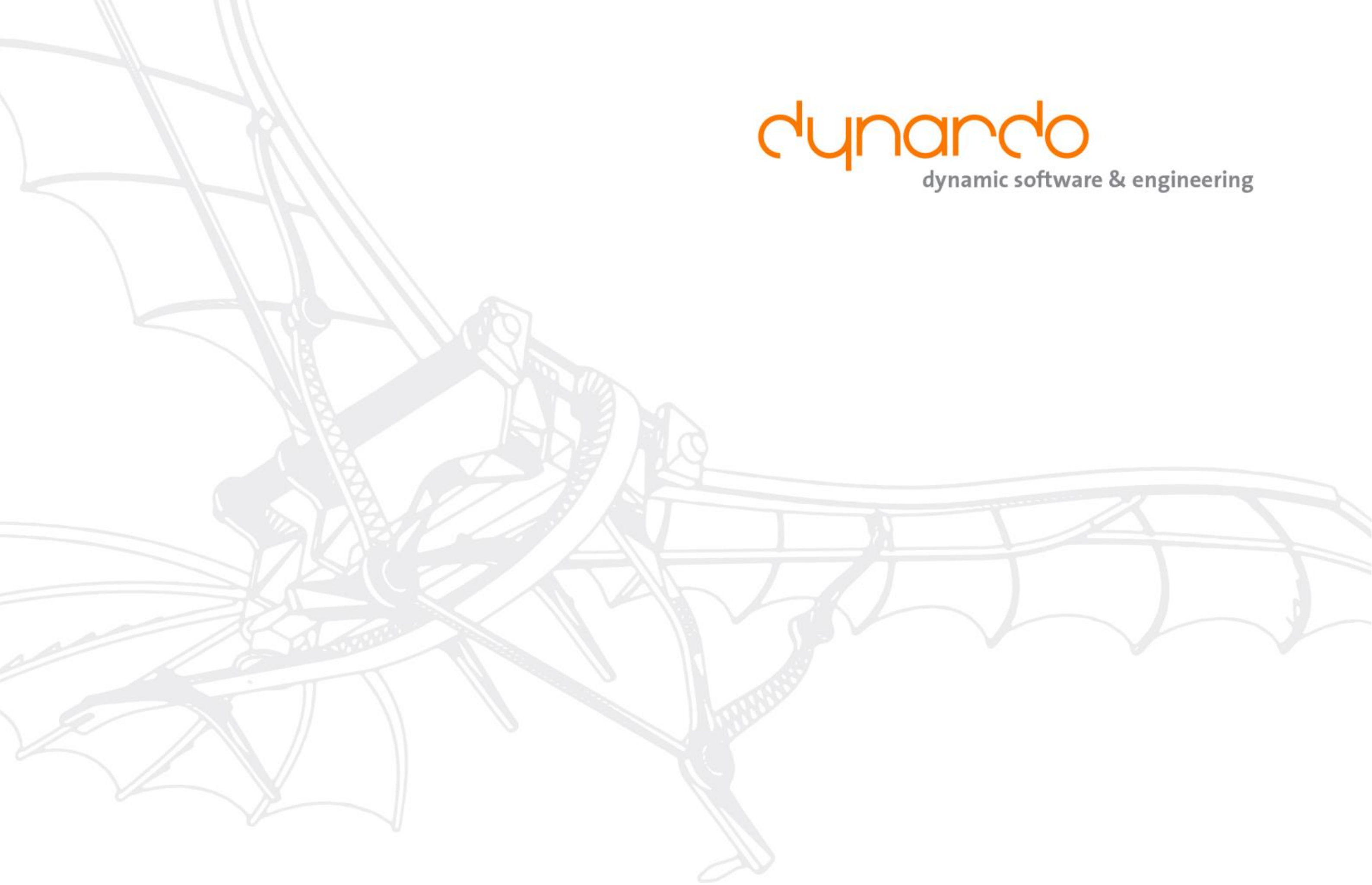

Lectures

Efficient Methods for Industrial Robust Design Optimization

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Summary:

In this paper suitable methods for robust design optimization are presented and discussed. Starting with an initial sensitivity analysis, the important design parameters can be identified and the optimization task can be significantly simplified. Taking into account uncertainties, the optimization task becomes more challenging. Instead of deterministic response values, uncertain model responses need to be analyzed. For a successful implementation this analysis requires the estimation of the probabilities of rare events. With help of a variance-based and reliability-based robustness evaluation, the required safety level can be implied in the optimization process and verified for the final design. Starting with the accompanying paper, we introduced the overview of robust design optimization and illustrated practical application. In this paper we continue with a more detailed view on the methodology.

