Fillet_Radius	2-4 mm
I	Thickness	15-25 mm
	Depth	15-25 mm

Initial nominal values

- Mass 1100 g
- Maximum stress 270 MPa
- Slipping height 28 mm
- Opening width 64 mm

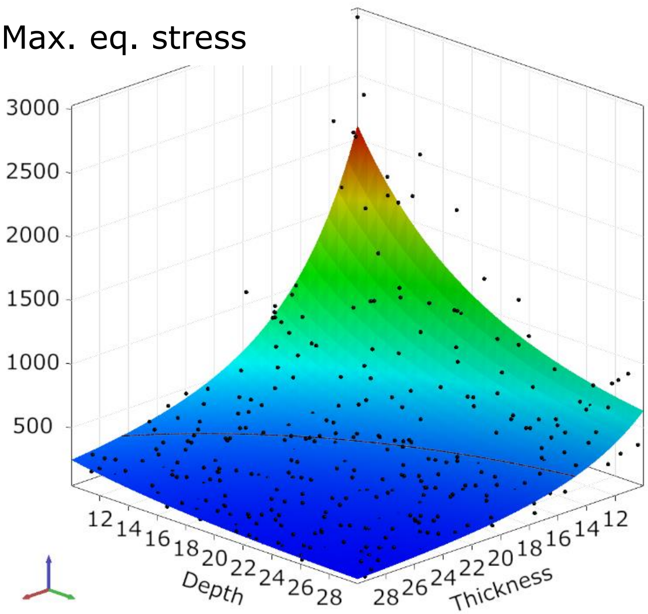
Random parameters

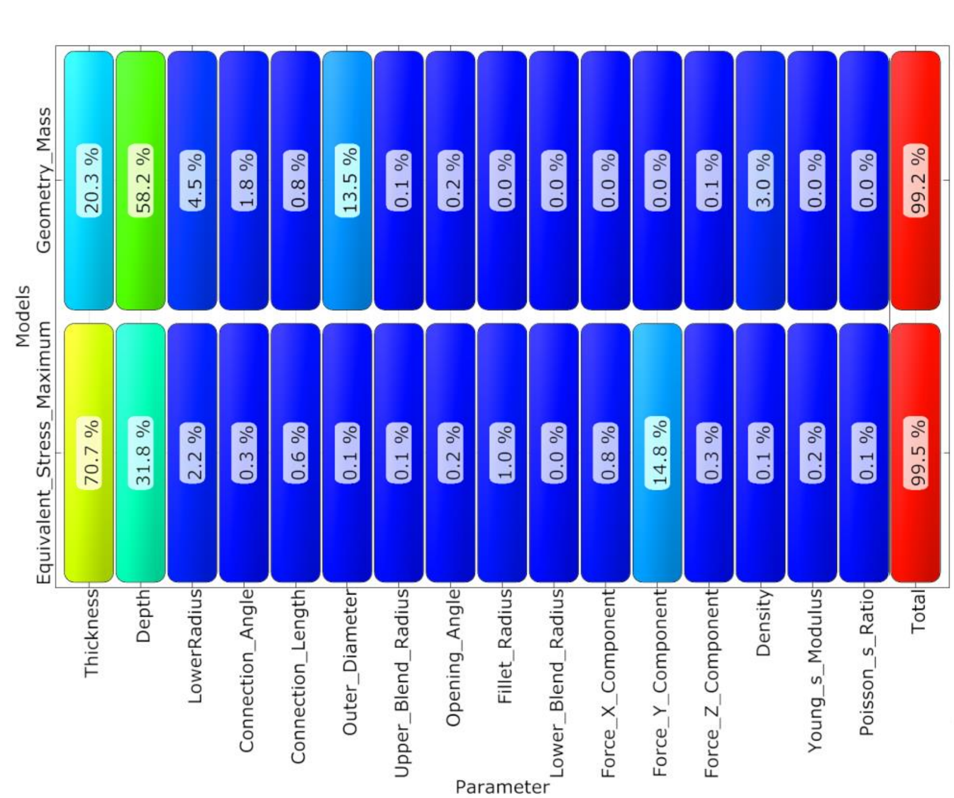
	Distribution	Mean value	Standard deviation
Outer diameter	normal	nominal	1 mm
Connection length	normal	nominal	1 mm
Opening angle	normal	nominal	2°
Upper blend radius	normal	nominal	1 mm
Lower blend radius	normal	nominal	1 mm
Connection angle	normal	nominal	2°
Lower radius	normal	nominal	1 mm
Fillet radius	normal	nominal	0.2 mm
Thickness	normal	nominal	1 mm
Depth	normal	nominal	1 mm
Young’s modulus	log-normal	2e11 N/m^2	1e10 N/m^2
Poisson’s ratio	log-normal	0.3	0.015
Density	log-normal	7850 kg/m^3	157 kg/m^3
Force x-direction	normal	0 N	100 N
Force y-direction	normal	6000 N	600 N
Force z-direction	normal	0 N	1.

