Robustness evaluations of the NVH comfort using full vehicle models by means of stochastic analysis

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Summary

Qualitative and quantitative robustness evaluation of the dispersion of important NVH comfort criteria is becoming an important part of the digital car development process. This scattering results from unavoidable scatter of design parameters. Using Latin-Hypercube Sampling strategies, practical applications show that also large numbers of scattering design parameters can be investigated with moderate effort. With correlation analysis and principal component analysis, the transfer behaviour of design parameter dispersion to important NVH performance criteria is investigated. As a result, the design parameters responsible for the main scatter of responses are identified. Furthermore, robustness analysis can be used to select the important design parameters for following optimization and if necessary reliability analysis. Especially in case of reliability analysis, where commonly only a few stochastic variables can be handled efficiently, a robustness evaluation may become a necessary preliminary task.

1.0 Introduction

At DaimlerChrysler AG, as a part of the digital car development process, the NVH behaviour of new car types is numerically evaluated and optimised by means of complex finite element full vehicle models. Herein, vibrations as well as the sound pressure are taken into account for acoustic comfort. One of the aims is to reduce the maximum values in the frequency responses. Up to now, mainly deterministic approaches have been applied to achieve this aim.

In practice, the vehicle design parameters scatter from the nominal values within acceptable ranges. If one aims to design a vehicle to be robust, i.e. as insensitive to such scattering as possible, the influence of the existing variations has to be taken into account in the car design. It is necessary to apply probabilistic calculation methods to quantify the robustness. In the following, robustness evaluations by means of stochastic calculations are presented. That means, they determine the sensitivity of the system responses regarding unavoidable existing scatter of input variables.

If the scattering of the responses due to existing variation of input parameters is high, these input parameters have to be taken into account in the optimization of the comfort behavior. A reduction of the input variables' scatter, a shift of the mean values, or influencing the transfer mechanisms could be considered. Certainly, this implies that the sensitive input variables,

Presented at VDI Congress Berechnung und Simulation im Fahrzeugbau, 2004
VDI-Berichte 1846, pp. 505-525

the transfer behaviour, and the degree of scatter of important response variables are thoroughly known. That is why, besides the quantitative evaluation of the scatter of important response variables, the identification of the responsible input parameters is of great importance. Starting from the important features of the correlation and variation structures, the interrelationships within the variables are analysed. The translation of the statistical results into comprehensible features of the vehicle model is hereby a necessary prerequisite to understand the transfer mechanisms.

It is marked that an increasing virtual prototyping itself augments the need for stochastic calculations. If any testing shall be replaced by simulations, the scatter of design parameters existing in the testing equipment needs to be considered in the calculation. When the calculations are executed only for a few configurations of vehicle states, assured statements regarding the robustness of the systems are often impossible to make. On the other hand, the increasing application of structural optimization augments the need for robustness verification of “optimized” designs, too. Optimization often leads to structures with less and less room for tolerances or uncertainties that can lead to considerable robustness problems in nonlinear system behavior. Thus, even small variations of input variables can result in important changes of the response variables.

2.0 Robustness evaluations

It is the aim of a robustness evaluation to investigate the sensitivity of the system responses on scattering input variables. Deterministic as well as statistical measures are used to evaluate the robustness. These may be exceeding of limit values or occurrence of system instabilities (buckling, resonance and so on) on one hand, and shift of mean values or, last but not least, large variational coefficients of the system responses, on the other hand.

Besides the evaluation of single response variables, the interconnections of cause and effect can be determined from the correlation structures, i.e. which scattering input variables are responsible for which scattering response variables.

Classical sensitivity studies investigate the variation of system responses in respect of the variation of input variables by means of calculation of extreme parameter combinations, analytical derivatives, or Design of Experiments (DOE). Additionally, the robustness evaluation is considering the probability of occurence of the parameter combinations in the sensitivity space. Frequent events are weighted higher, impossible or extremely rare events (such with a very small probability of occurrence) are not regarded. If necessary, to assure the significance of the robustness evaluation for rare events, a great number of samples is needed, or methods of reliability analysis [2] have to be used.

Often, robustness evaluations are the beginning of further stochastic calculations. When the most important stochastic variables are identified by means of robustness evaluation, calculations of probabilities of occurence using stochastic reliability analysis ([2] /

FORM/SORM, Importance Sampling, Directional Sampling) can be performed with only a few important scattering parameters.

2.1 Generation of the sample set

A statistically significant set of possible realizations (samples) is the basis of a robustness analysis.

2.2 Stochastic sampling methods

Methods of stochastic analysis for generating the sample set are based on variants of the Monte Carlo simulation. Especially easy to realise is the so called “Plain Monte Carlo” method which generates the random numbers following a given distribution function mainly unsystematically using a random number generator. However, this method yields relatively high statistical uncertainties for few samples and, in large parameter spaces, needs an extremely high number of samples to give reliable results. For robustness evaluation, the Latin Hypercube Sampling is especially suitable. This method generates the samples in a way that the variation widths are met as good as possible while in the same time undesired correlations between input variables are avoided. This results in a smaller statistical scattering of the results. Of course, for numerically expensive problems, the computing time has to be limited. By applying Latin Hypercube Sampling instead of Plain Monte Carlo, the number of solver runs can be significantly reduced.

2.3 Recommended number of samples

For a statistical evaluation of single variables (mean value, histogram, variational coefficient), a minimum sample number of ten is recommended, disregarding the number of variables.

For a statistical coverage of the linear correlation structure, in the worst case, one sample per matrix element of the triangle matrix is needed. In well posed problems, the correlation structure regarding “normal”, non rare events can be approximated with good accuracy using a sample number of 2*(input variables + response variables), for Latin Hypercube Sampling. Therefore, in the following examples, solely this sampling method has been used.

To reduce the necessary CPU expense for calculating the samples, robustness evaluations using response surface approximations have occasionally been proposed. In this case, the samples are no longer calculated in the original space but much faster in the approximation space. The use of response surface approximations for robustness evaluations is to be judged problematical as the response surfaces are not suitable for an appropriate reproduction of robustness problems. It is assumed that nonlinear effects, which often lead to robustness problems, are often but insufficiently described. Therefore, the use of global response surface approximations cannot be recommended for nonlinear problems.

2.4 Description of the input variable scattering

Besides an appropriate sampling method, the characteristics of important scattering parameters have to be described with sufficient quality by means of distribution functions. Typical distribution functions are e.g. normal distribution, lognormal distribution, Weibull distribution, equal distribution, or discrete distribution. The definition of realistic distribution functions for all important parameters is a natural prerequisite for the reliability of the results of robustness evaluations.

