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Robust Design and Reliability Analysis of an Electromagnetic Actuator System

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Kurzfassung

In diesem Artikel wird der robuste Entwurf eines elektromagnetischen Aktuatorsystems vergestellt. Einige Prozessparameter, wie der magnetische Fluss und die Federsteifigkeit werden innerhalb einer Optimierungschleife angepasst. Zusätzlich zur Reduktion der deterministischen Zykluszeit wird die Zuverlässigkeit des Systems betrachtet. Unter Berücksichtigung zusätzlicher Unsicherheiten in den mechanischen und elektromagnetischen Eigenschaften wird der erfolgreiche Prägevorgang als Nebenbedingung eingeführt. Unter Anwendung effizienter Methoden zur Zuverlässigkeitsanalyse wird eine schnelle gekoppelte Robust Design Optimierung ermöglicht.

Abstract

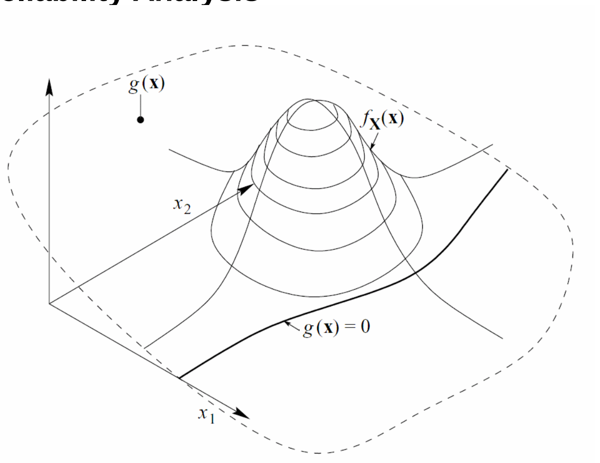
In this paper a robust design of an electromagnetic actuator system is presented, where some process parameters such as magnetic stroke and spring stiffness are modified within an optimization procedure. Additionally to reduction of the deterministic cycle time, the reliability of the cycle is considered. Based on additional random uncertainties in the mechanical and electromagnetical properties, the success probability of the simulated cycle is used as optimization constraint. With help of sophisticated reliability methods a fast and efficient Robust Deisgn Optimization is enabled.

Robust Design

Due to target-oriented, automatic optimization of virtual products new design possibilities are explored. However, highly optimized designs lead to high imperfection sensitivities and tend to loose robustness. Often the deterministic optimum is pushed to the boundaries of the feasible design space. As a result the optimized design, which was found by assuming deterministic model properties, may not be realizable in a production process. For this reason, it is necessary to investigate, how the optimized design is affected by scattering model input variables, which could be e.g. geometry and material parameters, boundary conditions and loads. The scattering inputs can be represented by means of scalar random variables having a certain dependence between each other. Random variables have the advantage compared to other

uncertainty models, that efficient methods of the well-developed probability theory can be applied.
A robust design may be characterized intuitively in that way, that its performance is largely unaffected by random perturbations of the model inputs. A possible measure is the variance indicator, where the relative variations of the critical model responses are compared to the relative variation of the input variables. If certain model responses are limited with respect to an undesired performance, the safety margin can be quantified as the interval between the mean value of the model response and the limit. Alternatively the probability that a certain limit is exceeded can be quantified and proven to be less than an acceptable value. This probability indicator can be evaluated by the probability-based robustness analysis, often called reliability analysis.

Reliability Analysis



$$
\begin{aligned}
P_F &= P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right] \\
&= \int_{g(\mathbf{x}) \leq 0} \cdots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
&= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I\left(g(\mathbf{x})\right) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}
\end{aligned}
$$

Figure 1: Limit state function and multivariate distribution analyzed in the reliability analysis and relation to the failure probability.

In the reliability analysis the integral of the failure domain is evaluated explicitly, as shown in Figure 1. This requires in the case of classical integration methods, such as Monte Carlo Simulation, a large number of simulation runs to estimate small failure probabilities. In our study we compare different methods, such as First Order Reliability Method (FORM) [1], directional sampling [2] and a recently developed adaptive response surface method [3] with respect to their efficiency. Further details about these methods can be found in [4].

