



# Sensitivity Analysis of a Turbo Compressor with Physical Parameterization

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# Turbo Optimization Strategies with Meta-Model

## Traditional Design/Optimization Strategy

Primary Design (Empiric)

Manual Improvement, CFD

optiSLang – small Parameter Range

## “Optimization does it all”

Primary Design (Empiric)

optiSLang – large Parameter Range

Works excellent, since  
Engineer + optiSLang = perfect Combination  
Key: small Parameter Range  
Draw back: needs to be done for each new Design

Strategy usually fails, bad Results  
Problem: large Parameter Range  
Benefit (if it works): investment upfront, fast  
Optimization for each new Design

# Agenda

- Turbomachinery Design Principles
- Traditional Parameterization and Limitations
- Physical Parameters
- Sensitivity Analysis

# Turbomachinery – Conservation Principals

Mass Conservation

$$\sum \int \underbrace{\rho \vec{c} \vec{n}}_{\dot{m}} dA = 0$$

Momentum Conservation

$$\sum \int \underbrace{(\rho \vec{c} \vec{n})}_{\dot{m}} \vec{c} dA = \sum \int (\vec{p} \vec{n} + \vec{\tau}) dA$$

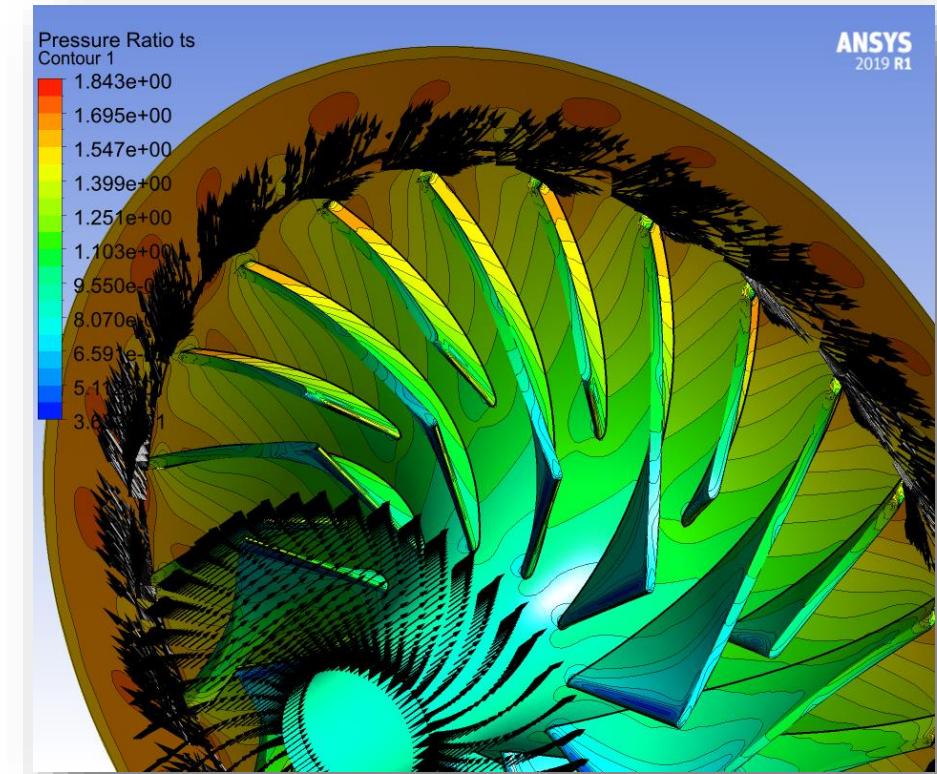
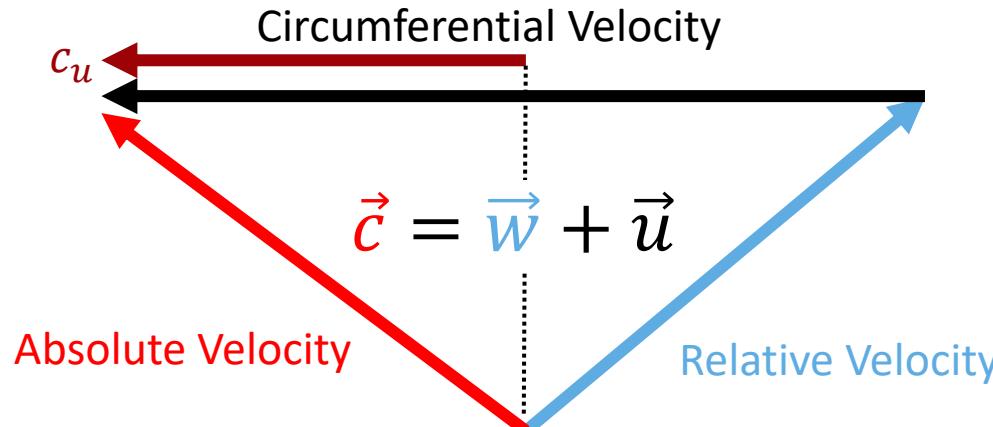
Energy Conservation – Euler Equation

$$\Delta h_t = P_{Mech} + Q_{Therm}$$

$$\Delta h_t = \Delta(uc_u)$$

Total Enthalpy Gain = Work due Rotation

Kinematics – Velocity Triangle



Thermodynamics, ideal Gas

Equation of State  $p = \rho RT$

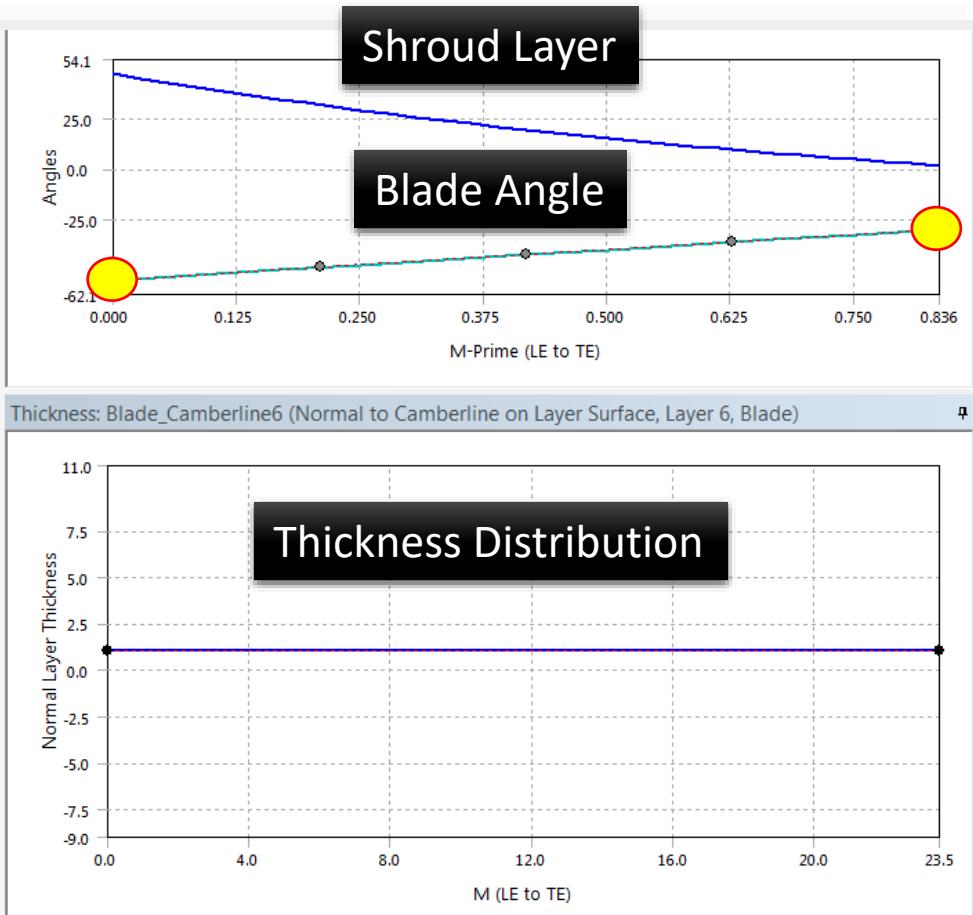
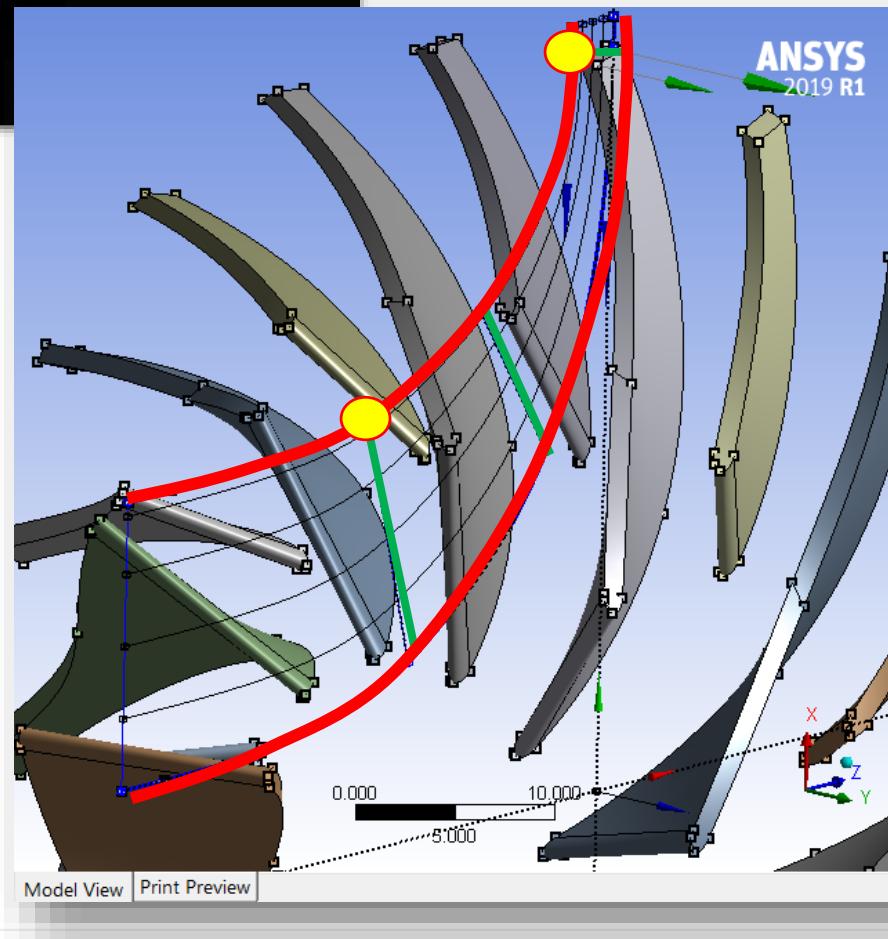
Caloric Equation  $h = c_p T$

