presented at the 12th Weimar Optimization and Stochastic Days 2015
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Statistical Analysis of Mistuned Compressor Wheels by Model Order Reduction

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Outline

What is Mistuning?

Simulation & FSI-Coupling

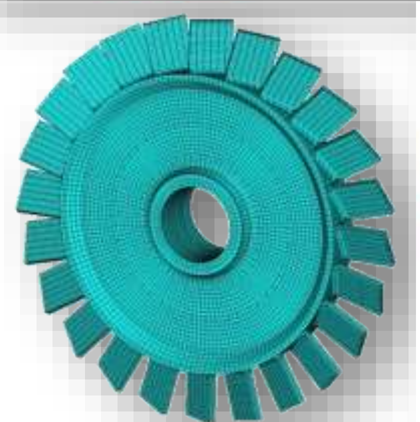
Mistuning - Modelling

Mistuning - Application

What is Mistuning?

- Why does Blade x break?
  - Local Production Error?
  - Local Material Error?
  - Local Overload?
  - Local Erosion?
  - …
- Non cyclic System due to
  - Allowed Production Tolerances
  - Small Erosion
  - …
- → Mistuned System

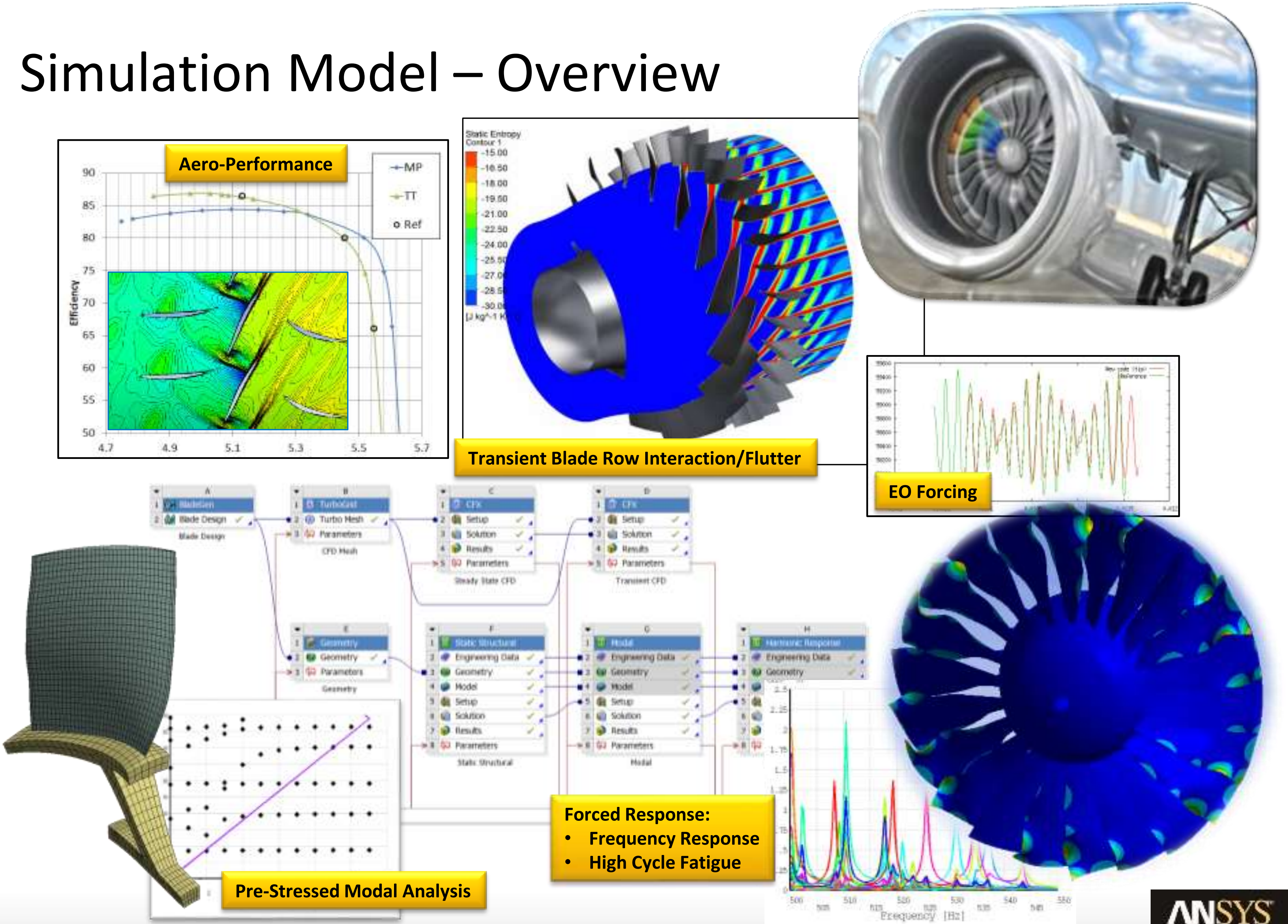
Rotor Damage at Blade x



CAD-Model (=Tuned System)

Real-Model (=Mistuned)

Simulation Model – Overview



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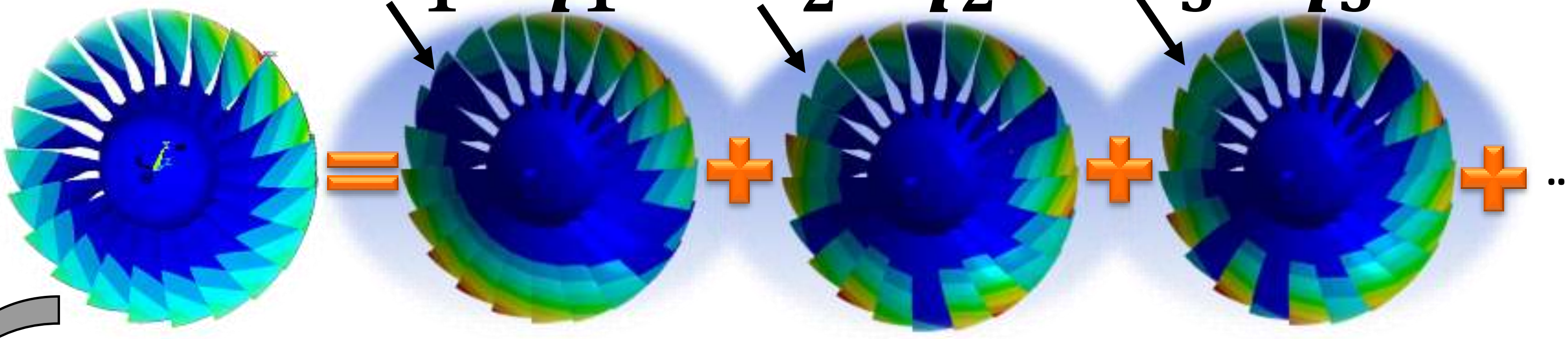
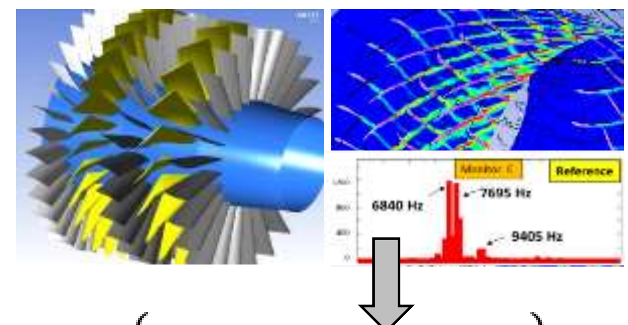
Mechanical - Reduced Order Model