For practical applications, the selection of parameters and the definition of the distribution function forms a serious obstacle.

The selection of the important parameters is a main objective of robustness evaluations. Here it is not recommended to prematurely restrict the variable space and the response space but to utilize existing CPU resources as completely as possible in order to consider rather some variables, load cases, or response variables too much than too less.

To approximate distribution functions, it is often sufficient to „translate“ available knowledge of possible scattering into appropriate distribution functions. If the mean values and expected maximum scatters are known, e.g. cut-off normal distributions can be approximated. Then, the expected maximum scatters are assigned to a probability of occurence (e.g sigma-values).

Therefore, in the normal distribution, it is assumed that
1-sigma values of 68.3 % of all samples
2-sigma values of 95.4 % of all samples (this corresponds approximately to the 5 % fractile)
3-sigma values of 99.73 % of all samples
are not exceeded.

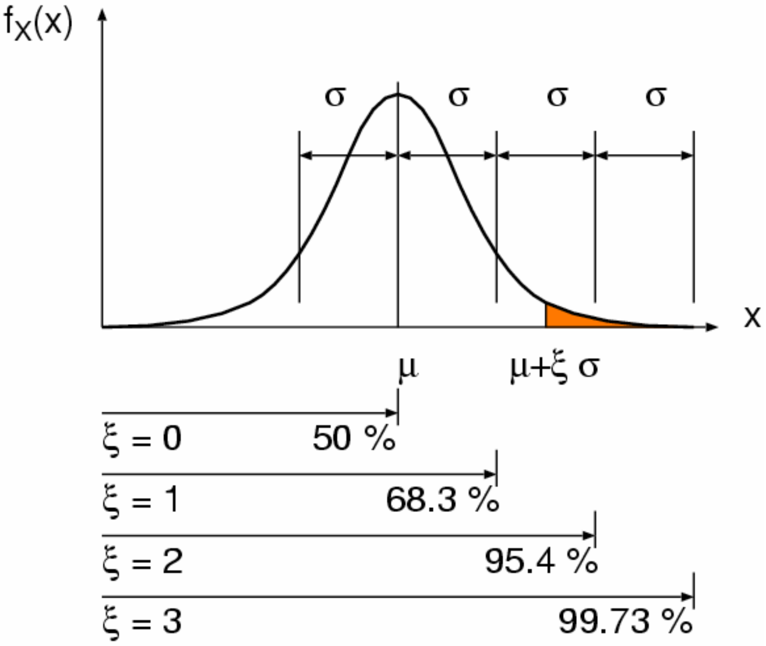

Fig.1: normal distribution with sigma values

In this case, the variational coefficients can be approximated by dividing the standard deviation by the mean value. For example, 20 % scatter with a probability of 2-sigma yields a variational coefficient of 0.2/2 = 0.10. Additionally, the gaussian distributions are generally cut off at the sigma-values of the expected maximum scatter. Thus, the resulting histograms of the input variables yield slightly smaller variational coefficients. Of course, such approximated distribution functions should in the following be verified for the most important scattering variables.

2.5 Statistical Evaluation

Using statistical methods, the samples are investigated regarding correlation and variation properties as well as regarding stability of the system response against the scatters of the input parameters.

2.5.1 Statistical measures of single input and response variables

Generally, the histograms, the mean values, and the variational coefficients of the input and response variables are calculated. The evaluation of the input variables serves to statistically ensure the drawn samples. The distributions should be compared to the nominal distributions. The evaluation of mean value and variational coefficient of the response variables allows for an assessment of the sensitivity of the system reaction. Hereby, the variational coefficient depicts the degree of scatter. Variational coefficients that are much larger than those of the associated input variables are to be regarded as conspicuous. In this case, it can be assumed that the transmission mechanisms in the system amplify the scatter of the input variables. Many engineering tasks aim at rather dissipating systems in which the variational coefficients of the response variables are smaller than those of the input

variables. The histograms of the single response variables should be investigated regarding clustering and bifurcations, which indicate system instabilities.
Besides the evaluation of single scattering response variables, the most important interconnections of input and response variables can be determined from the correlation structures.

2.5.2 Correlation structures

Correlation structure of parameter pairs

It is convenient to display linear correlation coefficients between input and response variables in a linear correlation matrix. From this matrix, a possible linear relation between the variables can be detected. Here, the correlation coefficients indicate the degree of interconnection or dependence of two variables. It is normalized to values between +1 and –1.

Parameter combinations with correlation coefficients ≥ 0.90 show nearly linear correlation between input and response variable. Parameter combinations with correlation coefficients ≥ 0.70 are generally denoted as conspicuous linear correlated. In this study all parameter combinations with correlation coefficients ≥ 0.50 there investigated. It can frequently be observed that only a small number of input variables have a significant linear correlation (≥ 0.50) to the response variables. These input variables generally have significant correlations to more than one response variables. These are indicated by bands in the correlation matrix.

To each matrix field, anthill plots are associated in pairs. In these plots, all realisations of a sample set are displayed in the twodimensional parameter space, ressembling an anthill in uncorrelated cases. Anthill plots allow for the visual evaluation of the correlation.
Additionally, in the anthill plots, clusterings or nonlinear relations between the variation of the input variables and of the response variables (which may e.g. be in our example a symptom of resonances) can be identified.

Correlation structures of parameter groups

While linear correlation structures show the explicit relation between the variation of two variables, the so called **Principal Component Analysis (PCA)** of the linear correlation matrix is used to investigate correlations of higher dimensions, i.e. the correlations of an input variable group to a response variable group. The principal component analysis yields the dominating correlation modes. It may be compared to a modal analysis, where the first eigenvalues dominate the global dynamic structural behaviour. Equivalently, the first correlation modes dominate the global highdimensional correlation behaviour. Therefore, generally only the first principal components are investigated. Advantageously, the principal components are normalized and sorted and displayed in matrix form as dyadic products. This matrix form shows the contributions of the variable groups to the scatter of the entire input-/output data set as well as their linear dependencies. That means that the dependencies in the high dimensional spaces are projected to small, manageable dimensions (variable groups).

3.0 Examples

It shall be shown on a typical finite element full vehicle model of a passenger car, how the influence of the input variables' scatter on important response variables can be measured by means of stochastic analyses and evaluated by means of statistical methods.