2<sup>nd</sup> Law  $dh = Tds + \frac{dp}{\rho}$

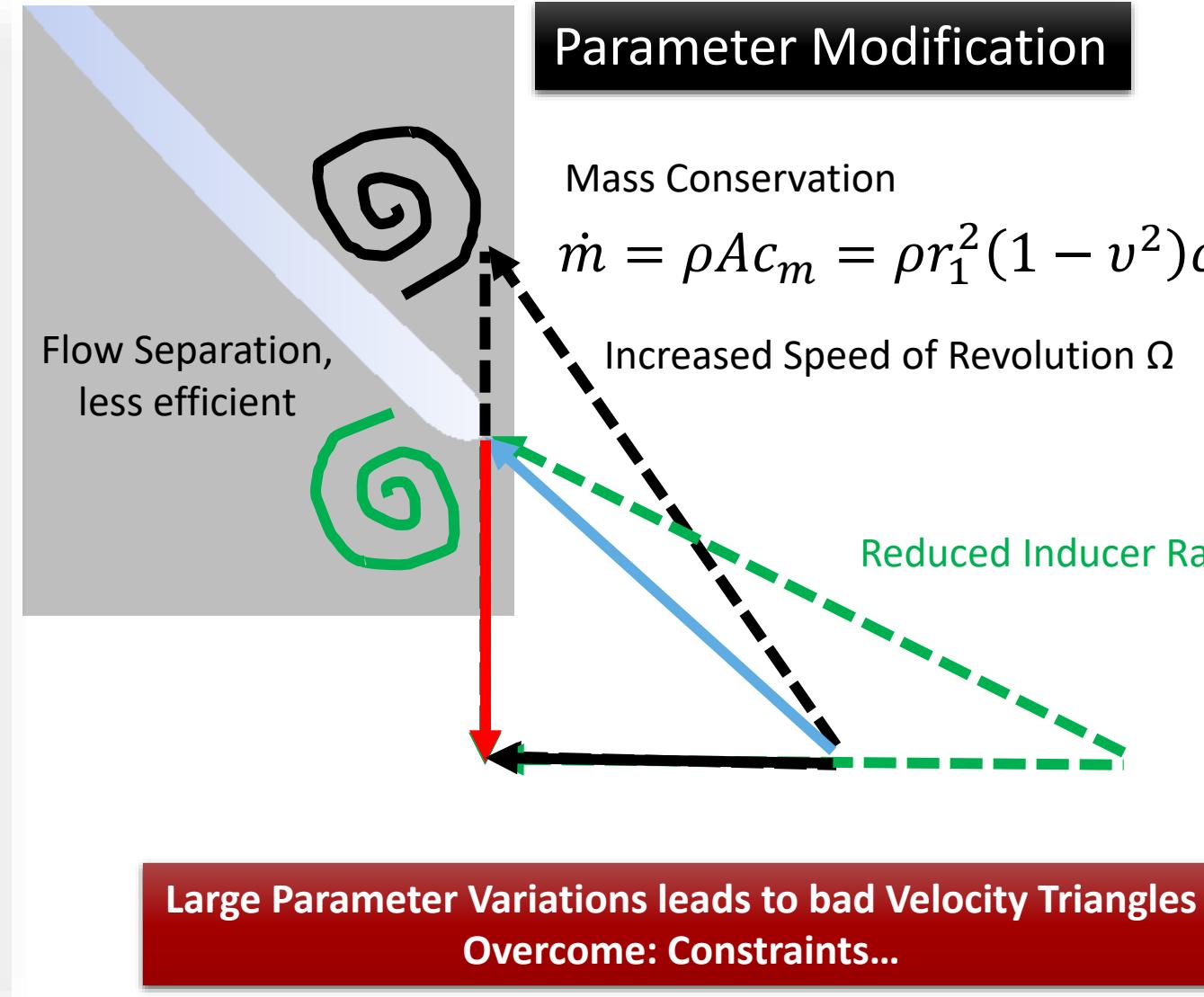
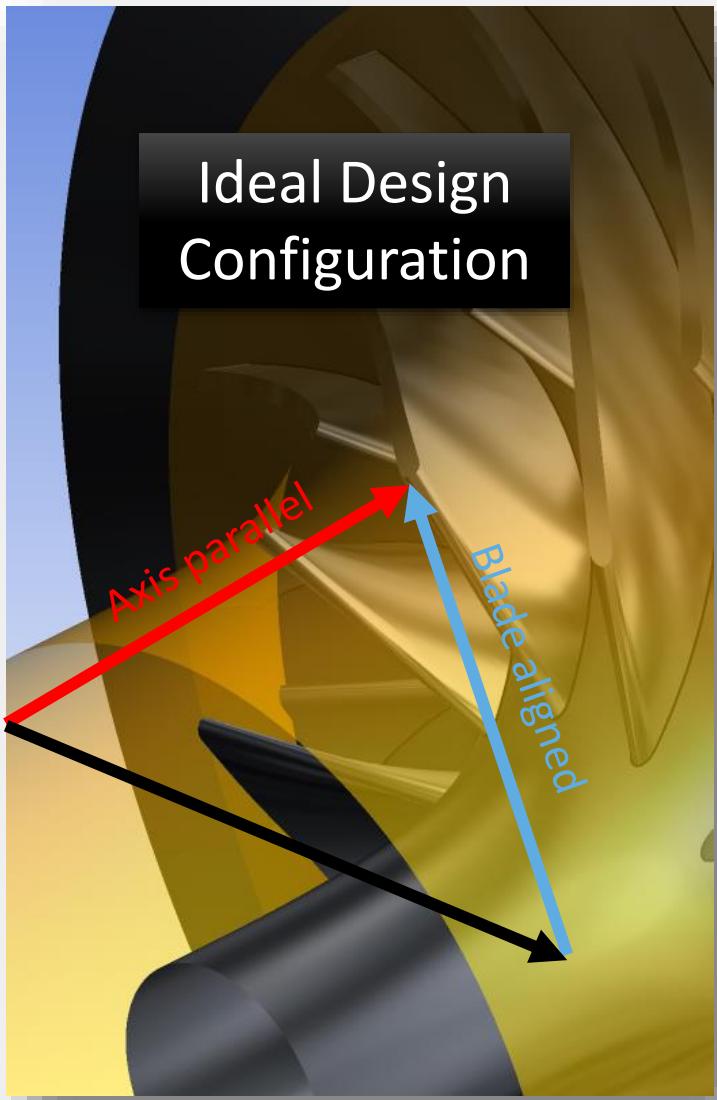
# Geometry Parameter

## Traditional Turbomachinery Parameterization

- B-Spline for Hub and Shroud
- Line/Curve for Leading- and Trailing Edge
- Blade Angles on n Layers
- Thickness Distribution
- Boundary Conditions