Keywords:

Stochastic analysis, sensitivity, optimization, robustness, uncertainty

1 Introduction

CAE-based optimization has a long tradition in engineering. Goal of optimization is often the reduction of material consumption while pushing the design performance to the boundaries of allowable stresses, deformations or other critical design responses. At the same time safety margins are asked to be reduced and products should be cost efficient and not over engineered. Of course a product should not only be optimal under one possible set of parameter realizations. It also has to function with sufficient reliability under scattering environmental conditions. In the virtual world we can proof that e.g. with a stochastic analysis, which leads to CAE-based robustness evaluation. If CAE-based optimization and robustness evaluation is combined, we are entering the area of Robust Design Optimization (RDO) which is also called Design for Six Sigma (DFSS) or just Robust Design (RD).
The main idea behind that methodology is that uncertainties are considered in the design process. These uncertainties may have different sources like, in the loading conditions, tolerances of the geometrical dimensions and material properties caused by production or deterioration. Some of these uncertainties may have a significant impact to the design performance which has to be considered in the design optimization procedure.
Before entering the discussions of suitable and efficient algorithms for CAE-based RDO we would like to point out some main points how to enhance from CAE-based optimization to CAE-based RDO.

In this paper different strategies to search for a robust design are presented and investigated with respect to their efficiency and applicability to time consuming numerical models.
Talking about the combination of robustness evaluation and optimization the frequency of coupling and interaction of both tasks has to be defined. We call it an iterative Robust Design Optimization (RDO) when deterministic optimization is combined with variance-based robustness analysis at certain points during the optimization process. Of course this requires the introduction of safety factors, which should assure that a sufficient distance to the failure criteria is given during the deterministic optimization. These safety factors may be adjusted iteratively during the iterative RDO process and a final robustness and reliability proof is mandatory at least at the end of the procedure. This procedure is the state-of-the-art in the majority of publications on real world RDO projects [11,12].
If the safety margins fluctuate within the optimization domain, e.g. due to several interacting failure phenomena, an iterative procedure may require a large number of iterations. In such a case, an automatic approach where the robustness criteria are estimated for every candidate in the optimization domain, a so-called nominal design, may be more efficient with respect to the CPU requirements. That procedure we call simultaneous RDO approach. Since the robustness evaluation is performed as an internal loop within the global optimization loop, this approach is sometimes also called “loop in loop” RDO.

2 Sensitivity Analysis

In contrast to local derivative based sensitivity methods, global variance based approaches quantify the contribution with respect to the defined total variable variation. Unfortunately, sufficiently accurate variance based methods require a huge numerical effort due to the large number of necessary simulation runs. Therefore, often meta-models or simplified regressions are used to represent the model responses by surrogate functions in terms of the model inputs. However, many meta-model approaches are available and it is often not clear which one is most suitable for which problem. Another disadvantage of meta-modeling is its limitation to a small number of input parameters. Due to the so-called "curse of dimensionality" the approximation quality decreases for all meta-model types dramatically with an increasing dimension. As a result, an enormous number of samples is necessary to represent high-dimensional problems with sufficient accuracy. In order to overcome these problems, Dynardo developed the Metamodel of Optimal Prognosis [5,6]. In this approach the optimal input parameter subspace together with the optimal meta-model type are automatically determined with help of an objective and model independent quality measure, the Coefficient of Prognosis (CoP).

3 Multidisciplinary Optmization

Besides using the sensitivity analysis to reduce the number of optimization parameters to the most important ones, sensitivity analysis can be used to learn more about the character of an optimization problem:

- How non-linear and noisy are the response functions?
- Can we improve the result extraction of the solver outputs in order to achieve a better representation of the responses by the optimization parameter?
- Should we adjust the variation range of the optimization parameters?
- How many responses are in conflict with each over?
- Did we already find well performing designs during the scan of the design space, which we could introduce as start designs to the following optimization algorithm?

