Introduction to Stochastic Analysis using Robustness and Reliability Methods

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Development team Ansys optiSLang

WOST Workshop 2023

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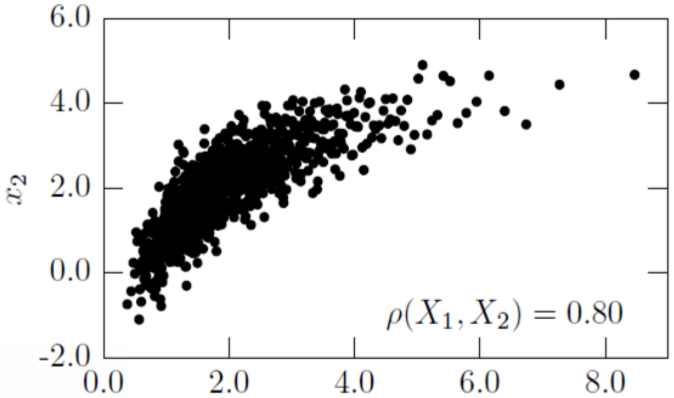
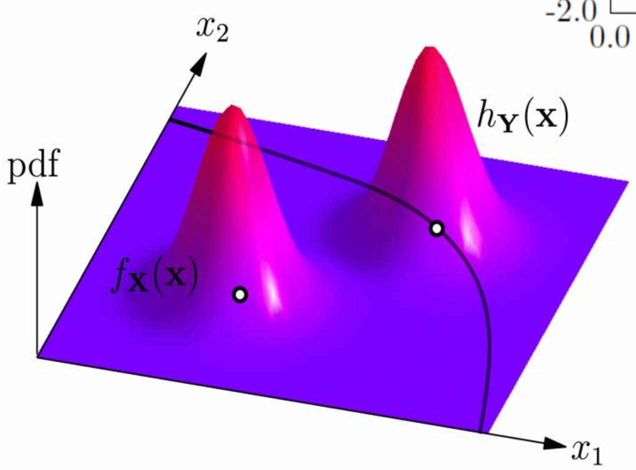
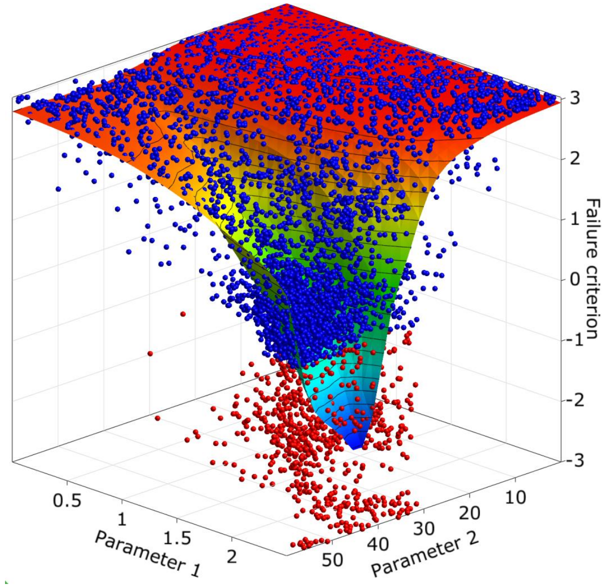
Outline



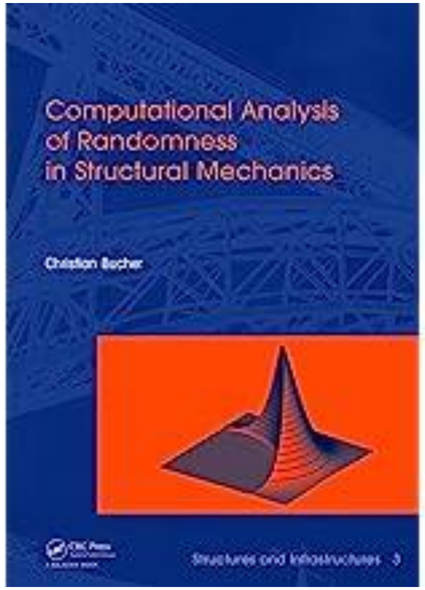




- **Definition of Uncertainties**
- **Robustness Evaluation**
- **Reliability Analysis**
- **Best practise**
- **Safety assessment of autonomous vehicles**

In Memory of Prof. Christian Bucher

- **1994 -2007** Professor for Structural Mechanics at the Bauhaus-University Weimar
- **2007-2023** Professor for Structural Mechanics at the TU Vienna
- **2001** Co-Founder of the Dynardo GmbH
- **2007** Co-Founder of the Dynardo Austria GmbH



- More than 300 academic publications in structural mechanics, dynamics, reliability, optimization and system identification
- Key algorithmic development in structural reliability and Robust design optimization

Definition of Uncertainties

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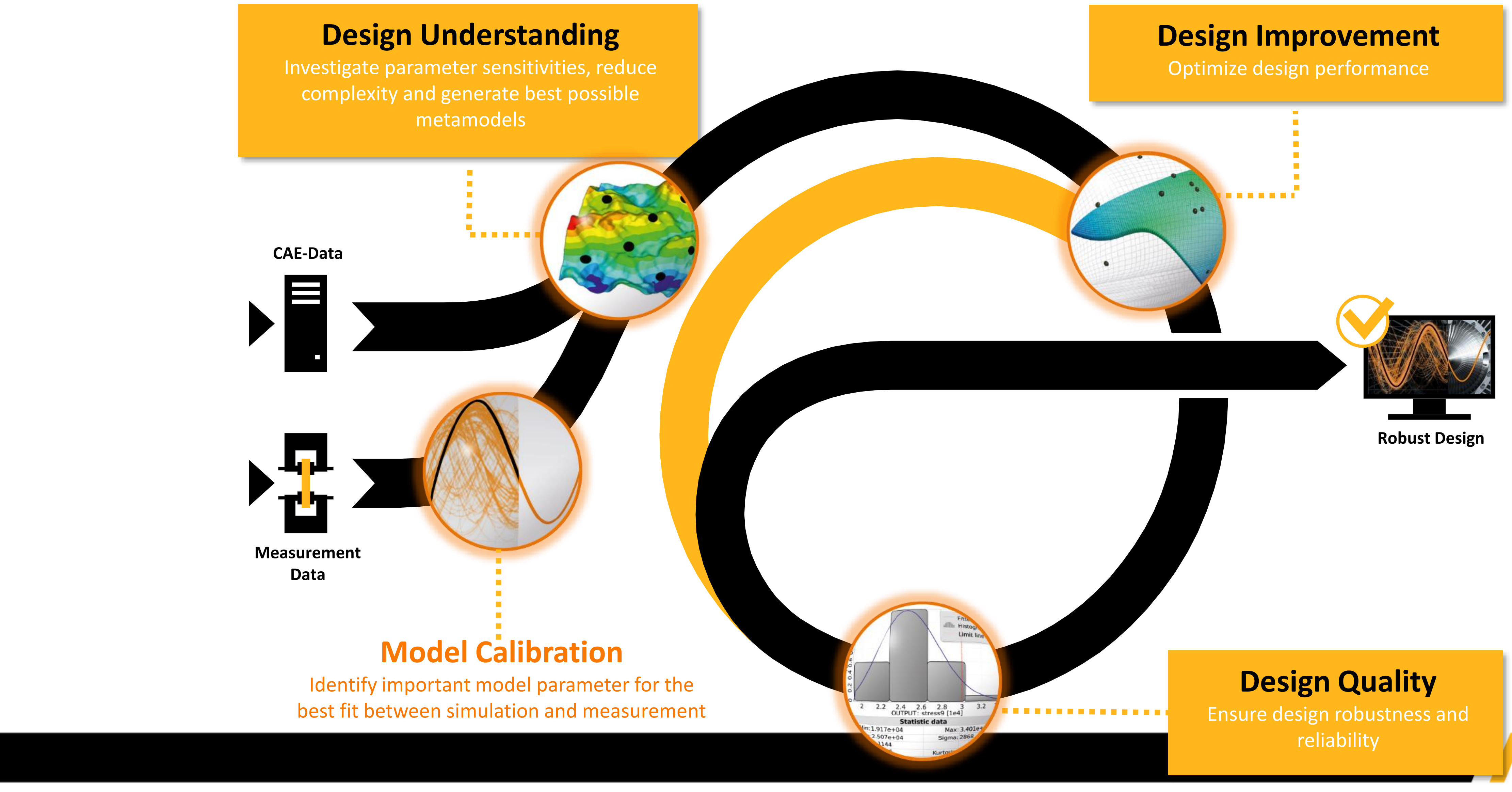
Robust Design Optimization

Best practice guideline for virtual product development



How to Define the Robustness of a Design?

- **Intuitively:** The performance of a robust design is largely unaffected by random perturbations

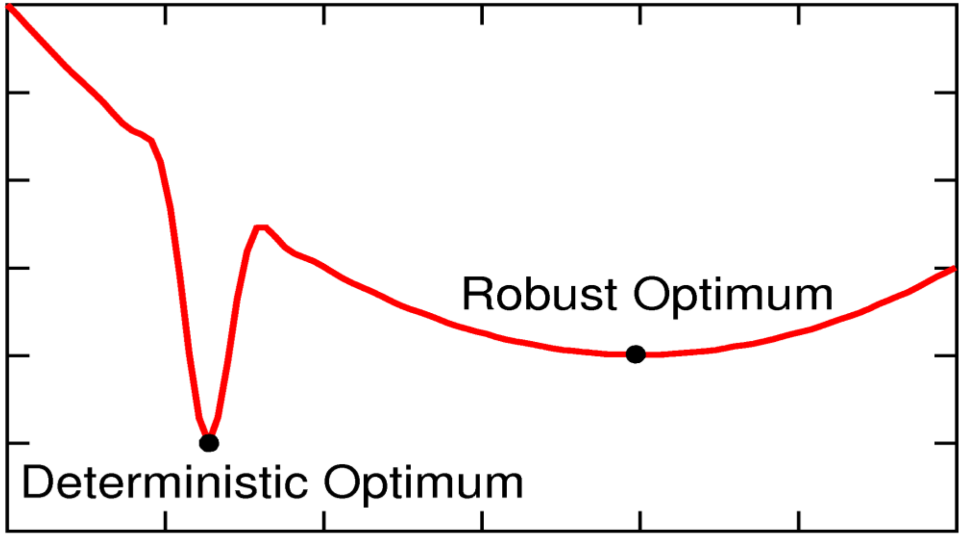
- **Variance indicator:** The coefficient of variation (CoV) of the objective function and/or constraint values is not greater than the CoV of the input variables

- **Sigma level:** Keep an undesired performance outside an interval of mean +/- sigma level (e.g. design for six-sigma)

- **Probability indicator (Reliability analysis):** The probability of reaching undesired performance is smaller than an acceptable value

How to Define the Robustness of a Design?