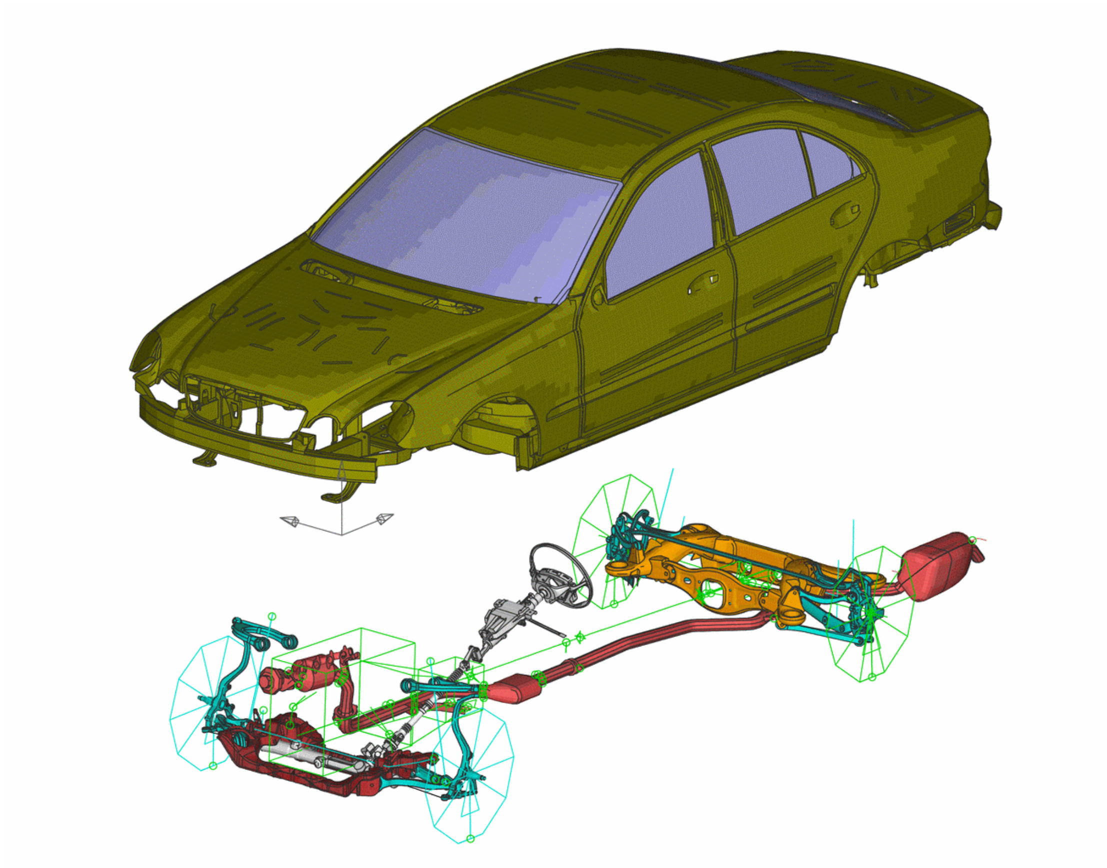

Fig. 2: FE full vehicle model

Herein, mainly the correlation structures, the variation of the response variables as well as nonlinearities in the anthill plots are analyzed. It is thus of interest which scattering input variables create which scattering of the responses, how strong the scattering of the responses is, and nonlinearities in the anthill-plots (the nonlinear transmission behaviour between input variables and responses).

The scattering of selected input variables is estimated from the maximum percentage of deviation associated to a sigma value of a normal distribution. This deliberately conservative estimation is performed as follows:

- +-30 % scatter about the mean value as 2 sigma value for scalar stiffnesses,
- +-20 % scatter about the mean value as 2 sigma value for scalar damping parameters,
- +-6 % scatter about the mean value as 2 sigma value for sheet thicknesses.

3.1 Robustness evaluations of bushings and tire characteristics in the suspension with relation to vibrations

Robustness evaluations are performed for 96 scattering characteristics of suspension bushings, tires and dampers regarding the peak values of selected frequency bands for the important comfort points driver's seat and steering wheel. All in all, 54 response variables are considered. The load cases wheel unbalances, bounce und idle speed are evaluated. A total of 377 samples is calculated.