Metamodel of Optimal Prognosis in RDO Space

- Sampling in RDO space
  Design parameters: Range ± 5σ
  Stochastic variables: Mean ± 5σ
- Advanced Latin Hypercube Sampling with 500 samples
- Global CoP ≈99% for all responses

Max. eq. stress

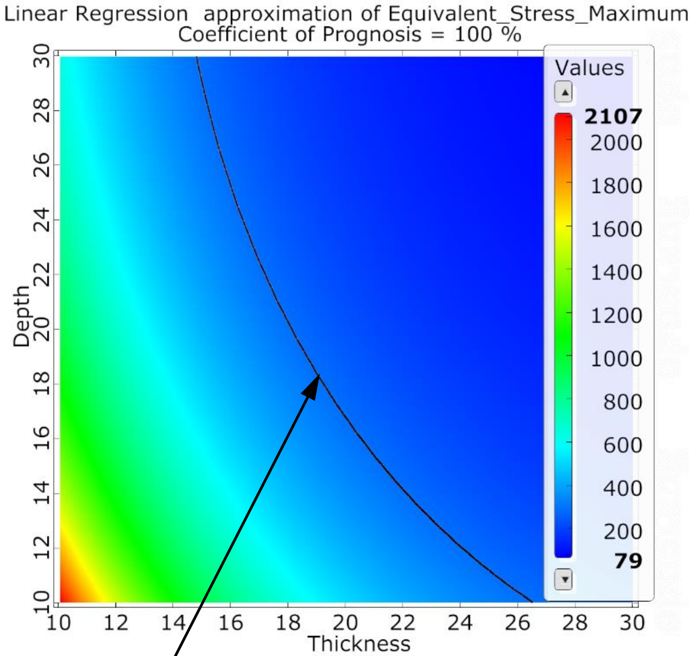


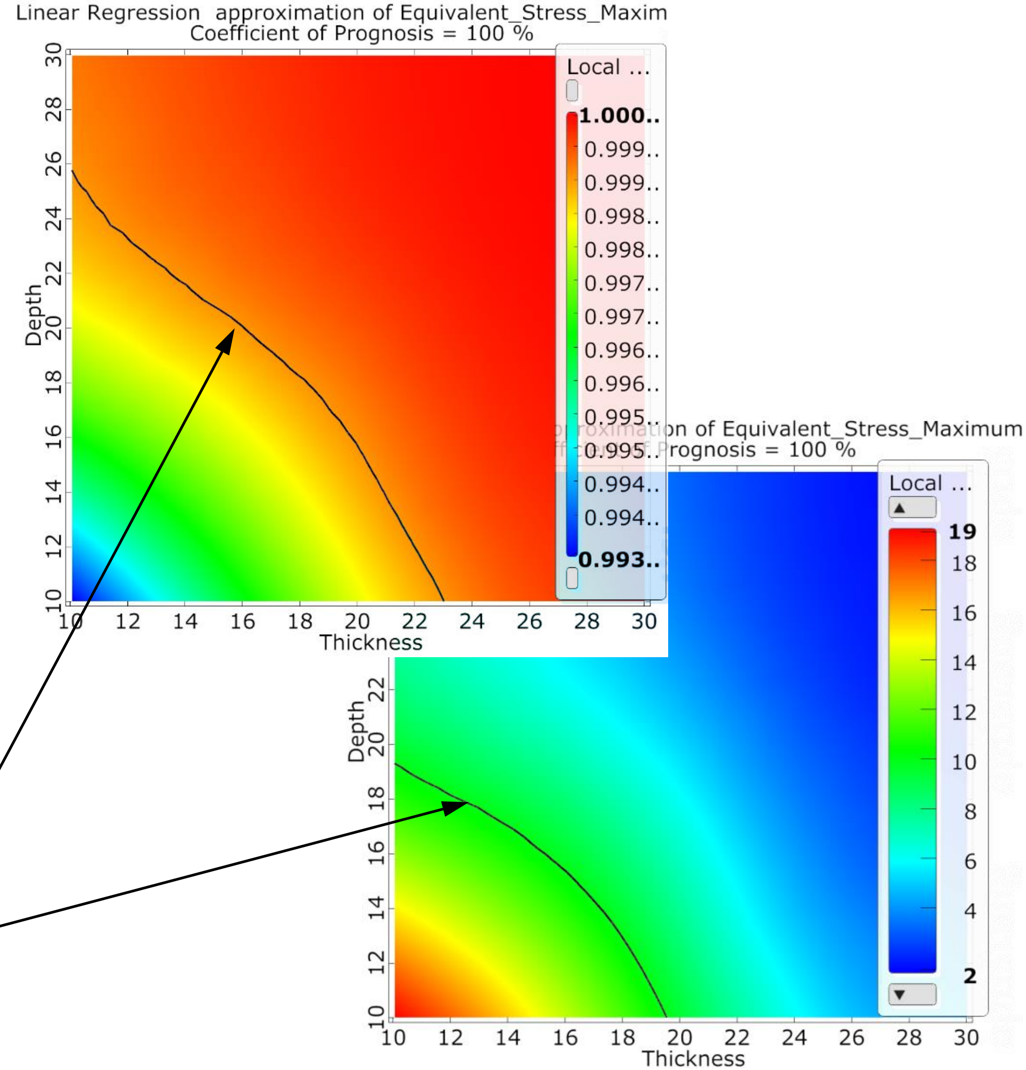


CoP values

dynardo

