Robust Design and Reliability-based Design Optimization

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Summary A large number of problems in manufacturing processes, production planning, finance and engineering design require an understanding of potential sources of variations and quantification of the effect of variations on product behavior and performance. Traditionally, in engineering problems uncertainties have been formulated only through coarse safety factors. Such methods often lead to overdesigned products. Furthermore, the deterministic optimization algorithms tend to push an optimized design towards the boundaries of the design space.

This paper reviews theories and methodologies that have been developed to solve optimization problems under uncertainties. In the first part the paper gives an overview over the state of the art in stochastic optimization methods such as robust design and reliability-based design optimization.

In addition, global response surface techniques as well as genetic programming in combination with first order reliability methods in reliability-based optimization are discussed. Two numerical examples from structural analysis under static and dynamic loading conditions show the applicability of these concepts. The probabilistic and structural analysis tasks are performed with ANSYS DesignXplorer and OptiSLang software packages.

Keywords stochastic optimization, robust design, reliability-based design optimization, design for six sigma, genetic programming, response surfaces, first order reliability methods

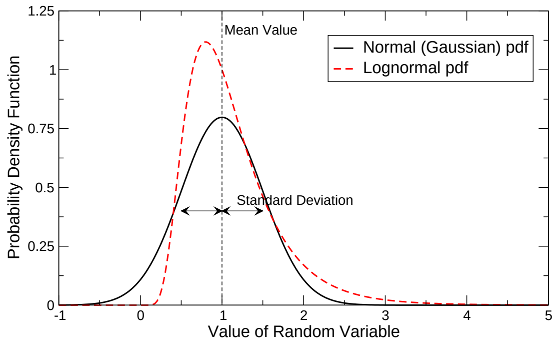


Figure 1: Probability density function $f_X(x)$ of the normal and lognormal distribution.

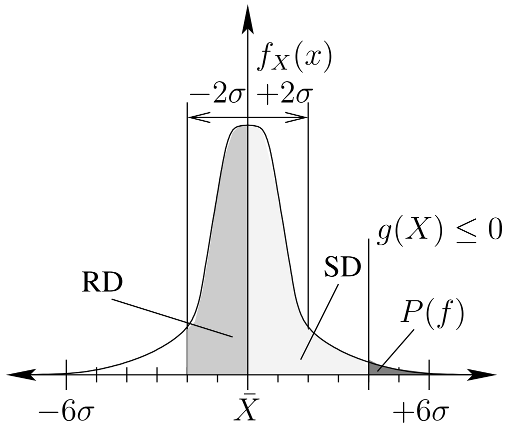


Figure 2: Normal distribution $f_X(x)$ with lower and upper specification limit on 2σ and 6σ level. Robust design (RD) and safety design (SD) ($\geq \pm 2\sigma$) depending on choosen limit state function $g(X) \leq 0$, e.g. stress limit state.

1 Introduction

1.1 Challenges on virtual prototyping and multidisciplinary optimization

Methods of multidisciplinary optimization have obtained an increasing importance in the design of engineering problems for improving the design performance and reducing costs. The virtual prototyping is an interdisciplinary process. Such a multidisciplinary approach requires to run different solvers in parallel and to handle different types of constraints and objectives. Arbitrary engineering software and complex non-linear analyses have to be connected. Resulting optimization problems may become very noisy, very sensitive to design changes or ill-conditioned for mathematical function analysis (e.g. non-differentiable, non-convex, non-smooth).

During the last years, many challenges on virtual prototyping have occurred. Product life cycles are expected to last for as little as a few months, and more and more customized products are developed, e.g. 1700 car models compared to only 900 ten years ago. The engineer's focus is more and more on "built-in-quality" and "built-in-reliability". The products are developed in the shortest amount of time, and, inspite of that, they have to be safe, reliable and robust. Some markets require optimized product designs to be robust, e.g. defense, aerospace, jet engine, nuclear power, biomedical, oil industry and other mission critical tasks.

At the same time, the structural models become increasingly detailed and numerical procedures become more and more complex. Substantially more precise data is required for the structural analysis. The optimized designs lead to high imperfection sensitivities and tend to loose robustness. Using a multidisciplinary optimization method, the deterministic optimum design is frequently pushed to the design space boundary. The design properties have no room for tolerances or uncertainties. So the assessment of structural robustness, reliability and safety will be more and more important. Because of that, an integration of optimization and stochastic structural analysis methods is necessary.

1.2 Design for six sigma and stochastic optimization

Probabilistic analysis typically involves two areas of statistical variability as shown in Table 1. The first group consists of the uncontrollable uncertainties and tolerances. These include material property variability, manufacturing process limitations, environmental variability, such as temperature, operating processes (misuse) and result scatter arising from deterioration. The second group – the controllable parameters – involves design configurations, geometry, loads, constraints and manufacturing process settings.