$$\mathbf{u} = \boldsymbol{\Phi}_1 \cdot q_1 + \boldsymbol{\Phi}_2 \cdot q_2 + \boldsymbol{\Phi}_3 \cdot q_3 + \ldots$$

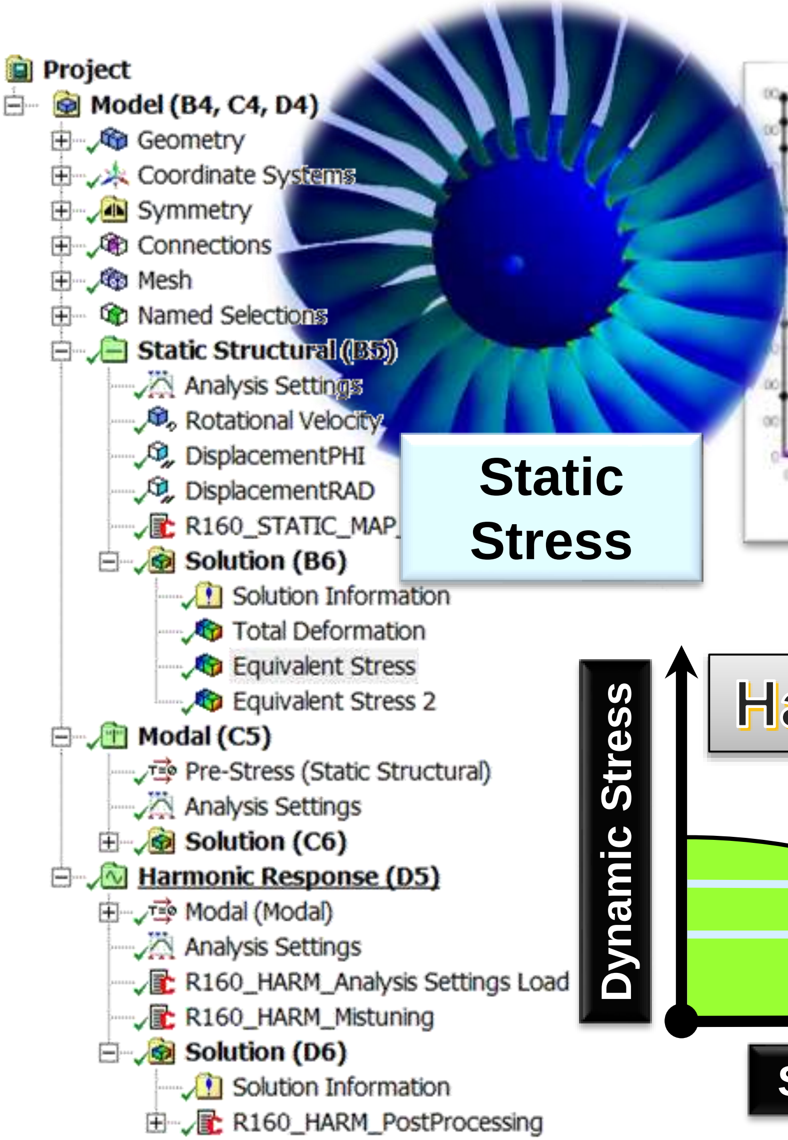


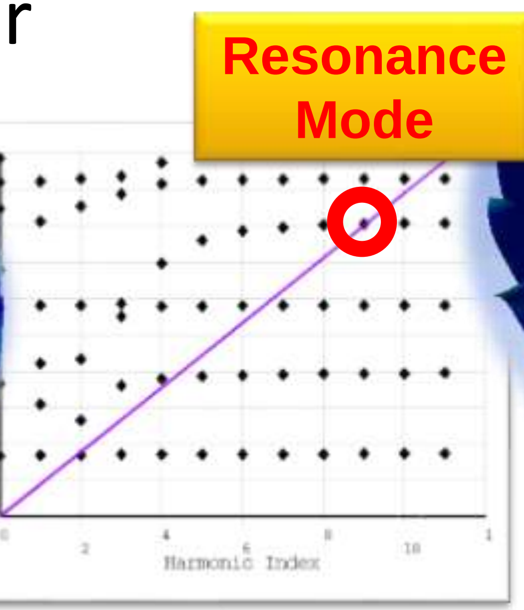


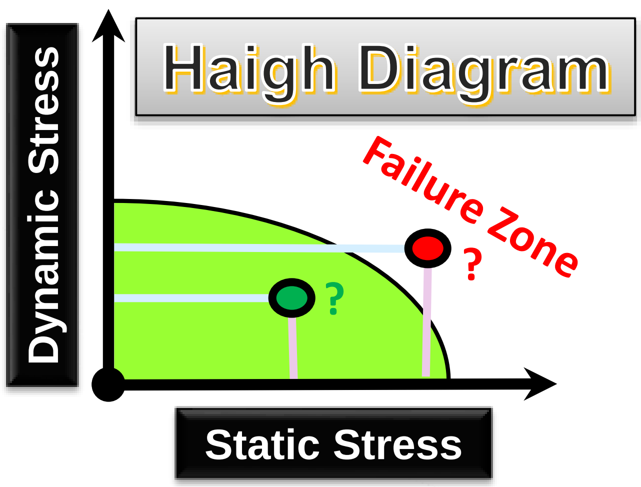
$$\left\{\underbrace{-\Omega^2}_{\text{Excitation frequency}}\underbrace{\mathbf{I}}_{\text{Reduced mass}} + \underbrace{i\Omega\left[\left(\beta+\frac{1}{\Omega^2}g\right)\boldsymbol{\Lambda}^s+\alpha\mathbf{I}\right]}_{\text{Reduced damping}} + \underbrace{\boldsymbol{\Lambda}^s}_{\text{Reduced stiffness}} + \underbrace{\mathbf{K_a}}_{\text{Aero stiffness}}\right\}\mathbf{a}e^{i\omega t} = \left\{\underbrace{\sum_n F^T_{n,EO}\tilde{\Phi}^T_h}_{\text{Projection to modal space and expansion from cyclic domain}}\underbrace{\mathbf{F}e^{i\phi(n-1)}}_{\text{Single sector engine order forcing}}\right\}e^{i\omega t}$$