Metamodel of Optimal Prognosis in RDO Space



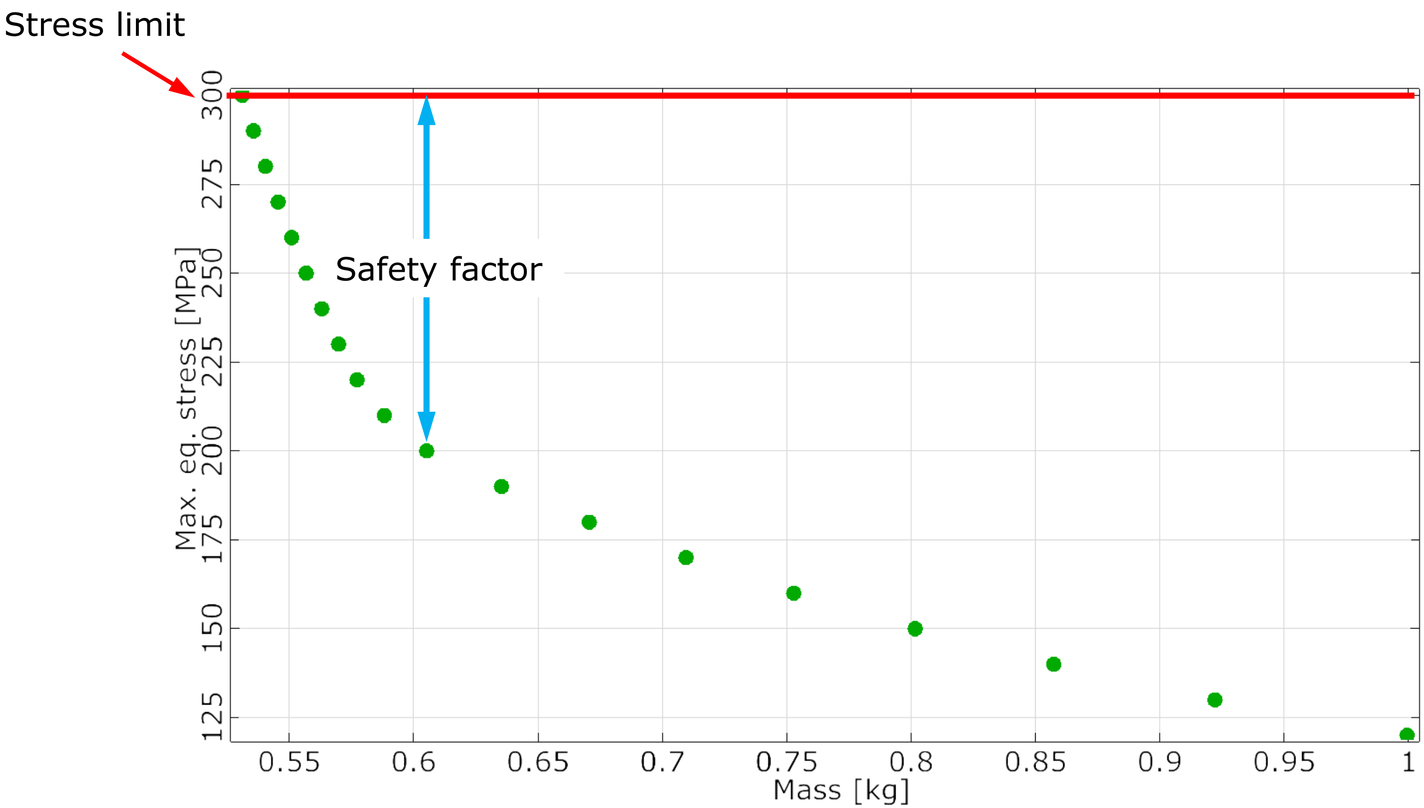


Equivalent Stress Maximum

- Limit 300 Mpa
- Local CoP 99,9%
- Absolute error 10 MPa

1st Study: Deterministic Pareto Optimization

- Criteria: minimize mass and max. stress simultaneously
- Obvious contradiction between mass and max. stress
- Each point is a valid optimum: choose safety distance