Property	SD/Mean %
Metallic materiales, yield	15
Carbon fiber composites, rupture	17
Metallic shells, buckling strength	14
Junction by screws, rivet, welding	8
Bond insert, axial load	12
Honeycomb, tension	16
Honeycomb, shear, compression	10
Honeycomb, face wrinkling	8
Launch vehicle , thrust	5
Transient loads	50
Thermal loads	7.5
Deployment shock	10
Acoustic loads	40
Vibration loads	20

Table 1: Sources of uncertainties (Klein *et al.* (1994)) given by standard deviation (SD) and mean value as shown in Figure 1.

Sigma level	Percent variation	Probability of failure $P(f)$	Defects per million (short term)	Defects per million (long term)
$\pm 1\sigma$	68.26	$3.17 \cdot 10^{-1}$	317400	697700
$\pm 2\sigma$	95.46	$4.54 \cdot 10^{-2}$	45400	308733
$\pm 3\sigma$	99.73	$2.7 \cdot 10^{-3}$	2700	66803
$\pm 4\sigma$	99.9937	$6.3 \cdot 10^{-5}$	63	6200
$\pm 5\sigma$	99.999943	$5.7 \cdot 10^{-7}$	0.57	233
$\pm 6\sigma$	99.9999998	$2.0 \cdot 10^{-9}$	0.002	3.4

Table 2: Sigma level depending on the variation of the normal distribution, defects per million and associated probability of failure $P(f)$. A probability of 3.4 out of 1 million is achieved when the performance target is 4.5 σ away from the mean value (short term). The additional 1.5 σ (long term) leading to a total of 6 standard deviations are used as a safety margin to allow for “drift of the mean value” in the properties and environment which the product can see over its lifetime.

Six Sigma is a quality improvement process to optimize the manufacturing process in a way that it automatically produces parts conforming to the six sigma quality level, as shown in Figure 2. Motorola documented more than $16 Billion in savings as a result of their Six Sigma efforts[1]. Since then, hundreds of companies around the world have adopted Six Sigma as a way of doing business.

In contrast, Design for Six Sigma optimizes the design itself such that the part conforms to Six Sigma quality even with variations in manufacturing. Design for Six Sigma is a concept to optimize the design such that the parts conform with six sigma quality, i.e. quality and reliability are explicit optimization goals. Robust design is often synonymous to “Design for Six Sigma” or “reliability-based optimization”. The possible sigma levels start at 1,2 σ (robust design optimization) and go up to 6 σ (reliability-based design optimization) (Koch *et al.* (2004)), as shown in Table 2.

Within the stochastic optimization, the statistical variability of the design parameter is considered. The most general method for solving stochastic optimization problems is the well established Monte Carlo simulation method. However, the major shortcoming of this approach is its vast need of computational resources (the number of solver runs required), and these cannot be presumed in general situations.

1.3 Robust design optimization

Optimized designs within the sigma level $\leq \pm 2\sigma$ are characterized as robust design (RD). The objective of the robust design optimization (e.g. Hwang *et al.* (2001); Ben-Tal & Nemirovski (2002); Doltsinis & Kang (2004)) is to find a design with a minimal variance of the scattering model responses aroung the mean values of the design parameters (see Byrne & Taguchi (1987); Phadke (1989)).

Other approaches for an evaluation of the design robustness, e.g. the linear approximation of "scattering" solver responses (see e.g. Abspoel *et al.* (2001)) or the variance estimation in genetic programming (see e.g. Pictet *et al.* (1996); Branke (1998)), independently of given parameter distributions will not be subject of the following remarks as they are not to be counted to stochastic optimization methods in a stricter sense.

1.4 Reliability-based optimization

In the reliability-based optimization, the optimization problem can be enhanced by additional stochastic restrictions ensuring that prescribed probabilities of failure can not be exceeded. Furthermore, the probability of failure itself can be integrated into the objective function. Frequently, the search for the optimum by means of deterministic optimization is combined with the calculation of the failure probability, e.g. using the first- order second-moment analysis (FOSM) (e.g. Melchers (2001)). A more promising combination may under certain circumstances involve the first and second order reliability methods (FORM/SORM) (e.g. Choi *et al.* (2001); Allen *et al.* (2004); Allen & Maute (2004)).

Within the deterministic optimization, a calculation of the failure probability of individual designs has to be performed in order to be able to properly evaluate these designs. Therefore, special attention has to be paid to the cost efficiency of this calculation. As an example, for smooth and well-scaled objective functions with few continuous design parameters, the deterministic optimization as well as the determination of the failure probability that is included within the optimization iteration loop may be performed by means of gradient based programming (e.g. Sequential Quadratic Programming, vgl. Schittkowski (1985)).

