

**Lectures**

**Robust Design Optimization  
Methods for Industrial  
Applications**

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# Robust Design Optimization Methods for Industrial Applications

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## Abstract

In this paper the optimal design of structures, processes or systems under consideration of uncertainties is addressed. Based on a probabilistic approach we represent the possible scatter of several input parameters as scalar random variables. In the presented Robust Design Optimization framework the possible amplitudes of the input scatter within the compromise with optimal nominal values are investigated. Within the application example where the nominal mass of a structure should be minimized we considered scatter of the geometry as well as of the material properties and loading conditions. As result of our analyses we can recommend a suitable compromise between the optimal mass and the requirements with respect to the scatter of the geometry parameters, which may support the decisions in engineering design of structures.

## 1. Robustness Evaluation

In order to match production quality requirements of designs, it is necessary that the scatter of all important responses caused by scattering material and geometrical properties and fluctuating environmental and operational conditions stays within acceptable ranges. With help of the robustness analysis this scatter can be estimated. Within this framework, the scatter of a response itself described by mean value and standard deviation or its safety with respect to a failure limit has to be quantified. The safety can be formulated variance-based with help of the safety margin between failure and the mean value and probability-based using the probability that the failure limit is exceeded. In figure 1 this is shown in principle.

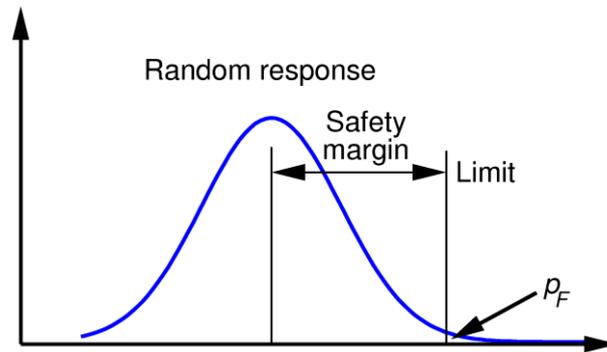


Figure 1: Scatter of a response with safety margin (distance between mean and the failure limit) and the corresponding probability of failure  $p_F$

In the variance-based approach the safety margin is often given in terms of the corresponding standard deviation of the response. The six sigma concept requires a minimum safety margin of 4.5 times the standard deviation for short term analysis. The 4.5 sigma margin of a normal distribution corresponds to a failure rate of 3.4 defects out of one million design realizations. In order to extrapolate the short term analysis to long term estimates an additional, purely empirically 1.5 sigma safety margin was introduced which results in the well-known six sigma safety margin for short term analysis.

However, the assumption of a normally distributed response may be not valid if non-linear effects dominate the mechanisms of failure or disoperation. In such cases the extrapolation of rare event probabilities like 3.4 out of a million just from the estimated mean value and standard deviation may be strongly erroneous. Thus, the assumption of a normal distribution should be verified at least at the final nominal design or the probability of failure should be estimated with the more qualified reliability analysis. This question is discussed in detail in (Will 2017).

Besides these fundamental questions, approaches for variance-based robustness evaluation need to estimate the necessary statistical measures as mean value and standard deviation with a sufficient confidence. Based on the initial uncertainty definition, several approaches for their determination are available.

In case of a linear dependence between input variables and responses the mean value and the standard deviation can be calculated analytically in a closed form. Therefore, some methods use a linearization around the mean for these estimates (e.g. First Order Second Moments), which is very efficient for a small number of input parameters. However, in case of non-linear dependencies such a procedure may obtain strongly erroneous statistical estimates. Similar approaches by using a global linear or quadratic response surface have the same limitations: if the assumed linear or quadratic dependence is not valid, the estimated safety level may be far away from the real value.

For industrial applications with a larger number of scattering inputs and non-linear dependencies Monte Carlo based methods are more suitable (Will 2007). The Latin Hypercube Sampling (LHS) is one approach, where the distribution of the samples is optimized with respect to small errors in the statistical estimates. This method does not assume any degree of model behavior and can handle also discontinuous responses. Furthermore, it works independently of the number of input parameters. Rough estimates of mean and standard deviation are possible with just 20 solver runs. More precise estimates of mean and standard deviation can be obtained by using 50 to 100 samples, but of course pure sampling strategies need a very high number of samples for reliable estimates of rare event probabilities. Based on the evaluated data and the estimated scatter of the responses, variance-based sensitivity measures can be evaluated in order to further analyze the source of uncertainty. For this purpose we recommend the Metamodel of Optimal Prognosis approach (Most & Will 2008, 2011), where an optimal meta-model is generated for the available samples and variance based sensitivity indices (Saltelli 2008) are estimated using these meta-models.