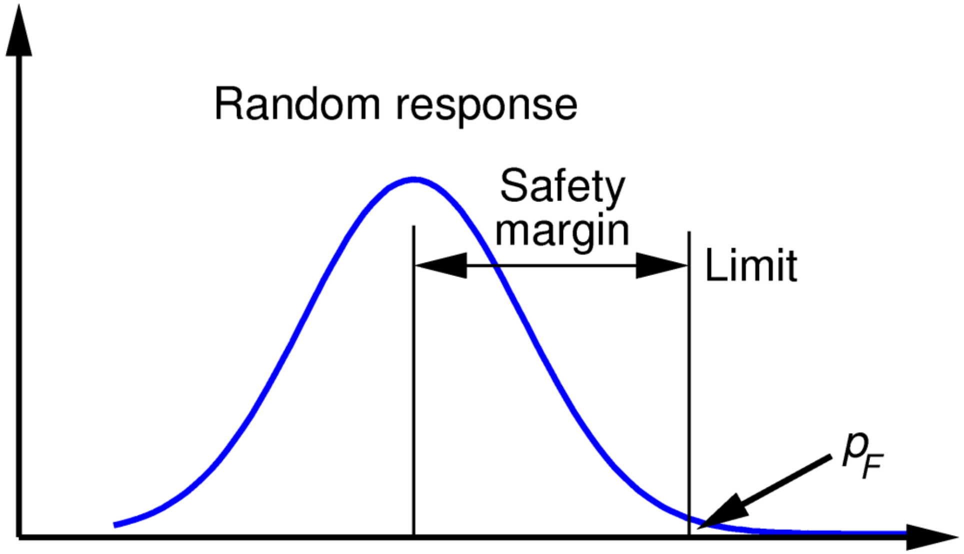
Robustness in terms of stability



- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Robustness in terms of requirements

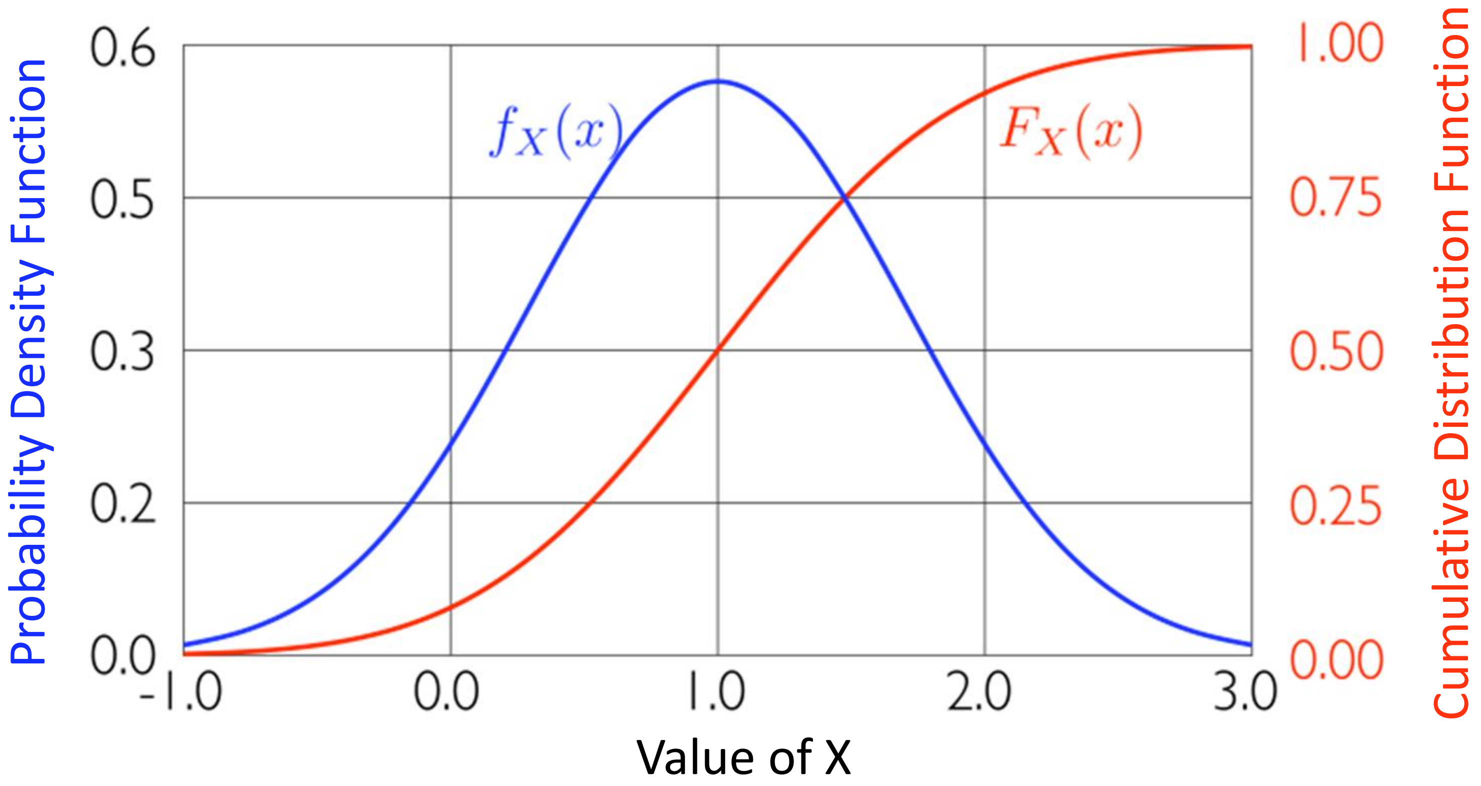


- Safety margin (sigma level) of one or more responses y:

$$(y_{limit} - \mu_Y)/\sigma_Y \geq a$$

- Reliability (failure probability) with respect to given limit state: $p_F \leq p_F^{target}$

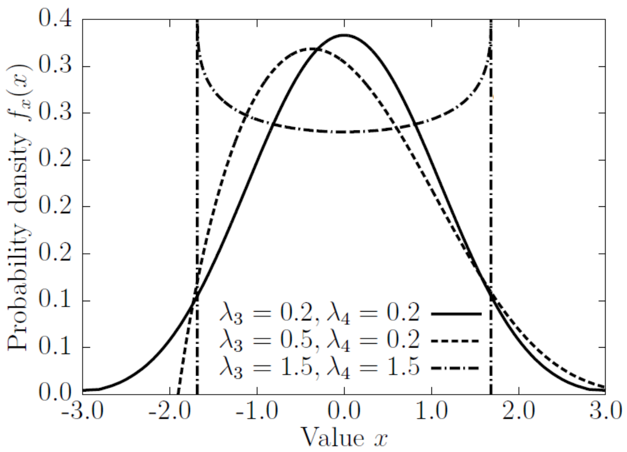
Probability Distribution and Density Function



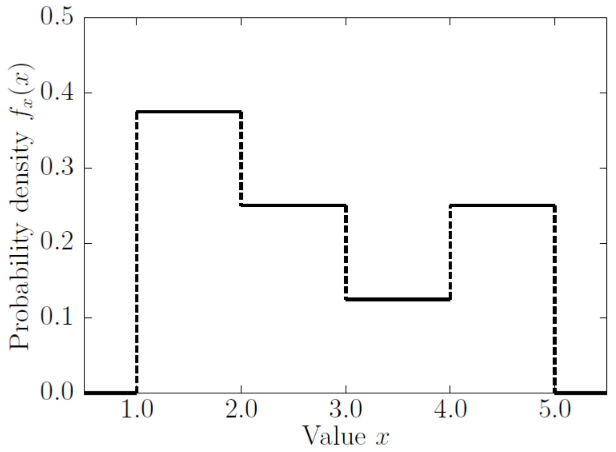
$$F_X(x) = P[x \leq X] \quad f_X(x) = \frac{d}{dx} F_X(x) \quad \int_{-\infty}^{\infty} f_X(x)dx = 1$$

Distribution Types

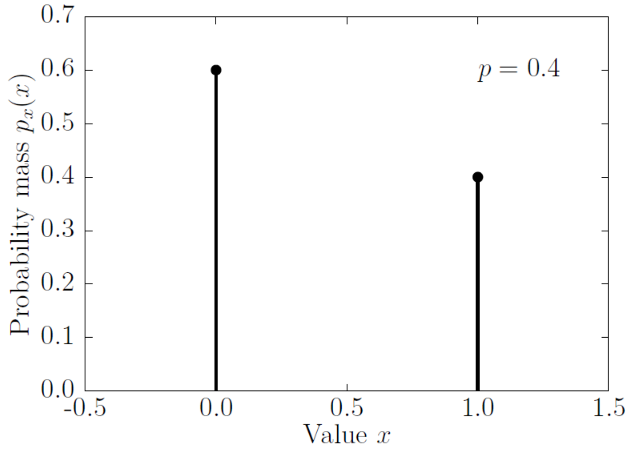
Generalized Lambda



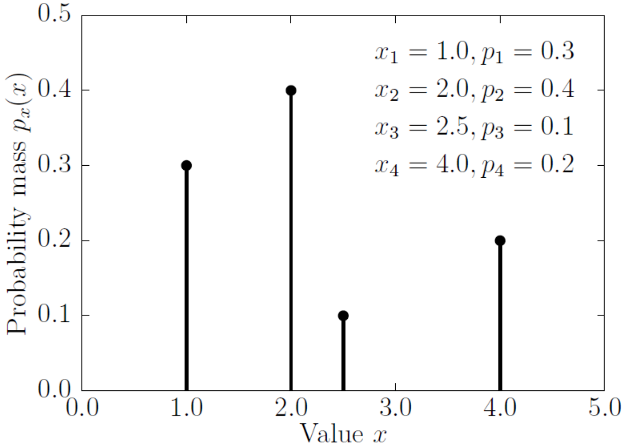
Multi-Uniform

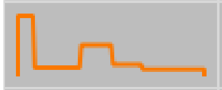
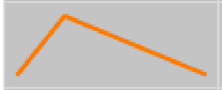
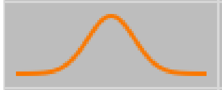
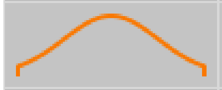
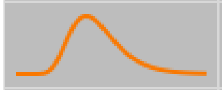
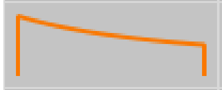
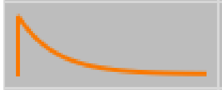
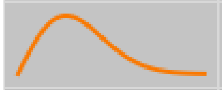
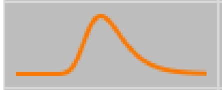
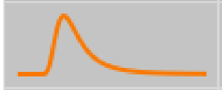
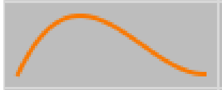
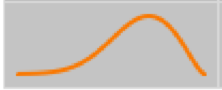