In Kharmanda *et al.* (2002) a decrease of the numerical expense of these two nested iterations is attempted by substituting the deterministic objective function as well as the limit state function on which the point of largest probability density is searched within FORM by a single objective function in a hybrid design space. However, this leads to an enlargement of the design space for the gradient based programming.

1.5 Approximation methods

In the reliability-based optimization, frequently approximation function are applied that at the same time approximate the design space and the space of random parameters by means of a meta-model, e.g. in Choi *et al.*; Youn *et al.* (2004); Yang & Gu (2004); Rais-Rohani & Singh (2004). Successful industrial applications of these methods can amongst others be found in Youn & Choi (2004).

In Royset & Polak (2004), a linear approximation of the limit state function serves as a constraint of the optimization problem. An improvement of the optimization result is tempted in Royset *et al.* (2003) by taking into account the gradients of the limit state function.

However, in the robust optimization (see Chen *et al.* (2004); Wilson *et al.* (2001)) as well, different approximation models in combination with an appropriate variance determination are used, e.g. global polynomial approximations and Kriging models. Their use is restricted to problems with few random variables and few optimization variables ($n \leq 10$).

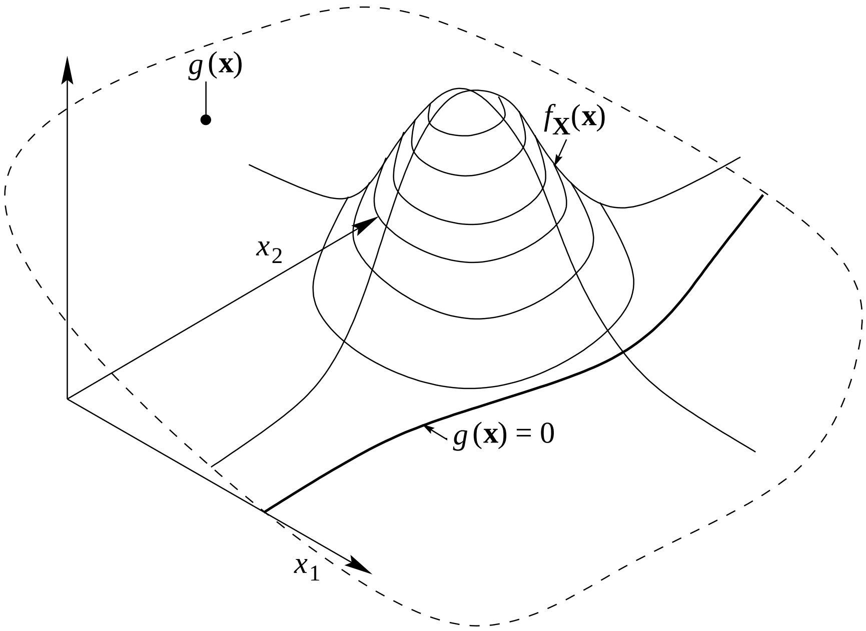


Figure 3: $f_{\mathbf{X}}(\mathbf{x})$: joint probability density function, $g(\mathbf{x})$: structural response function, $g(\mathbf{x}) = 0$: limit state function.

2 Stochastic optimization

2.1 Introduction

2.1.1 Reliability-based design optimization

In reliability-based design optimization, the deterministic optimization problem

$$\begin{aligned} f(d_1, d_2, \dots d_{n_d}) &\rightarrow \min \\ g_k(d_1, d_2, \dots d_{n_d}) &= 0;\ k = 1, m_e \\ h_l(d_1, d_2, \dots d_{n_d}) &\geq 0;\ l = 1, m_u \\ d_i &\in [d_l, d_u] \subset \mathbb{R}^{n_d} \\ d_l &\leq d_i \leq d_u \\ d_i &= E[X_i] \end{aligned} \quad (1)$$

with n_r random parameters $\mathbf{X}$ and n_d means of the design parameters $\mathbf{d} = E[\mathbf{X}]$ is enhanced by additional m_g random restrictions

$$\int \overset{n_r}{\cdots} \int_{g_j(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} - P(\mathbf{X} : g_j(\mathbf{X}) \leq 0) \leq 0;\ j = 1, m_g \quad (2)$$

with the joint probability density function of the basic random variables $f_{\mathbf{X}}(\mathbf{x})$ and m_g limit state functions $g_j(\mathbf{x}) \leq 0$ (see Figure 3). The probability of failure in (2) is calculated applying the reliability analysis.