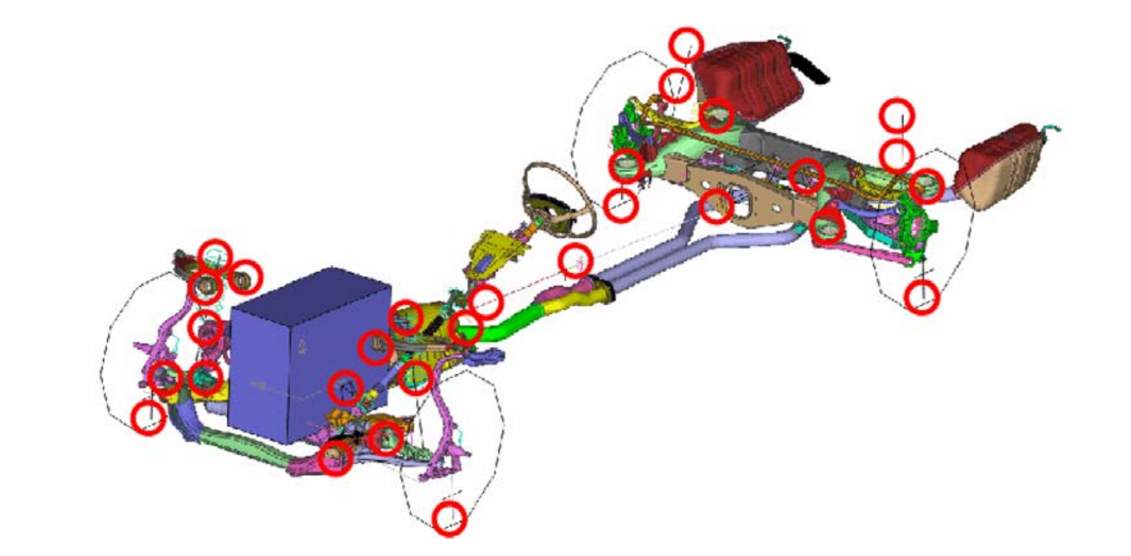

Fig. 3: Varied parameters in the suspension

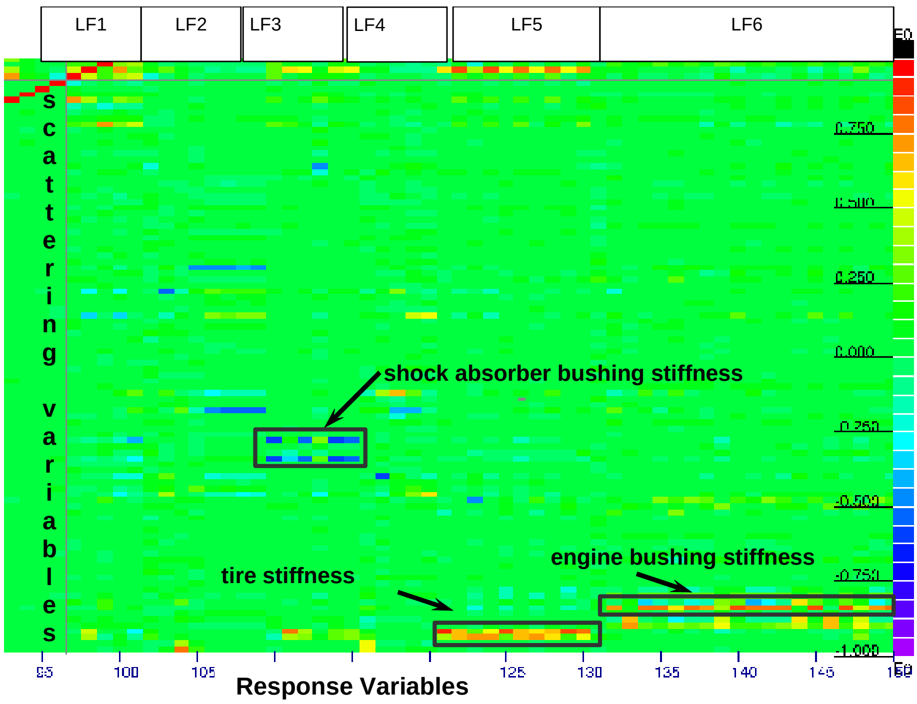

Fig. 4: Matrix of linear correlation

All in all, only 12 significant input variables can be observed in the linear correlation matrix (fig. 4). The most significant correlations in the load case bounce are related to the stiffness of the front wheels, and in the load case idle speed to the stiffness of the engine mount. These two variables have the biggest influence on the scatter in these load cases. The results are consistent with hitherto existing knowledge. Furthermore, a quantitative assessment of the phenomena is possible.

Fig. 5: First PCA

The advantage of the PCA (fig. 5) compared to the interpretation of the linear correlation structure is the connection of the engine mount stiffness and the front wheel stiffness to the scatters in the load case idle speed. The correlation coefficients show slight scatter amplification effects only for a few response variables. Thus, the suspension mainly dampens the transmission of the scattering of mount characteristics to the resulting scatter of response variables.

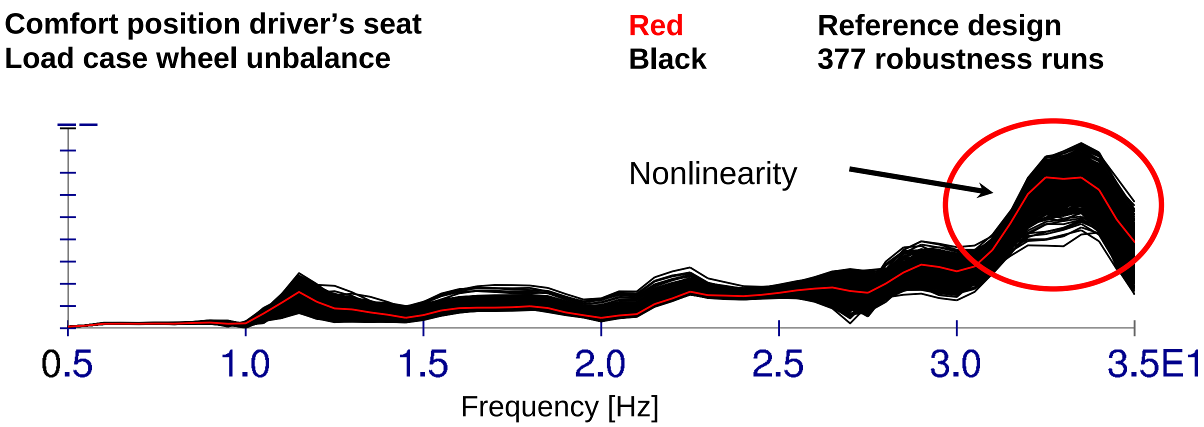

Fig. 6: Visualization of the acceleration's scatter

In the loadcase wheel unbalance, a nonlinearity can be observed (i.e. a nonlinear relation between the variation of an input variable and a response variable). If, in reality, the stiffness of the shock absorber bushing decreased below a certain value the response variables would rise considerably. These nonlinearities can be identified in the anthill plot (fig. 7) as well as in the curve progression of all designs of the comfort point (fig. 6). The physical background was found to be a resonance effect of a vehicle vibration. Inspite of this nonlinearity, the scatters of the concerned response variables remain moderate, i.e. the variational coefficient of the response variables does not increase relative to the variational coefficient of the shock absorber bushing, and the maximum values lie below undesired amplitudes. Thus, by evaluation of the correlation and variation structures, three important input variables could be identified.

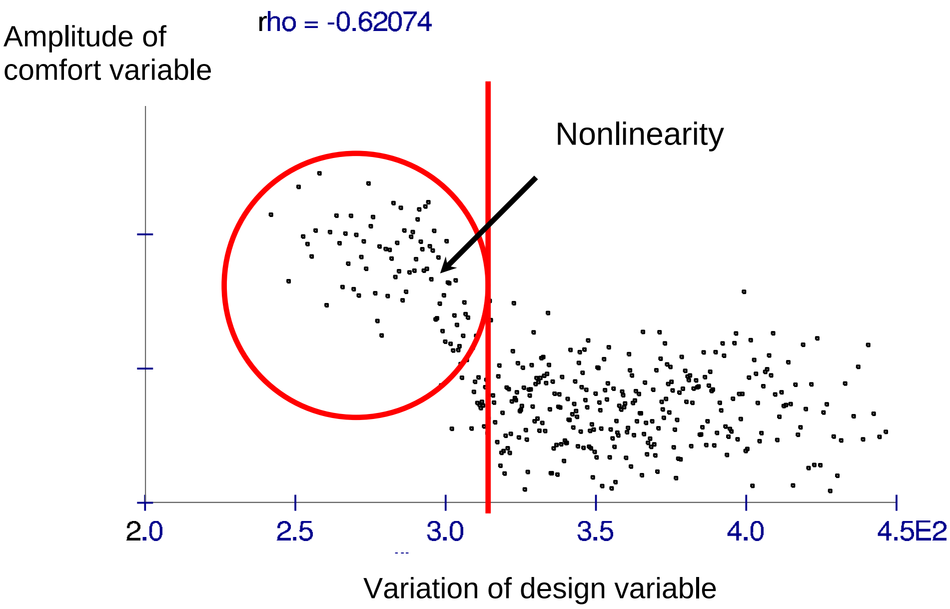

Fig. 7: Anthill-plot stiffness of shock absorber bushing versus response
* rho = correlation coefficient between both variables

3.2 Robustness evaluations of bushing stiffnesses in the suspension with relation to the sound pressure

Robustness evaluations are performed for 76 scattering bushing stiffness values regarding sound pressure levels at four different positions in the passenger compartment. This leads to a total of 40 response variables. Two excitations of the engine are investigated. A total of 199 samples is calculated.