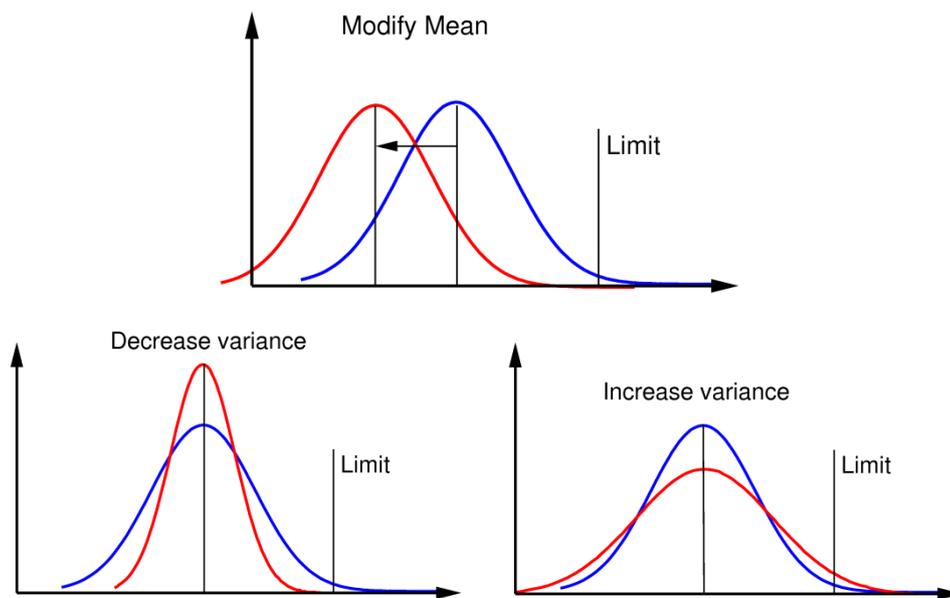


Figure 2: Modification of a nominal design and corresponding input scatter: increase of a safety margin by moving the mean (top), decreasing the input scatter or decreasing a safety margin by increasing the input scatter (bottom)

## 2. Robust Design Optimization

If an investigated nominal design does not fulfill the robustness requirements, different strategies for an improvement are possible. Such a procedure is e.g. the automatic Robust Design Optimization (Most & Will 2012, Most &

Neubert 2013), where deterministic optimization methods are extended by considering uncertainties of specific input variables. With help of a statistical evaluation of the no longer deterministic objective function and constraint conditions, the design is driven to a region where the robustness requirements are fulfilled while the desired performance is optimal.

Different procedures within an RDO framework are possible as shown in figure 2: One approach modifies the nominal values while keeping the assumed scatter constant until the required safety margin is reached. This can be done by a fully automatic approach or more efficient by an iterative (decoupled) procedure, where deterministic constraints are adapted until the specific robustness criteria are fulfilled (Will 2017). Another possible way would be the reduction of the input scatter while keeping the nominal values. This procedure is straight forward since we get the sensitivity of the input scatter as a result of the variance based robustness analysis.

If a nominal design fulfils the robustness requirements an increase of the specified scatter of specific input parameters may lead to a cheaper production due to the weaker requirements with respect to the manufacturing tolerances. Since often the scatter amplitudes are estimated quite roughly an investigation of optimal scatter amplitudes may be in the point of interest. All of these different robustness requirements can be addressed quite comfortable with the fully coupled RDO framework, which usually needs a significant number of model evaluations. In order to accelerate these procedures a suitable meta-model approximation of the original solver responses might be reasonable. Such a meta-model has to be built in the coupled robustness-optimization space considering the bounds of the design variables as well as the possible bounds of the scatter of the pure random input parameters. However, since a meta-model is just a mathematical approximation of the physical phenomena, the final nominal design should be verified with a qualified robustness evaluation using the original simulation model.