Furthermore the objective function can be enhanced by additional criteria such as minimization of the probability of failure $P(f)$

$$f(d_1, d_2, \dots d_{n_d}, P(f)) \rightarrow \min \quad (3)$$

[1] source: www.isixsigma.com/library/contentc020729a.asp

with

$$P(f) = \int_{g_j(\mathbf{x}) \leq 0} \overset{n_r}{\cdots} \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{4}$$

2.1.2 Robust design optimization

Within the robust design optimization, the objective (1) is enhanced by the requirement to minimize the variances $\sigma^2_{X_i}$

$$f(d_1, d_2, \ldots d_{n_d}, \sigma^2_{X_1}, \sigma^2_{X_2}, \ldots \sigma^2_{X_{n_r}}) \rightarrow \min \tag{5}$$

with

$$\sigma^2_{X_i} = \frac{1}{M-1} \sum_{k=1}^{M} \left(x_i^k - \mu_{X_i} \right)^2$$

2.2 Genetic programming in combination with the first-order reliability method (FORM)

For a multitude of optimization problems in structural mechanics, the precision of the input data must be doubted. The deviations from the target values or nominal values can often be reasonably described by random variables. Especially such problems without any overlapping of design variables and random variables permit to choose completely different strategies for optimization and stochastic analysis in order to ably exploit the their advantages.

Genetic programming is reasonably used in cases when the objective function's and or the restrictions' dependency on the design parameters is not differentiable or not even continuous. Furthermore, genetics can yield good results for problems that are characterized by not contiguous areas ("islands").

Typically, the failure probabilities of well designed systems are small. Therefore, a reliability method has to be applied that provides these value at a reasonable expense. This can be with good success the first-order reliability method (FORM) Rackwitz & Fießler (1978) for problems the restrictions of which (usually including the failure probability in some form) are depending on the stochastic variables in a differentiable way.

In this context it has to be considered that genetic algorithms generally implement the restrictions in the form of penalty terms. Thus, the choice of an appropriate penalty method is of a certain importance.

2.3 Response surfaces on design and random space

Normally, the response function $g(\mathbf{x})$ of a structural system is described implicitly, e.g. through an algorithmic procedure within finite element analysis. Alternatively, the original structural response function can be approximated by a response surface function $\tilde{g}(\mathbf{x})$ of a polynomial form (Faravelli (1986); Engelund & Rackwitz (1992); Rajashekhar & Ellingwod (1993)).

A commonly used method for response value approximation is the regression analysis. Usually, the approximation function is a first order or second order polynomial (Box & Draper (1987); Myers (1971)). As an example in the $(n = 2)$-dimensional case, k-responses $(k = 1, ..., m)$ will be approximated using a least square quadratic polynomial in the following form:

$$\tilde{g}_k(\mathbf{x}) = \beta_1 x_{1k} + \beta_2 x_{2k} + \beta_{11} x^2_{1k} + \beta_{22} x^2_{2k} + 2\beta_{12} x_{1k} x_{2k} + \epsilon_k \tag{6}$$

Herein the term ϵ_k represents the approximation errors. The approximate coefficients β can be calculated using the least square postulate

$$S = \sum_{k=1}^{m} \epsilon^2_k = \boldsymbol{\epsilon}^T \boldsymbol{\epsilon} \rightarrow \min$$

One of the major advantages of the response surface method lies in its potential to selectively determine the number of structural analyses for each support point. This is especially helpful if some overall knowledge on the system behavior - particularly near to the failure region - is a priori available. By such means the computational effort can be substantially reduced.

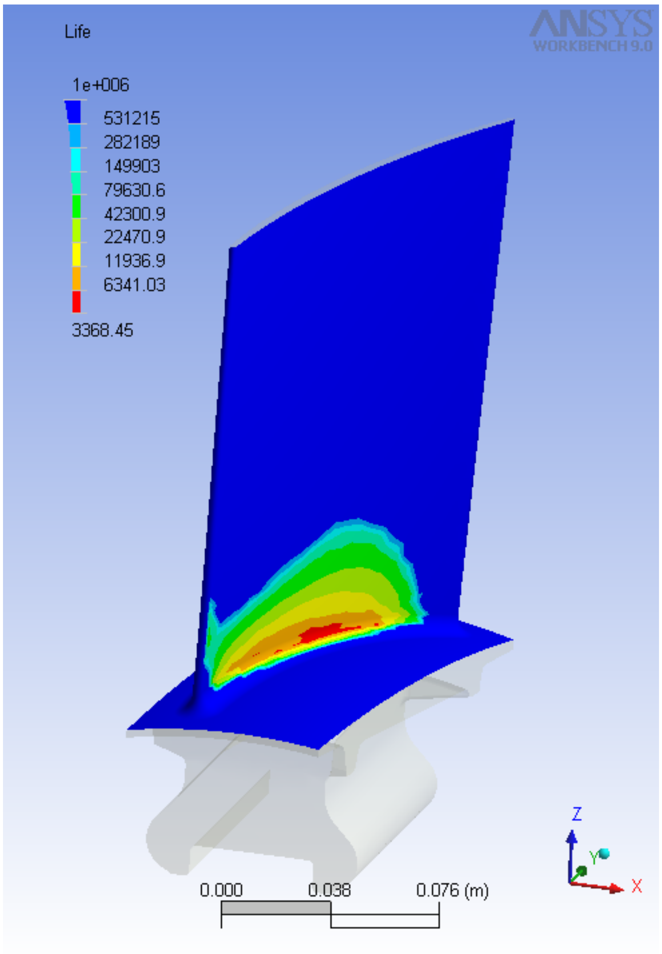


Figure 4: Turbine blade with rotational velocity and cycle fatique life.