Robust design optimization

In robust design optimization, the optimization task is formulated under the consideration of uncertainties. For this purpose we model the uncertainties with scalar random variables with a given distribution type and correlations between each other. In the RDO framework the optimization variables itself (e.g. geometry parameters of a structure) and even additional variables (e.g. material properties) may be assumed as random. This may result in pure optimization, pure stochastic and mixed optimization-

stochastic variables. Additionally to the deterministic objective and constraint functions, the robustness of a design is considered within the RDO procedure.
Generally two different approaches are possible for this purpose: the fully coupled robust design optimization, where the robustness or reliability constraints are evaluated for every optimization design. Alternatively, the iterative robust design optimization reduces the numerical effort by introduction safety factors to the constraints and solving the RDO task by a deterministic optimization. Both procedures are explained and compared in [5].

The electromagnetic actuator system

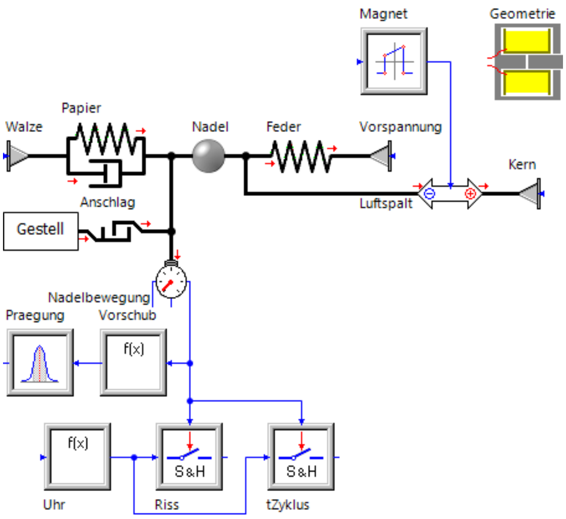


Figure 2: Investigated electromagnetic actuator system.

The investigated system in this paper is an electromagnetic actuator used to drive a Braille printer [6]. In Figure 2 the corresponding Simulation X model is shown. As optimization and random variables the initial needle displacement, the spring stiffness, the diameter of magnetic armature and the magnetic stroke are considered. As optimization goal the cycle time should be minimized under the consideration of a successful embossing. In a deterministic optimization the obtained optimum is located very close to the constraint condition. Since the constraint function is one for successful embossing and smaller one, if the needle does not emboss the paper, this function is dominated by a strong kink, as shown in Figure 3. Therefore, it is not possible to estimate the safety margin by using a robustness analysis with a small number of samples. For the reason an iterative robust design optimization is not applicable and a fully coupled reliability-based robust design optimization is performed. In this analysis the failure probability is evaluated for each optimization design. As safety requirement, a 99.9% embossing rate for the optimal design is requested.