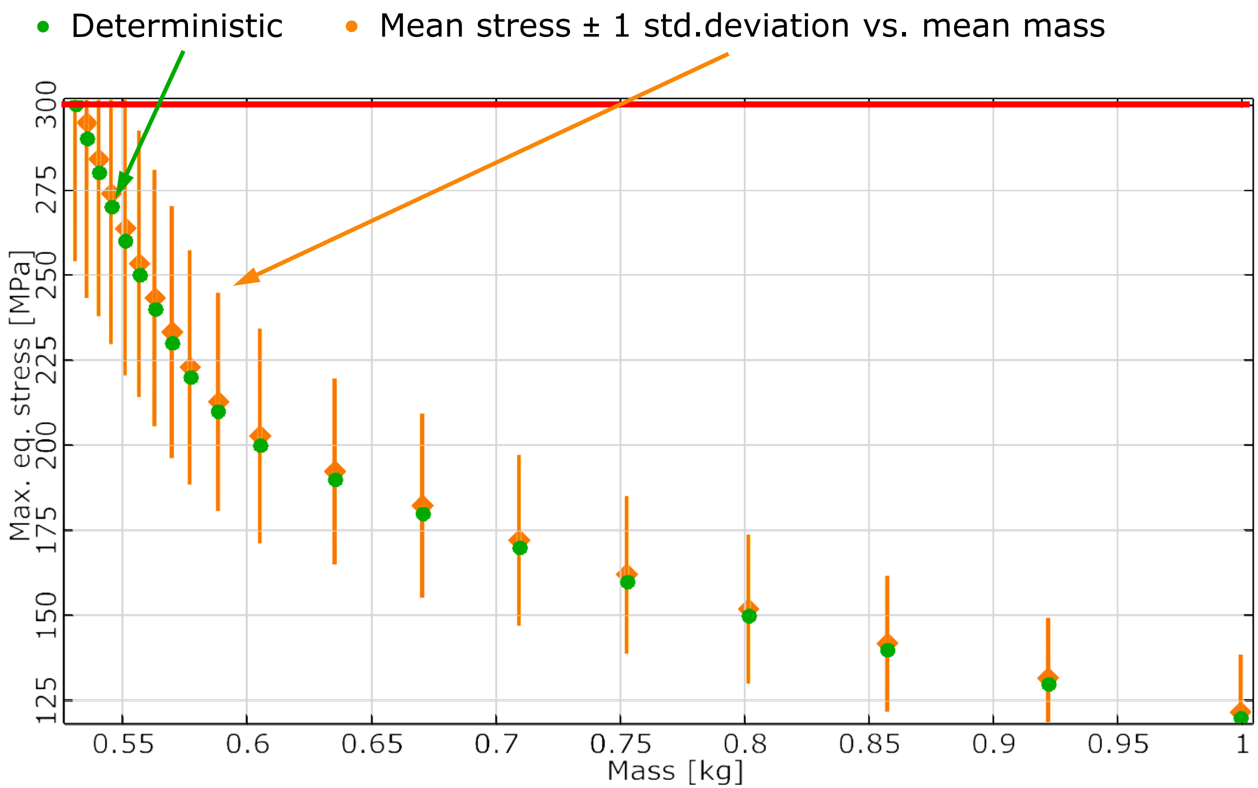


2nd Study: Robustness of Pareto Designs

- Apply stochastic sampling centered at each pareto-optimal design
- Evaluate means and standard deviations of mass and stress

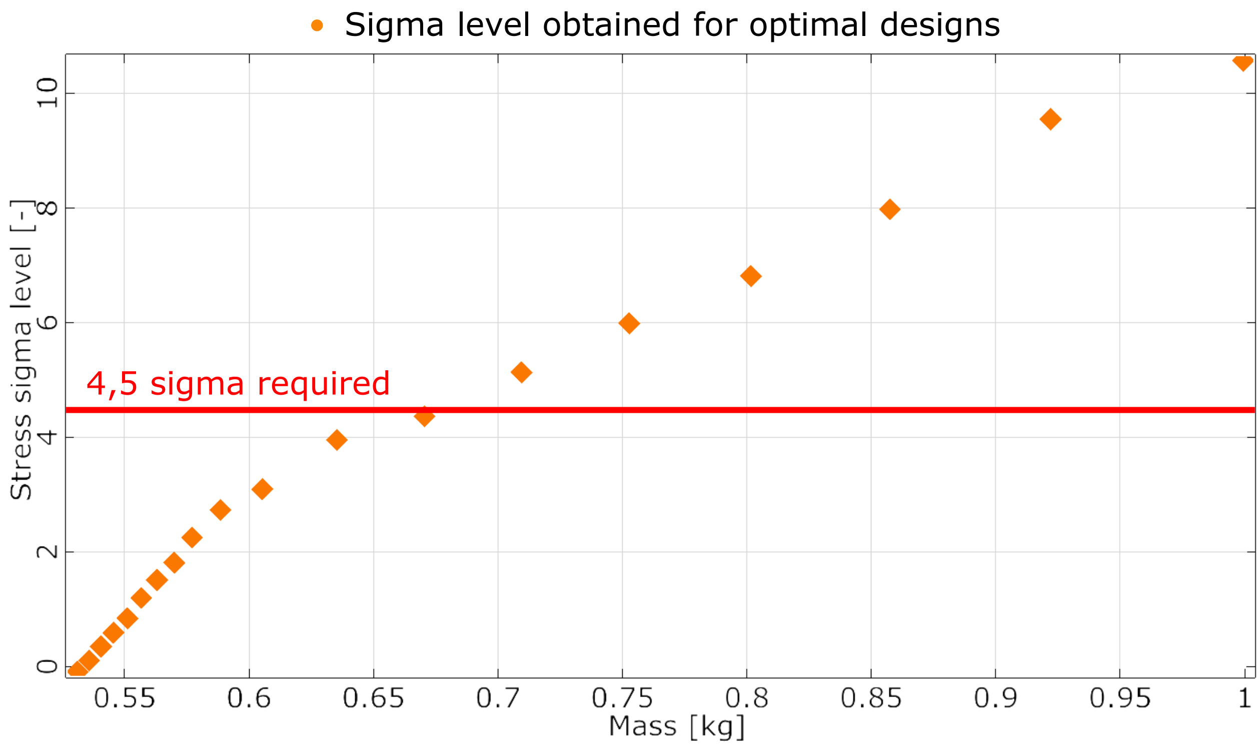
➔ Non-constant scatter over the observed range

➔ Observable shift of mean stress w.r.t. deterministic result



2^{nd} Study: Robustness of Pareto Designs

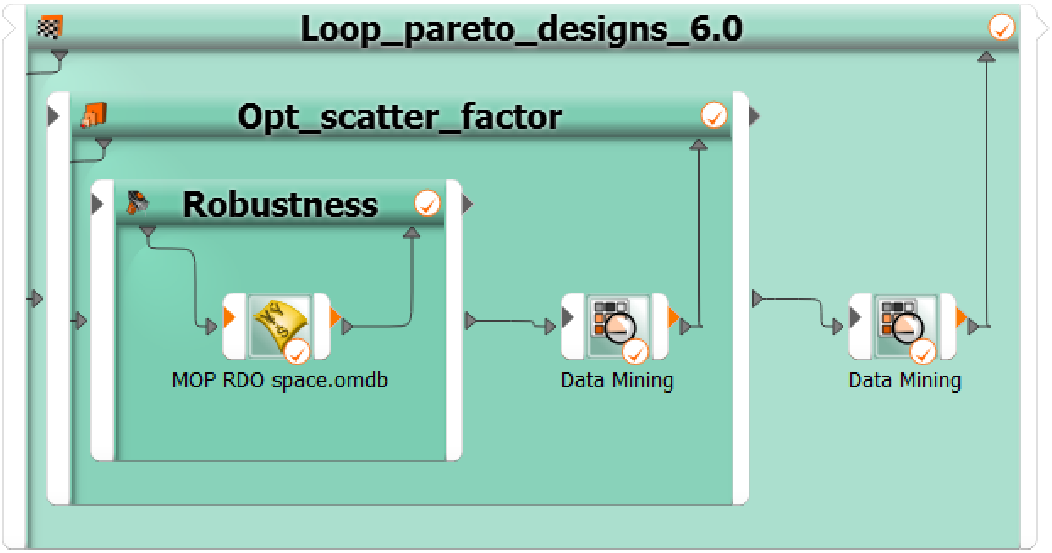
- With help of obtained sigma level, a more meaningful safety margin can be chosen
- Stress constraint for deterministic optimization can be derived
- Result not guaranteed due to varying scatter: iteration required