Hence, approximations become important which can be based e.g. on the response surface method. Unfortunately, the global approximation schemes widely used in the application of the response surface method can be quite misleading due to the lack of information in certain regions of the random variable space. It is therefore required to avoid such undesirable interpolation errors at reasonable computational effort.

3 Numerical examples

3.1 Robust optimization of cycle fatique life

An example serves to demonstrate the applicability of the global response surface method. The computational probabilistic and structural analysis tasks were performed with the software package ANSYS DesignXplorer.

The DesignXplorer solutions provide the capability to create a robust design optimization by permitting to define both design variables and uncertainty variables, and then to optimize a set of reliability aims for quantities such as fatigue life, stress, or deflection. DesignXplorer, which is based on Design of Experiments (DOE), works from within the Workbench environment to perform DOE analyses of any Workbench simulation, including those with CAD parameters.

The mechanical system is a turbine blade, as shown in Fig. 4, subjected to rotational velocity. The initial design is a minimal fatique life of 3368 cycles. It is assumed that the tangential and axial leaning

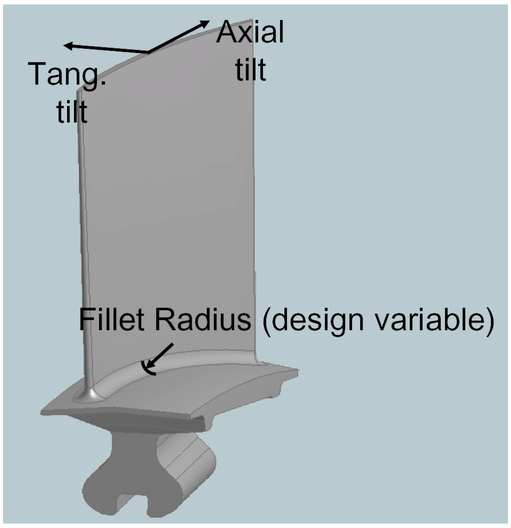


Figure 5: Turbine blade with random variables tangential and axial leaning and the design parameter fillet radius.

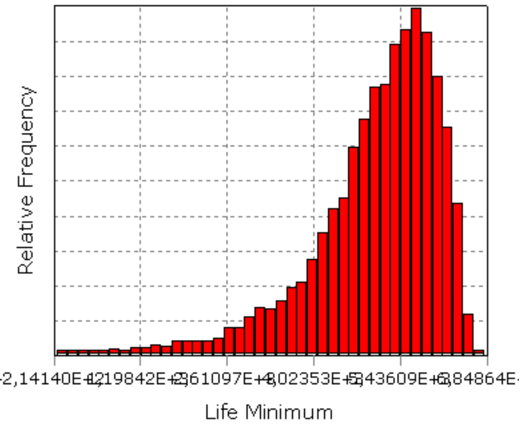


Figure 6: Probability density function of the fatique life response.

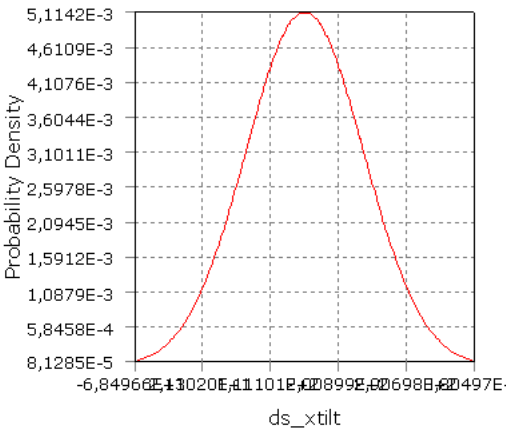


Figure 7: Probability density function of the tangential leaning.