Both procedures, the double loop and the iterative approach, require a qualified knowledge of the uncertainty of the scattering input parameters. If no qualified information is available, a sophisticated reliability analysis could not serve a better result quality as the much faster variance based robustness analysis. Furthermore the question of finding a good compromise between a system performance and input scatter requirements is a typical challenge. Often not only the nominal values of the input parameters itself but also the requirements to the deliverer may have a significant influence on the production cost.

In the following example, the double-loop RDO approach is applied to analyse the influence of fixed input scatter to the safety margin as well to find the optimal input scatter within a compromise between scatter and nominal values.

### 3. Application example

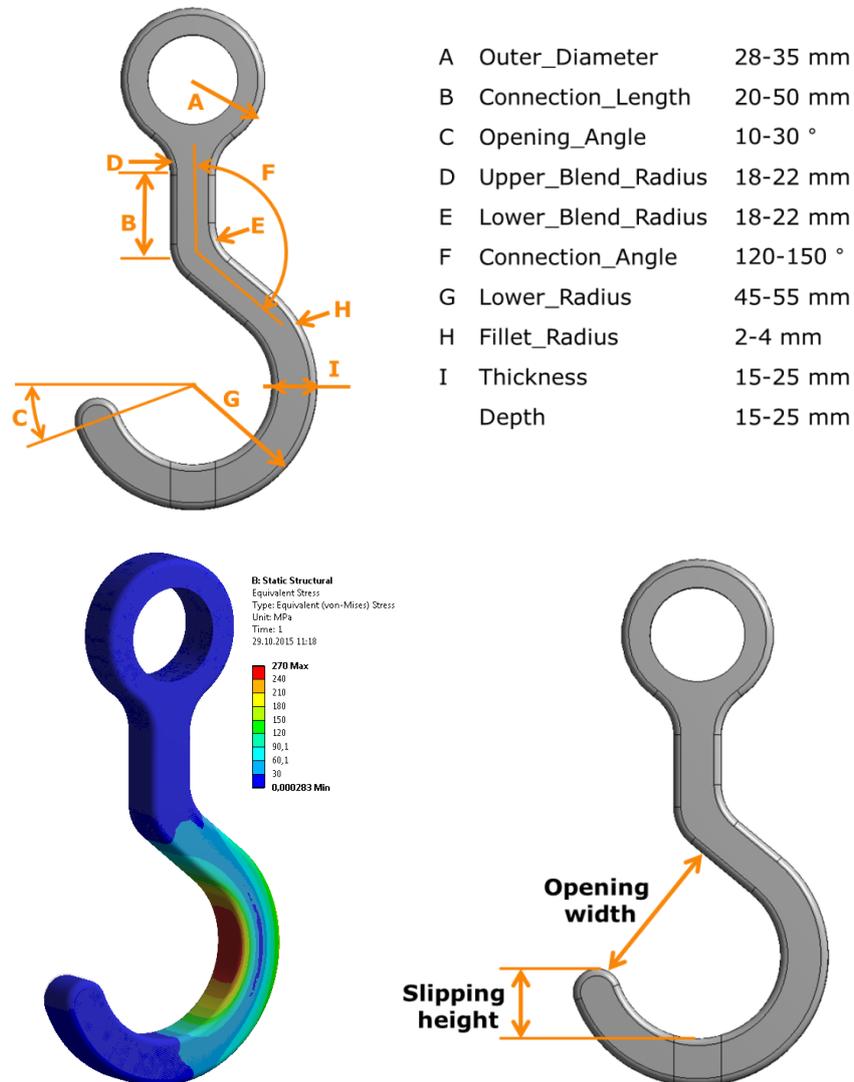


Figure 3: Investigated steel hook: optimization parameter (top), v. Mises stresses (bottom left) and the two further constraints (bottom right)

In this example a steel hook subjected to a vertical load of 6000 N is investigated. The hook is analyzed by linear finite elements and has a cylindrical support at its head, which can perform free rotations. The numerical simulation including geometry modelling and automatic meshing is done using ANSYS Workbench. The Robust Design Optimization task in this example is to minimize the mass while the maximum stress should not exceed a failure limit of 300 MPa. The required safety level is defined with a 4.5 sigma safety margin corresponding to a failure rate of 3.4 out of a million. For the optimization 10 geometry properties are considered as design parameters. They are illustrated with their ranges in figure 3. As further constraints the nominal

design should have an opening width of 50 mm and its slipping height in the deformed structure should not exceed 10 mm.