4th Study: Optimization of Scatter

- Motivation: adaptation of constraints to find robust design changes performance (here: weight)
- Control of input scatter changes costs (manufacturing precision)
- Where is the balance?

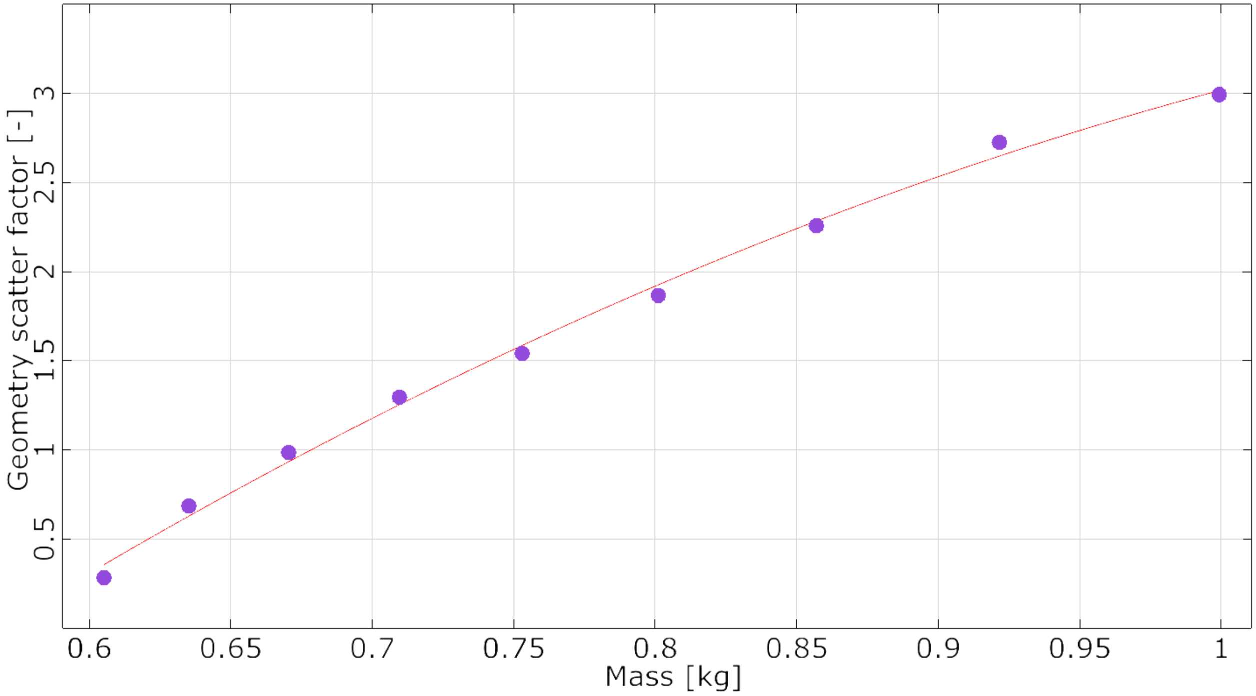
➔ Introduce scatter factor for geometry parameters (without angles)
➔ Optimize scatter factor to given sigma level



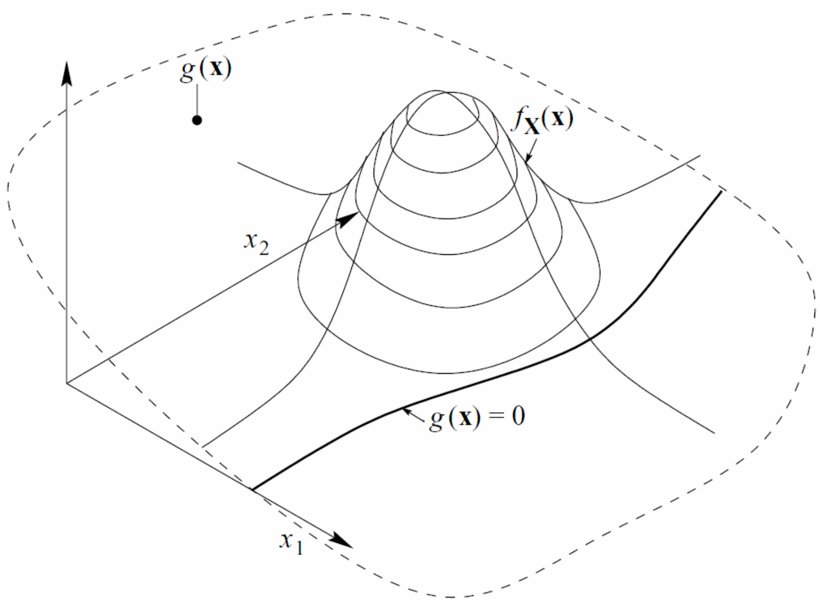
4th Study: Optimization of Scatter

- Optimized scatter factors for geometry parameters (without angles)
- For each deterministic pareto design
- Required sigma level: 4.5