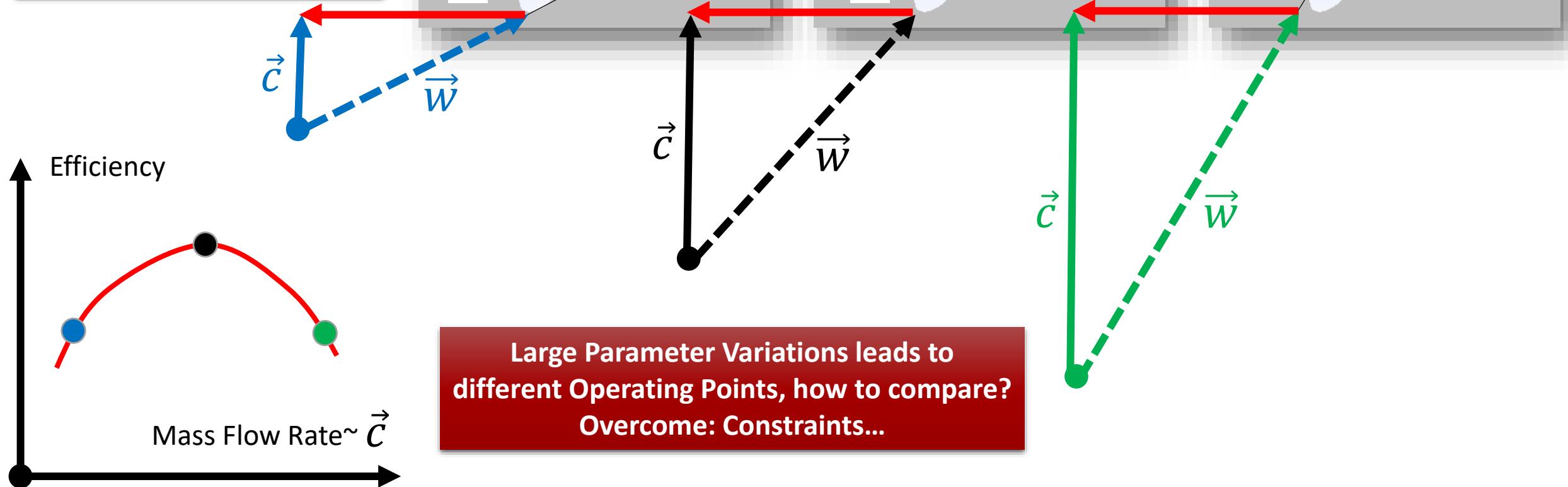


# Inducer Geometry

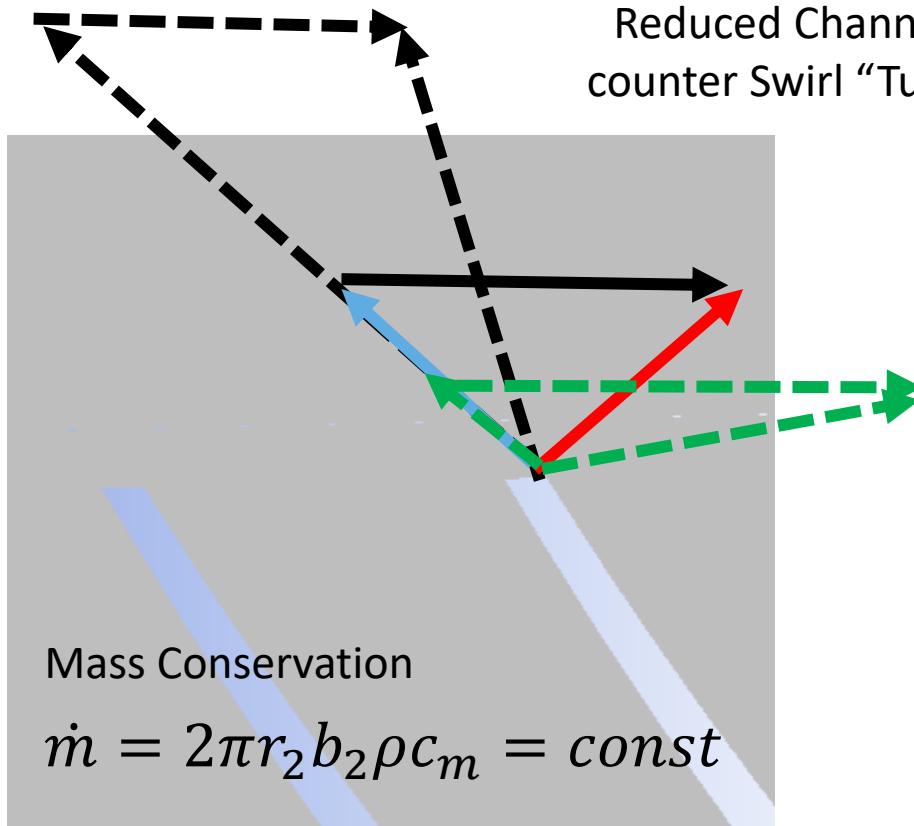
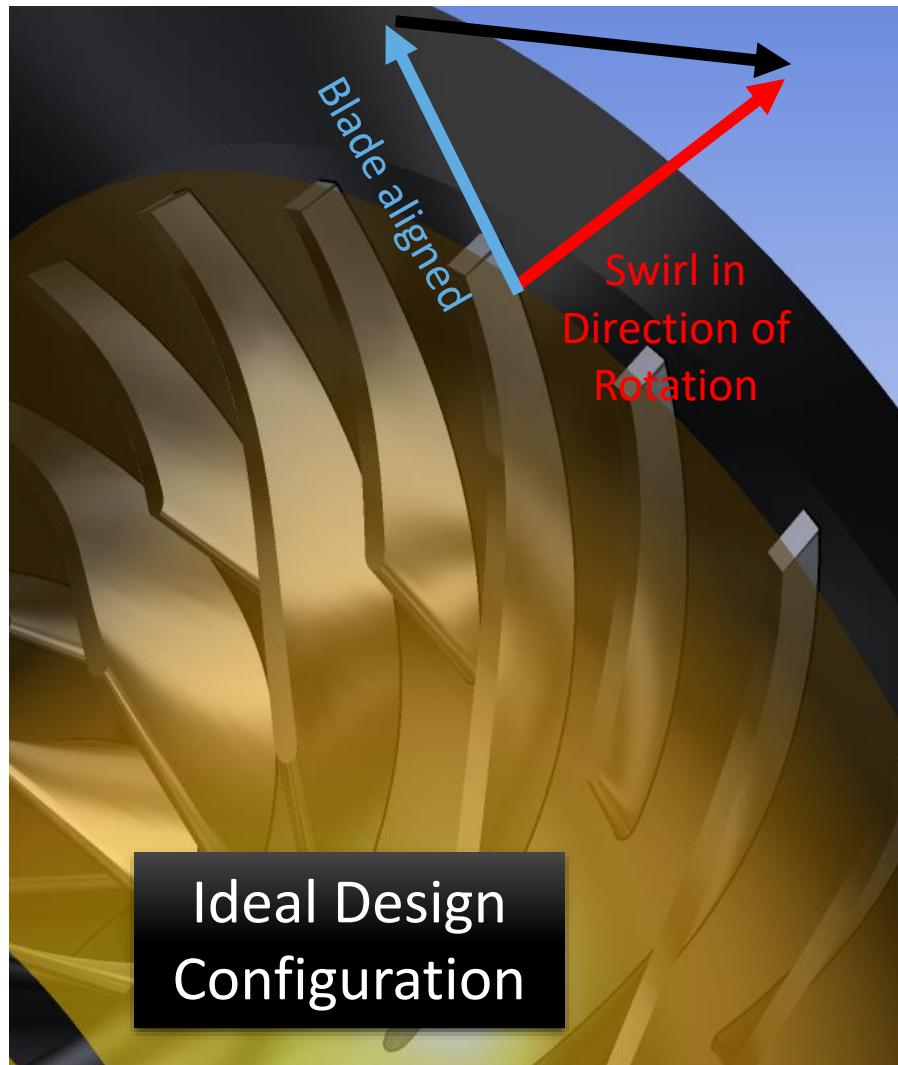


# “Hidden” Impact of Geometry Change

In general: Geometry Change will “shift” Operating Point!  
Operating Point Change has similar Impact like Geometry Change!



# Discharge Geometry



Increased Impeller Radius  $r_2$ :  
→ low  $c_m$   
→ high  $uc_u$   
→ high  $dp/dr$   
→ Flow Separation

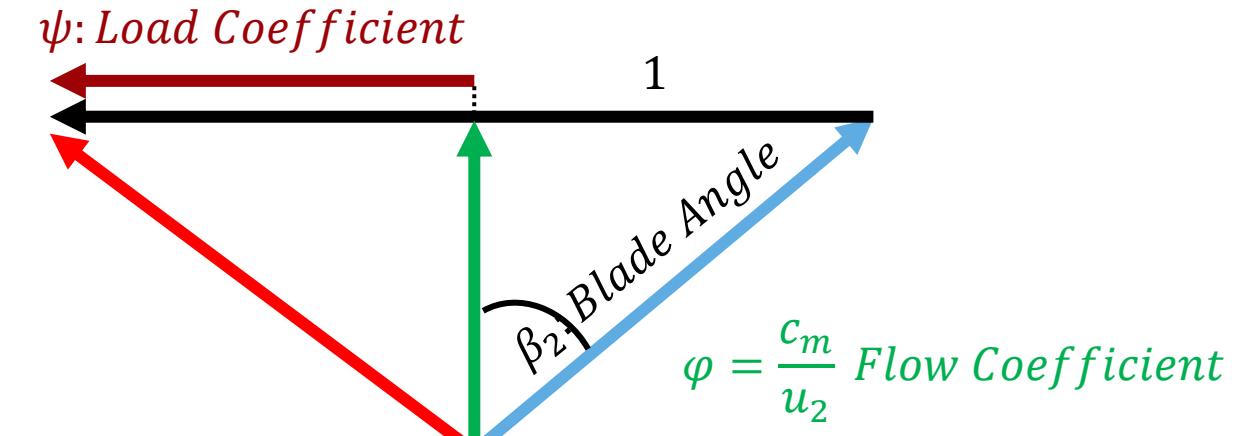
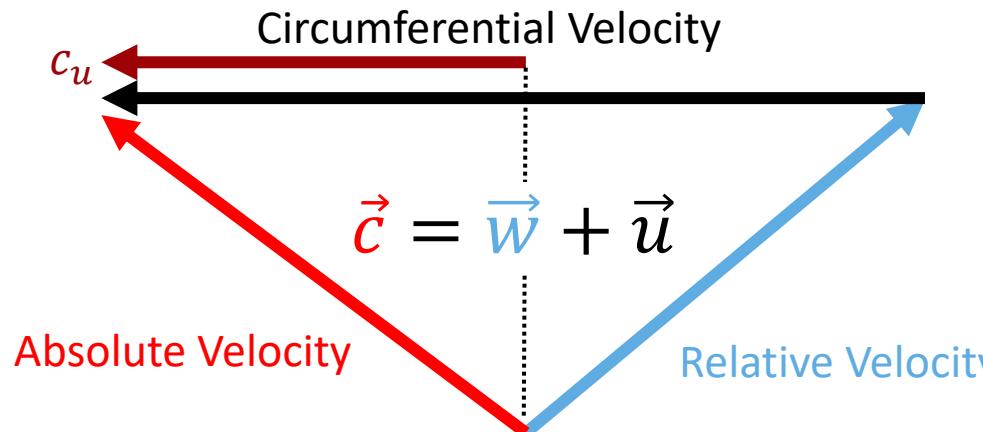
Large Parameter Variations leads to bad Velocity Triangles  
Overcome: Constraints...