Bernoulli

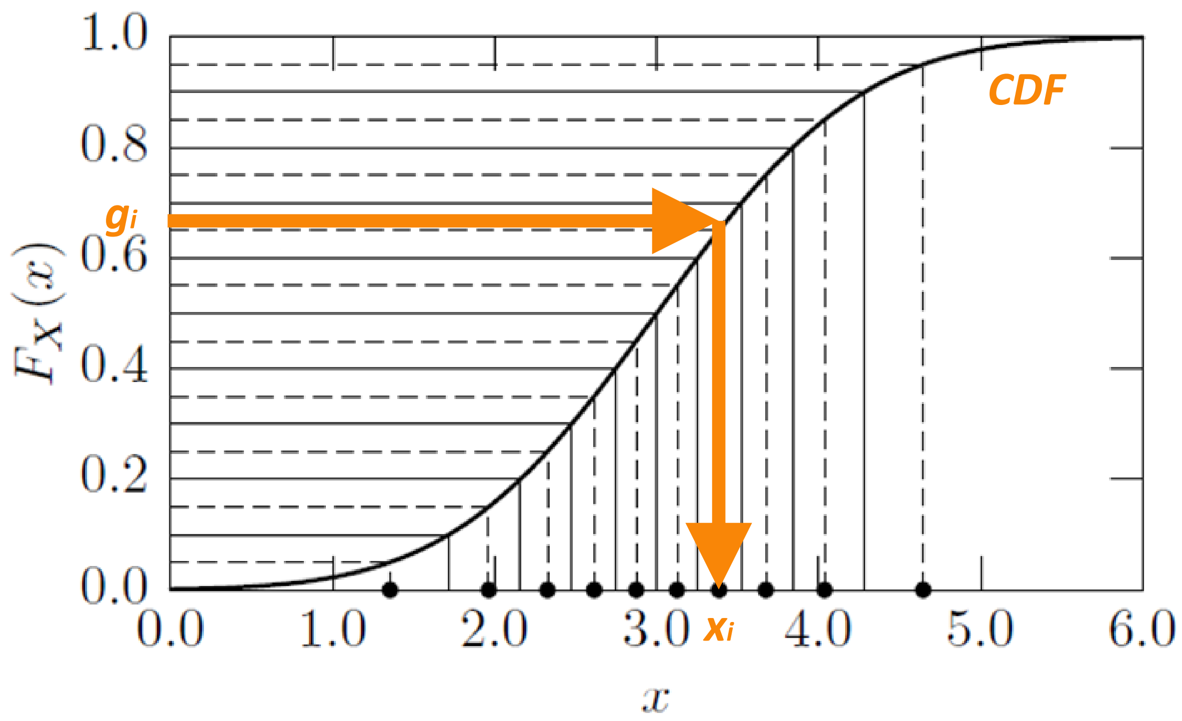


General discrete



PDF	Type
	UNIFORM
	MULTIUNIFORM
	TRIANGULAR
	NORMAL
	TRUNCATEDNORMAL
	LOGNORMAL
	LOGUNIFORM
	EXPONENTIAL
	WEIBULL
	GUMBEL
	FRECHET
	BETA
	LAMBDA

Transform Uniform to Target Distribution



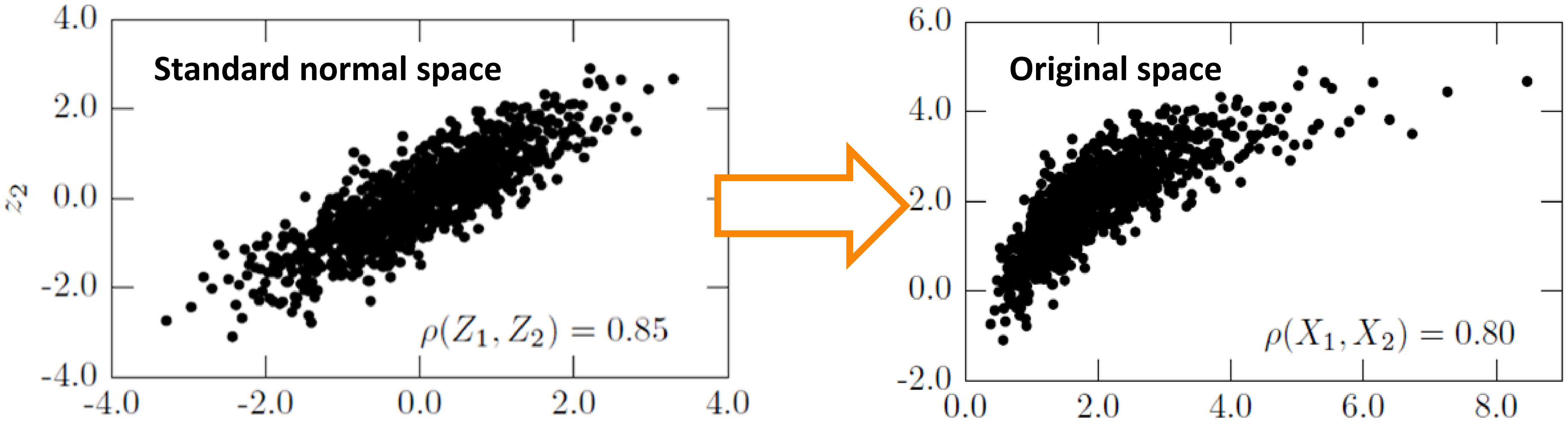
$$x_i = F_{X_i}^{-1}(g_i)$$

- Generation of uniformly distributed samples g_i between 0 and 1
- Samples of uncorrelated random numbers can be generated using the inverse cumulative distribution function
- CDF and its inverse should be available as closed formula

Modeling of Input Correlations by the Nataf model

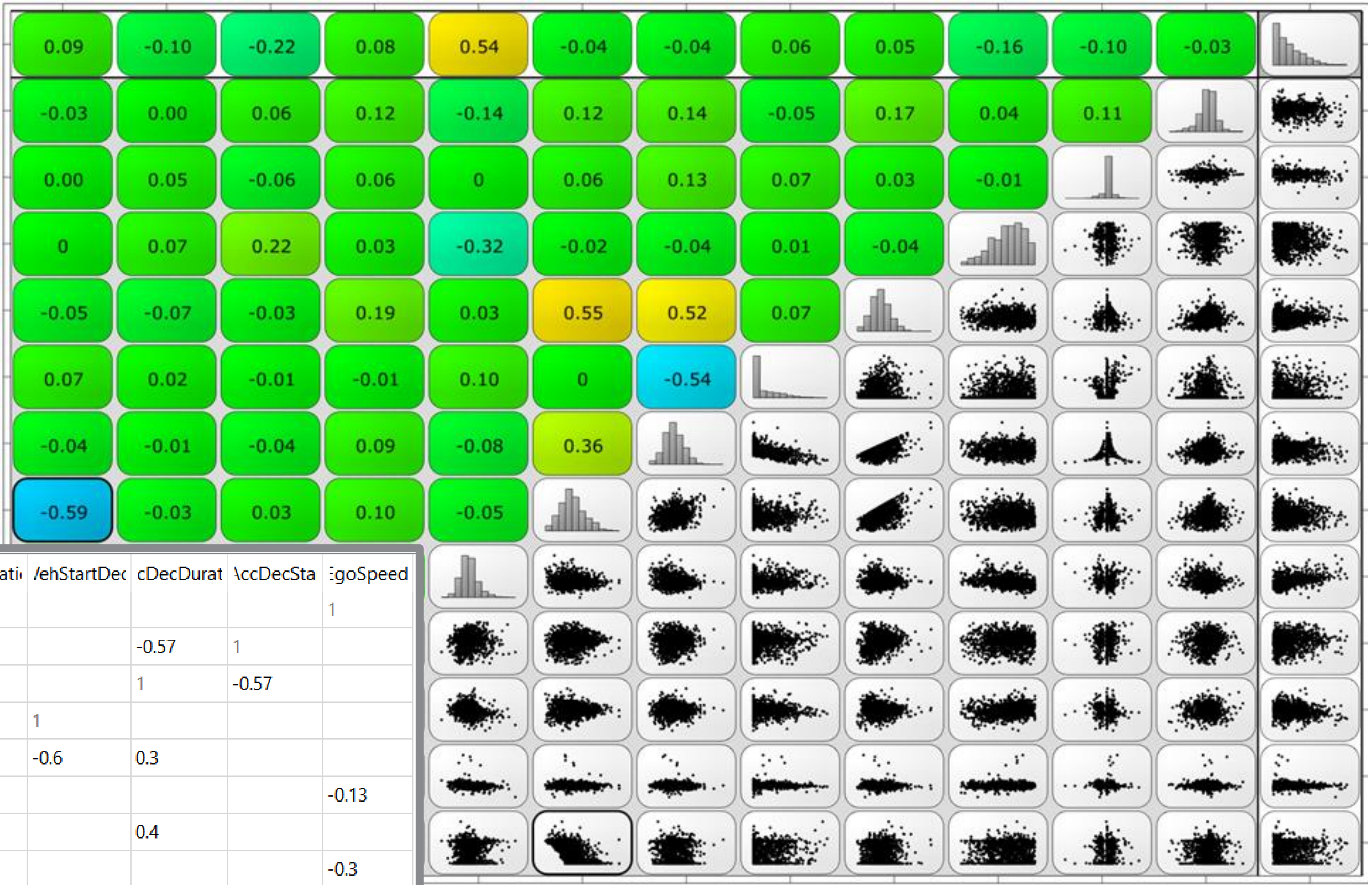
- Samples are generated according to a multi-dimensional standard normal distribution
- For each random variable the original marginal distribution is obtained by using the inverse distribution function
- Required linear correlation coefficients in standard normal space are iteratively obtained from correlations in original space

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^k|\mathbf{C}_{\mathbf{XX}}|}} \exp\left[-\frac{1}{2}(\mathbf{x}-\bar{\mathbf{X}})^T\mathbf{C}_{\mathbf{XX}}^{-1}(\mathbf{x}-\bar{\mathbf{X}})\right] \qquad x_{i,j} = F_{X_i}^{-1}[\Phi(z_{i,j})]$$