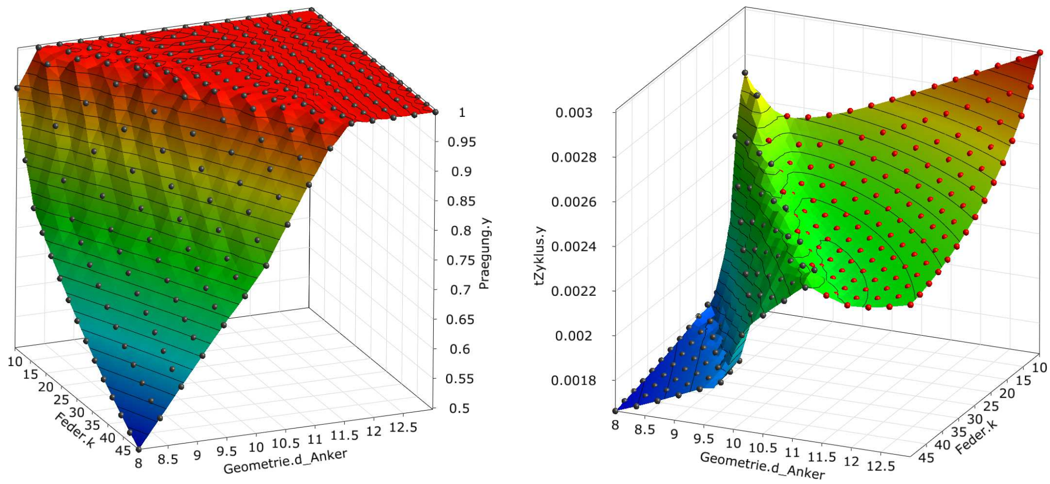
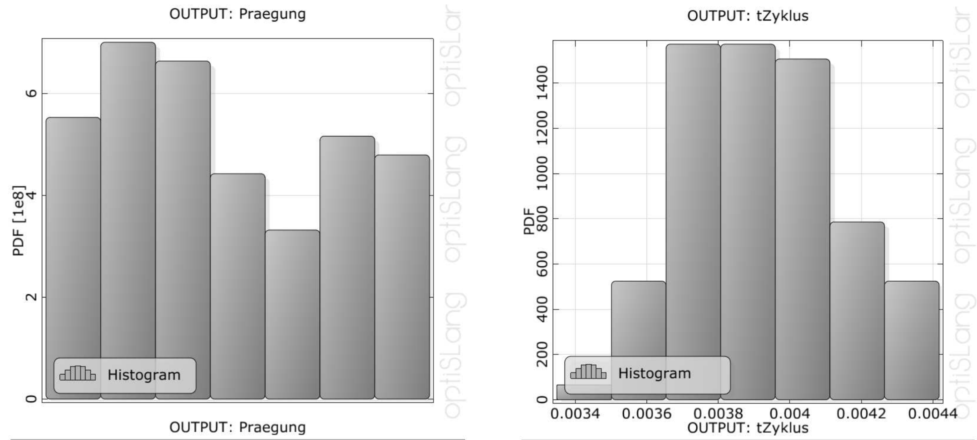


Figure 3: Deterministic constraint function (left) and objective function (right) of the electromagnetic actuator system.

Step 1: Robustness and reliability analysis at the initial reference design

In a first step the robustness and reliability is evaluated at the reference design, where spring stiffness is 50 N/mm, the initial needle displacement is 0.15 mm, the diameter is 15 mm and the magnetic stroke is 0.002 s. The variation is assumed to be 10% for needle displacement, stiffness and stroke and 5% for the diameter. All inputs are assumed to be normally distributed.

The robustness analysis is performed using 100 Latin Hypercube designs with the defined reference as mean and the given variation of the input variables. All 100 designs lead to a successful embossing as shown in Figure 4. The cycle time varies between 3.3 and 4.4 ms.
Furthermore a reliability analysis is performed at the reference design. First the classical FORM algorithm was investigated, which does not work due to the missing gradients of the embossing at the reference design. Therefore, an evolutionary algorithm is used to search for the most probable point. For comparison directional and adaptive sampling is applied. The results are given in table 1. The table indicates similar results for all three methods. The accuracy of the adaptive sampling seems to be the best, but the FORM approach does not enable an error estimation. However, all three methods indicate a failure probability of approximately 10^{-8}, which is much less as the required probability of 10^{-3} (equivalent to a success rate of 99.9%).



Statistic data			
Min:	1	Max:	1
Mean:	1	Sigma:	5.51e-010
CV:	5.51e-010		
Skewness:	0.2602	Kurtosis:	1.878

Statistic data			
Min:	0.003348	Max:	0.004417
Mean:	0.003928	Sigma:	0.0002173
CV:	0.05532		
Skewness:	0.1085	Kurtosis:	2.626

Figure 4: Statistical data of the embossing (left) and the cycle time (right) at the initial reference design

Method	**FORM with Evolutionary Algorithms**	**Directional sampling**	**Adaptive sampling**
Probability of failure	$1.6 * 10^{-8}$	$2.6 * 10^{-8}$	$1.2 * 10^{-8}$
Estimation error	-	$1.3 * 10^{-8}$	$0.3 * 10^{-8}$
Reliability index	5.53	5.44	5.57
Number of model runs	500	607	900

Table 1: Estimated probabilities of failure at the initial reference design

Step 2: Deterministic optimization and robustness analysis at the optimal design

In a second step, a deterministic optimization is performed. Within the parameter ranges given in table 2, the minimum cycle time is searched while a successful embossing is required as constraint. Before performing the optimization, a sensitivity analysis is evaluated in the design parameter ranges. With help of the Metamodel of Optimal Prognosis [7] the functional dependence between input parameters and reponses is be assessed quantitatively and qualitatively based on 100 Latin Hypercube samples within the deterministic ranges. Figure 5 indicates the results of the sensitivity analysis. The functional dependence between the design parameters and the responses seem to be almost monotonic and clearly interpretable. As important variables the stiffness, the stroke and the diameter have been identified. The initial needle displacement seems to be minor important.