# Dimensionless Analysis

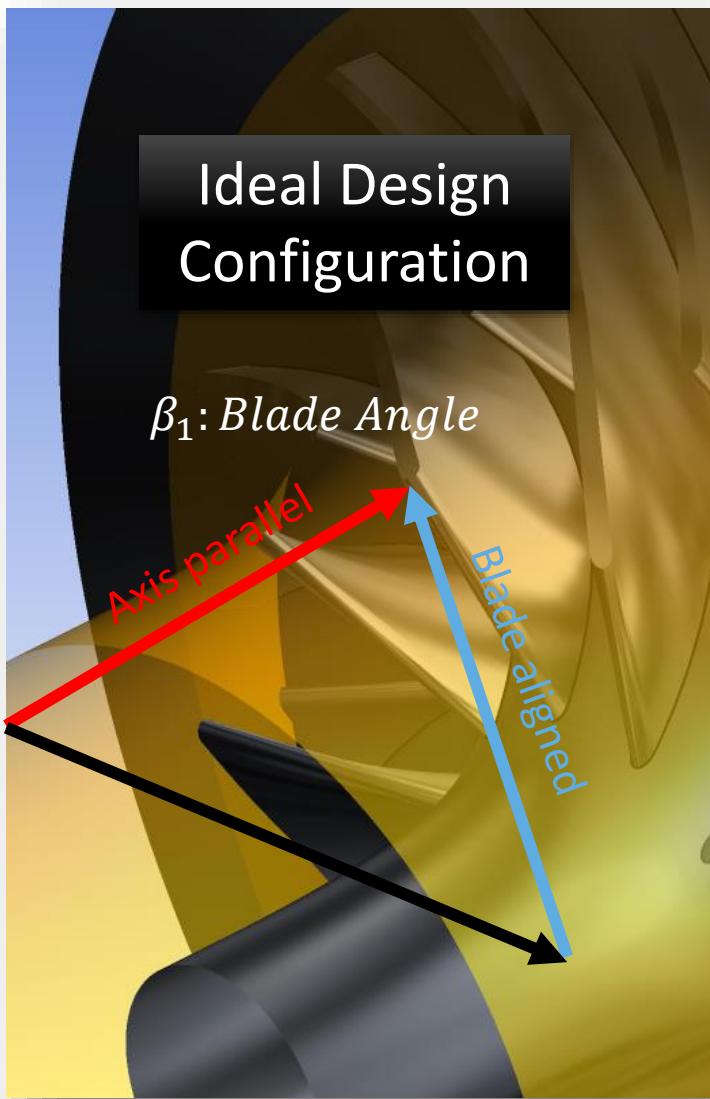
- Ambient Conditions → Reference Speed of Sound  $a_t$
- Impeller Tip Speed → Reference Velocity  $u_2$
- Dimensionless Velocity Triangle and Euler Equation

$$\frac{h_{t2}}{h_{t1}} = 1 + (\kappa - 1) \underbrace{\frac{u_2^2}{\kappa R T_{t1}}}_{M_{t,u}^2} \underbrace{\frac{c_u}{u_2}}_{\psi}$$

Dimensionless Velocity Triangle by  $\Psi$ ,  $\varphi$  and  $\beta$ .  
For all: meaningful upper and lower Bounds  
possible, without Constraint

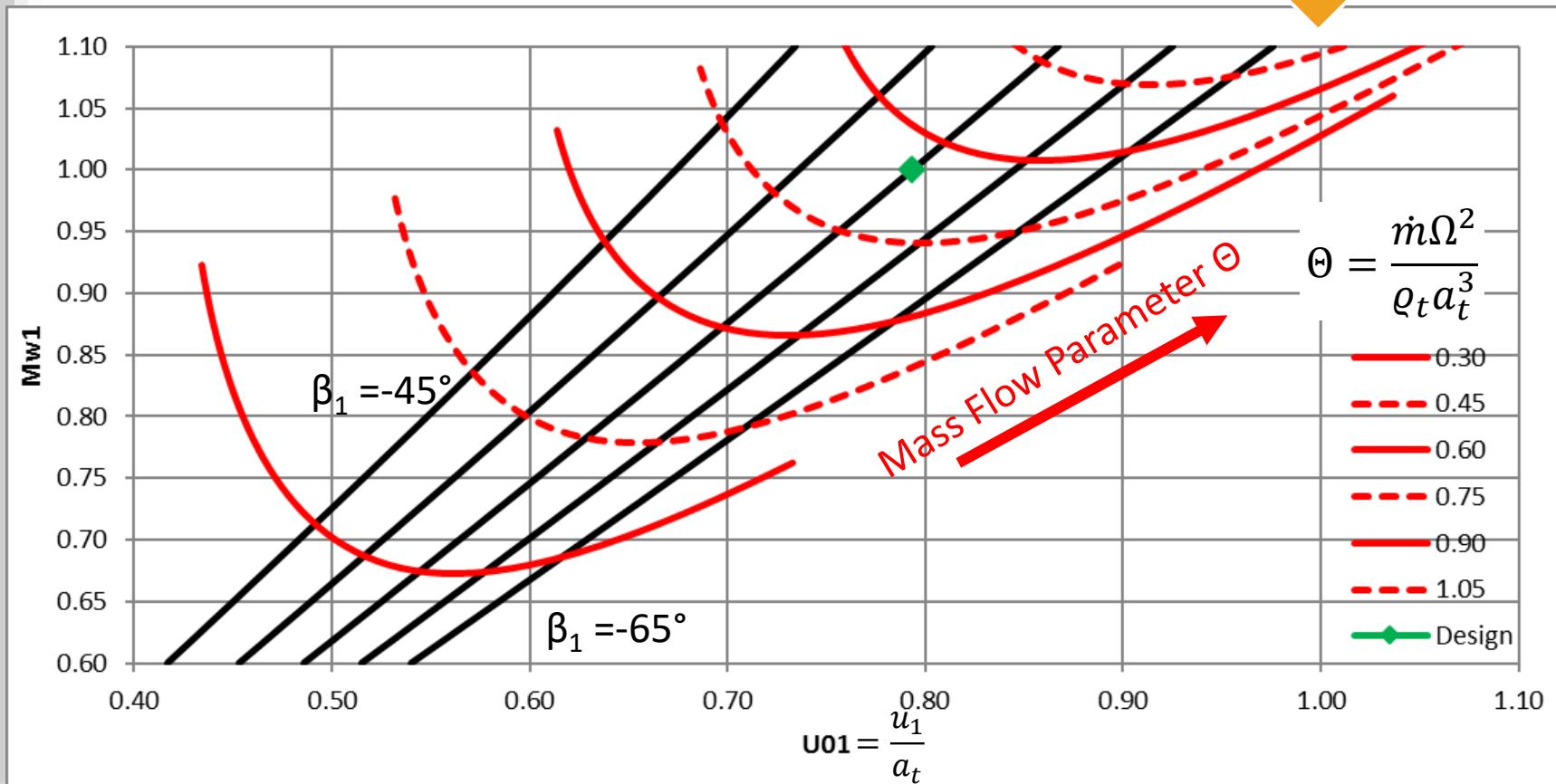


# Inducer Design

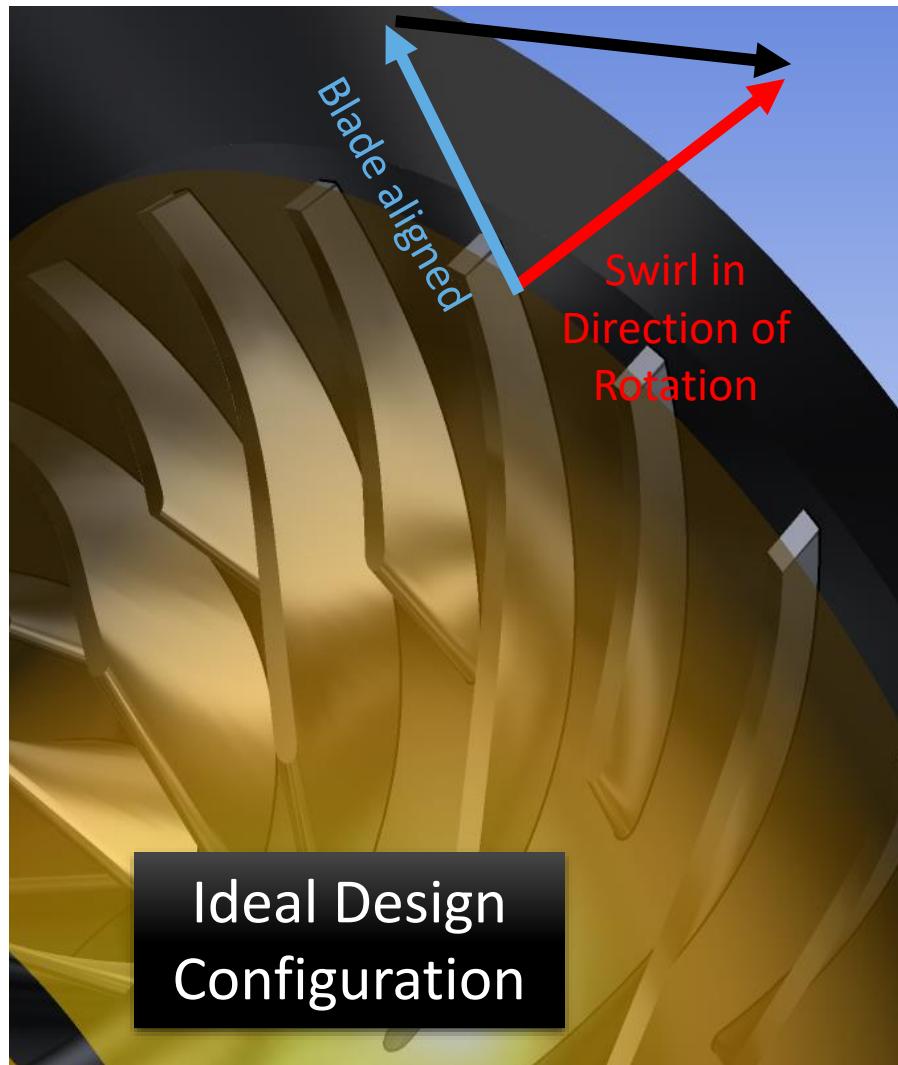


Native Parameters:  $c_1, w_1, u_1, \beta_1, r_1, \Omega, T_{t1}$   
Dimensionless: Velocity Triangle + Mass Flow Definition  
Design Requirement: low  $M_{w1} \rightarrow$  low Friction