Definition of Input Correlations in optiSLang

- Definition of pairwise linear input correlations in original distribution space
- Small correlation coefficients between -0.2 and 0.2 observed in data should be neglected



	taVehicleSp	itInDuratic	cutInOffset	hDecDurati	/ehStartDec	cDecDurat	\ccDecSta	EgoSpeed
firstEgoSpeedS_t0	-0.3		-0.13					1
firstEgoAccDecStartTime_1						-0.57	1	
firstEgoAccDecDuration_1_orig		0.4		0.3		1	-0.57	
cutInVehStartDecTime				-0.6	1			
cutInVehDecDuration_orig		0.49		1	-0.6	0.3		
cutInOffset	0.25		1					-0.13
cutInDuration		1		0.49		0.4		
cutInDeltaVehicleSpeedX_t0	1		0.25					-0.3

Robustness Analysis

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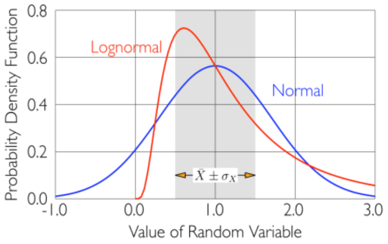
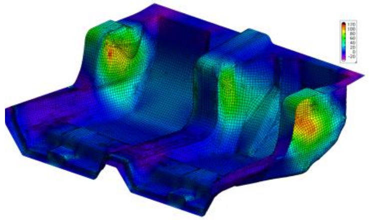
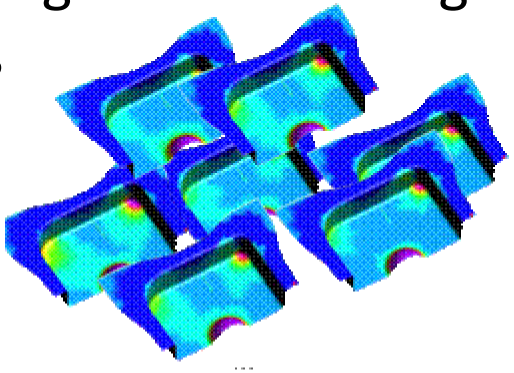
Variance based Robustness Analysis

1) Define the robustness space using scatter range, distribution and correlation

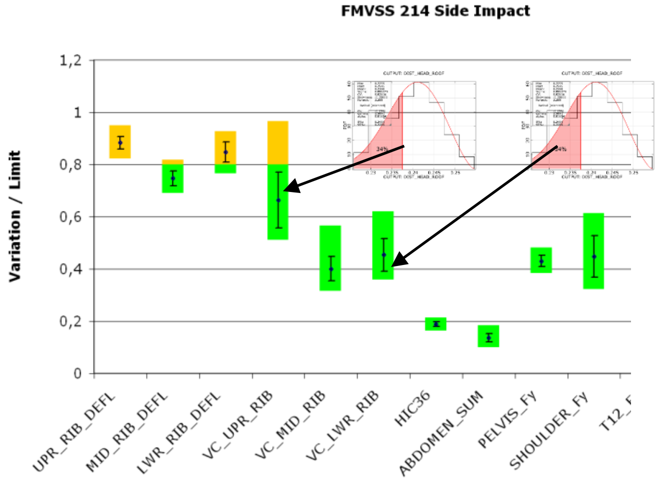




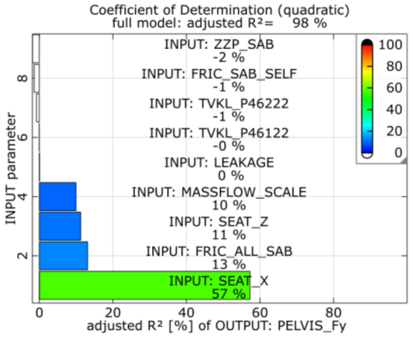
2) Scan the robustness space by producing and evaluating *n* designs



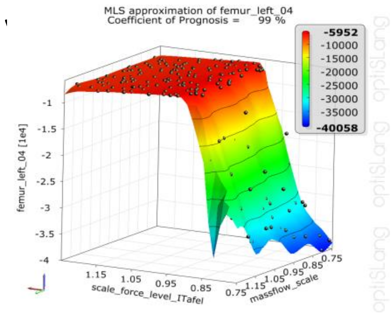
3) Check the variation



4) Check the explainability of the model

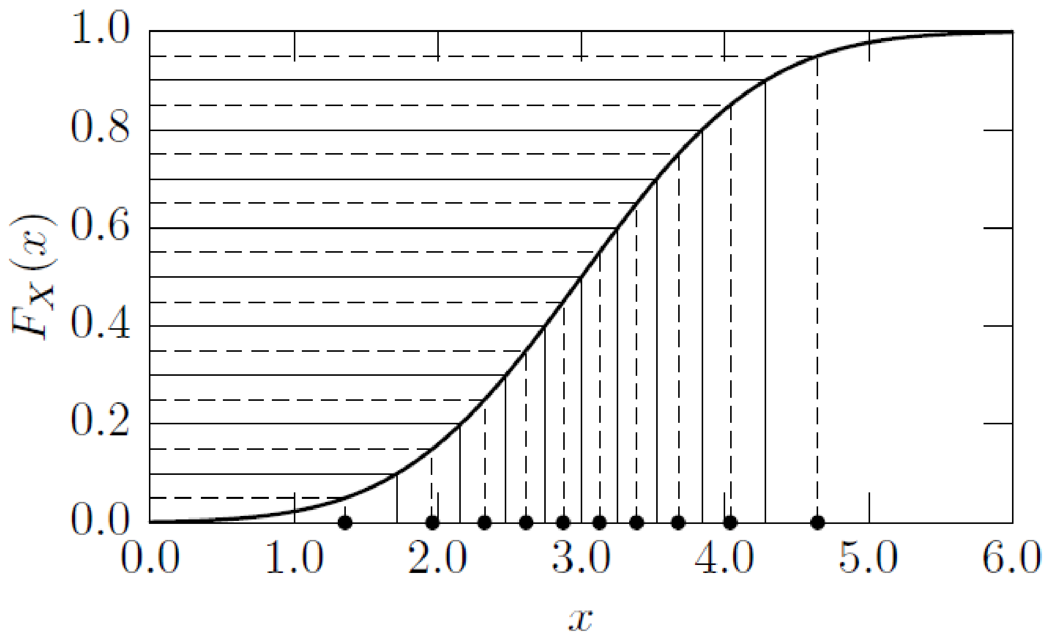


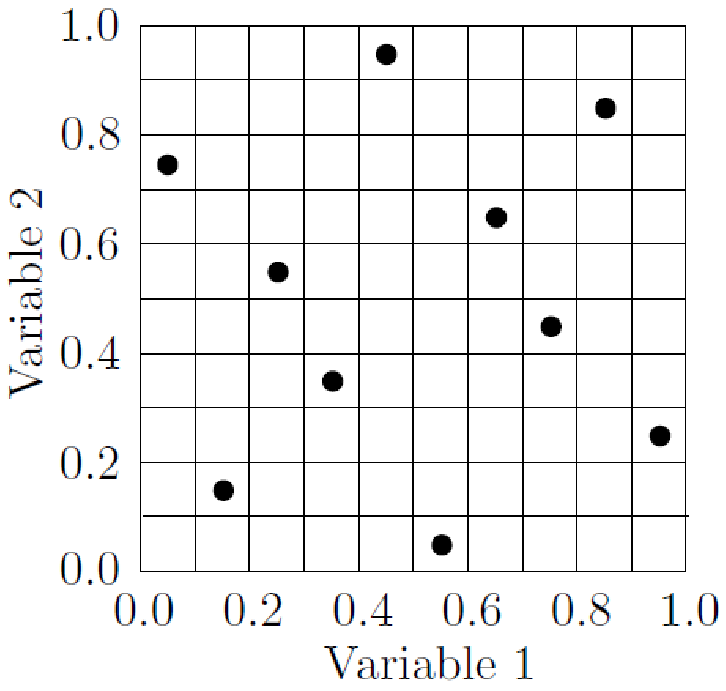
5) Identify the most important scattering



Advanced Latin Hypercube Sampling

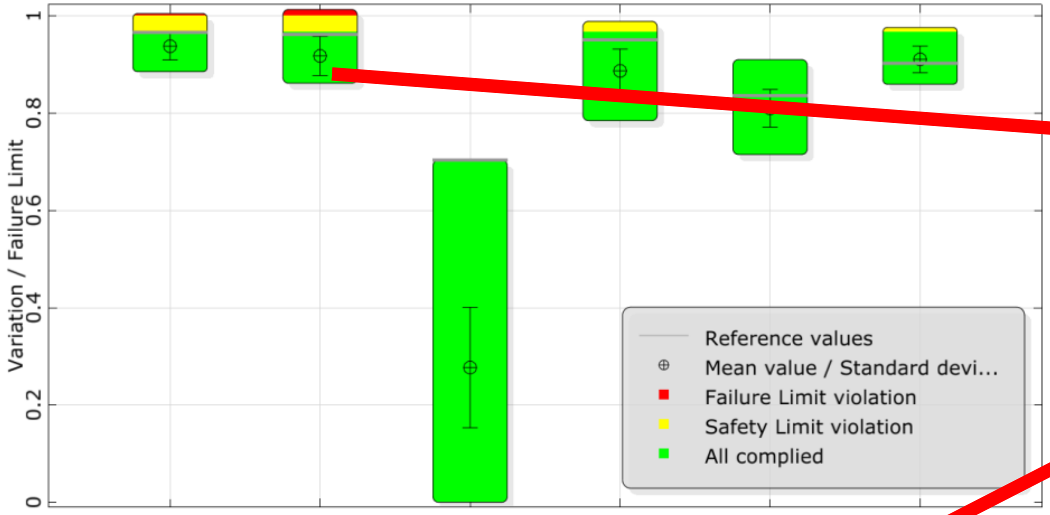
- Very efficient Monte Carlo Simulation
- Distribution function is subdivided into N classes of equal probability
- Reduced number of required samples for statistical estimates
- Reduced unwanted input correlations