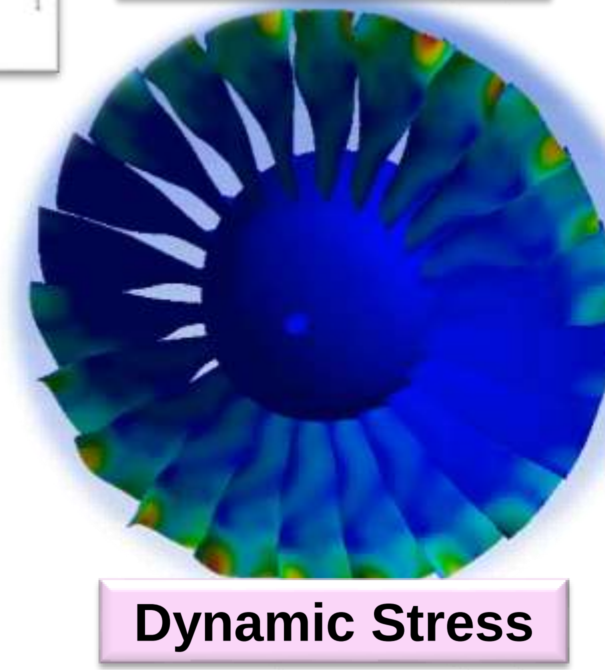
CFD-Flutter Simulation→

Results – Tuned Rotor









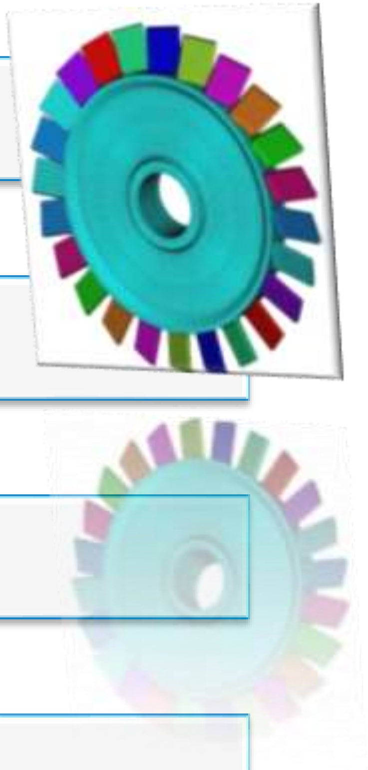
Mistuning - Modelling

1 DOF Minimal Modell

2 Sector Modell with Point Mass

CMS Modell (*<u>on 1 Sector!</u>*)

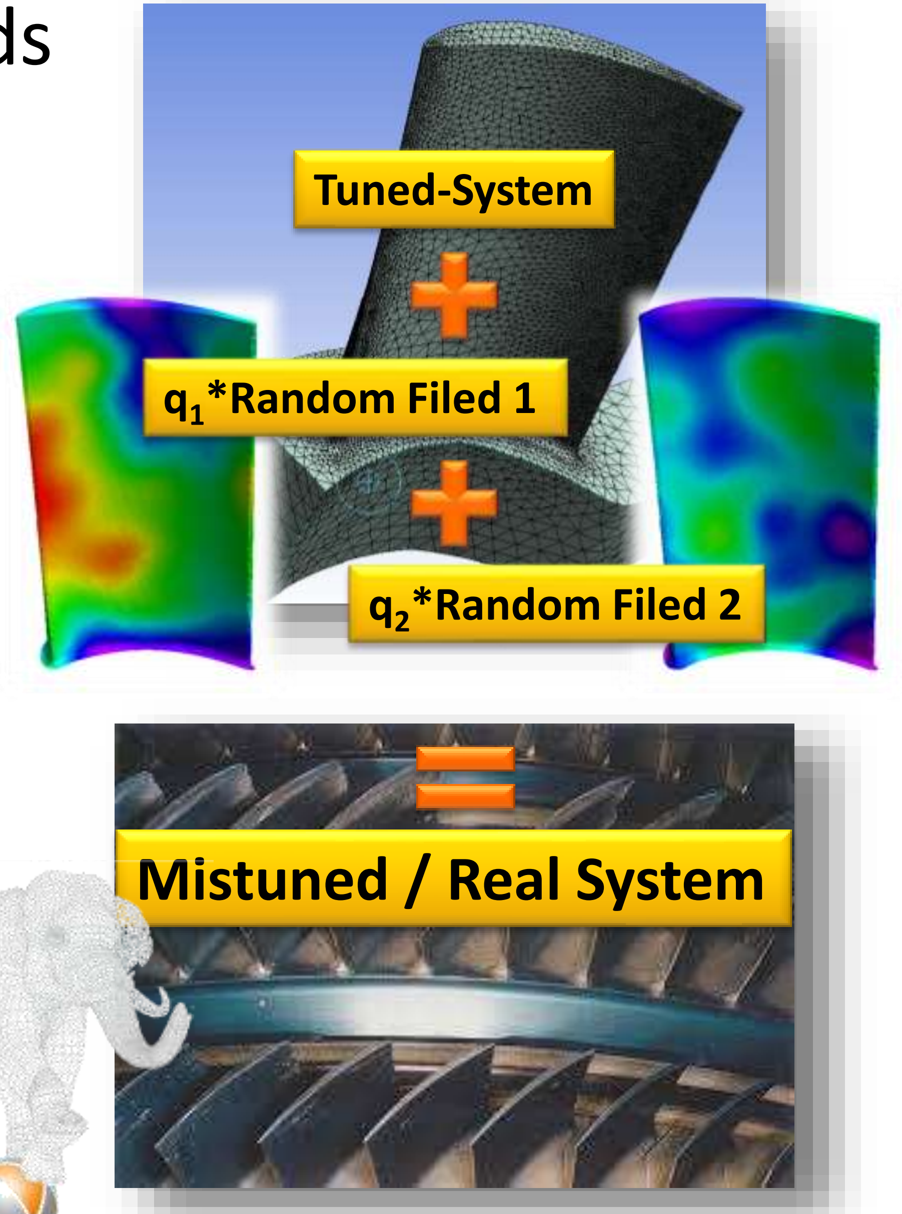
Full Model with Random Fields



Full Model - Random Fields

- Measured / Assumed Variation → Random Fields (=Eigenvectors of Correlation Matrix)
- Random Field is morphed on tuned System Mesh