Velocity Triangle  
+  
Mass Flow Rate

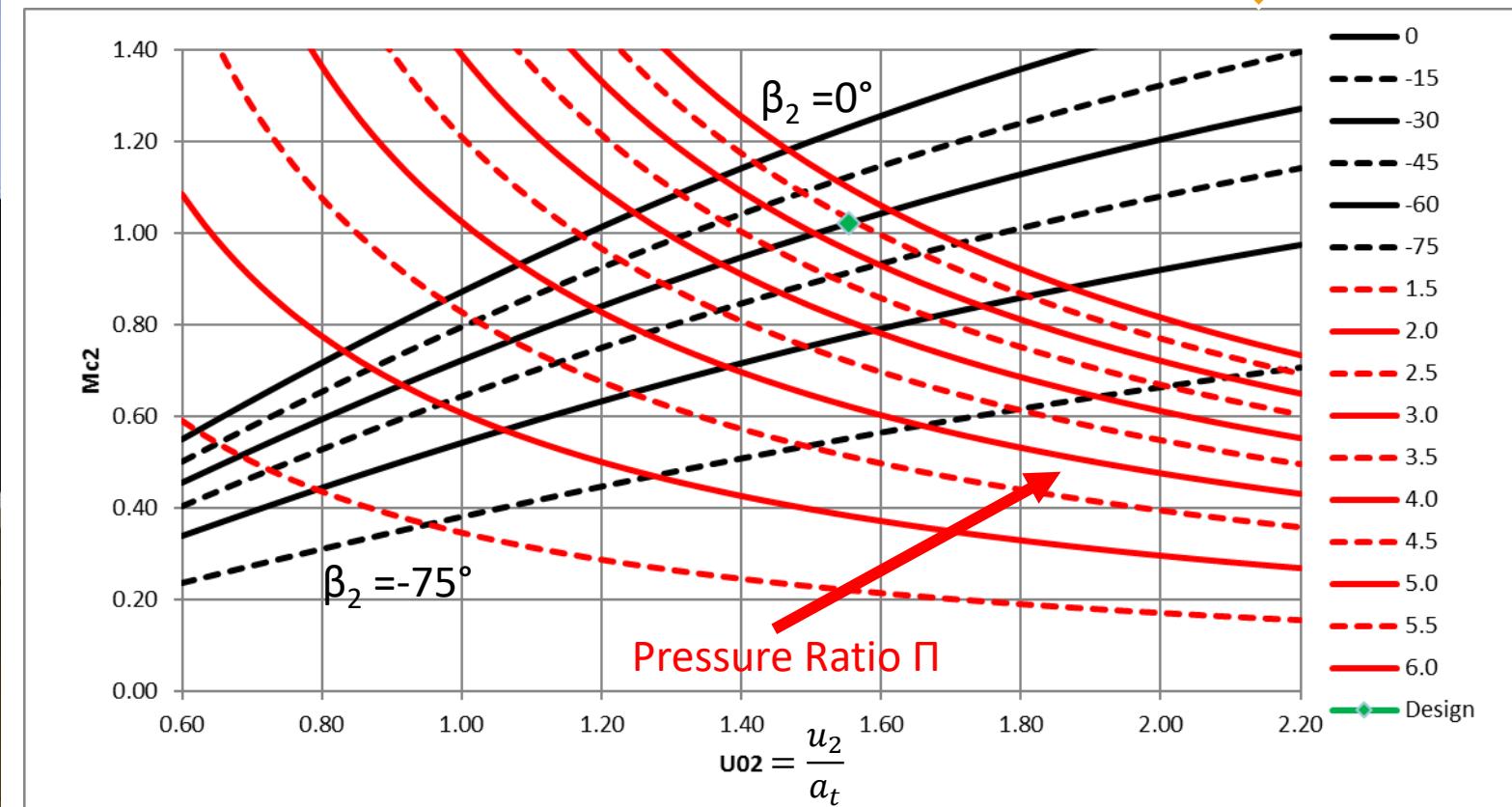


# Discharge Design



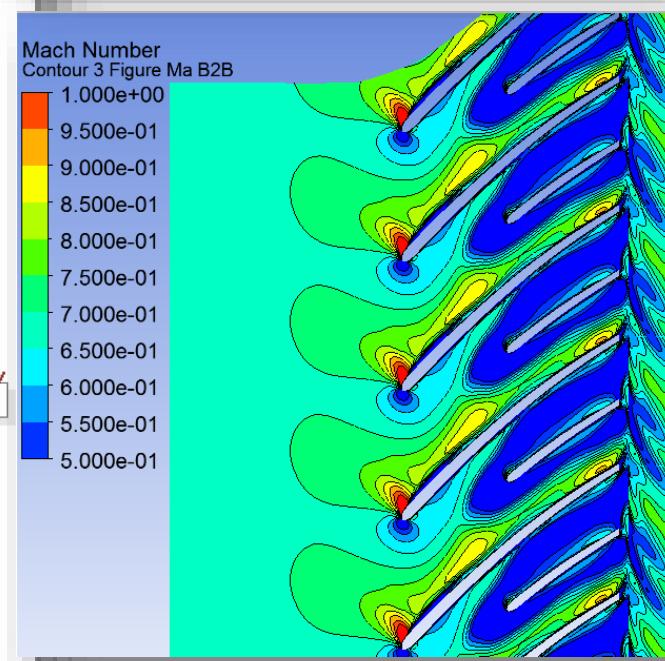
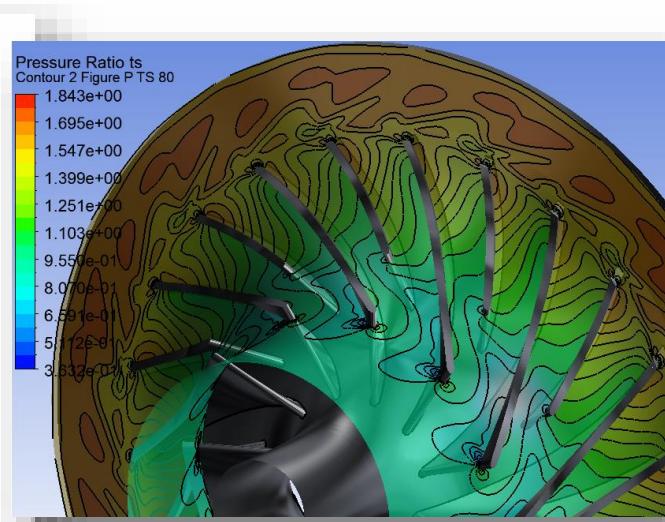
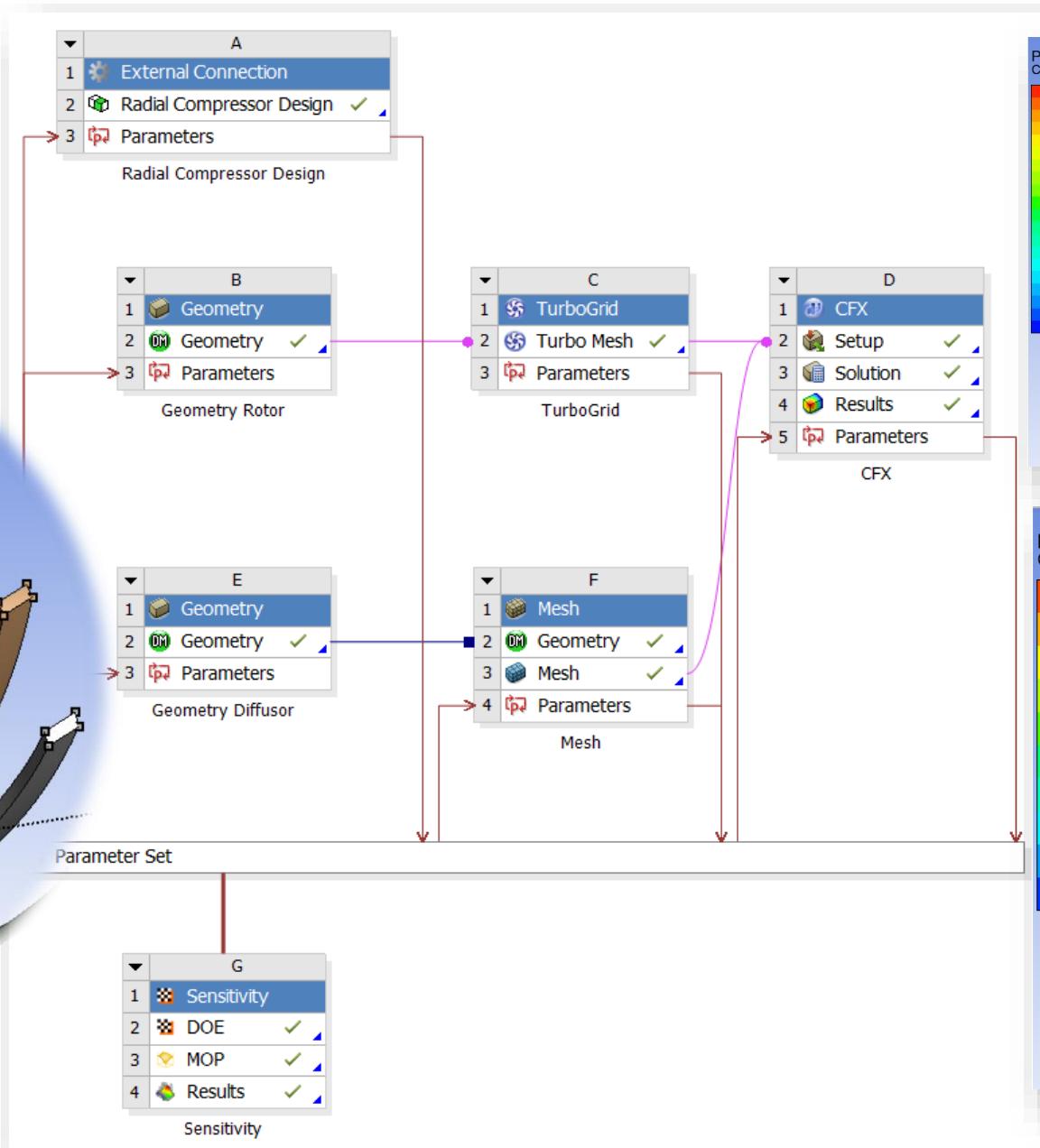
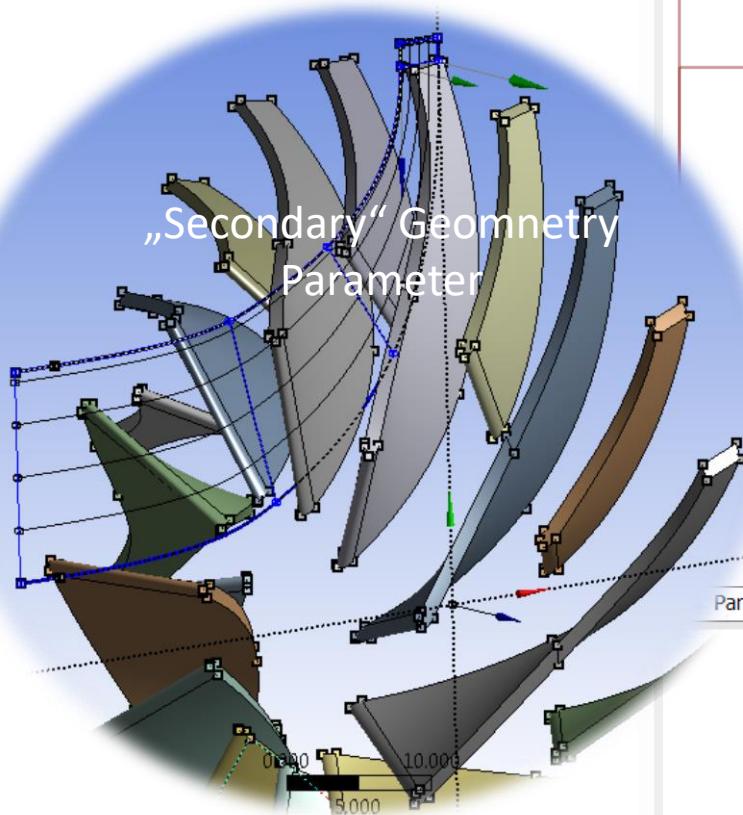
Native Parameters:  $c_2, w_2, u_2, \beta_2, r, \Omega, T_{t1}, T_{t2}, p_{t1}, p_{t2}$   
Dimensionless: Velocity Triangle + Euler Equation  
Additional Correlation:  $\Pi = f(\Theta, n_s)$