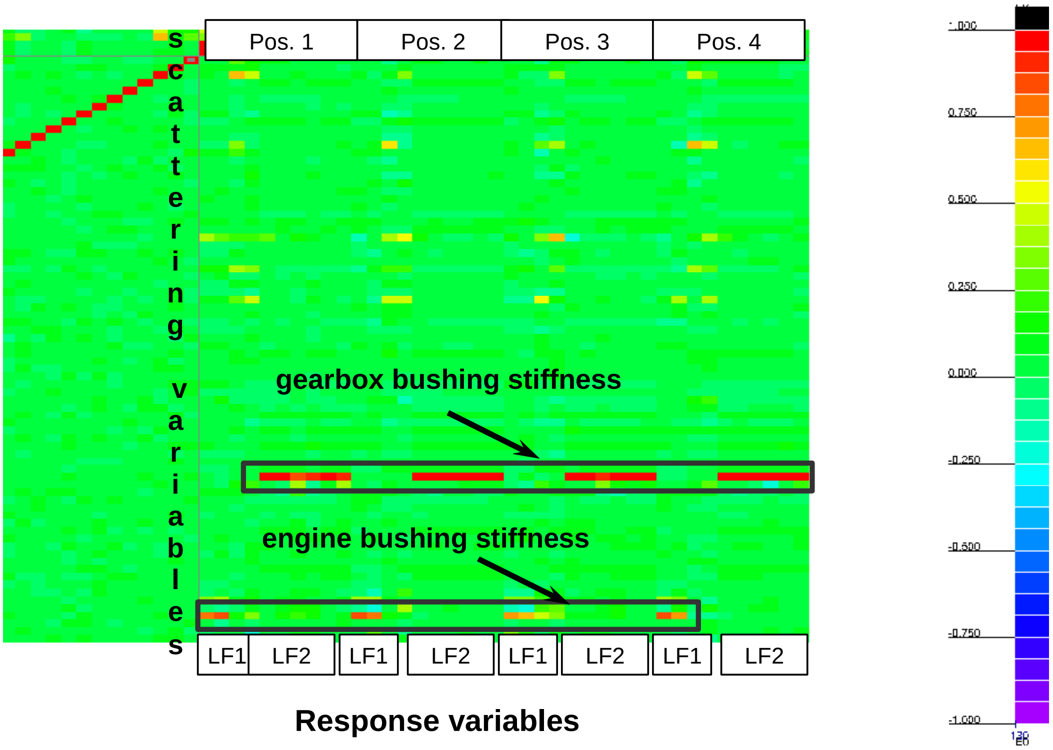

Fig. 8: Matrix of linear correlation

As was expected, the resulting correlation or variation structures are mainly determined by the stiffnesses of the engine and the gearbox mount (fig. 8). All correlation coefficients of the response variables relative to the gearbox mount correspond in the first loadcase approximately to 1.0 and thus show the linear relation of the scatters of all response variables to the scatter of the gearbox mount (fig. 9).

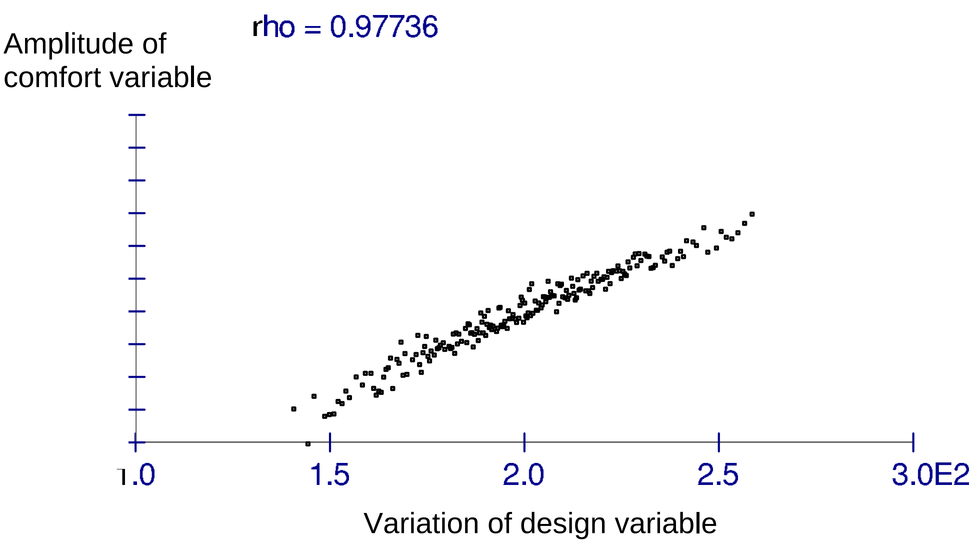

Fig. 9: Anthill-plot stiffness of gearbox mount versus response variable

In the second loadcase, the correlation coefficients of the response variables relative to the stiffness of the engine mount lie between 0.6 and 0.9 (fig. 10). This shows that additional input variables are influencing the scatters of the response variables, besides the relation of the scatters of all response variables to the stiffness of the engine mount.