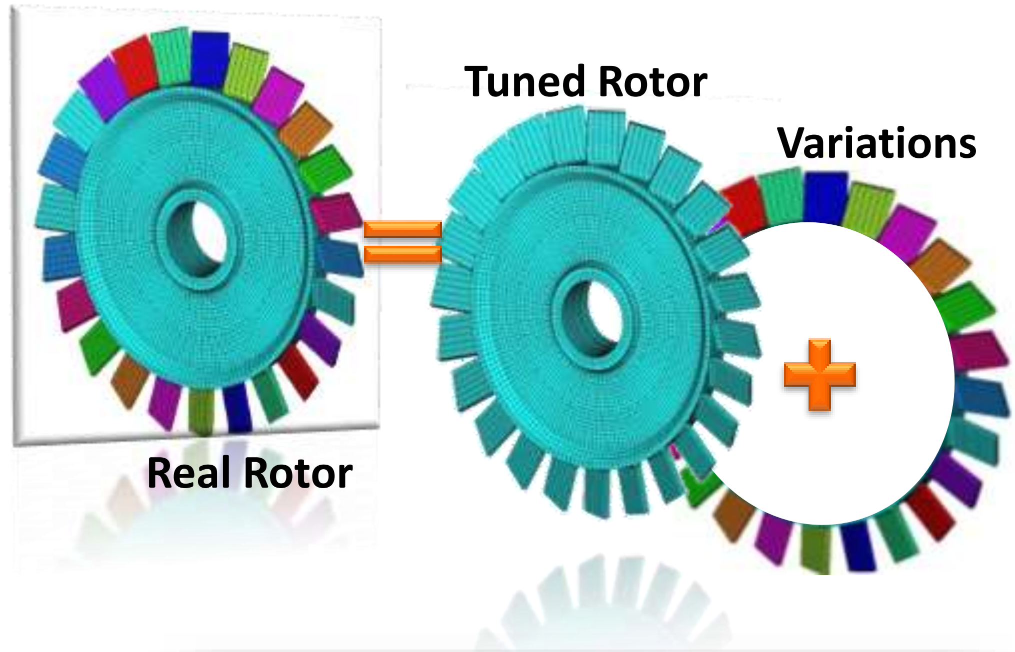
- Pro:
  - Model for large Mistuning
  - efficient handling of statistics
- Con:
  - computational effort

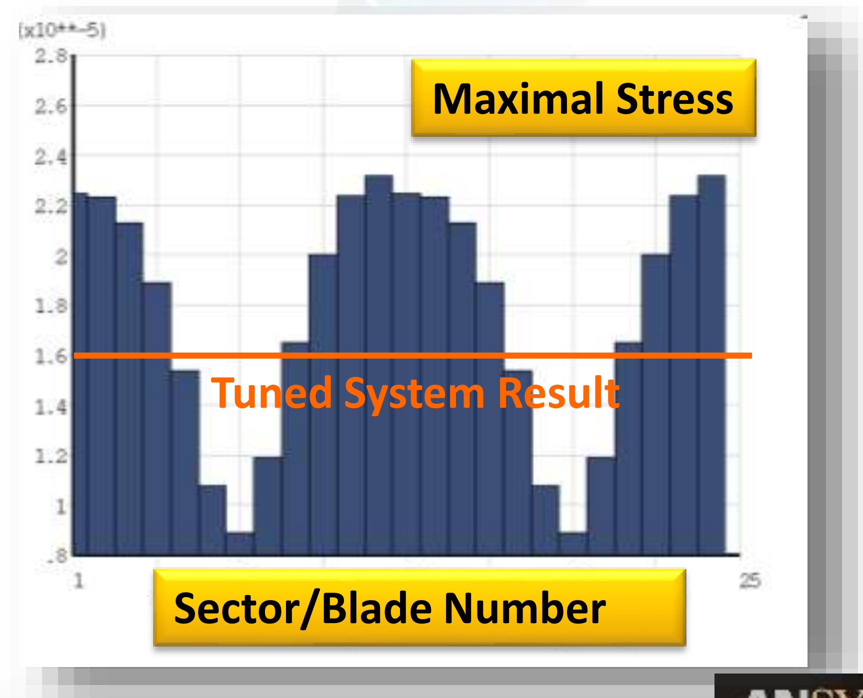


CMS Model

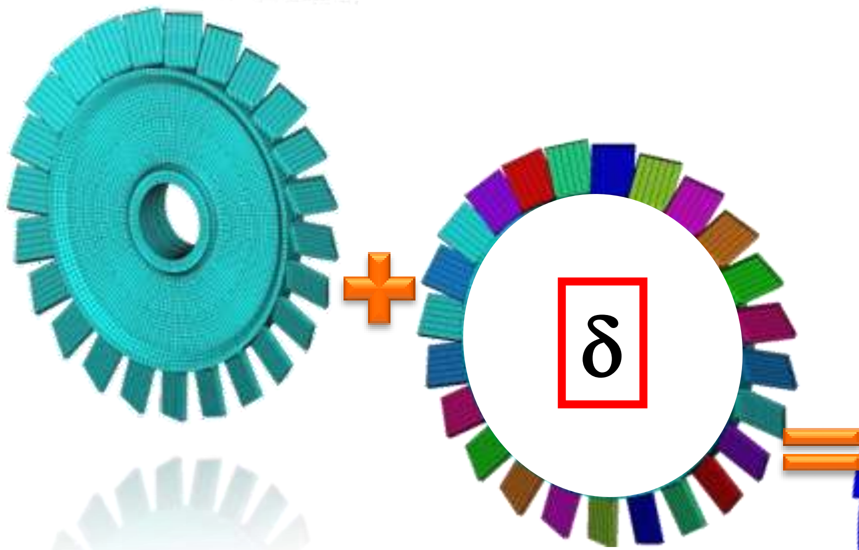
- Cyclic Model + non-cyclic Mistuning
  – Proportional Mistuning
  – Intended Mistuning
  – …
- Different Stress Levels on Sectors/Blades