Velocity Triangle  
+  
Euler Equation

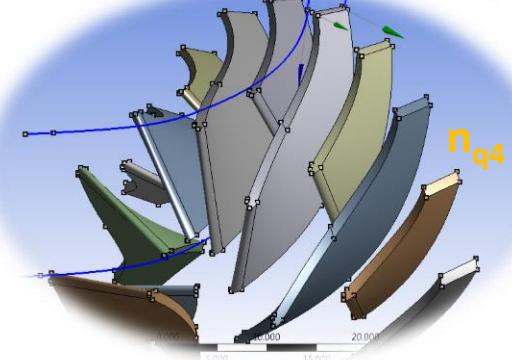
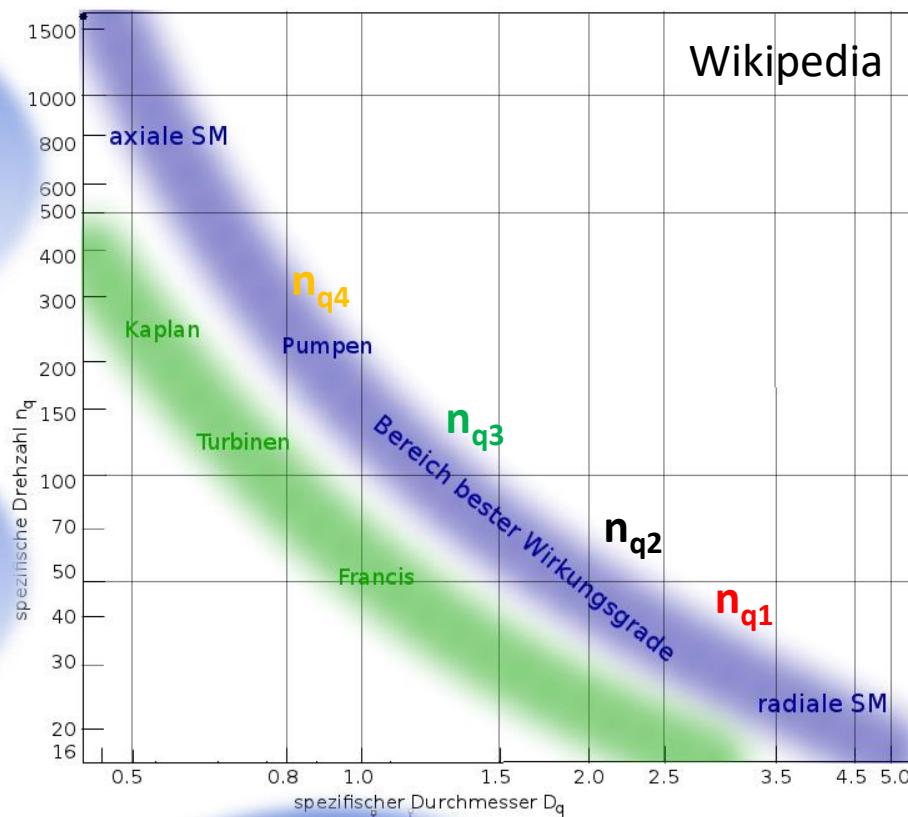
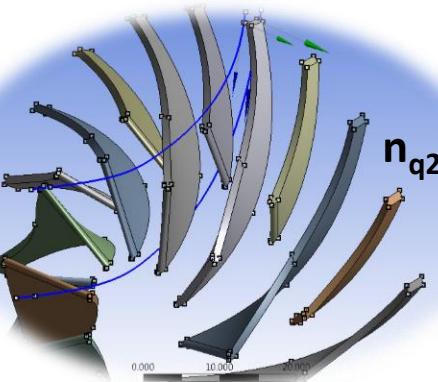
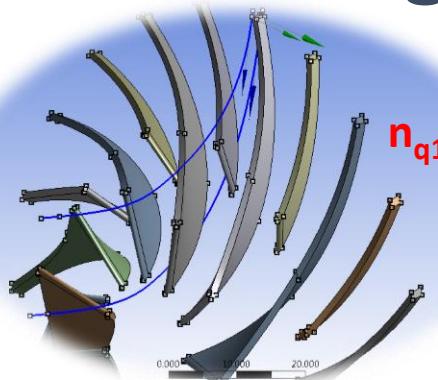


# Workflow

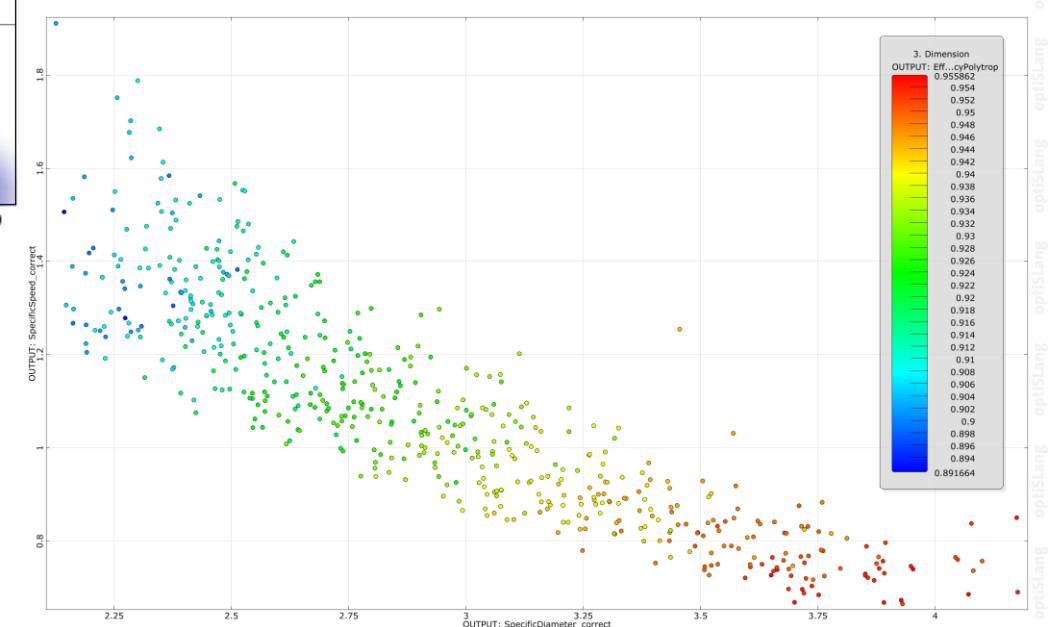
WB-Python Script to compute Main Dimensions wrt to Input Parameter