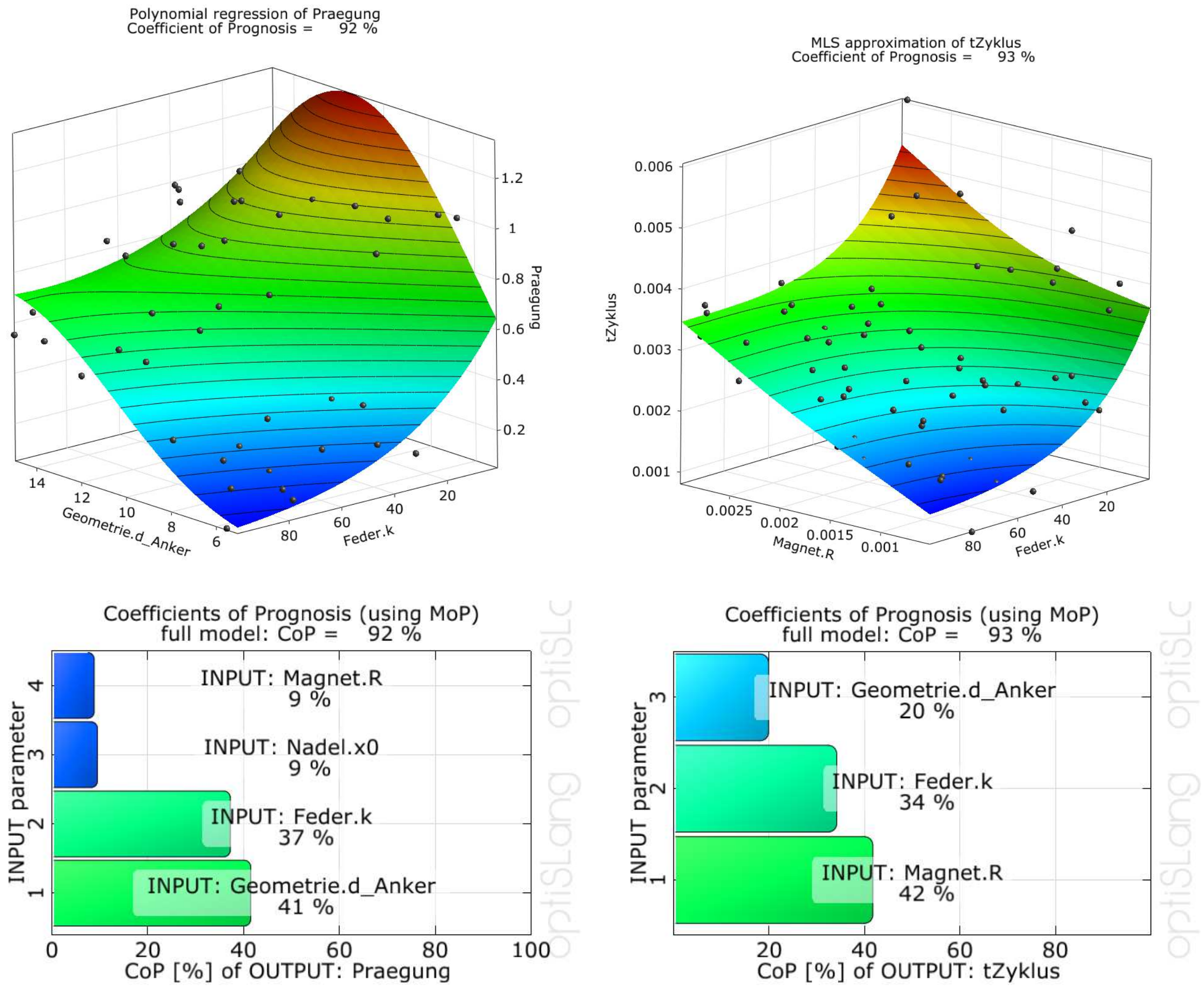


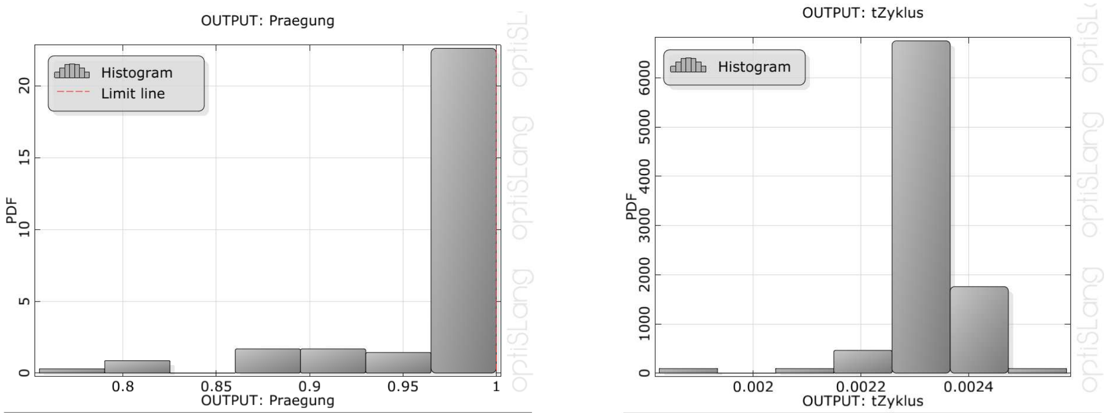
Figure 5: Functional dependence and sensitivity indices of the embossing (left) and the cycle time (right) in the deterministic design space

Based on the results of the sensitivity analysis different optimization strategies are investigated: a gradient-based method (NLPQL), the Simplex Nelder Mead method, both using the best design of the sensitivity analysis as start design, an Adaptive Response Surface (ARSM) and a global Particle Swarm Optimization (PSO), both starting globally. Details about the optimization methods can be found in the optiSLang documentation [8]. In table 2 the results of the different methods are given. The table indicates that although the methods find similar optimal cycle time values, the design parameter value vary significantly. This is caused by a value of the objective function around the optimum with very small gradients. In such a situation, methods without gradient use (explicitly or implicitly), such as the simplex method and PSO, are more accurate. The minimum cycle time is obtained by the simplex method, which is a reduction of the reference value of 27%.

	Range	Start	NLPQL	Simplex	ARSM	PSO
Spring stiffness	0.5 – 100 N/mm	50.0	35.10	**39.96**	27.37	47.66
Needle displacement	0.15 – 2.0 mm	0.15	0.15	**0.15**	0.15	0.15
Magnetic stroke	0.5 – 3.0 ms	2.0	1.12	**1.21**	1.24	1.23
Diameter	5.0 – 15.0 mm	15.0	11.08	**11.16**	10.52	11.78
Cycle time		3.13 ms	2.293 ms	**2.288 ms**	2.324 ms	2.292
Number of model runs			125 (+100)	**159 (+100)**	210	400

Table 2: Deterministic optimization: design parameter ranges and results of the different optimizers