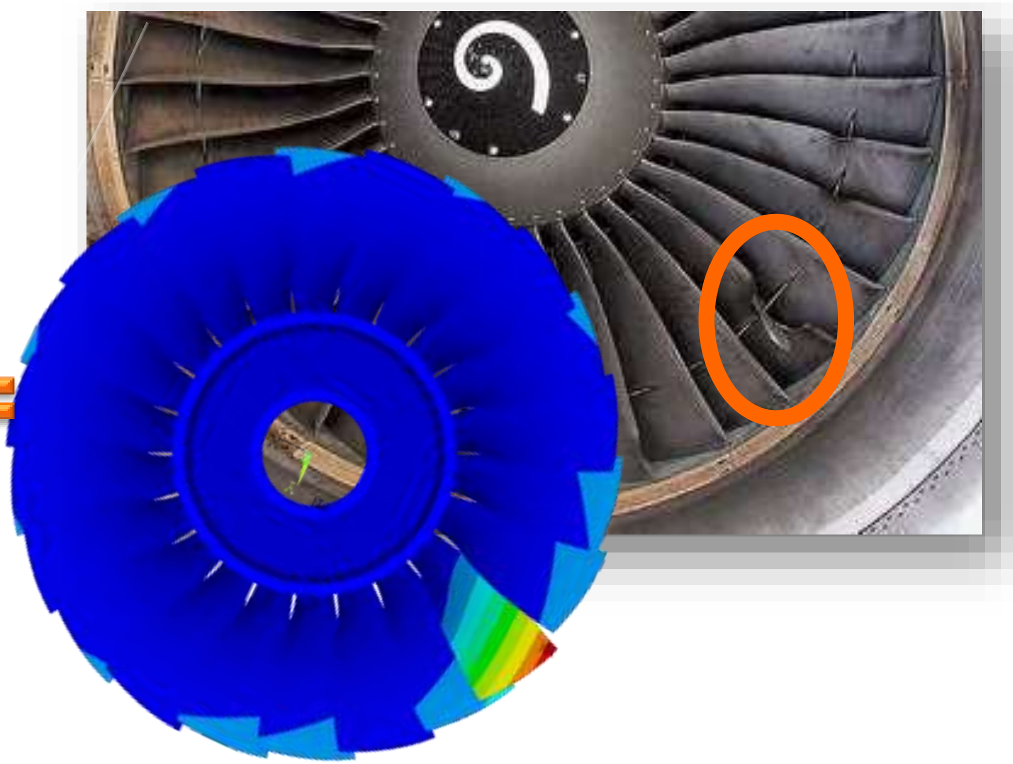
- Pro:
  – efficient: single sector mesh required
- Con:
  – small Mistuning





Mistuning - Reduced Order Model





$$\boxed{\tilde{\mathbf{q}}_{\phi,h}} = [\boldsymbol{\Lambda}^B]^{-1} \begin{bmatrix}\boldsymbol{\phi}^B \\ 0\end{bmatrix}^T \mathbf{K}_o \, {}^{\Gamma}\tilde{\boldsymbol{\Phi}}_h$$

Cyclic Modes are approximated by cantilevered Modes→Modal Participation Factors q

$$\Bigg\{ \underbrace{-\Omega^2}_{\text{Excitation frequency}} \underbrace{\mathbf{I}}_{\text{Reduced mass}} + \underbrace{i\Omega\left[\left(\beta + \frac{1}{\Omega}g\right)\boldsymbol{\Lambda}^s + \alpha\mathbf{I}\right]}_{\text{Reduced damping}} + \underbrace{\boldsymbol{\Lambda}^s}_{\text{Reduced stiffness}} + \underbrace{\sum_n \boxed{\mathbf{q_n}}^T \boxed{\delta_n} \mathbf{K}_o \boxed{\mathbf{q_n}}}_{\text{Mistuning terms}} + \underbrace{\mathbf{K_a}}_{\text{Aero stiffness}} \Bigg\} \mathbf{a} e^{i\omega t} = \Bigg\{ \underbrace{\sum_n F^T_{n,EO} \tilde{\Phi}^T_h}_{\text{Projection to modal space and expansion from cyclic domain}} \underbrace{\mathbf{F} e^{i\phi(n-1)} \Bigg\} e^{i\omega t}}_{\text{Single sector engine order forcing}}$$

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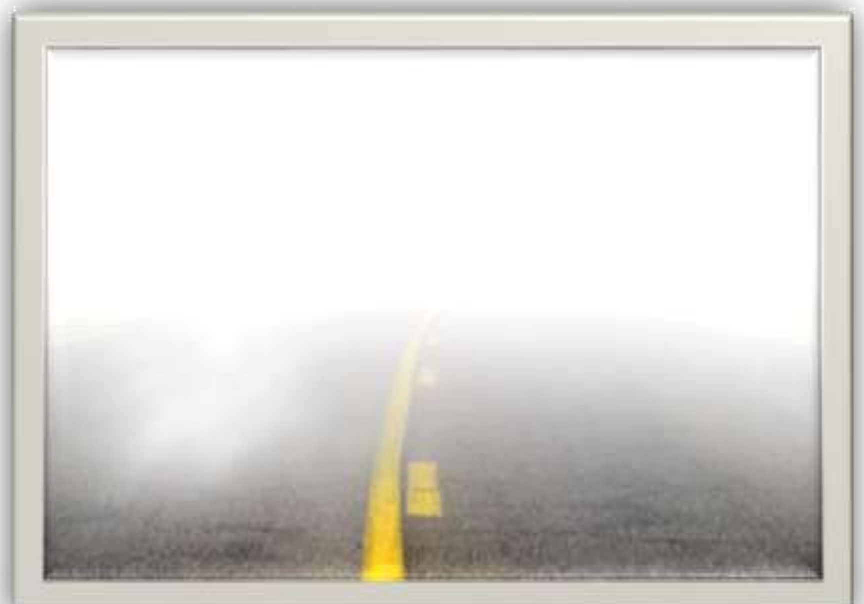
optiSLang Strategy