Consequently, the findings from the sensitivity study will result in a qualified definition of the input parameter space and of the objectives and constraints and even enables an appropriate choice of a suitable optimization algorithm including the corresponding settings.

For the search of an optimal design on an approximation function or even with direct solver runs, a huge number of optimization algorithms can be found in literature. They can be classified in gradient-based algorithms like steepest descent and Newton methods [4], heuristic gradient-free approaches like grid or pattern search, adaptive response surfaces and simplex optimizers and furthermore nature inspired search methods like genetic and evolutionary algorithms [1], particle swarm optimization [3] and simulated annealing. In Table 1 a set of representative methods is given and assessed with respect to its field of applications.

Often, several objectives have to be addressed in the optimization task. In case that these objectives are in conflict with each other and if a suitable weighting of these objectives is not possible with the available knowledge, the application of single-objective optimization methods may be difficult. In such cases, the optimization can be performed by searching simultaneously for different possible compromises between the conflicting objectives, a so-called Pareto frontier. Once this frontier is obtained, more qualified decisions are possible in order to specify further requirements for the selection of a suitable design out of this frontier or its further improvement by single-objective methods. Today, genetic and evolutionary algorithms as well as particle swarm optimization methods are often the methods of choice for an efficient Pareto search.

Algorithm	**global search**	**local search**	**No. of parameter**	**constraints**	**failed designs**	**solver noise**	**discrete parameters**
Gradient based (NLPQL/ SQP)	no	ideal	<= 20	many	no	no	no
Downhill simplex method	yes	ideal	<= 5	few	yes	minor noise	ordered discrete
Response surface methods	ideal	ideal	<= 20	many	yes	yes	ordered discrete
Evolutionary & genetic algorithms	ideal	yes	many	yes	many	yes	yes
Particle swarm optimization	ideal	yes	many	yes	yes	yes	yes
Simulated annealing	yes	no	<= 20	few	yes	yes	yes

Table 1: Comparison of common optimization methods: the maximum number of parameters is defined regarding the efficiency of the optimization method compared to the other algorithms.

4 Robustness Evaluation

In order to match production quality requirements of designs it is necessary, that the scatter of all important responses caused by scattering material and geometrical properties and fluctuating environmental and operational conditions stays within acceptable ranges. With help of the robustness analysis this scatter can be estimated. Within this framework, the scatter of a response itself descripted by mean value and standard deviation or its safety with respect to a failure limit has to be quantified. The safety can be formulated variance-based with help of the safety margin between failure and the mean value and probability-based using the probability that the failure limit is exceeded. In Figure 1 this is shown in principle.

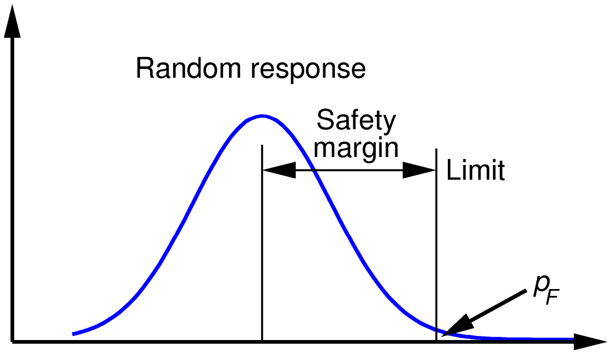

Figure 1: Scatter of a fluctuating response with safety margin (distance between mean and the failure limit) and the corresponding probability of failure p_F

In the variance-based approach the safety margin is often given in terms of the corresponding standard deviation of the corresponding response. A “six sigma” design should fulfill a safety margin of six times the standard deviation. Assuming a normally distributed response, the classical six sigma concept considers an additional safety of 1.5 times the standard deviation. The 4.5 sigma margin of a normal distribution corresponds to a failure rate of 3.4 defects out of one million design realizations. The assumption of a normally distributed response may be not valid if non-linear effects dominate the mechanisms of failure or disoperation. In such cases the extrapolation of rare event probabilities like 3.4 out of a million just from the estimated mean value and standard deviation may be strongly erroneous. Thus, the assumption of a normal distribution should be verified at least at the final RDO design. If this is not the case, the probability of failure should be estimated with the more qualified reliability analysis.