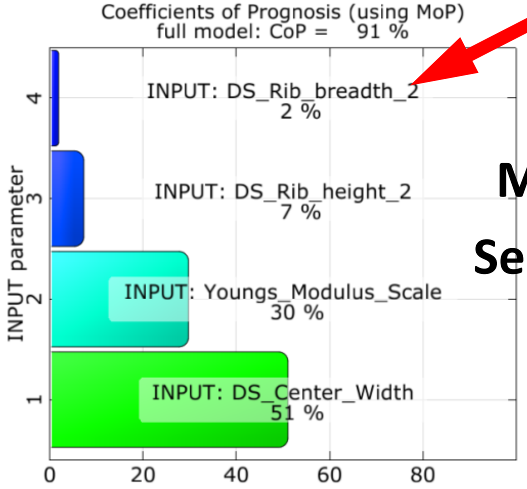




Robustness Postprocessing

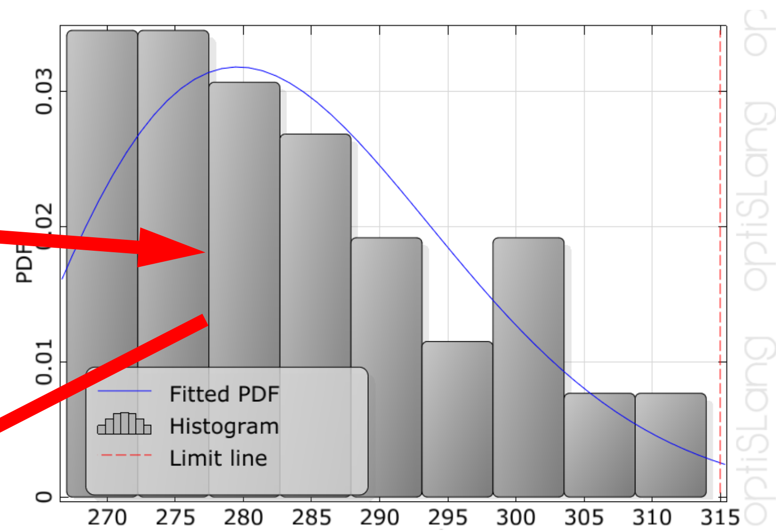
Traffic light plot





MOP/CoP Sensitivities

Histogram & Statistical Data



Statistic data			
Min:	267	Max:	314
Mean:	284.4	Sigma:	12.49
CV:	0.04393		
Skewness:	0.5633	Kurtosis:	2.338
Fitted PDF: Rayleigh			
Mean:	284.4	Sigma:	12.49
Limit x = 315			
P_rel:	1	1 - P_rel:	0
P_fit:	0.983168	1 - P_fit:	0.016832
Sigma-Level:	2.4496		

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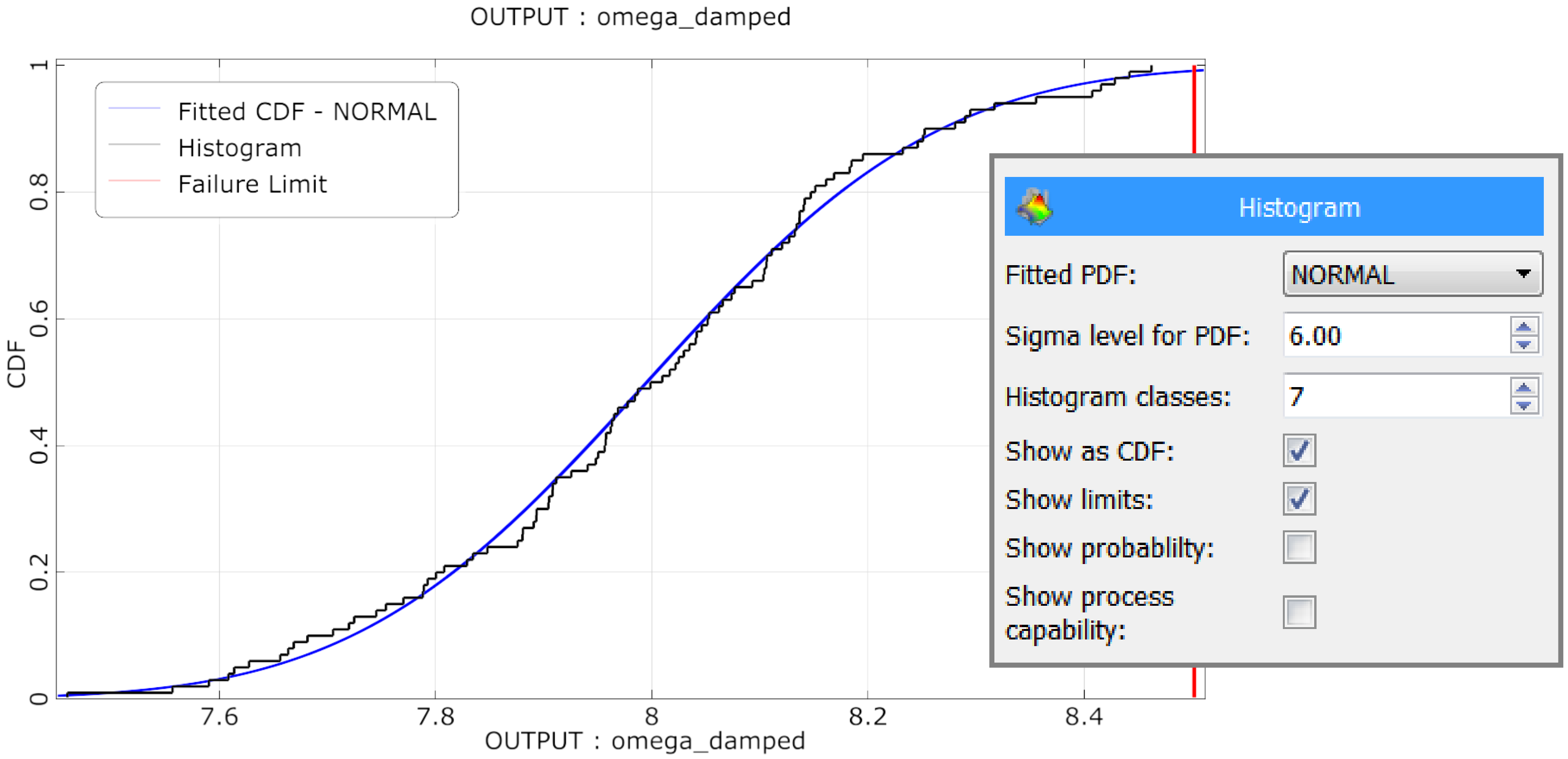
Distribution Fit

- Automatic fit compares deviation of empirical (sample) distribution function with analytical CDF of candidate distribution types

→ Recommended distribution type has minimum sum of squared errors

- Single distribution type is fitted via moments to data points

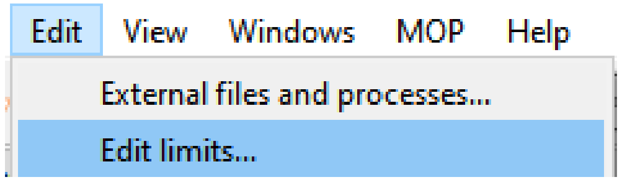


Limits

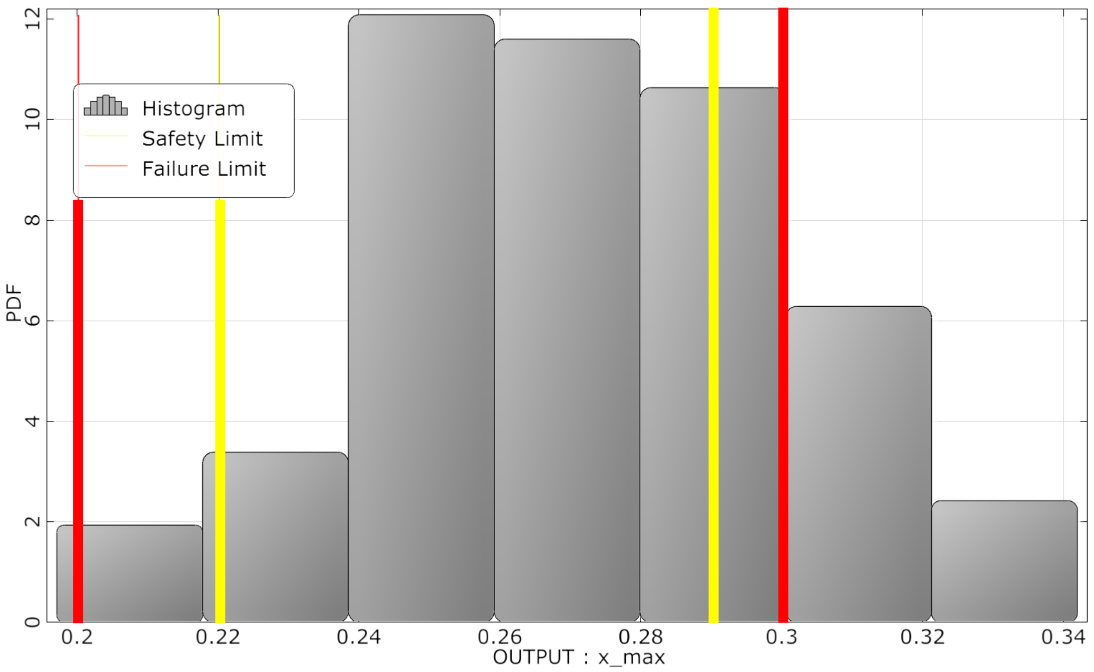
- Define lower and/or upper safety and/or failure limits

→ Limits are indicated in the histogram, box-whisker and traffic light plots

→ Probabilities of violating the limits are calculated



	Dimension	Safety Limit		Failure Limit		
1	omega_damped		8.3	Lower	8.5	Target
2	x_max	0.22	0.29	0.2	0.3	



Limit : Safety Limit			
	Lower value = 0.22	Upper value = 0.29	Total
P_rel:	0.05	0.32	0.37
Sigma-Level:	1.74967	0.616846	
Limit : Failure Limit			
	Lower value = 0.2	Upper value = 0.3	Total
P_rel:	0.01	0.18	0.19
Sigma-Level:	2.42582	0.954919	

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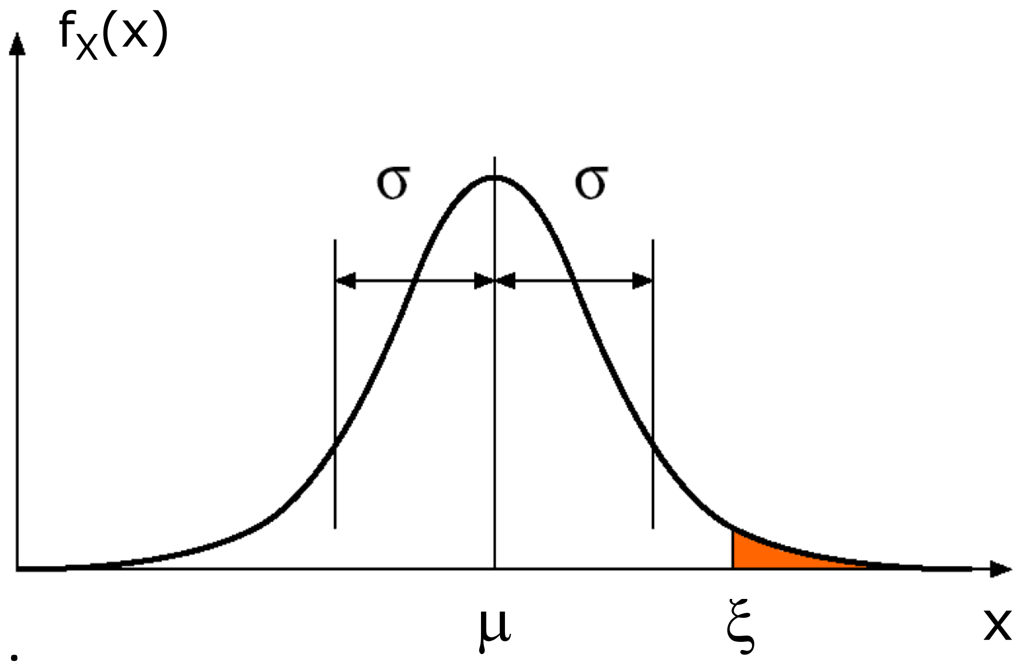
Exceedance Probability