➔ Conflict between mass and scatter



Reliability Analysis

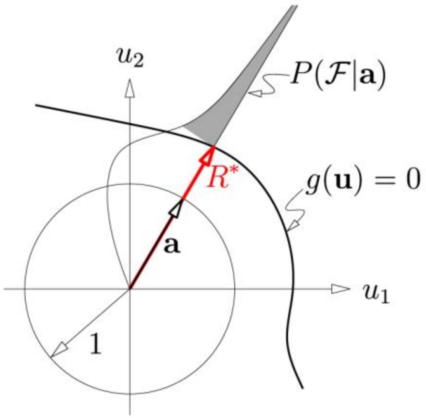


$$P_F = P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right]$$
$$= \int \cdots \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I\left(g(\mathbf{x})\right) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

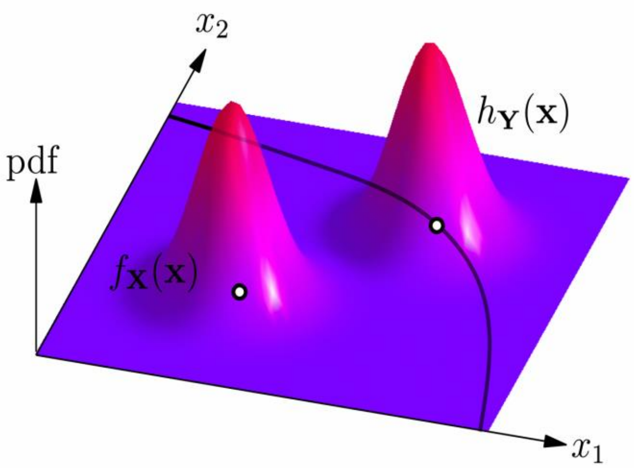
- Limit state function $g(\mathbf{x})$ divides random variable space $\mathbf{X}$ in safe domain $g(\mathbf{x})>0$ and failure domain $g(\mathbf{x}) \leq 0$
- Multiple failure criteria (limit state functions) are possible
- Failure probability is the probability that at least one failure criteria is violated (at least one limit state function is negative)
- Integration of joint probability density function over failure domain

Advanced Methods for Reliability Analysis

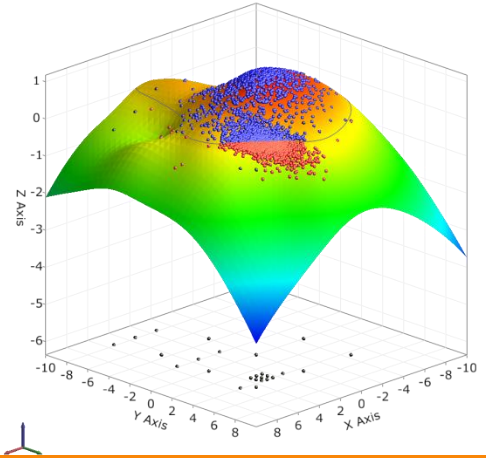
Directional Sampling



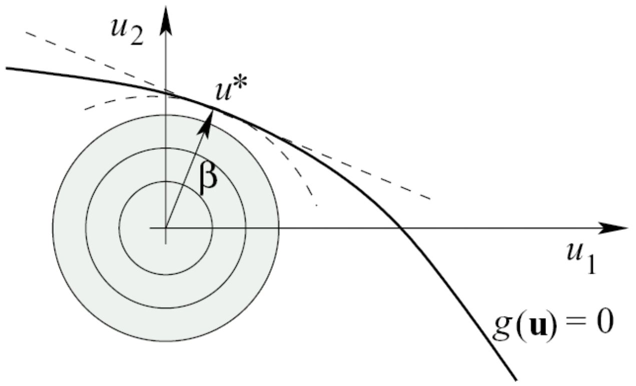
Importance Sampling



Adaptive Response Surface Method



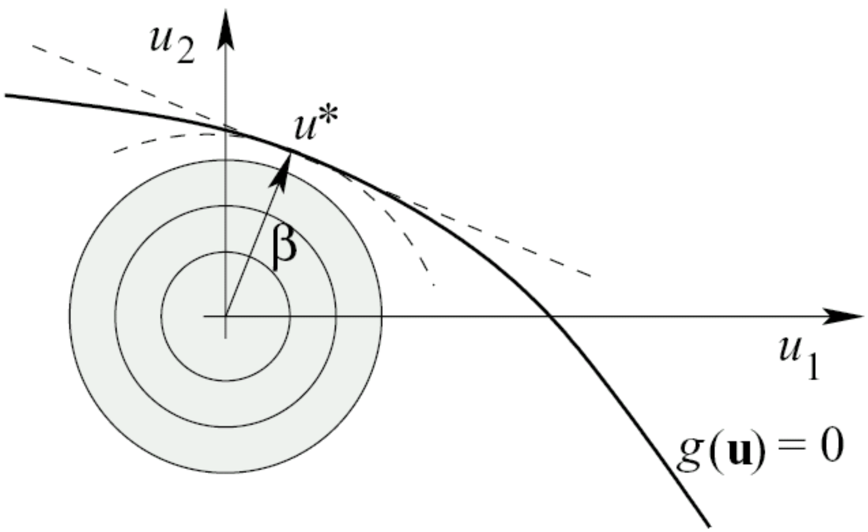
First Order Reliability Method



First Order Reliability Method (FORM)