1.1. Variance based robustness analysis

Today, the majority of RDO approaches uses variance-based robustness measures in order to minimize the variation of a response with or without constraining the mean, which is known as Taguchi approach, or to reach a certain level of safety quantified by the safety margin or sigma levels, where Design for Six Sigma is one possible concept.
Since the Taguchi approach is very renowned in the industrial Six Sigma community, we would like to point out that the application of this strategy to the virtual product development field often needs some extensions. The main idea of the Taguchi approach is to reduce the scatter of a final product outcome in case that scatter is not acceptable. At a production line that limit can usually be measured with high accuracy. If the limit is exceeded, a sensitivity analysis is performed in order to detect the responsible scattering input variables. With this information some of the important input sources may be reduced in order to result in smaller product scatter. But the reduction of the input scatter is only one possibility to reduce the output scatter. This reduction requires a higher quality with respect to the materials and the production process. This might extremely increase the production costs. In virtual prototyping it is usually not accepted that just the quality requirements are increased. We are asked to reduce the sensitivity of the designs with respect to the input scatter or to redesign the product in such a way, that the same input scatter does not result in a violation of the robustness requirements. Often, Taguchi based RDO strategies try to reduce the sensitivity to the input scatter only and aim for designs with the lowest standard deviation. But a design with a very low standard deviation can still violate the safety constraints. On the other hand, the Taguchi approach may result in designs which are over engineered and not cost effective. Therefore, we recommend in the framework of variance based RDO not only to target for designs with low standard deviation, but also to consider a sufficient safety margin to failure or operation limits. As a consequence the proof of a sufficient safety margin to failure

and operational limits should be the primary goal of RDO in virtual prototyping. In case of high requirements with respect to the safety level like in Design for Six Sigma a final reliability proof should be considered.

1.2. Reliability-based robustness analysis

Often variance-based measures are used within the RDO workflows due to their efficiency with respect to the number of solver runs. Therefore, it is very important that at least for the final design the targeted probability of exceeding a failure limit is verified. In engineering applications reliability levels of at least 3 sigma (1.3 out of 1000) are usually required for non-critical products like high end consumer goods, while up to 5 sigma (less than one failure in one million designs) is required for safety relevant critical components.
Usually, the kind of rare events is connected to the non-linearity of the structures. Thus, we cannot estimate the probability distribution of relevant response values (e.g. maximum stresses) with respect to rare events with a sufficient accuracy, if only variance based methods or low-order approximations based on linearization or series expansions are applied. Therefore, the estimates of small probabilities have to be verified with a qualified reliability analysis.

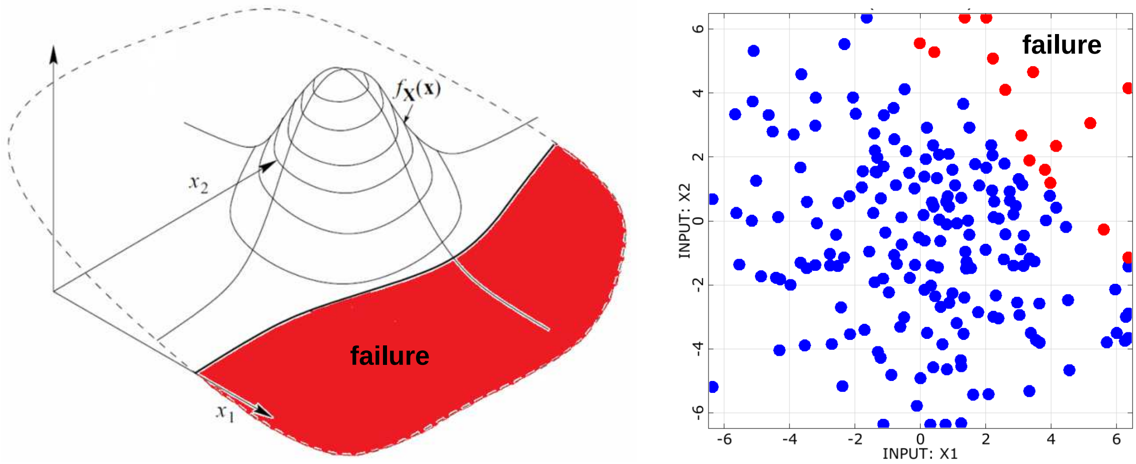

Figure 2: Reliability analysis as multi-dimensional integration of the probability density of the inputs uncertainties over the failure domain (left) and integration by Monte Carlo Simulation (right)