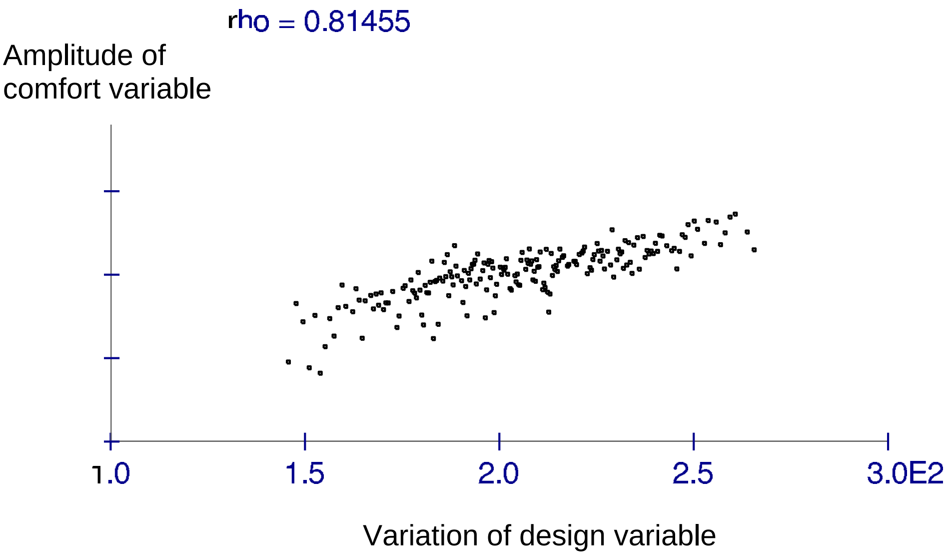

Fig. 10: Anthill-plot stiffness of engine mount versus response variable

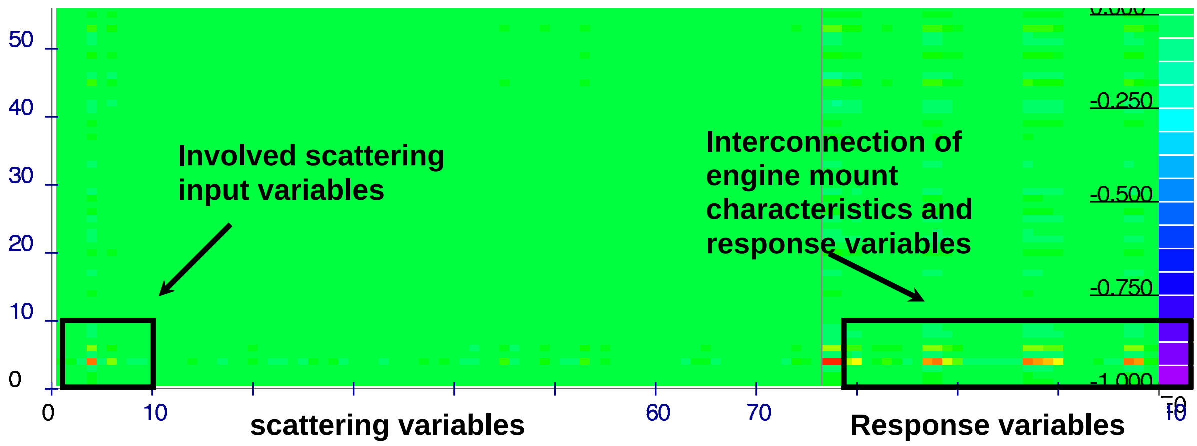

Fig. 11: Second PCA

For one loadcase, the first principal component confirms the dominance of the gearbox mount. In the other loadcase, the second principal component (fig. 11) shows that the stiffness characteristics of the engine mount are dominant.

All in all, the scatter of the sound pressure levels is moderate and lies below undesired amplitudes (fig. 12). By means of the evaluation of the correlation and the variation structures, the two dominant input variables could reliably be identified. Thus, these two load cases can significantly be influenced by scatter or variation of these few characteristics.

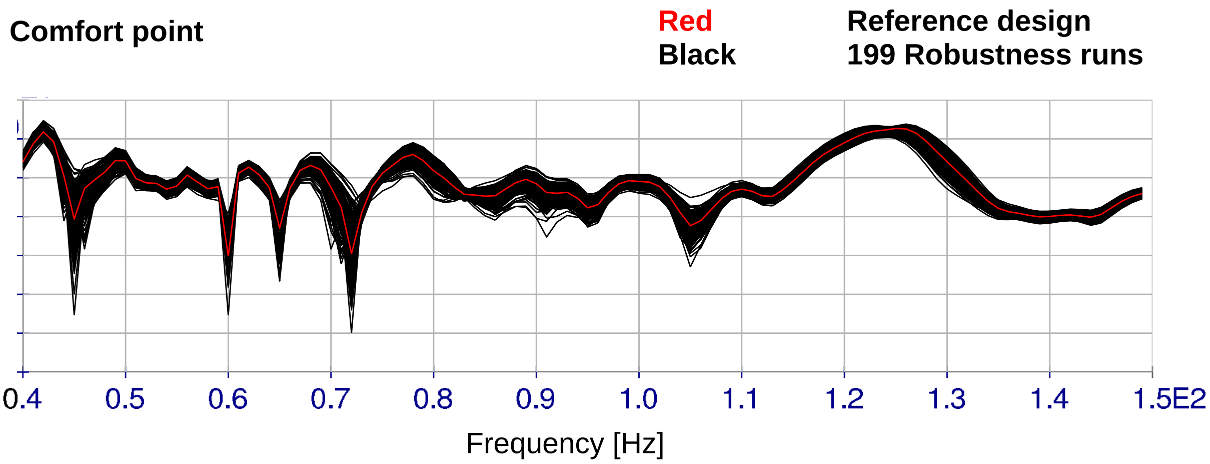

Fig. 12: Visualization of the sound pressure level's scatter

3.3 Robustness evaluations of sheet thicknesses of the car body with relation to the sound pressure

Robustness evaluations for 265 selected scattering sheet thicknesses are performed. The sheets are located in the power transfer path for engine excitation towards the passenger compartment. Sound pressure levels at four comfort points in the passenger compartment are investigated. This results in a total of 48 response variables. Again, the two engine excitations of the previous example are investigated. A total of 483 samples is calculated.