### Deterministic optimization

In a first step a deterministic optimization is performed in order to analyse the possible relation between the minimization of the mass and the stress constraint. For this purpose a Metamodel of Optimal Prognosis is built in the design space using 500 samples. In figure 4 the estimated sensitivity indices are shown. Furthermore the total model quality in terms of the Coefficient of Prognosis is given. Here 100% indicates perfect prediction quality of the individual meta-models.

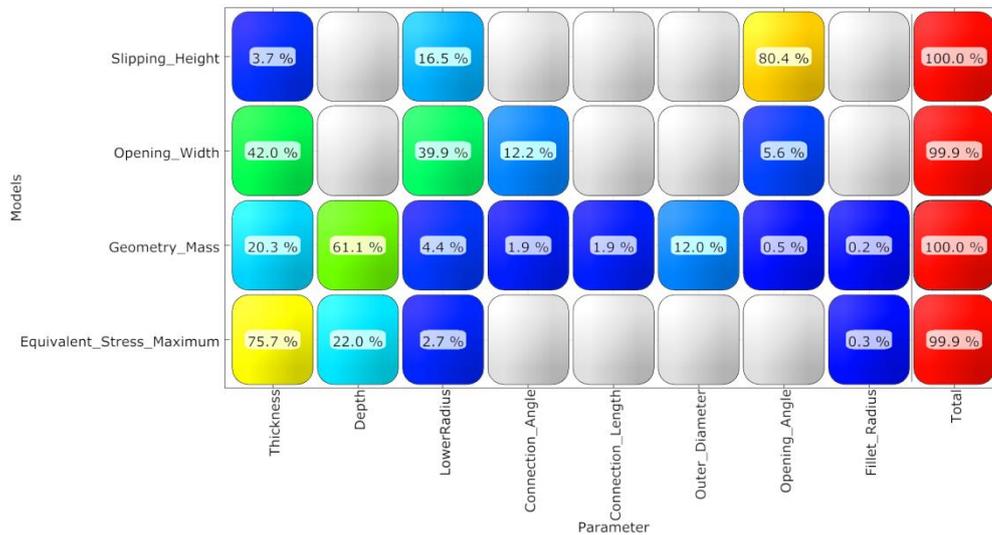


Figure 4: Estimated variance based sensitivity indices in the deterministic design space using the Metamodel of Optimal Prognosis

By using the MOPs in the deterministic design space a nested optimization is performed by analysing the possible minimum mass with respect to different values for the stress constraint. The two other constraints are considered with the given limits. In figure 5 the results are illustrated. The figure indicates, that for example a safety factor of the stress of 1.5, which is equivalent to a stress limit of 200 MPa in the deterministic case, would lead to a possible mass of about 760g. In order to quantify the robustness of the individual solutions in the following step the input scatter is considered.

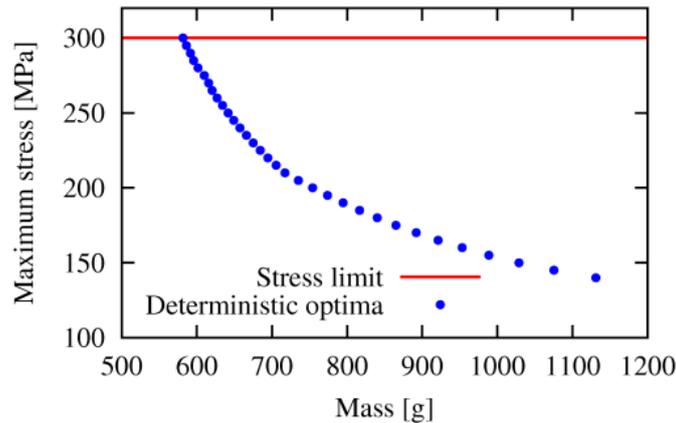


Figure 5: Observed conflict between the minimum mass and the allowed maximum stress of the hook structure using the MOP in the deterministic space