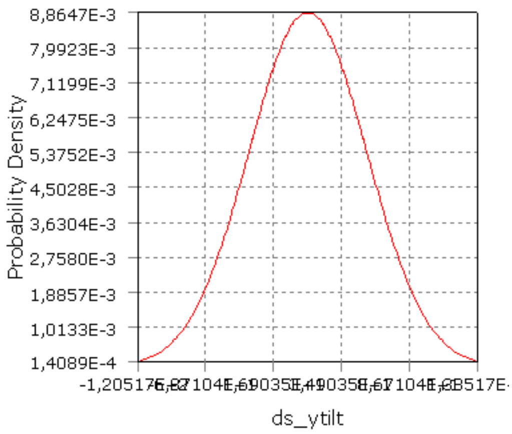
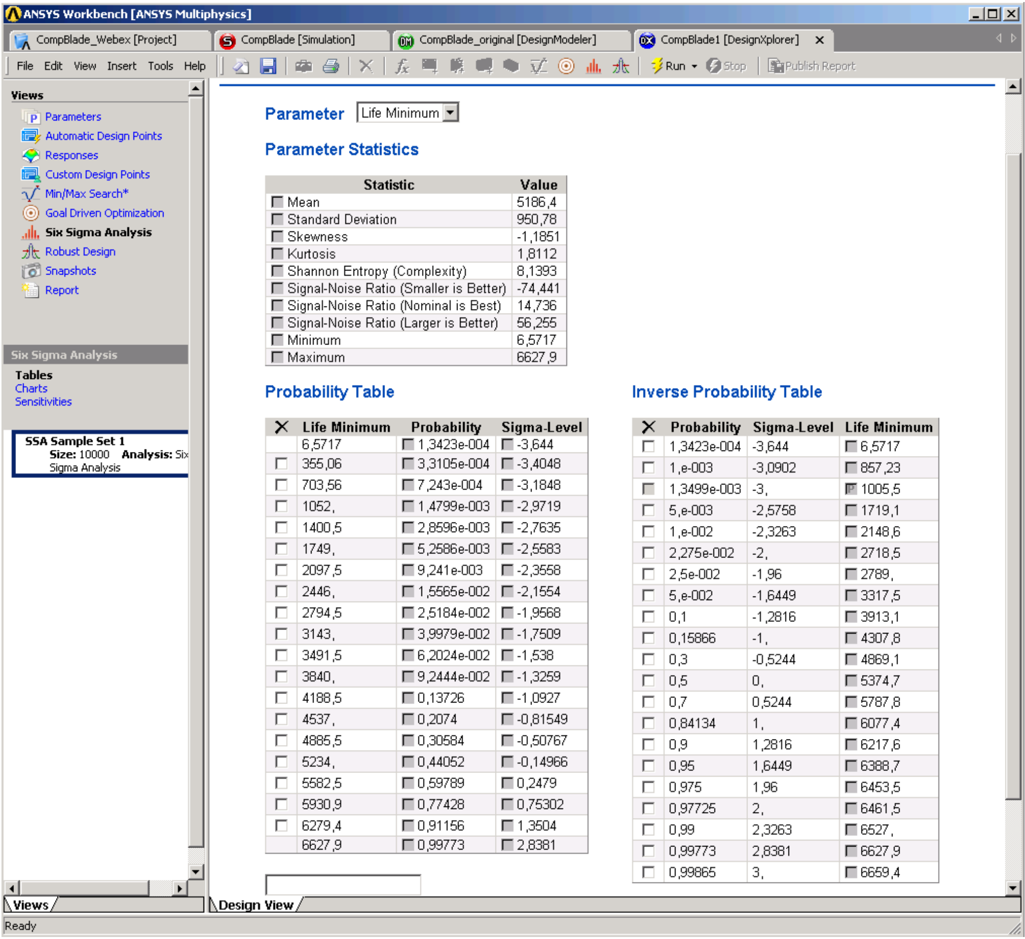


Figure 8: Probability density function of the axial leaning.



Statistic	Value
Mean	5186,4
Standard Deviation	950,78
Skewness	-1,1851
Kurtosis	1,8112
Shannon Entropy (Complexity)	8,1393
Signal-Noise Ratio (Smaller is Better)	-74,441
Signal-Noise Ratio (Nominal is Best)	14,736
Signal-Noise Ratio (Larger is Better)	56,255
Minimum	6,5717
Maximum	6627,9

Life Minimum	Probability	Sigma-Level
6,5717	1,3423e-004	-3,644
355,06	3,3105e-004	-3,4048
703,56	7,243e-004	-3,1848
1052,	1,4799e-003	-2,9719
1400,5	2,8596e-003	-2,7635
1749,	5,2586e-003	-2,5583
2097,5	9,241e-003	-2,3558
2446,	1,5565e-002	-2,1554
2794,5	2,5184e-002	-1,9568
3143,	3,9979e-002	-1,7509
3491,5	6,2024e-002	-1,538
3840,	9,2444e-002	-1,3259
4188,5	0,13726	-1,0927
4537,	0,2074	-0,81549
4885,5	0,30584	-0,50767
5234,	0,44052	-0,14966
5582,5	0,59789	0,2479
5930,9	0,77428	0,75302
6279,4	0,91156	1,3504
6627,9	0,99773	2,8381

Probability	Sigma-Level	Life Minimum
1,3423e-004	-3,644	6,5717
1,e-003	-3,0902	857,23
1,3499e-003	-3,	1005,5
5,e-003	-2,5758	1719,1
1,e-002	-2,3263	2148,6
2,275e-002	-2,	2718,5
2,5e-002	-1,96	2789,
5,e-002	-1,6449	3317,5
0,1	-1,2816	3913,1
0,15866	-1,	4307,8
0,3	-0,5244	4869,1
0,5	0,	5374,7
0,7	0,5244	5787,8
0,84134	1,	6077,4
0,9	1,2816	6217,6
0,95	1,6449	6388,7
0,975	1,96	6453,5
0,97725	2,	6461,5
0,99	2,3263	6527,
0,99773	2,8381	6627,9
0,99865	3,	6659,4

Figure 9: Example of the postprocessing of the robust design optimization using ANSYS DesignXplorer.

are independent Gaussian random variables with statistics given in Figure 5.