At the final optimum of the simplex optimizer, a robustness analysis is performed using again 10% variation for stiffness, needle displacement and stroke and 5% variation for the diameter. Figure 6 shows the obtained results: a successful embossing is reached in only 67% of the samples, which is much smaller as the required 99,9%. Thus, the deterministic optimum does not fulfill the robustness requirement.



Statistic data			
Min:	0.7553	Max:	1
Mean:	0.9737	Sigma:	0.05215
CV:	0.05356		
Skewness:	-2.298	Kurtosis:	7.717
Limit x = 1			
P_rel:	0.33	1 - P_rel:	0.67
Sigma-Level:	0.503847		

Statistic data			
Min:	0.001826	Max:	0.002582
Mean:	0.002324	Sigma:	7.675e-005
CV:	0.03303		
Skewness:	-2.559	Kurtosis:	21.05

Figure 6: Statistical data of the embossing (left) and the cycle time (right) at the deterministic optimum

Step 3: Coupled reliability based robust design optimization

In this step a robust design optimization is performed using a deterministic optimization procedure by considering reliability constraints. The optimizer evaluates at each nominal design the deterministic cycle time and the probability to fail in the embossing.
As optimizer the simplex algorithm is applied and as reliability method the adaptive sampling strategy is used. In order to reduce the huge numerical effort, the adaptive sampling is performed with 2iterations, each having only 50 samples. This does not lead to a very accurate estimate of the failure probability, thus a final proof at the obtained optimum is necessary. During the optimization procedure a reliability index equal or larger 3 is used as constraint condition.