# Cordier Diagram

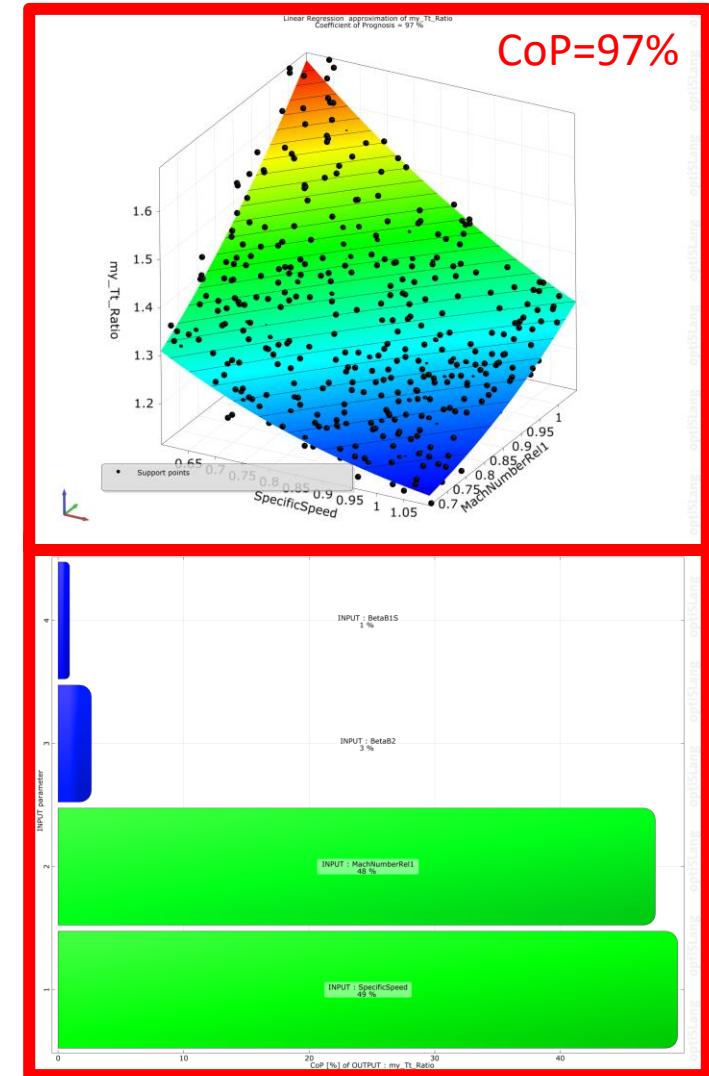
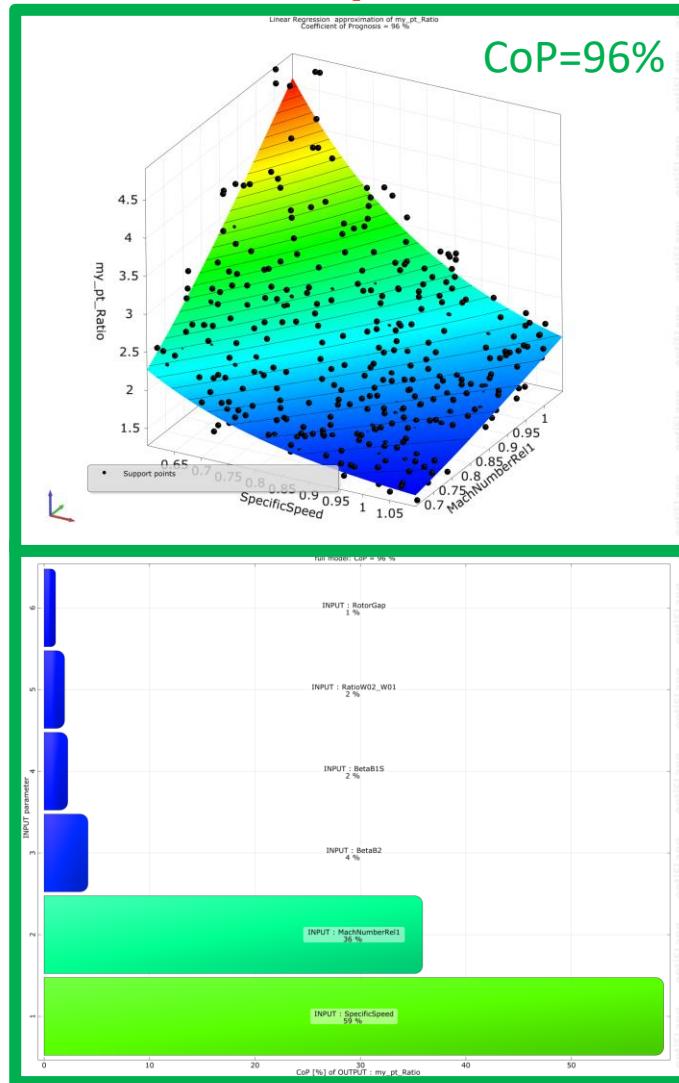
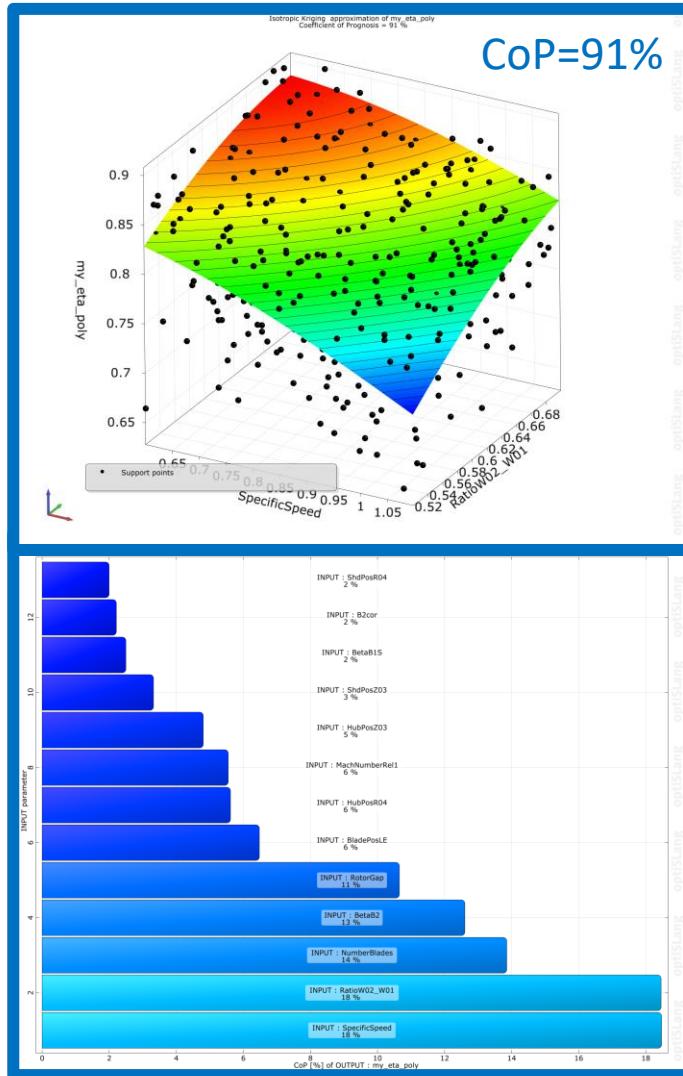


Parameterization respects Cordier Correlation  
Polytropic Efficiency in Meta-Model Sampling  
between 89.1% and 95.5%



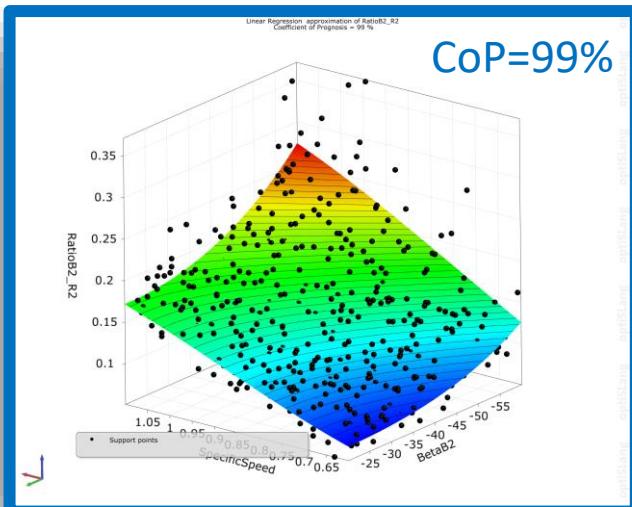
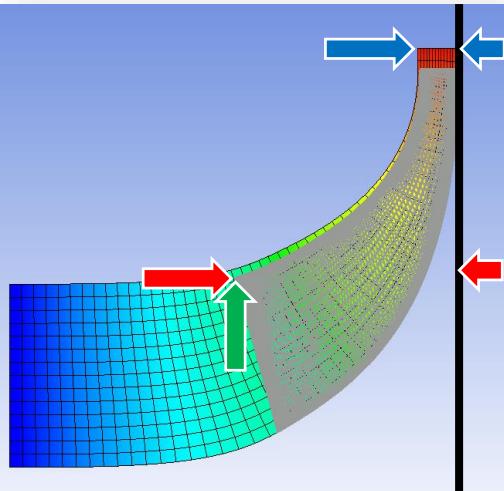
# Result – State Variables

$$\text{Polytropic Efficiency} = \frac{\kappa - 1}{\kappa} \frac{\ln(\text{Total Pressure Ratio})}{\ln(\text{Total Temperature Ratio})}$$