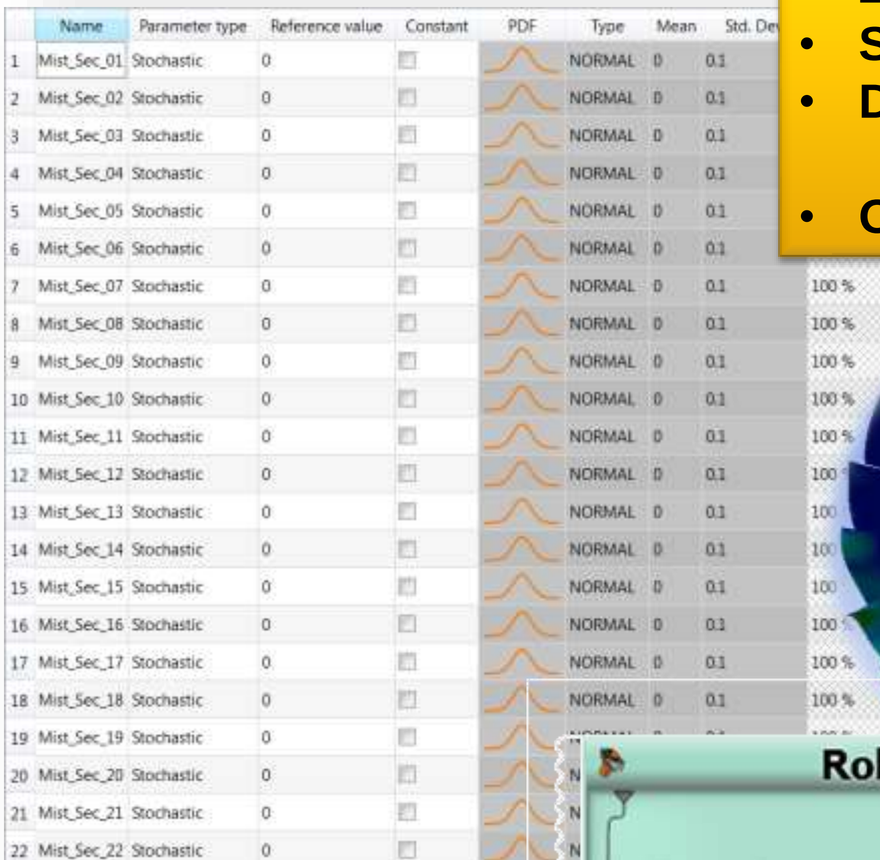
General Procedure:

- Design Optimization
  - Gradient Based
  - Generic
  - Evolutionary
  - …



- Design of Experiments
  - Data Sampling
  - Detecting Correlations
  - **Detecting Important Parameters**
  - **Parameter Space Reduction**
  - Response Surface: MoP
  - **Reliable Meassure of Prediction Quality: CoP**
- Design Optimization
  - …

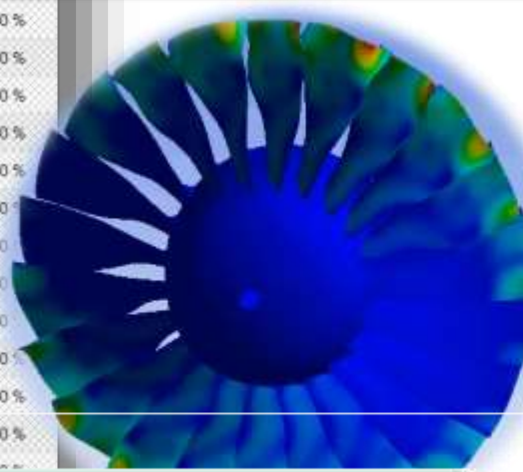
optiSLang Set-Up

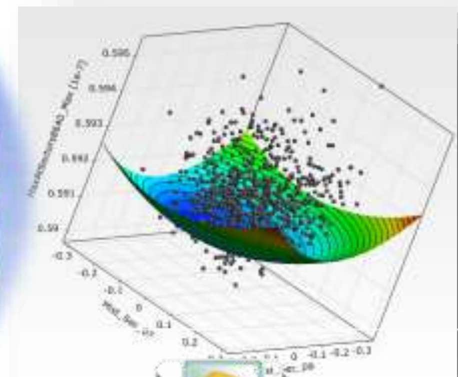


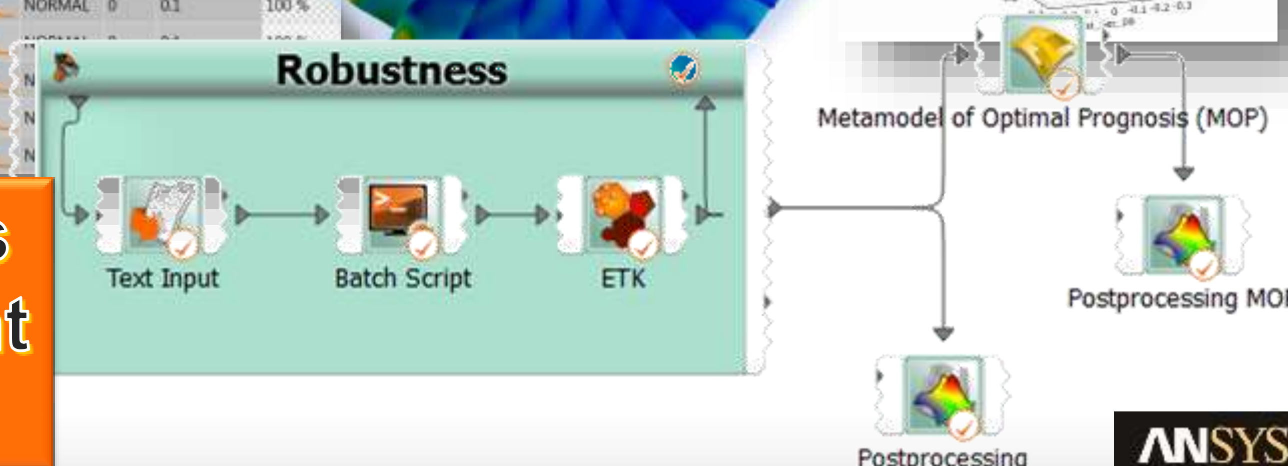
	Name	Parameter type	Reference value	Constant	PDF	Type	Mean	Std. De[v]	
1	Mist_Sec_01	Stochastic	0			NORMAL	0	0.1	
2	Mist_Sec_02	Stochastic	0			NORMAL	0	0.1	
3	Mist_Sec_03	Stochastic	0			NORMAL	0	0.1	
4	Mist_Sec_04	Stochastic	0			NORMAL	0	0.1	
5	Mist_Sec_05	Stochastic	0			NORMAL	0	0.1	
6	Mist_Sec_06	Stochastic	0			NORMAL	0	0.1	
7	Mist_Sec_07	Stochastic	0			NORMAL	0	0.1	100 %
8	Mist_Sec_08	Stochastic	0			NORMAL	0	0.1	100 %
9	Mist_Sec_09	Stochastic	0			NORMAL	0	0.1	100 %
10	Mist_Sec_10	Stochastic	0			NORMAL	0	0.1	100 %
11	Mist_Sec_11	Stochastic	0			NORMAL	0	0.1	100 %
12	Mist_Sec_12	Stochastic	0			NORMAL	0	0.1	100
13	Mist_Sec_13	Stochastic	0			NORMAL	0	0.1	100
14	Mist_Sec_14	Stochastic	0			NORMAL	0	0.1	100
15	Mist_Sec_15	Stochastic	0			NORMAL	0	0.1	100
16	Mist_Sec_16	Stochastic	0			NORMAL	0	0.1	100
17	Mist_Sec_17	Stochastic	0			NORMAL	0	0.1	100 %
18	Mist_Sec_18	Stochastic	0			NORMAL	0	0.1	100 %
19	Mist_Sec_19	Stochastic	0						
20	Mist_Sec_20	Stochastic	0			N			
21	Mist_Sec_21	Stochastic	0			N			
22	Mist_Sec_22	Stochastic	0			N			