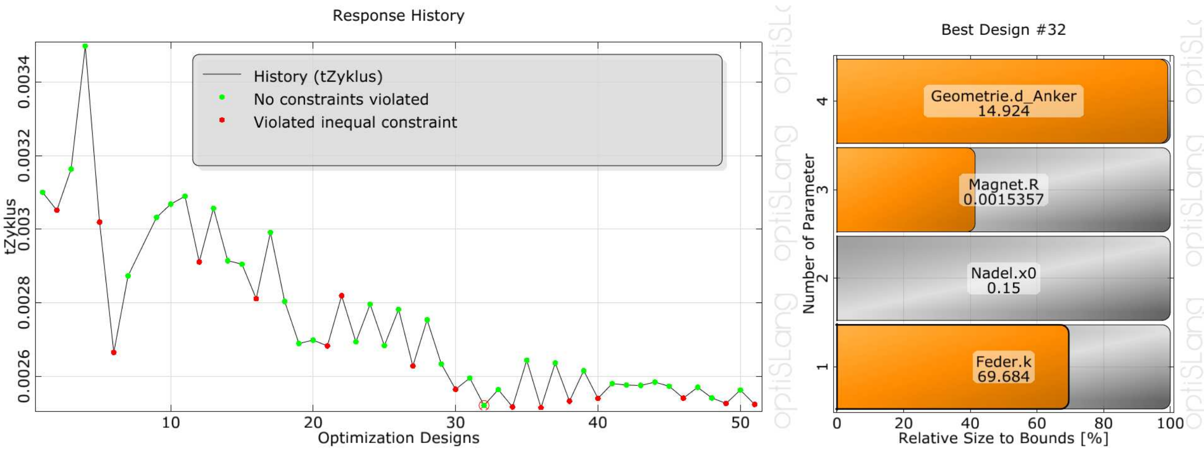


Figure 7: Convergence of the fully coupled robust design optimization

In Figure 7 the observed convergence of the optimizer is shown. After only 31 designs, the optimum is found. The optimum design has a cycle time of 2.52 ms, which is 0.23 ms larger as the deterministic optimum, but 19.5% better than the start design. The rough reliability estimate by the adaptive sampling method indicated a probability of failure of $2.6*10^{-4}$ which corresponds to a reliability index of 3.47. However, due to the small number of samples in the reliability method, a more accurate proof is necessary.

Step 4: Final reliability proof at the robust optimum

In order to verify the final optimum, again different reliability methods are performed:
A FORM method based on evolutionary algorithm and simplex optimizer, the directional sampling method and adaptive sampling. The results are given in table 3. The table indicate, that the estimated failure probability is approximately 0.2 % which corresponds to a embossing success rate of 99.8%, which is close to the initial requirement.

The presented results are obtained by using optiSLang, a software package for optimization and robust design evaluation, which enables the representation of many random variable distributions and even interactions between these random variables.

Method	FORM with EA+Simplex	Directional sampling	Adaptive sampling
Probability of failure	$1.3 * 10^{-3}$	$2.0 * 10^{-3}$	$1.9 * 10^{-3}$
Estimation error	-	$1.0 * 10^{-3}$	$0.6 * 10^{-3}$
Reliability index	3.02	2.88	2.90
Number of model runs	770	500	300

Table 3: Estimated probabilities of failure at the final robust design

Conclusions

In this paper an optimization of an electro-magnetic actuator system under consideration of reliability constraints is performed. Since the limit state function, which is the embossing, is a constant function for successful embossing, an approximation of the most probable failure point based on the results around the mean is not possible. Therefore, classical FORM and variance based robustness analysis (which is equivalent to First Order Second Moment methods) did not give useful results. Due to this fact, more sophisticated reliability methods have applied. With help of a fully coupled reliability based design optimization a robust design was found, having a large success rate close to the required value.

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