**Robustness evaluation with defined input scatter**

	<b>Distribution</b>	<b>Mean value</b>	<b>Standard deviation</b>
Outer diameter	normal	nominal	1 mm
Connection length	normal	nominal	1 mm
Opening angle	normal	nominal	2°
Upper blend radius	normal	nominal	1 mm
Lower blend radius	normal	nominal	1 mm
Connection angle	normal	nominal	2°
Lower radius	normal	nominal	1 mm
Fillet radius	normal	nominal	0.2 mm
Thickness	normal	nominal	1 mm
Depth	normal	nominal	1 mm
Young’s modulus	log-normal	2e11 N/m <sup>2</sup>	1e10 N/m <sup>2</sup>
Poisson’s ratio	log-normal	0.3	0.015
Density	log-normal	7850 kg/m <sup>3</sup>	157 kg/m <sup>3</sup>
Force x-direction	normal	0 N	100 N
Force y-direction	normal	6000 N	600 N
Force z-direction	normal	0 N	100 N

**Table 1: Assumed scatter for the robustness evaluation of the nominal designs**

The classical variance based robustness analysis considers a known scatter for a nominal design (Most & Will 2012). Now we investigate the influence of the scatter with respect to the observed optimal nominal designs of the previous section. In table 1 the assumed distribution type and scatter is given. For each of the optimal designs indicated in figure 5 a variance based robustness evaluation is performed on global meta-models spanned in the combined robustness-design-space. For this purpose the optimization parameter are

varied within their optimization bounds plus/minus 5 times the standard deviation. The pure stochastic parameters are varied just within plus/minus 5 sigma. Within these bounds a uniform LHS with 500 samples is generated and evaluated with the finite element model. Based on the obtained results a MOP is generated for each response. In Figure 6 the estimated sensitivity indices and the total model quality is given for MOPs of the 4 responses. The figure indicates again an excellent prediction quality.

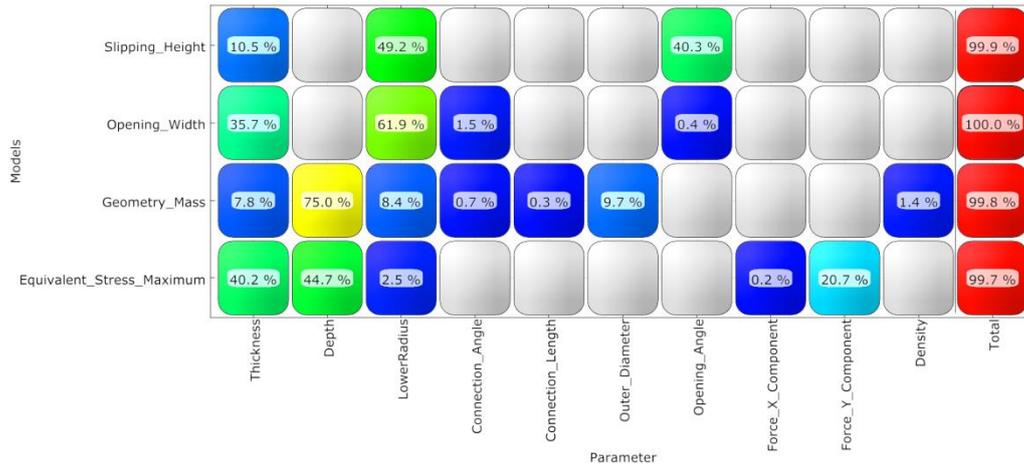


Figure 6: Estimated variance based sensitivity indices in the combined robustness-optimization-space using the Metamodel of Optimal Prognosis

Using the global MOPs in the RDO space then the robustness evaluation for each nominal design is performed using 200 samples considering the distribution in table 1. In figure 7 the observed mean value and scatter for the maximum stress is shown. It can be clearly seen, that with increasing mass the mean and the scatter of the stress decreases. Based on the observed mean value and standard deviation of each nominal design the safety margin with respect to the stress limit of 300 MPa can be estimated. In figure 8 the resulting sigma level is shown depending on the nominal mass. The figure indicates that for a mass of about 850g a sigma level of 4.5 could be reached. For a 6 sigma safety margin the mass could not be larger as 950g.