- Probability of reaching values above a limit



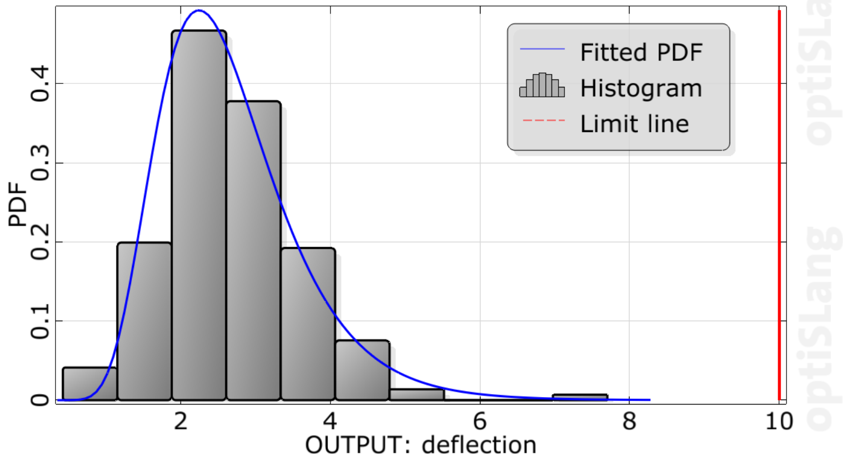
- For Gaussian distribution:

$$P_\xi = P[X \geq \xi]$$

ξ	μ	$\mu + \sigma$	$\mu + 2\sigma$	$\mu + 3\sigma$	$\mu + 4\sigma$	$\mu + 5\sigma$
P_ξ	$5.0 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-2}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-5}$	$2.9 \cdot 10^{-7}$

Variance-based Robustness Analysis

- Sufficient estimates of **mean** and **variance** with 50 to 100 samples
- Distribution fit and extrapolation of **small event probabilities** may be very inaccurate
- More precise **reliability methods** should be applied to verify small probabilities



Statistic data			
Min:	0.4254	Max:	7.704
Mean:	2.67	Sigma:	0.9357
CV:	0.3505		
Skewness:	1.017	Kurtosis:	6.465
Fitted PDF: Log-Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	0.999974	1 - P_fit:	2.56303e-005
Sigma-Level:	7.83397		

Fitted PDF: Normal			
Mean:	2.67	Sigma:	0.9357
Limit x = 10			
P_rel:	1	1 - P_rel:	0
P_fit:	1	1 - P_fit:	2.33147e-015
Sigma-Level:	7.83397		

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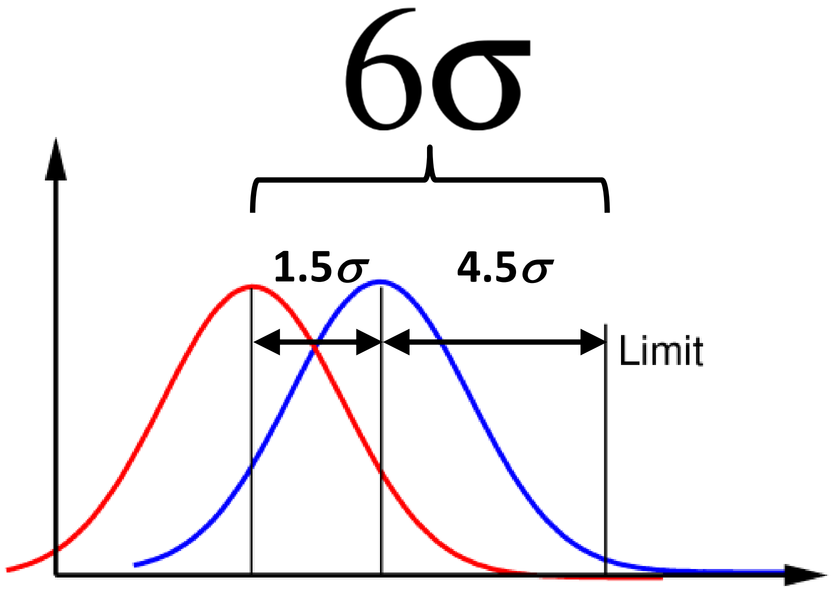
Six Sigma and Design for Six Sigma (DFSS)

- Methodology for **quality management** and **process improvement**
- Most common approaches:
  DMAIC - Define – Measure – Analyze – Improve – Control (existing process)
  DMADV – Define – Measure – Analyze – Design – Verify (new process)

- Six Sigma requires a failure level smaller than **3.4 defects per million** opportunities (DPMO)

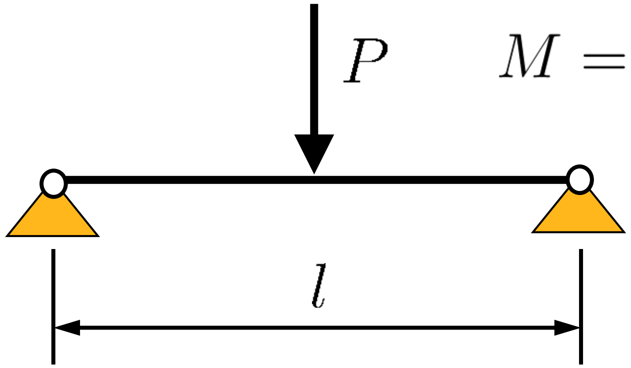
- Assuming a normal distribution a **4.5 sigma safety margin** is required
- An additional empirically based **1.5 sigma shift** was introduced of the mean value

- **Variance-based robustness analysis** is a suitable tool within a Six Sigma quality management process



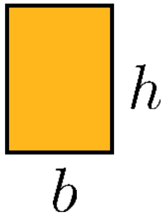
Example: Three-point Bending Beam

System



$$M = \frac{P \cdot l}{4}$$

Cross section



$$I = \frac{b \cdot h^3}{12}$$

Maximum stress

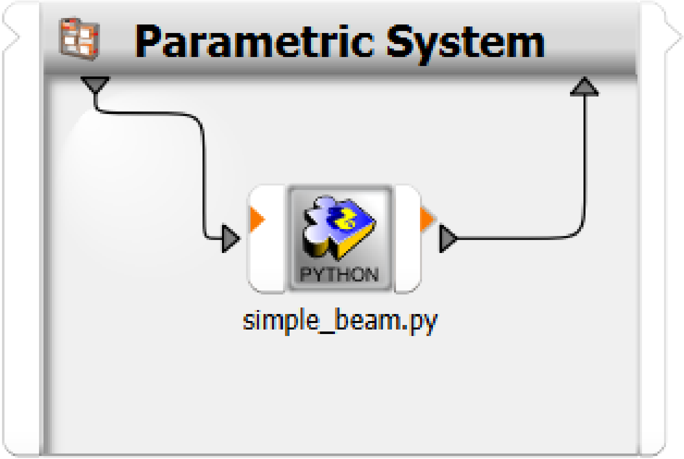
$$\sigma = \frac{M \cdot h}{2 \cdot I}$$

Deflection

$$w = \frac{P \cdot l^3}{48 \cdot E \cdot I}$$

	Distribution	Mean value	Standard deviation
Load P	Normal	700 N	180 N
Length l	Normal	2000 mm	20 mm
Width b	Normal	50 mm	2 mm
Heigth h	Normal	100 mm	2 mm
Young‘s modulus E	Lognormal	11000 N/mm²	2230 N/mm²
Failure stress σ_{fail}	Lognormal	22.0 N/mm²	4.4 N/mm²

Example: Three-point Bending Beam



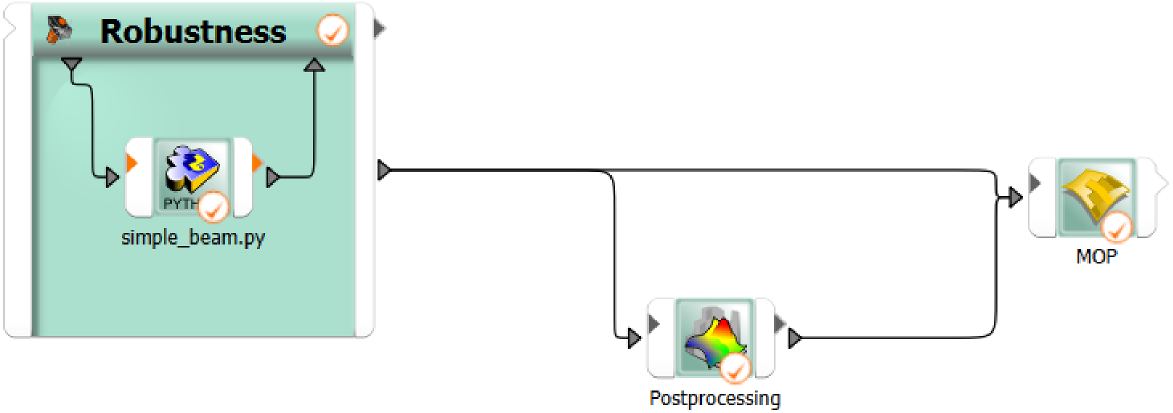
	Name	Parameter type	Reference value	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	Emod	Stochastic	11000		LOGNORMAL	11000	2230	20.2727 %	9.28551; 0.200689
2	height	Stochastic	100		NORMAL	100	2	2 %	100; 2
3	length	Stochastic	2000		NORMAL	2000	20	1 %	2000; 20
4	load	Stochastic	700		NORMAL	700	180	25.7143 %	700; 180
5	stress_fail	Stochastic	22		LOGNORMAL	22	4.4	20 %	3.07143; 0.198042
6	width	Stochastic	50		NORMAL	50	2	4 %	50; 2

Example: Three-point Bending Beam

Limits which should be considered in the safety assessment:

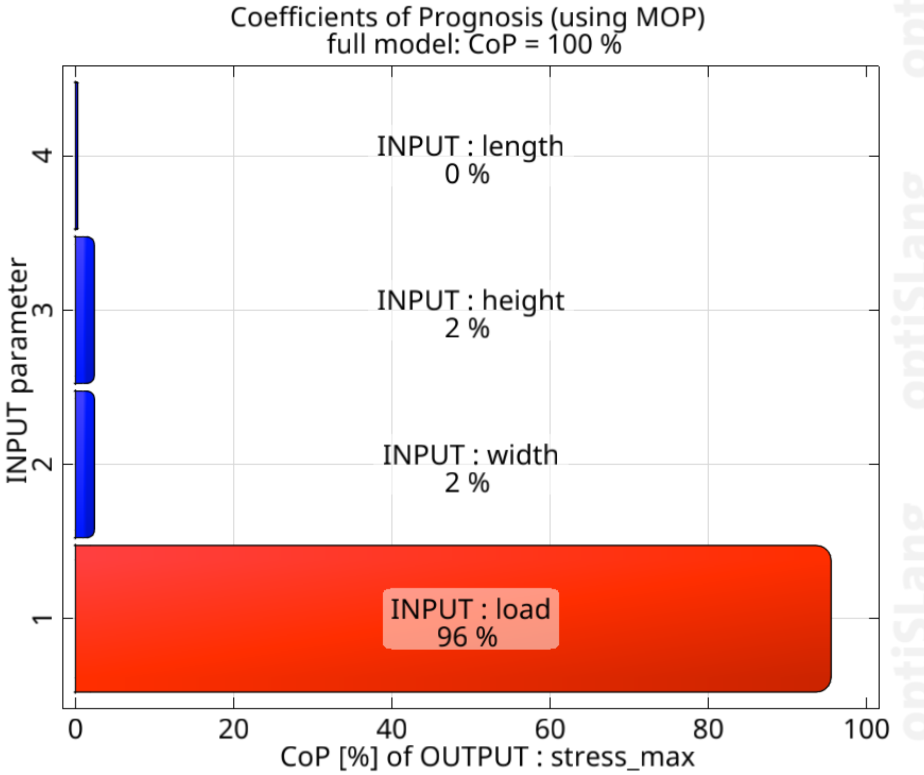
- Maximum stress should be smaller or equal than failure stress:
  load_ratio = maximum_stress/failure_stress ≤ 1
  safety_margin = failure_stress - maximum_stress ≥ 0
- Maximum deflection should not exceed l/200=10mm

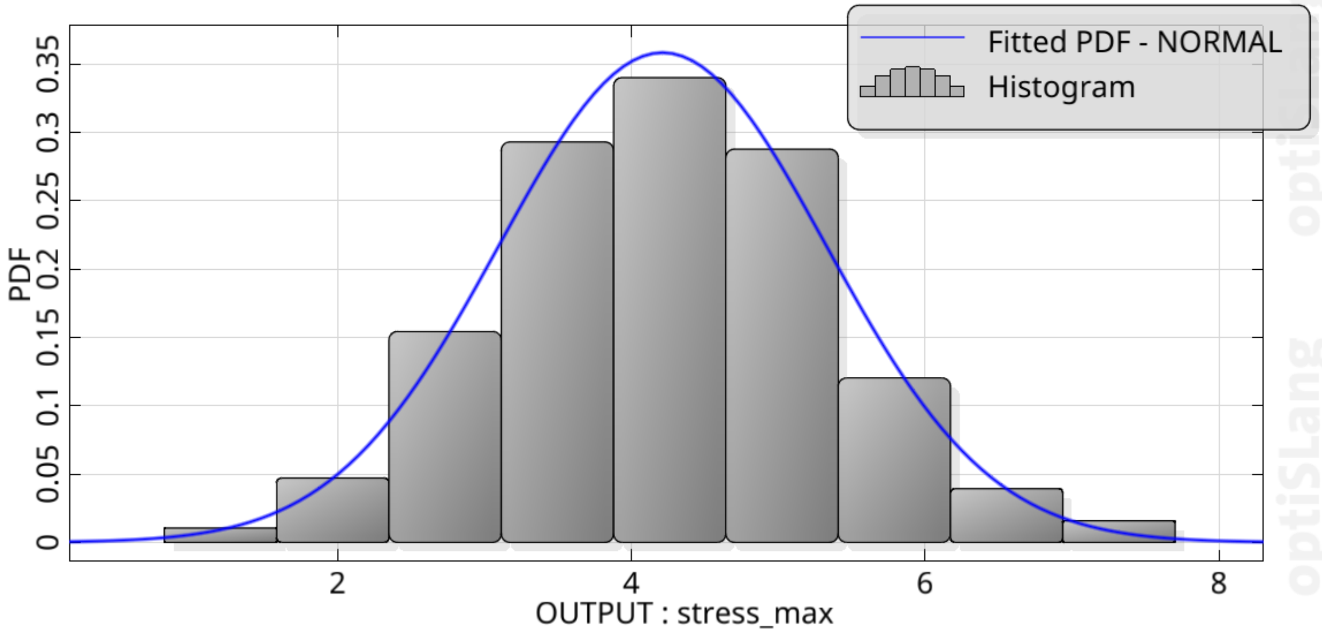
- 500 Latin Hypercube samples are evaluated in the Robustness workflow



Example: Three-point Bending Beam

- Maximum stress is approximately normally distributed
- Load is most important input

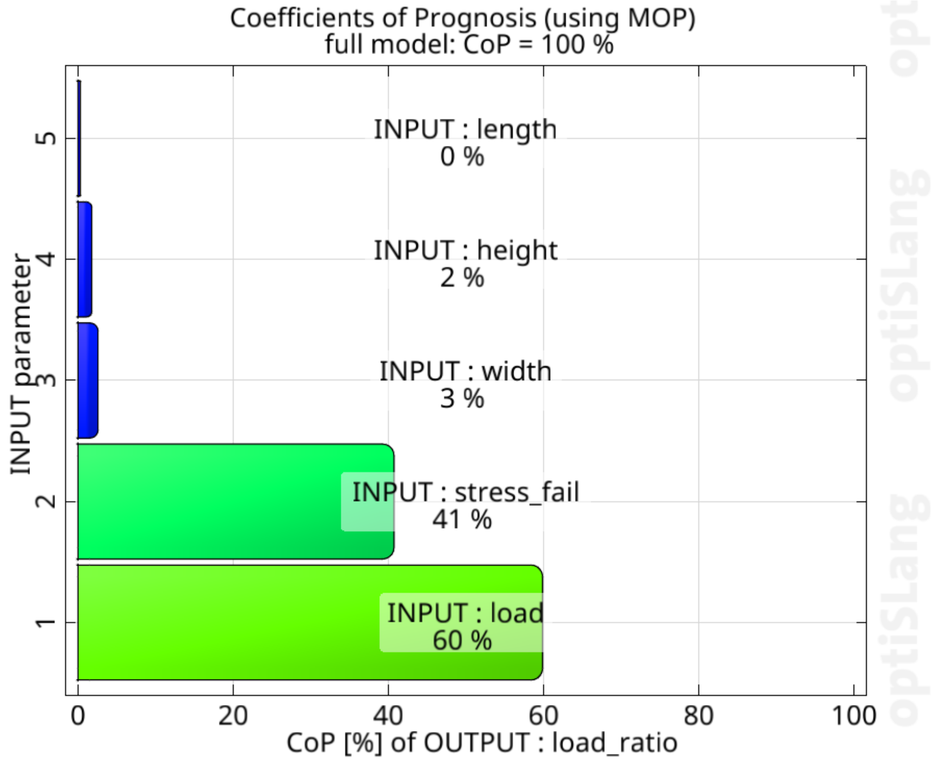


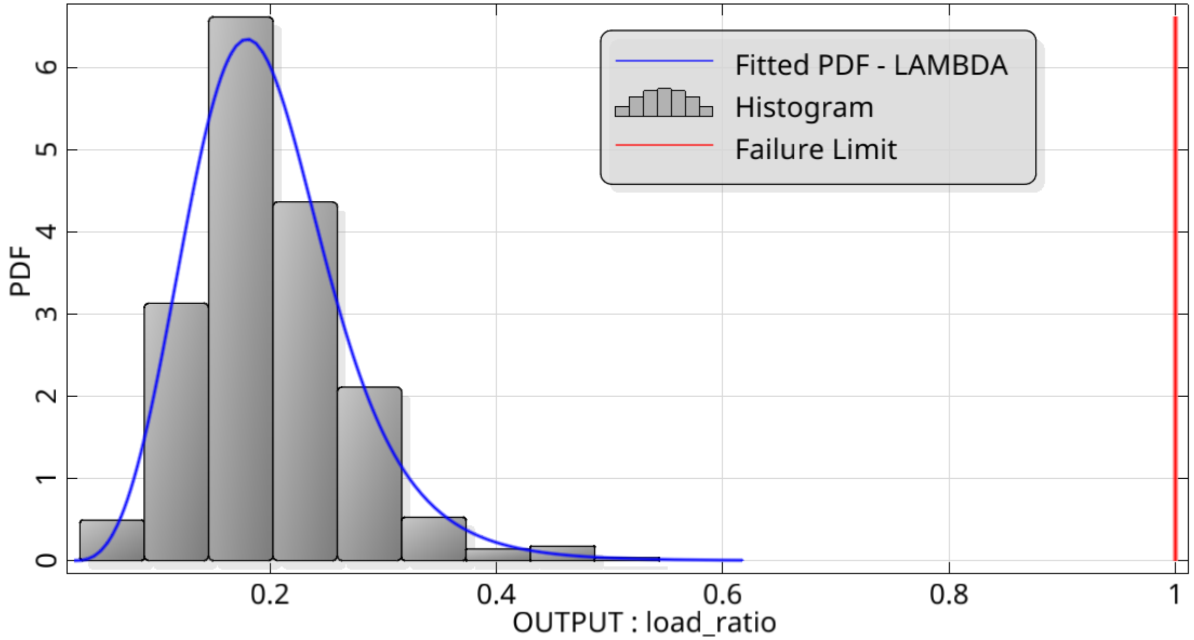


Statistical data			
Min:	0.821872	Max:	7.7026
Mean value:	4.21322	Standard deviation:	1.11325
CoV:	0.264229		
Skewness:	0.0635949	Excess kurtosis:	0.0750143
Fitted PDF : NORMAL			