Within the reliability method the probability of reaching a failure limit is obtained by an integration of the probability density of the uncertainties in the failure domain as shown in Figure 2. One well-known method is the Monte Carlo Simulation, which can be applied independently of the model non-linearity and the number of input parameters [9]. This method is very robust and can detect several failure regions with highly non-linear dependencies. Unfortunately, it requires an extremely large number of solver runs to proof rare events.
Therefore, more advanced sampling strategies have been developed like Directional Sampling, where the domain of input variables is scanned by a line search in different directions, or Importance Sampling, where the sampling density is adapted in order to cover the failure domain sufficiently and to obtain more accurate probability estimates with much less solver calls. Other methods like the First or Second Order Reliability Method (FORM & SORM) are still more efficient than the sampling methods by approximating the boundary between the safe and the failure domain, the so-called limit state. In contrast to a global low order approximation of the whole response, the approximation of the limit state around the most probable failure point is much more accurate. Nevertheless, only one dominant failure point can be found and evaluated. A good overview of these “classical” methods is given in [2].
For a successful application of global response surface methods, it is necessary to assure that the region around the most probable failure point is approximated sufficiently accurate. This can be reached by an iterative adaptation scheme, where new support points are generated in this region. With this improvement also two or three important failure regions can be represented with a small number of solver runs [10].
In reliability analysis where small event probabilities have to be estimated, we have to pay special attention that the algorithms obtain an acceptable level of confidence in order to detect the important regions of failure. Otherwise, they may estimate a much smaller failure probability and the safety assessment will be much too optimistic.

The available methods for an efficient reliability analysis try to learn where the dominant failure regions are and concentrate their simulation effort in those regions in order to drastically reduce the necessary CAE simulations. This is necessary to become candidates of reliability for real world applications. But unfortunately every time you try to learn something you are in danger to take wrong short cuts and all of that methods are in danger to miss the failure domain and to come up with much too optimistic estimations of failure probability. Therefore, we strongly recommend verifying the variance based estimates of the failure probability with at least two different reliability methods. That will produce a trustworthy window of failure probabilities in order to make reasonable design decisions based on CAE-models.

Approach	Non-linearity	Failure domains	No. parameters	No. solver runs
Monte Carlo Simulation	Arbitrary	arbitrary	many	>10^4 (3 sigma) >10^7 (5 sigma)
Directional Sampling	Arbitrary	arbitrary	<= 10	1000-5000
Adaptive Importance Sampling	Arbitrary	one dominant	<= 10	500-1000
First Order Reliability Method	Monotonic	one dominant	<=20	200-500
Adaptive Response Surface Method	Continuous	few dominant	<=20	200-500

Table 4: Comparison of different qualified reliability methods

5 Robust Design Optimization

In the framework of Robust Design Optimization the deterministic optimization methods are now extended by considering uncertainties of specific input variables. With help of a statistical evaluation of the no longer deterministic objective function and constraint conditions, the design is driven to a region where the robustness requirements are fulfilled while the desired performance is optimal.

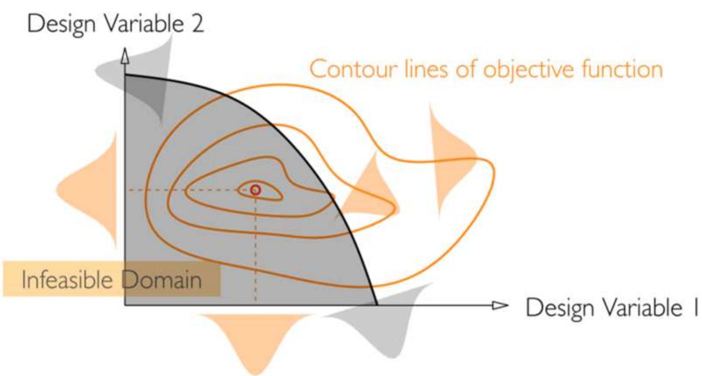

Figure 3: Robust Design Optimization: Optimization considering uncertain input parameters and responses as well as uncertain objective and constraint functions