The number of required Design of Experiments (finite element solver runs) amounts 15. The results of the robust design optimization is an filled radius of 0.2505 with life minimum value of 1006 cycles. The corresponding failure probability (see Figure 9) is equal to $1.35 \cdot 10^{-3}$ and the sigma level is -3.

3.2 Stochastic optimization of a dynamic structure

The aim of the classical optimization problem for structural elements is to minimize the mass while observing deformation or strength restrictions. The mass of the displayed simple beam with rectangular

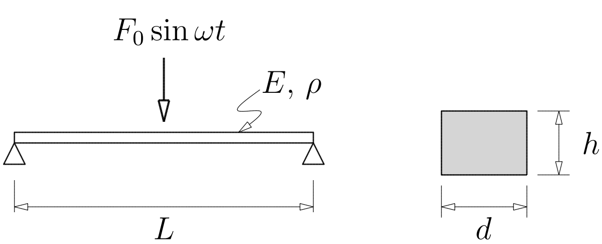


Figure 10: Beam with rectangular cross section

cross section (d, h) subjected to deadload and a harmonic load $F(t)$ shall be minimized. The following

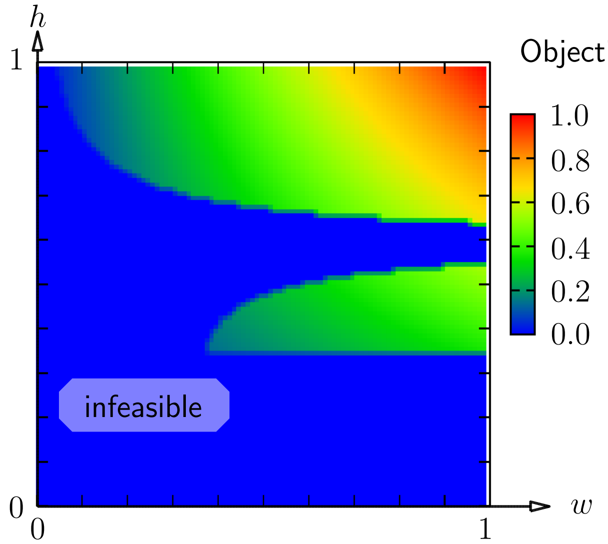


Figure 11: Deterministic objective and feasible design space.

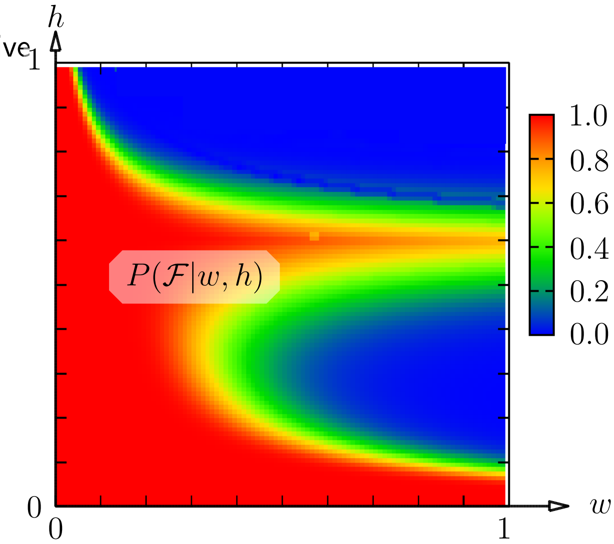


Figure 12: Failure probability depending on h and w.

restrictions hold:

- the central deflection w_g due to deadload is smaller than $5\,mm$.
- the additional central deflection w_d due to the dynamic load $F(t)$ is smaller than $10\,mm$

The computational probabilistic and multidisciplinary analysis tasks were done with the software package OptiSLang (Bucher *et al.* (2001)).

The objective function (i.e. the cross section area) and the admissible area are displayed in Figure 11 for assumed values of $F_0 = 20\,kN$, $\omega = 60\,rad/s$, $E = 3 \cdot 10^{10}\,N/m^2$, $\rho = 2500\,kg/m^3$, $L = 10\,m$ and $g = 9.81\,m/s^2$ This figure shows that two separate admissible areas exist. A gradient based optimizer generally encounters difficulties to override the area boundaries in order to find the global optimum.