- **Reference=Mean Value=0, i.e. Tuned**
- **22 Blades → 22 Random Variables**
- **Standard Deviation=0.1% 1.0% 10% 100%**
- **DoE with 400 and 800 dps**

- **Objective: Variation of Meximal Stress**

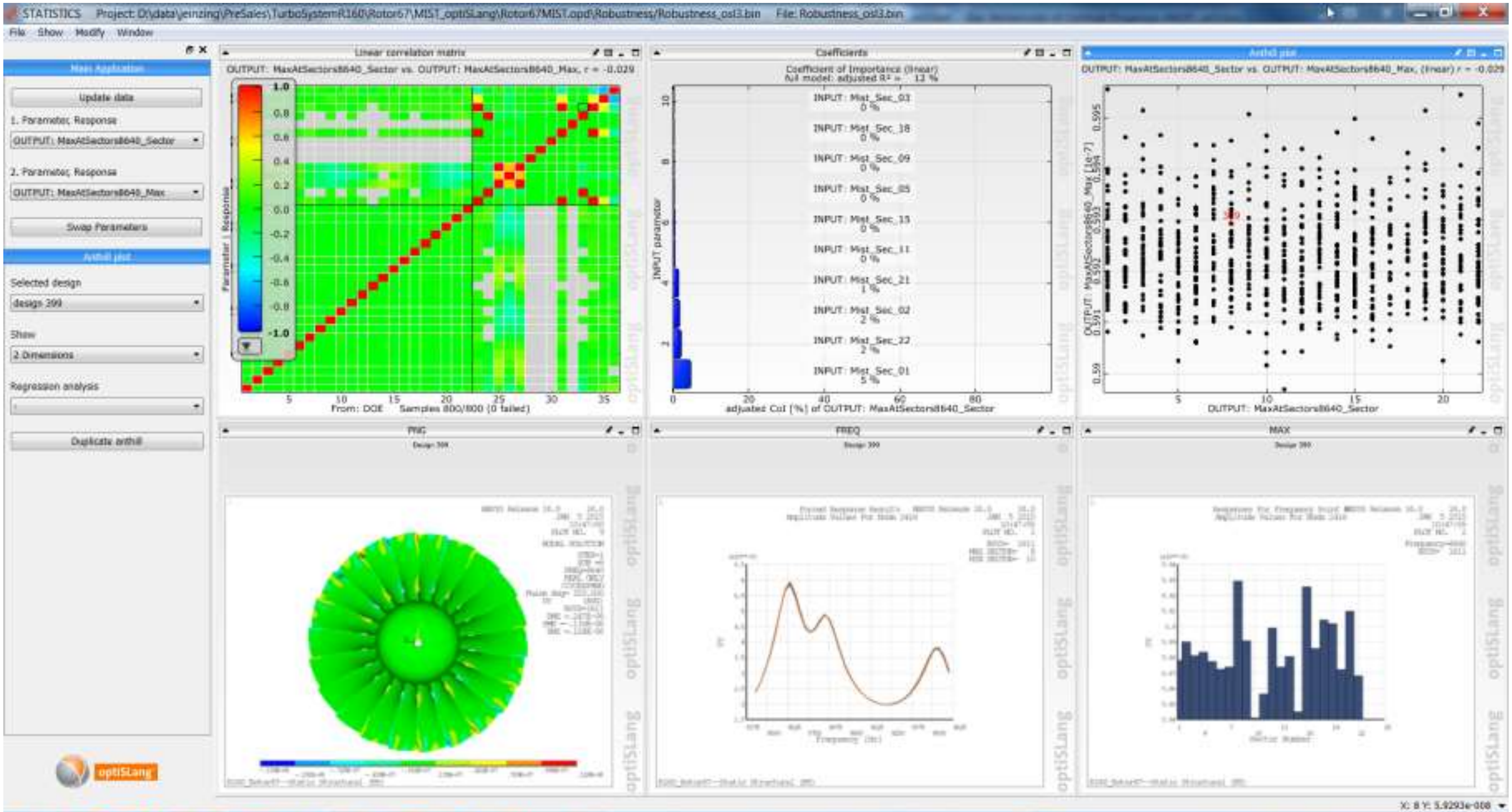






Which Blades have dominant impact???