The mathematically most accurate way to obtain a robust design which fulfills even the requirements of a small failure probability would be to couple a suitable deterministic optimizer with a reliability analysis and formulate the constraint conditions with respect to the required safety level. This approach is called reliability-based Robust Design Optimization and is applied for a fast simulation model in [8]. Due to the partially high numerical effort of a single reliability analysis and the required verification of the reliability estimates by a second algorithm, for the most real world application it is not possible to perform such an sophisticated analysis. On the other hand it is not necessary to proof the safety level for each design during the optimization process with high confidence. Therefore, simplified methods have been developed. A first step to improve the efficiency and the robustness of a simultaneous Robust Design Optimization is to estimate the safety level with variance-based robustness measures. Taking into account an additional safety margin like in case Design for Six Sigma, the coupled analysis can be performed by using only 20 to 50 samples for each nominal design in order to get reasonable estimates and to drive the optimizer in the right direction. Nevertheless, a final reliability proof should be considered again verifying the rough assumptions in the variance-based approach.

One important advantage of this approach is that the expensive reliability analysis has to be performed only at the end of the procedure. Furthermore, due to the dimension independent estimates of the variance based measures, a large number of uncertain inputs can be considered. However, still the numerical effort is 20 to 50 times larger than the deterministic optimization with safety factors. This limits this approach to computational less expensive simulation models.
To overcome this limitation global response surfaces are often applied. In case of simultaneous Robust Design Optimization the approximation function has to consider all design parameters and all of the uncertain inputs. For cases where the number of uncertain inputs is large, an efficient application of such an approach is not possible as shown in [7], since the number of necessary support points increases dramatically. On the other hand, a reduction of the number of inputs is dangerous since their influence may change when moving the nominal design through the design space. However, in some cases this approach may perform well, but always a final robustness evaluation with direct solver calls should be performed in order to verify the approximated safety level.
In our experience an iterative Robust Design Optimization procedure can be applied more general and stable to industrial applications than the simultaneous approach. In this procedure the deterministic optimization has to consider safety factors to assure a certain safety margin of the critical responses. After a first optimization step the robustness measures are evaluated and the safety level is assessed. These safety factors should be chosen in that way, that the robustness requirements are fulfilled. Generally the safety factors are not known *a priori*. In this case a suitable initial guess is specified and the initial deterministic optimization is performed. If the safety requirements are not fulfilled, the responsible safety factors have to be increased and the deterministic optimization has to be repeated. With this approach usually a robust design is found within 3 to 4 iterations. If the robustness criteria are expected to be fulfilled, again a reliability proof is necessary to verify higher safety levels.

6 Summary

Successful integration of RDO strategies into CAE-based virtual product development cycles needs a RDO strategy which is in balance with the available knowledge about uncertainties of scattering variables, with available criteria's to reliable quantify robustness or safety of designs as well as with the dimensionality and non-linearity of the RDO task.
For definition of successful objectives and criteria's for robust designs sensitivity analysis in the design space of optimization as well as in the space of scattering variables are very helpful. Any kind of design space defined by a hand full or hundreds of optimization parameter is a valid space to optimize the design. For real world RDO applications we have to expect that at least in the robustness space we have to start with a large number of potentially important scattering variables. In contrast to the design space of optimization the variable reduction in robustness space starting from all possible influencing variables is only possible with know how about unimportance of scattering variables.
If the RDO task is defined with appropriate robustness measures and safety distances multiple optimization strategies can be performed successfully to drive the design in the direction of being optimal and robust. If a design evaluation needs significant time the balance between the number of CAE design runs and the accuracy of robustness measures is a challenge for all RDO strategies, iterative or simultaneous. Then all of them try to minimize the number of design evaluations to estimate the robustness measures. If small failure probabilities (like smaller than 1 out of 100) need to be proven, algorithms of reliability analysis have to applied, at least at the end of an RDO process to prove the optimal design.

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