Therefore, the use of genetic optimization methods seems promising. Furthermore, in many application cases – especially concerning structural dynamics – the characterizing parameters are afflicted with stochastic uncertainties. In the present example it is assumed that the dynamic load amplitude F_0 and the excitation angular frequency ω are random variables with gaussian distribution. The mean values correspond to the aforementioned nominal values, and both variational coefficients have been assumed to be 10%. This yields that the restriction from the dynamic load can only be met with a certain probability < 1. Fig. 12 displays the probability of violation of the dynamic restriction (i.e. the conditional failure probability $P(\mathcal{F}|w, h)$) as a function of the design parameters w und h.

The subsequent optimization was started with the additional restriction that the conditional failure probability be $< 1\%$. In the framework of genetics, designs with a higher failure probability were punished by a penalty term S the value of which is independent from $P(\mathcal{F})$. Hence, the objective function writes

$$L = h \cdot w + SH[P(\mathcal{F}) - 0.01] \tag{7}$$

In this equation $H[.]$ designates the Heavyside function. The penalty parameter S has been assumed as 100. A genetics run with 30 generation with 50 individuals each yielded the following best individual: $h = 0.90617$, $w = 0.78035$. The failure probability in this case was 0.38%, which is significantly below the threshold of 1%. Fig. 13 illustrates the progression of the genetic algorithm by displaying the populations of the first, the tenth, and the 20th generation. The concentration on areas with acceptable failure probability is well distinguishable.

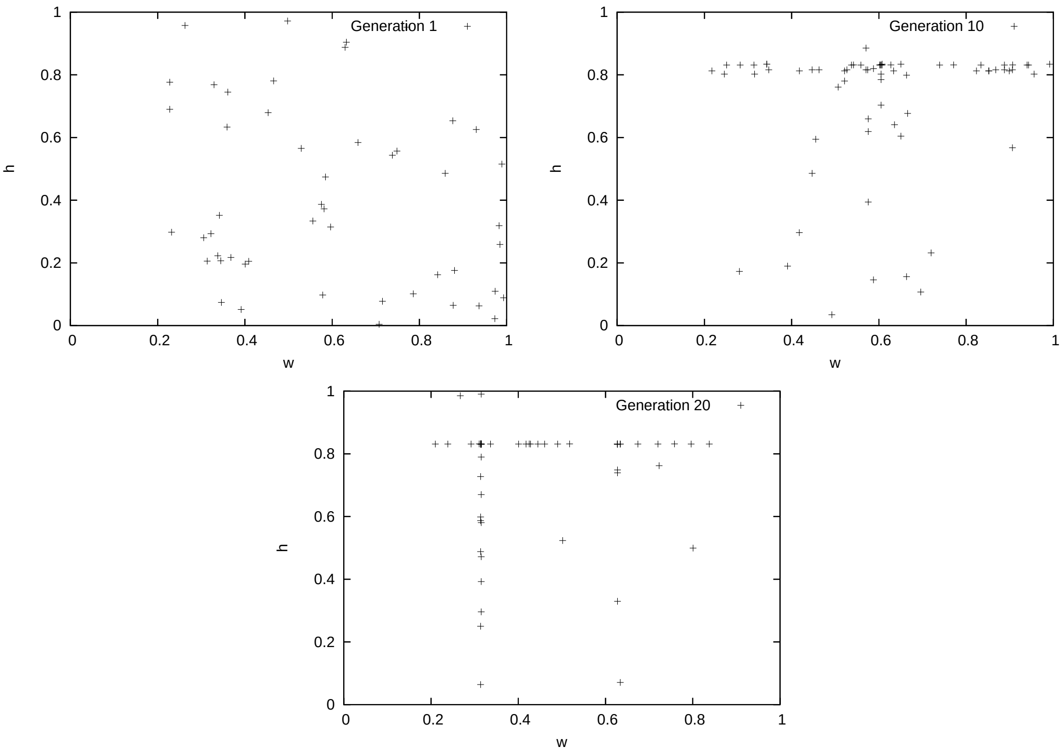


Figure 13: Evolution of the population in genetics

4 Concluding Remarks

Stochastic optimization can provide multiple benefits. It permits the identification of those design parameters that are critical for the achievement of a certain performance characteristic. A proper adjustment of the thus identified parameters to hit the target performance is supported. This can significantly reduce product costs.

The effect of variations on the product behaviour and performance can be quantified. Moreover, stochastic optimization can lead to a deeper understanding of the potential sources of variations. Hence, a minimization of the effect of variations (noise) is made possible, and appropriate steps to desensitize the design to these variations can be determined. Consequently, more robust and affordable product designs can be achieved.

In the framework of quality inspection, cost-effectiveness is increased as the inspection can be focused on the parameters that have been determined as critical for the performance.

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