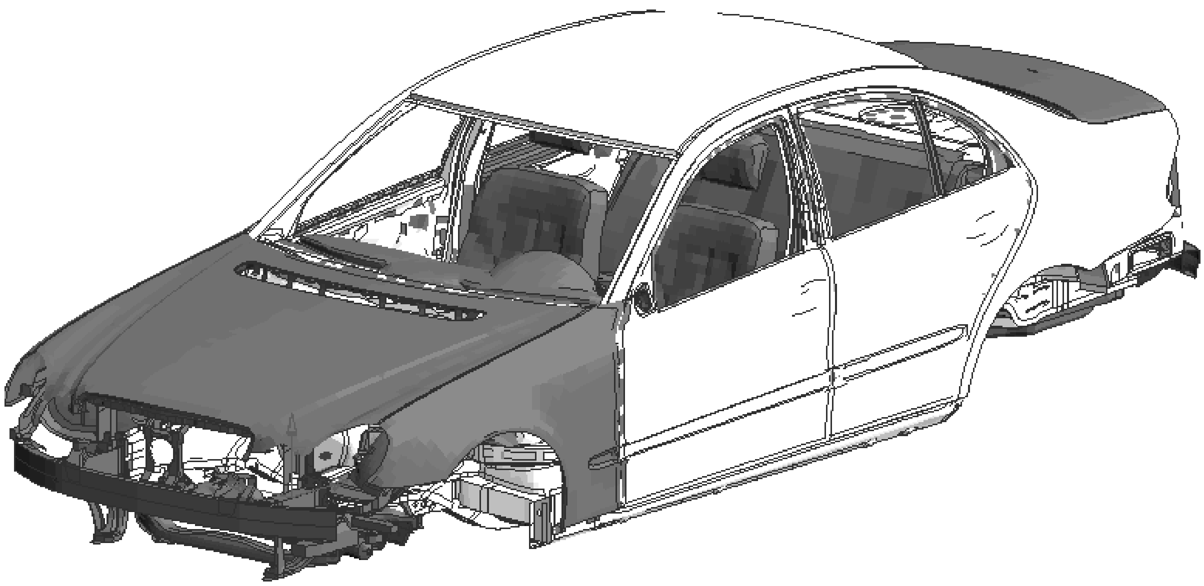

Fig. 13: Car body (varied sheets coloured white)

The maximum scatters of the sound pressure levels are of the similar order of magnitude as previously in the robustness evaluation of the bushing characteristics. This shows the need to consider the influence of the sheet thickness scatter as well.

Unlike in the previous examples, no dominance of any single variables for the correlation and variation structures could be observed. The reason for this is the much stronger interdependency of the 265 car body sheets. The important increases of response variables are thus always influenced by several input variables. Nevertheless, only 12 sheet components showed noteworthy correlations to the response variables with correlation coefficients ≥ 0.50 (fig. 14). Of these, two were especially noticeable, one located in the door area and the other in the rear.

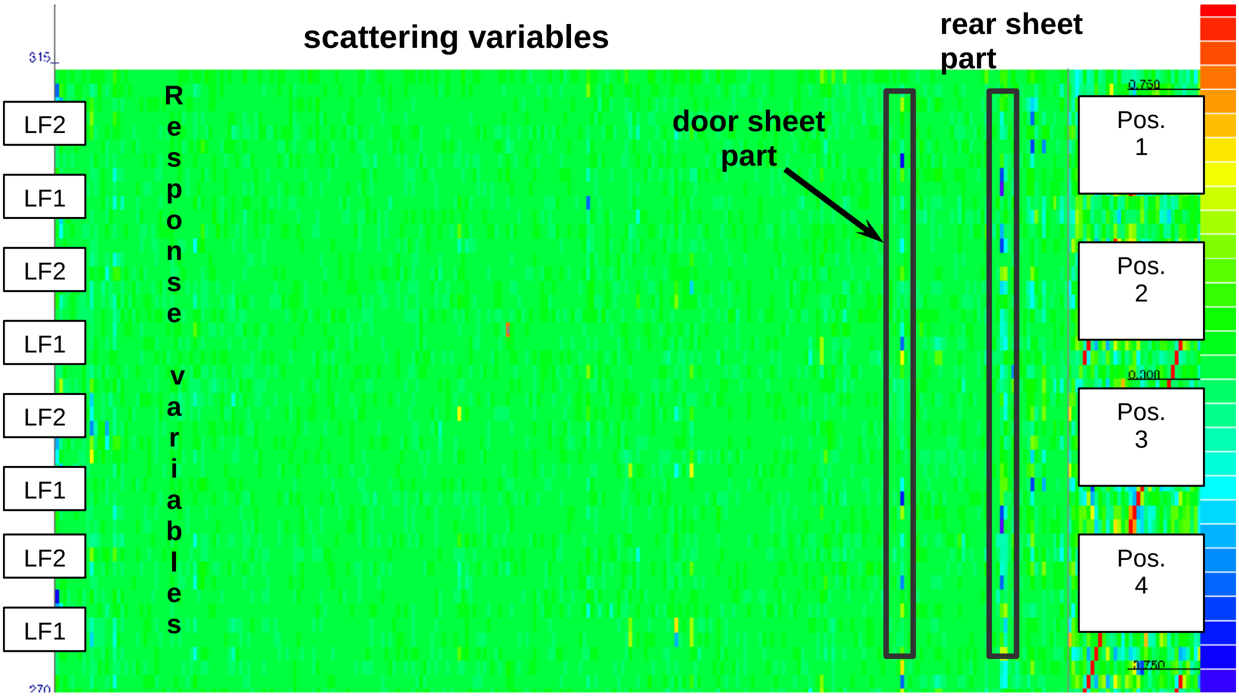

Fig. 14: Matrix of linear correlation

In fig. 15, the high correlation coefficient of the sheet component in the door shows an almost linear correlation to the corresponding response variable. In the anthill plot of the rear sheet component versus the correlated response variables in fig. 17, a nonlinearity can be observed: Starting from a certain sheet thickness, the amplitude values decrease significantly. This nonlinearity causes a visible increase of the scattering in the curve progression of all designs of the comfort point (fig. 16).