- Operates in the space of standardized Gaussian variables
- Search for failure point with maximum probability density (*design point*)
- Equals the point on the limit state surface with minimal distance to origin
- Default algorithm is gradient-based optimization
- Limit state function is linearized around design point
  - ➢ Requires continuously differentiable limit state function
  - ➢ Multiple design points (local minima) are not supported
  - ➢ Independent search for each limit state may be more robust

$$\mathbf{X} \to \mathbf{U} \sim \mathcal{N}(0;1) \qquad \rho_{i,j\neq i} = 0$$



$$\mathbf{u}^* : \frac{1}{2}\mathbf{u}^T\mathbf{u} \to \min, \qquad g(\mathbf{u}) = 0$$

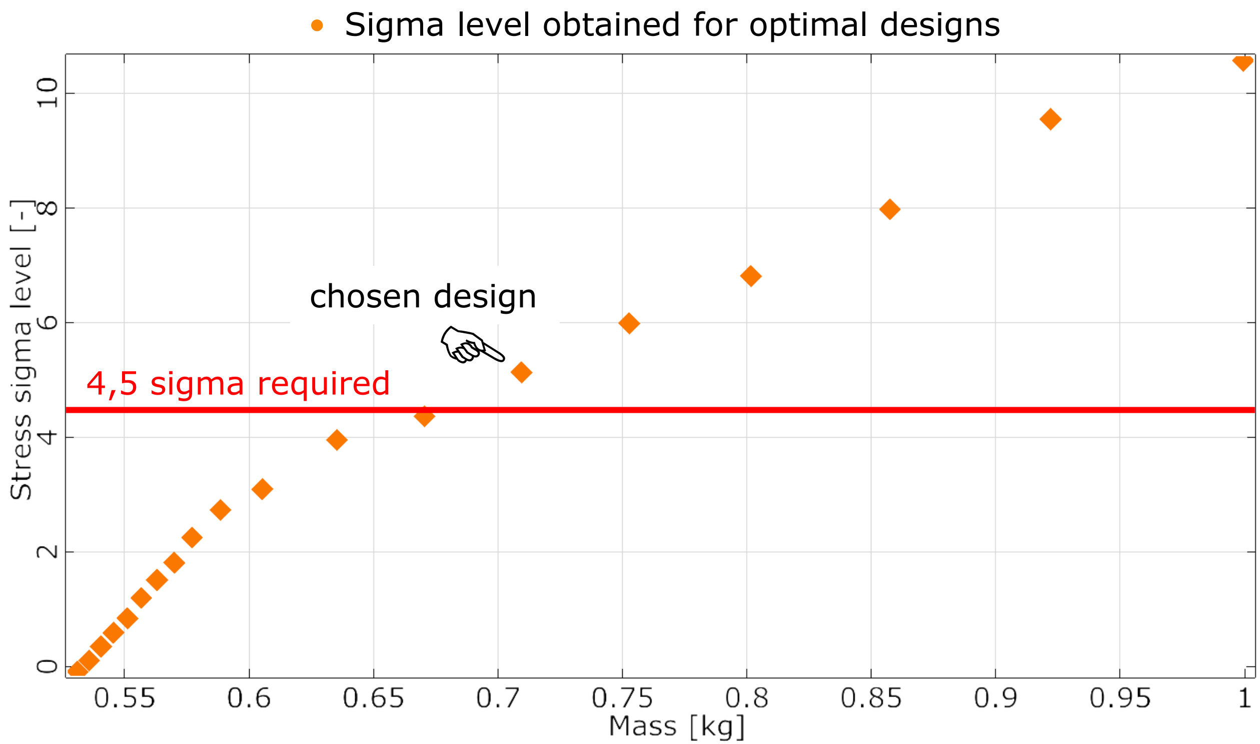
$$P_f = \Phi(-\beta)$$

5th Study: Reliability Analysis of Chosen Design

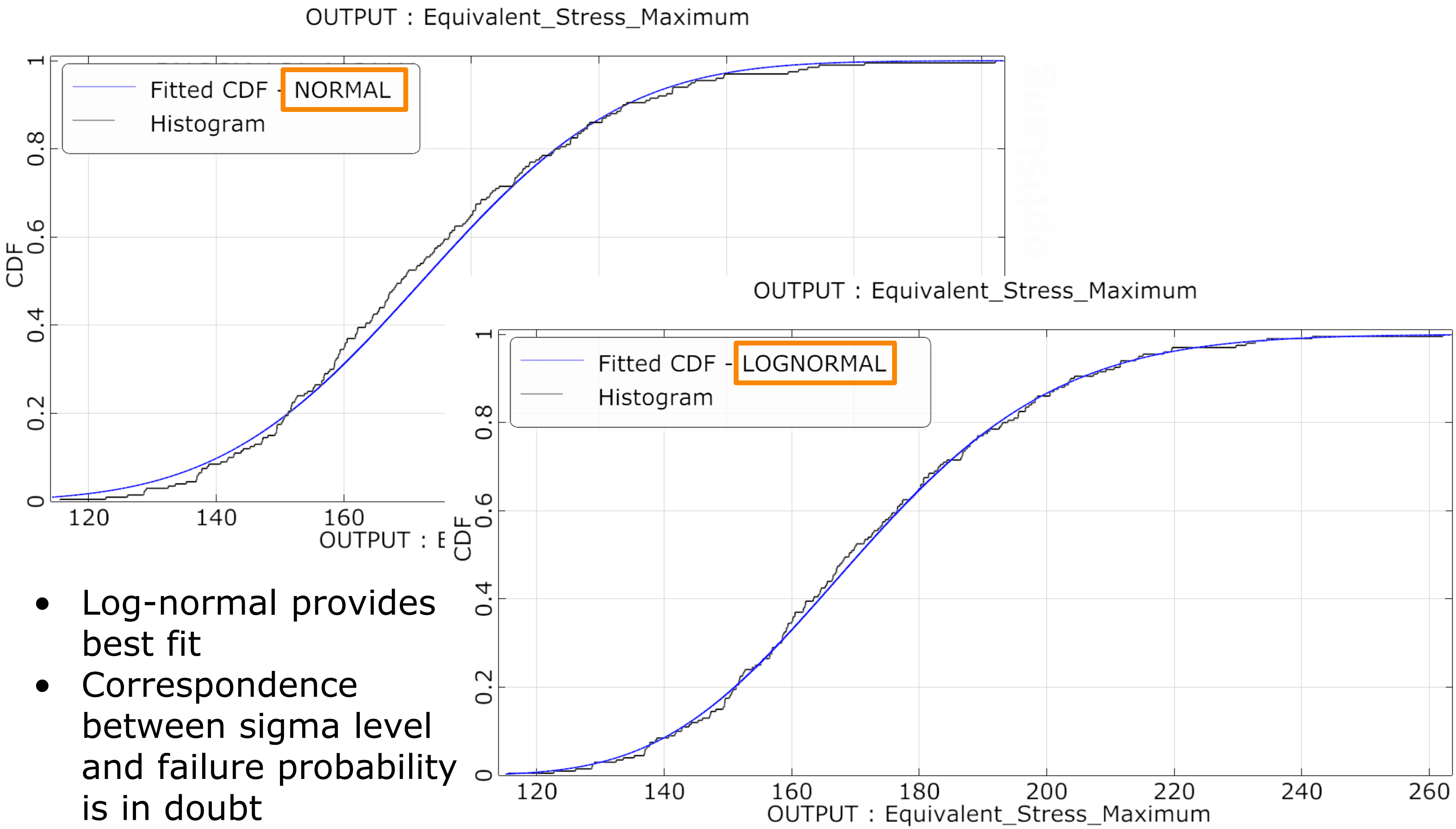
- Previous studies were performed on an approximation model
- Only variance-based analyses despite high sigma level demand

➔ Proof of safety is necessary

- Chosen design: Mass = 7.08kg; Stress sigma level = 5.13



Check for Normal Distribution

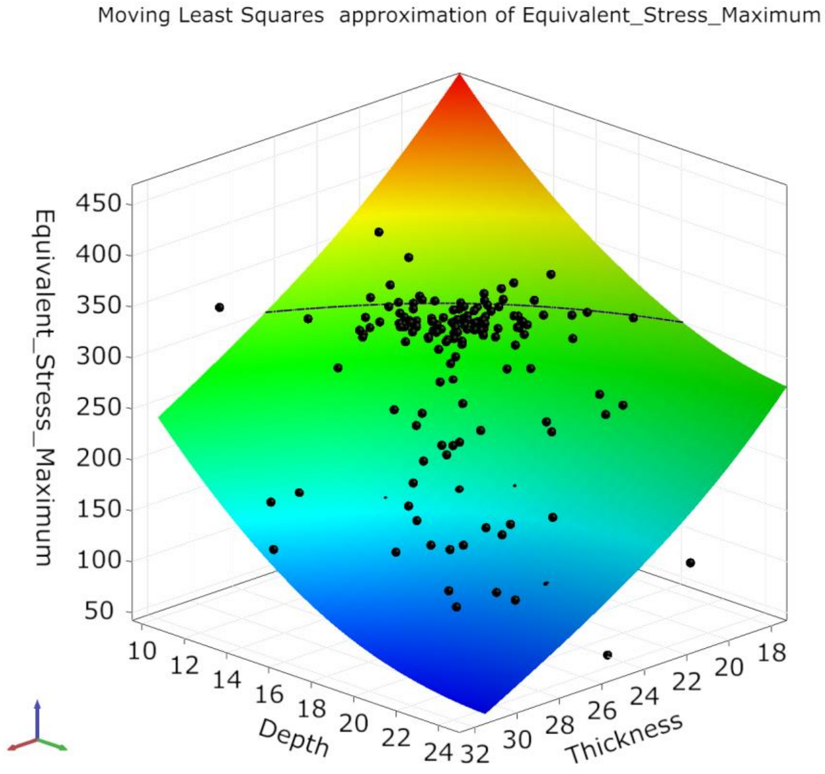


- Log-normal provides best fit
- Correspondence between sigma level and failure probability is in doubt

Results of Probability-Based Analysis

- Reliability analysis using Adaptive Response Surface Method combined with Directional Sampling (ARSM-DS)
- Limit state function: $300MPa - max.\ eq.\ stress = 0$
- 200 solver calls needed

- Failure probability $P_F = 8\text{E-}6$
- Reliablity index $\beta = 4.3$
smaller than sigma level 4.5!



Summary and Conclusions

- Robust Design Optimization (RDO) helps to develop designs that
  - Have optimal performance
  - Fulfill reliability (or quality) criteria without "over-design”
- The handles to influence robustness are
  - Optimization constraints (→ mean values of results)
  - Input scatter (→ scatter or sigma level of results)
- The optimal design shall provide a balance between
  - Performance (“product promise”)
  - Manufacturing costs (due to precision)
- Presented study was performed on an approximate model (Metamodel of Optimal Prognosis)
  - Fast analyses become possible
  - High demand on model accuracy

  ➔ For safety requirements: proof by reliability analysis is necessary