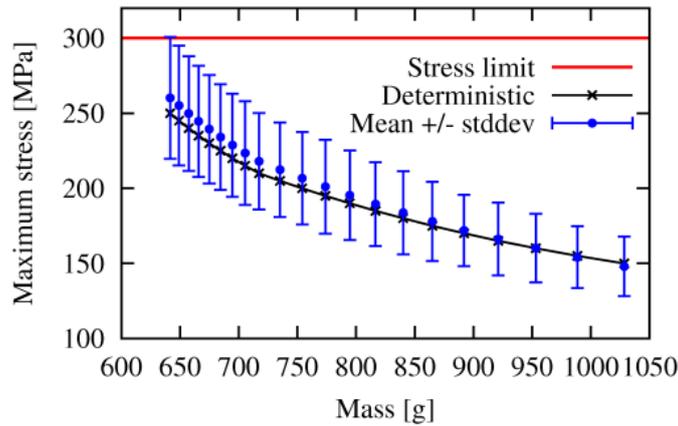


Figure 7: Observed scatter for the different nominal designs

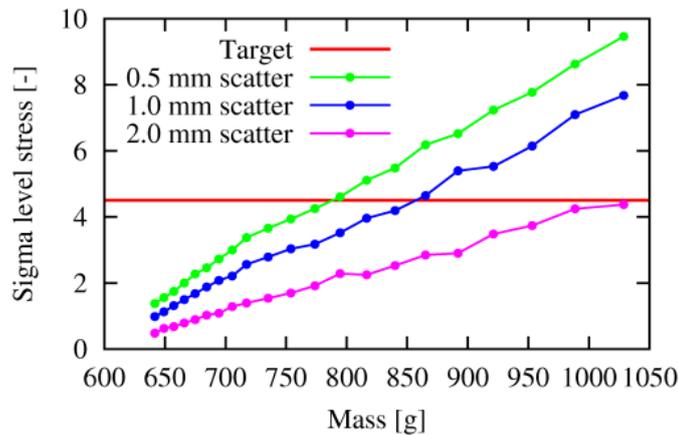


Figure 8: Estimated safety margin for the stress limit considering different standard deviations for the geometry scatter

### Robustness evaluation with modified input scatter

In the next step the scatter of the geometry parameters is modified in order to investigate its influence on the safety margin. For this purpose the standard deviations of all geometry parameters except the angles are multiplied by factor of 0.5 in a first analysis and of 2.0 afterwards and the robustness evaluations are performed in both cases for all nominal designs. The scatter of the pure stochastic parameters is kept constant as given in table 1. In figure 8 the resulting safety margins are shown additionally. It can be clearly seen, that by decreasing the input scatter of the geometry parameters the required mass for a 4.5 sigma safety margin can be decreased by about 70g. On the other hand, with the increased scatter even the heaviest nominal design does not reach a 4.5 sigma safety margin.

Finally, we investigate how we could increase the scatter for each nominal design in order to still obtain a defined safety margin. This has been done by using a coupled optimization with nested robustness evaluation using again the MOP in the RDO space. As optimization criteria the scatter factor has to be maximized and as constraint the required sigma level has to be fulfilled. In figure 9 the nested system implemented in the software optiSLang is shown: for each nominal design a nested robust design optimization is performed. The results of this analysis are shown in figure 10. The results show directly the conflict between the nominal mass value and the geometry scatter. Now the analyst could decide which compromise may lead to the suitable requirement w.r.t. deliverers.

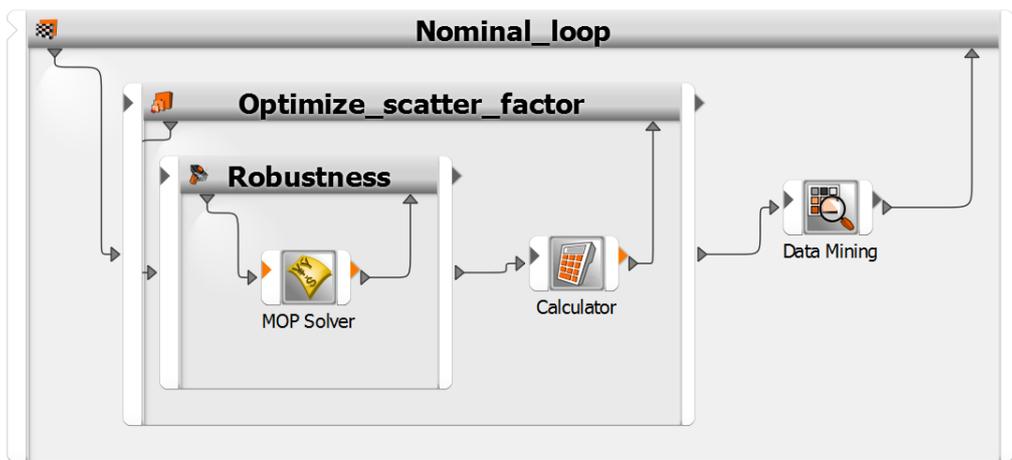


Figure 9: Nested loop of the robust design optimization to find the maximum possible scatter factor for each nominal design