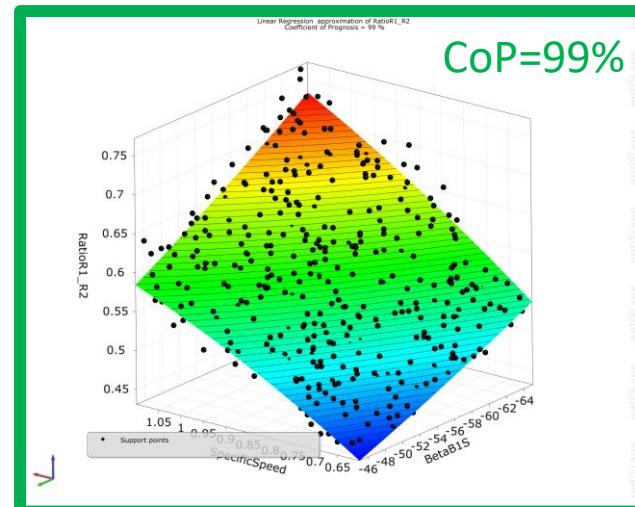


# Results – Design Correlations

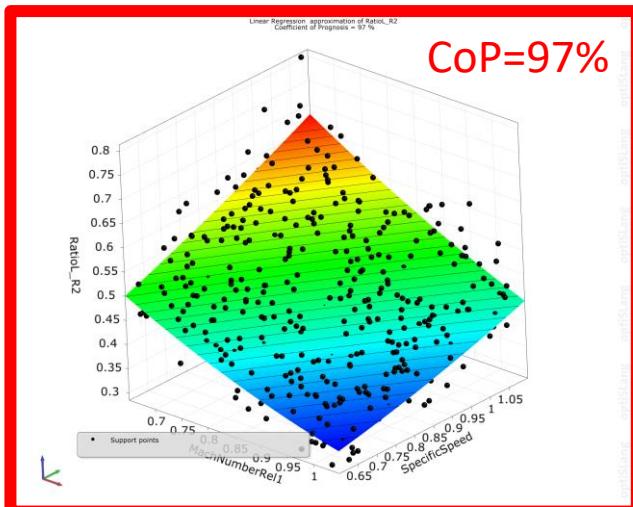
Channel Hight  $b_2/r_2$



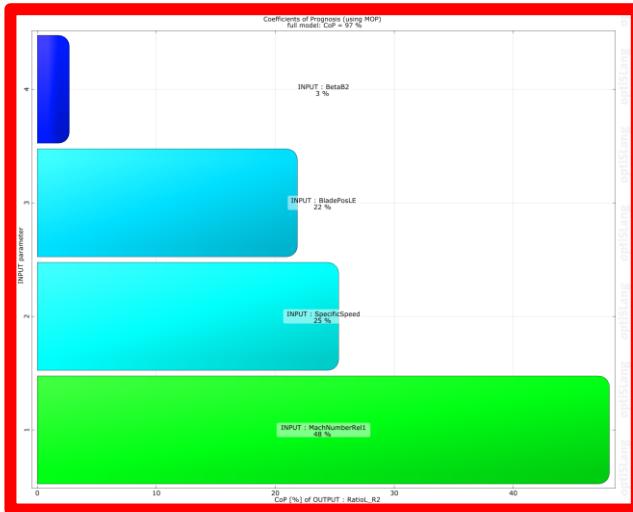
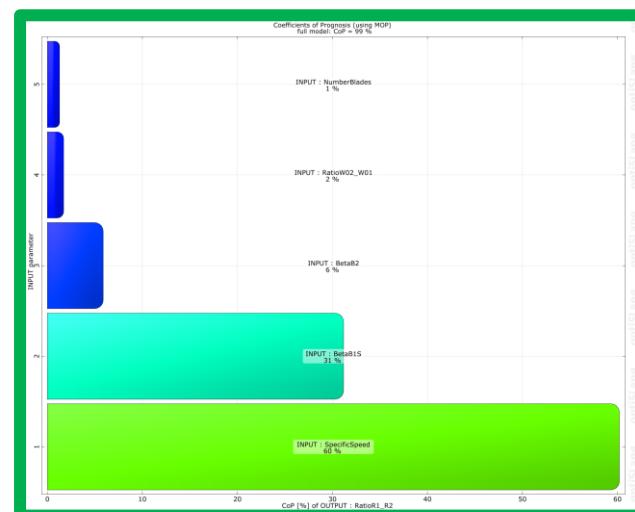
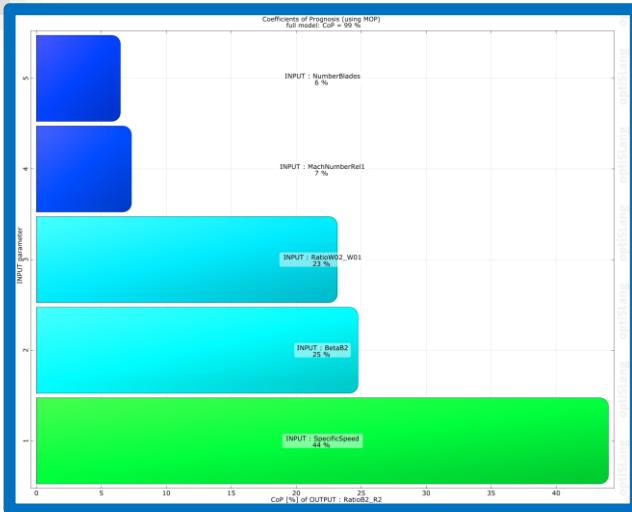
Inducer Radius  $r_1/r_2$



Blade Length  $L/r_2$

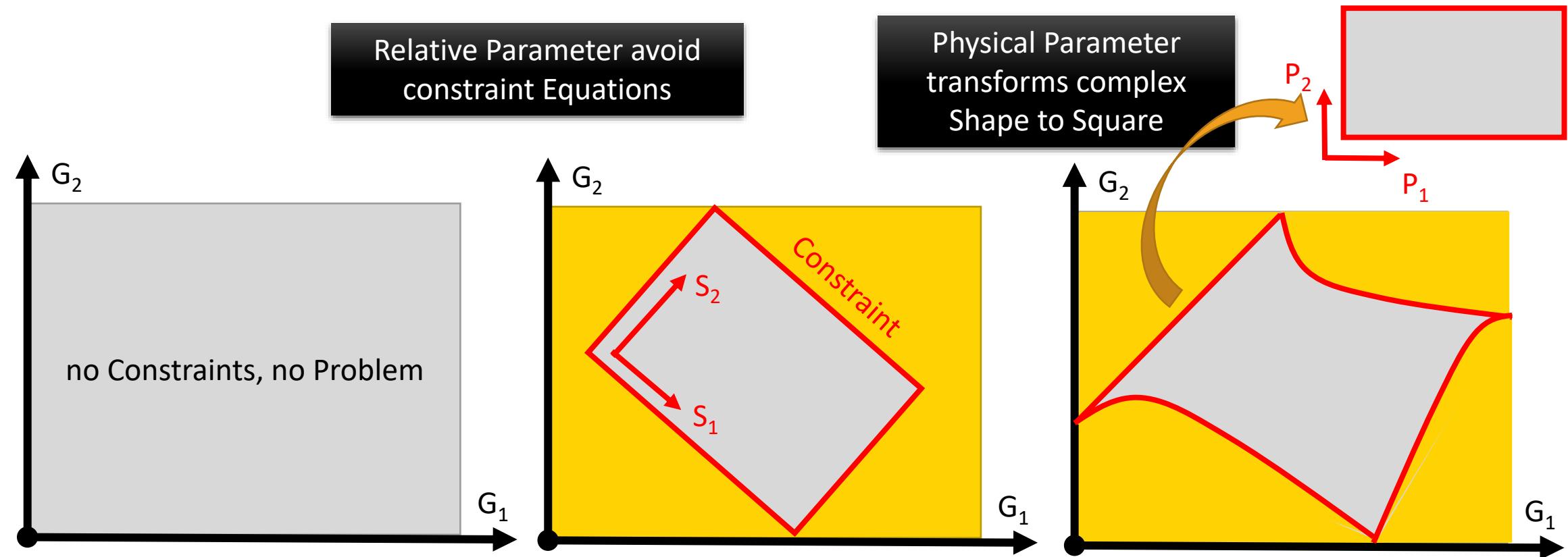


Usefull  
Information for  
further primary  
Design



# Summary – Parametrization

- Geometry Parameters  $G_i$  are very easy to define and vary
- Geometric and/or Physical Constraints might be needed → Use Relative Parameters  $S_i$
- Geometric and/or Physical Constraints can become complex → Use Relative Parameters  $P_i$



# Key to Successful Optimization