optiSLang DoE Monitoring

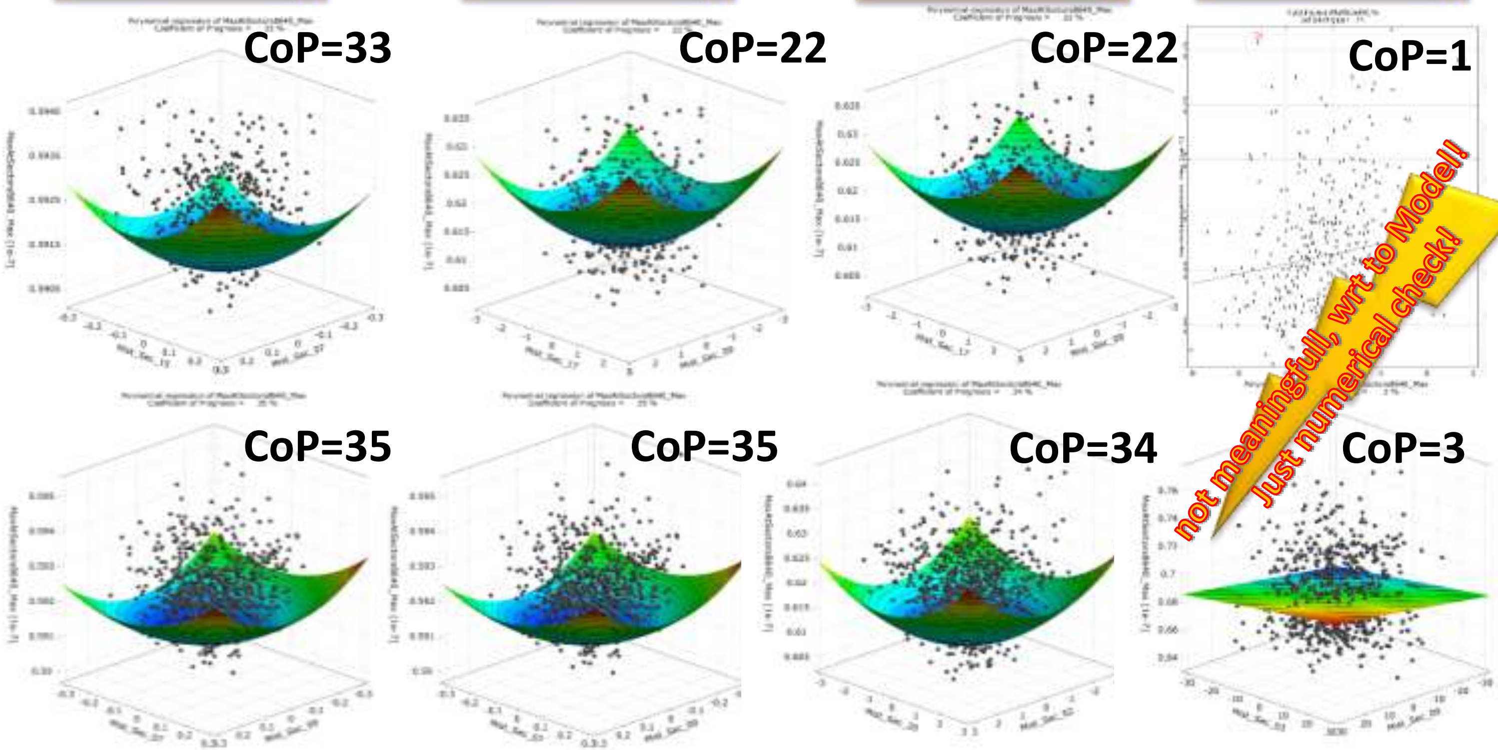


Meta Modell of Optimal Prognosis

Increasing Coefficient of Prognosis

Bad CoP!! WHY??

Std. Dev. δ =0.1%	**Std. Dev. δ =1.0%**	**Std. Dev. δ =10%**	**Std. Dev. δ =100%**



400 Design Points

800 Design Points

Important Parameters

Increasing Coefficient of Prognosis

Important Parameters not detected properly!! WHY??

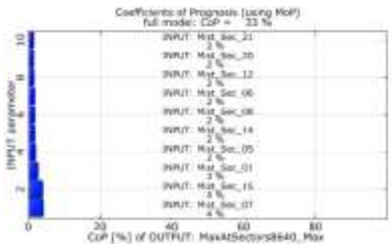
Std. Dev. δ =0.1%

Std. Dev. δ =1.0%

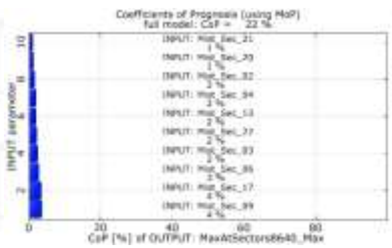
Std. Dev. δ =10%

Std. Dev. δ =100%

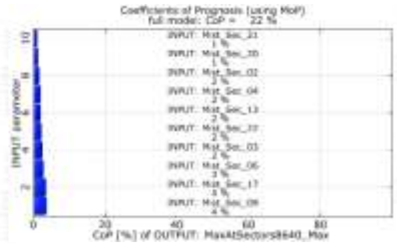
CoP=33



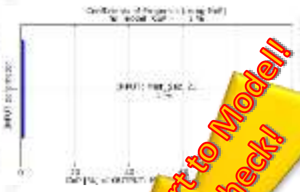
CoP=22



CoP=22



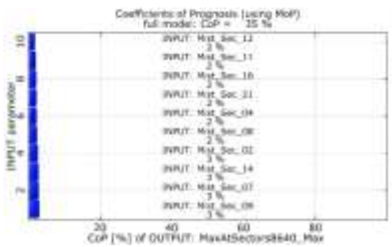
CoP=1



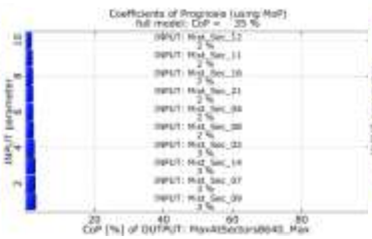
400 Design Points

not meaningfull, wrt to Model!! Just numerical check!

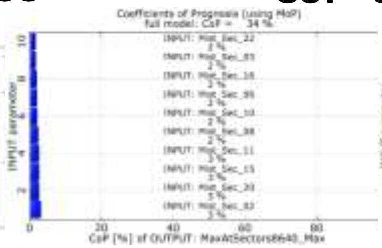
CoP=35



CoP=35



CoP=34



CoP=3

800 Design Points

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Best-Practice Guide Lines

- Reason for small Coefficient of Prognosis:
  - Number Design Points
  - Numerical Error
  - Model Error
  - Multiple Mechanism
  - Bad Parameterization (TWC vs. discrete)
  - Post-Processing Output (SOS vs. single Value or Signal)
- Number of Design Points for Meta-Model depends on:
  - Number of *important* parameters, *here: all parameter!!*
  - Nonlinearity of Response Surface

Objective for Meta-Model: Maximal Coefficient of Prognosis

Summary and Outlook

- Summary
  - Mistuning has significant Influence to Bladed-Rotors (and other cyclic structures)
  - Efficient Mistuning Models are available
- Outlook
  - Further Investigation with more Design Points
  - TWC Parameterization
  - Post-Processing by SOS
  - Large Mistuning: Application of Random Fields

Use Simulation and Statistical Analysis...

...to avoid:

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