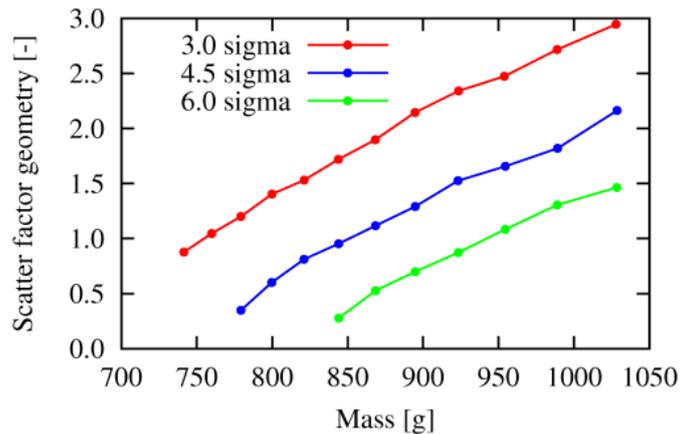
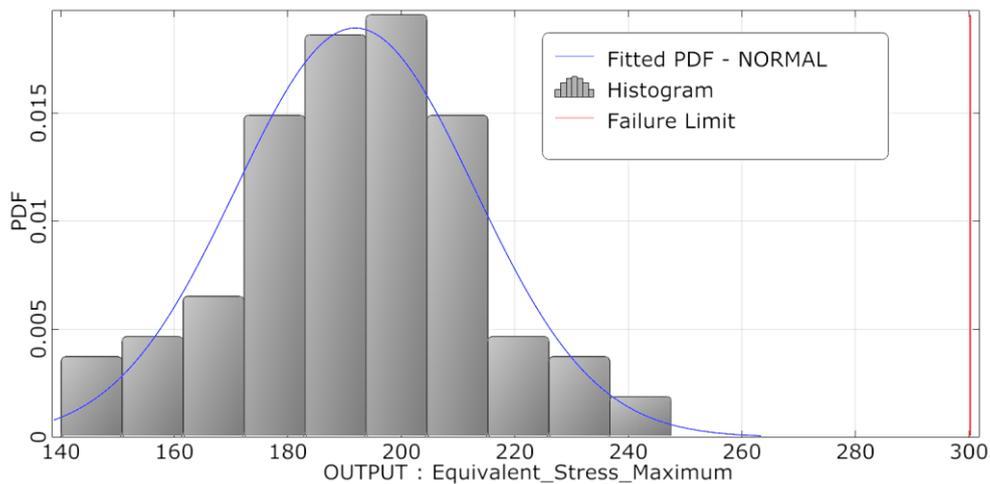


Figure 10: Estimated scatter factors for different nominal mass values with respect to different safety requirements

The presented analyses have been performed on a metamodel approximation in order to support quickly a decision on how to quantify the scatter requirements and how to choose a suitable nominal design for a given safety requirement.

After choosing a nominal design and a possible scatter level, a final robustness evaluation using the direct simulation model shall be performed in order to proof the estimated safety margin. In our example we choose a scatter factor of 0.5 and the nominal design with 795g deterministic mass and 190 MPa maximum stress. For the corresponding input parameters the robustness evaluation is performed using the finite element model with 100 LHS samples. In figure 11 the observed scatter of the maximum stress is illustrated. The robustness proof results in a safety margin of 5.1 sigma. Thus the estimated sigma level of the MOP based RDO could be verified. The full RDO analysis has been done by using the 500 model evaluations for the MOP approximation and 100 designs for the robustness proof.



*Figure 11: Robustness proof at the chosen nominal design using 100 LHS samples with the finite element model: mean value of stress = 191.6 MPa, standard deviation = 21.1 MPa, resulting safety margin = 5.1 sigma*

## Acknowledgement

This research has been partly supported by the German Federal Ministry of Education and Research (BMBF) within the project *Robust- und Zuverlässigkeitssimulation mechatronischer Systeme einschließlich Alterung und Verschleiß* (ROMESA), which is gratefully acknowledged by the authors.

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