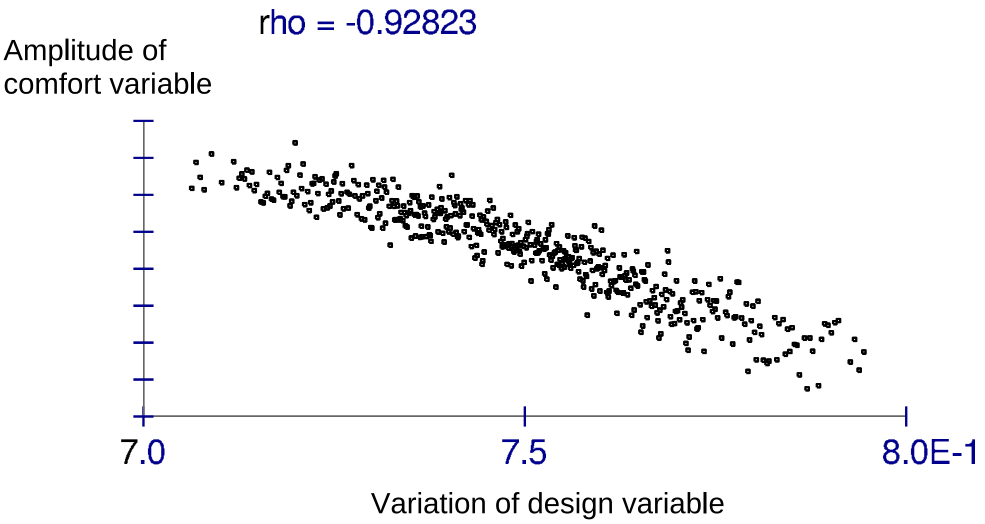

Fig. 15: Anthill-plot door sheet component versus a response variable

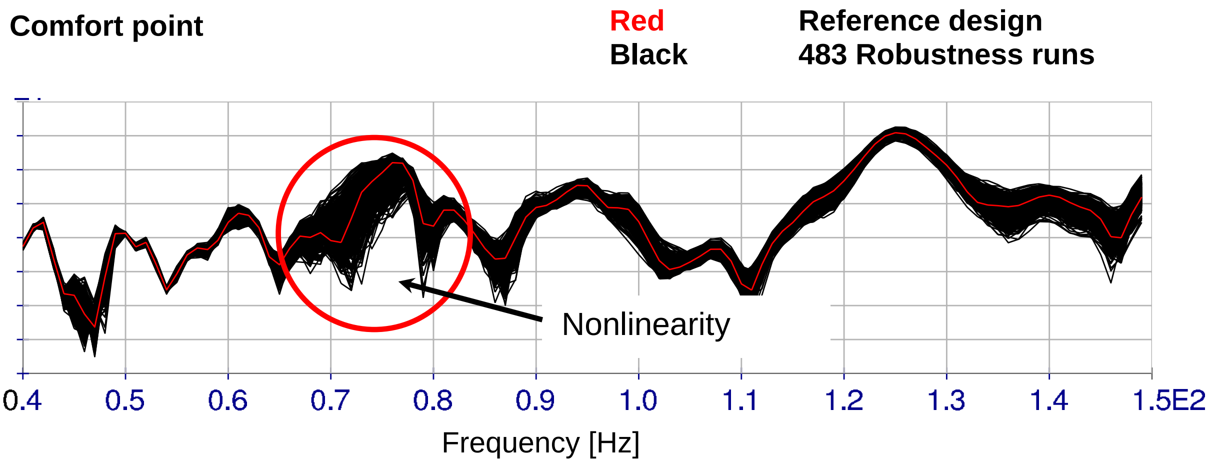

Fig. 16: Visualization of the sound pressure level's scatter

The evaluation of the correlation and variation structures thus yielded two important input variables.

To evaluate the admission of maximal amplitude values, the biggest deviations relative to the reference values of both cases have been superponed linearly, on one hand those resulting from the suspension characteristics based robustness evaluation, and, on the other hand, those from the sheet thickness based robustness evaluation.

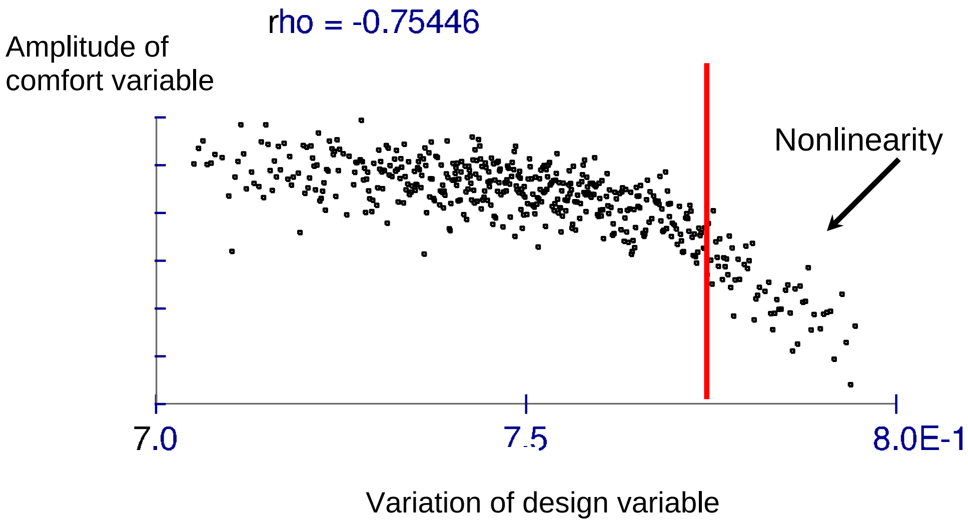

Fig. 17: Anthill-plot rear sheet component versus a response variable

3.4 Summary of all examples

In all examples, the robustness of the vehicle structure could be verified for the assumed scattering of suspension characteristics and car body sheet thicknesses for the investigated loadcases. The maximum scatters of important response variables lied within the tolerance range and did not exceed undesired amplitude levels.

Fortunately, very stable correlation structures could be observed. During the calculations, the correlation structures (i.e. linear correlation matrix and PCA) have been observed. Once no further significant variation could be noted with increasing number of calculations, the correlation structures were reliably determined. This means the required sample number to reliably determine the correlation structures were achieved and the statistical measures are trustworthy.

Please note that the reliable determination of the correlation structures is a necessary condition to achieve trustworthy results. If the used number of calculations is too small, the statistical measures are afflicted with high incertitudes. In the worst case, the correlations then are random and useless for an evaluation of the structure.

Often, only few variables are dominating the correlation and variation structures and only few nonlinearities in the transmission behaviour can be identified in the anthill plots. Thus, the robustness evaluations can reliably yield the most important scattering input variables. At the same time, the robustness evaluations yield valuable information about the transfer paths of the scatters and their optimization potential.

4.0 Outlook

After having identified the scattering input parameters responsible for the scatter of important response variables by means of robustness evaluations, optimizations in smaller parameter spaces can be executed. Please note that the optimization parameter spaces (i.e. all input values that constructively can be varied) and the parameter spaces of the robustness evaluation (all scattering parameters such as input variables, loads, boundary conditions) do have intersections but are not identical in realistic structures. A simultaneous treatment of both the optimization task and the reliability problem generally overstrains the computational resources. Therefore, an iterative procedure with optimization considering constraints of reliability/robustness and robustness analysis is recommendable [5]. In case of considerable variations of the parameters to optimize, it is recommendable to execute a final robustness evaluation of the optimized structure after the completed optimization.

If exceedances of allowable values are detected by the robustness evaluations, the related probabilities can be determined from the histograms. Here, the sampling of the robustness evaluation yields reliable statements up to probabilities of occurence of about 5 to 10 %, depending on the number of executed finite element simulations. If smaller probabilities of occurence shall be calculated, reliability analysis methods for small parameter spaces [2] should be used.

The presented frequency response analyses have been executed with the finite element program NASTRAN. All described stochastic and statistical algorithms are implemented within the software package Slang [2]. The robustness evaluations have been performed with the program OptiSLang [1].

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