Example: Three-point Bending Beam

- Load ratio indicates high safety
- Load and failure stress are most important

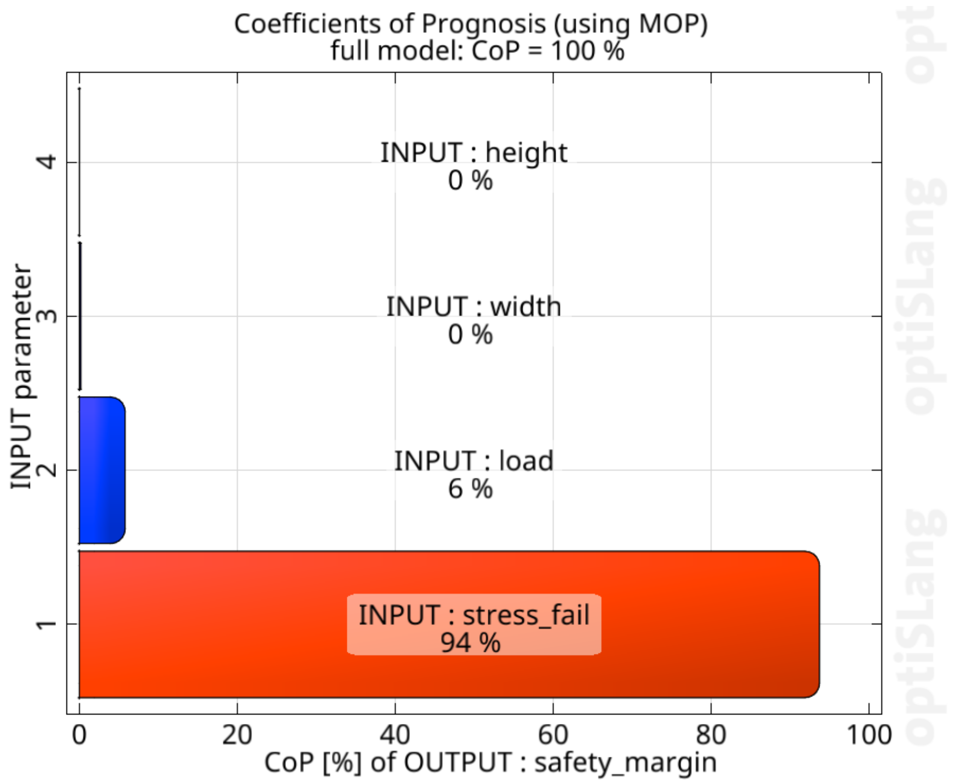


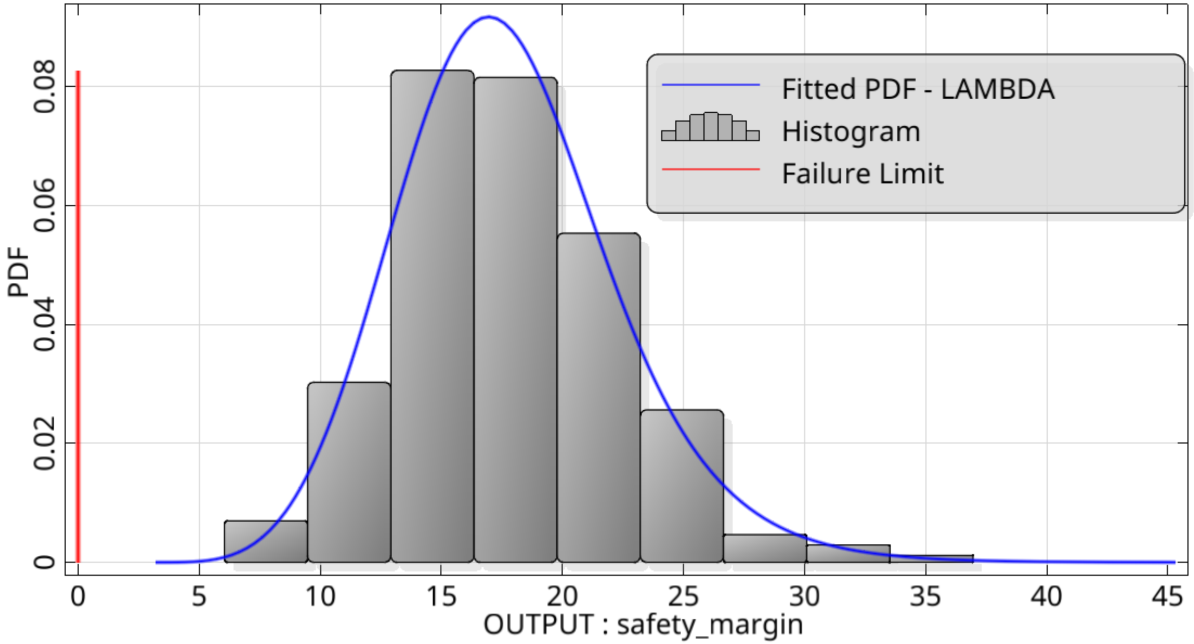


Statistical data			
Min:	0.03218	Max:	0.543876
Mean value:	0.199735	Standard deviation:	0.0695948
CoV:	0.348436		
Limit : Failure Limit			
	Lower value = not set	Upper value = 1	Total
P_fit:		1.039e-06	1.039e-06
Sigma-Level:		11.4989	

Example: Three-point Bending Beam

- Safety margin indicates too small safety
- Failure stress is much more dominant than for the load ratio

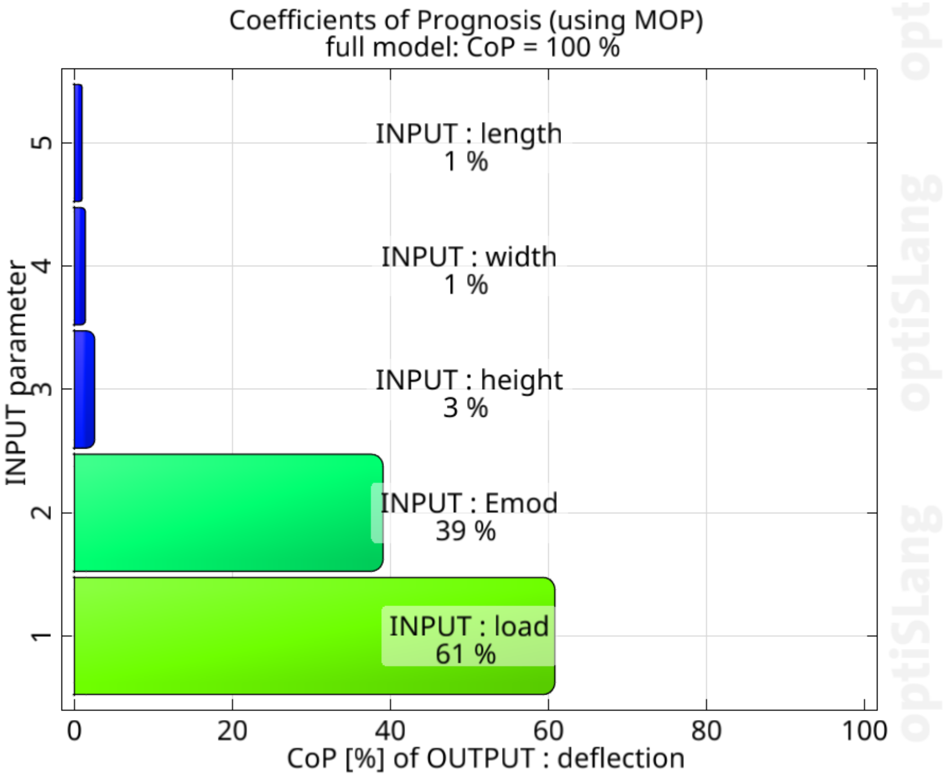


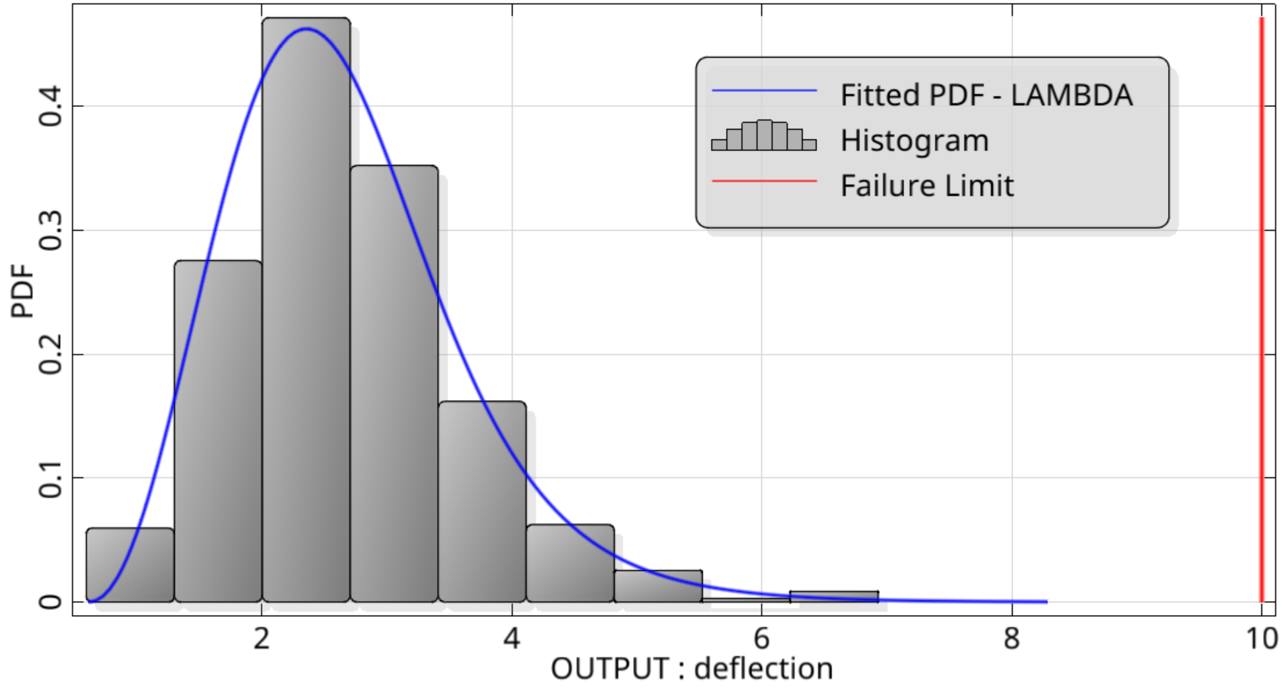


Statistical data			
Min:	6.04844	Max:	36.9558
Mean value:	17.7856	Standard deviation:	4.58565
Limit : Failure Limit			
	Lower value = 0	Upper value = not set	Total
P_fit:	0		0
Sigma-Level:	3.87854		

Example: Three-point Bending Beam

- Deflection limit indicates high safety
- Load and Young's modulus are most important

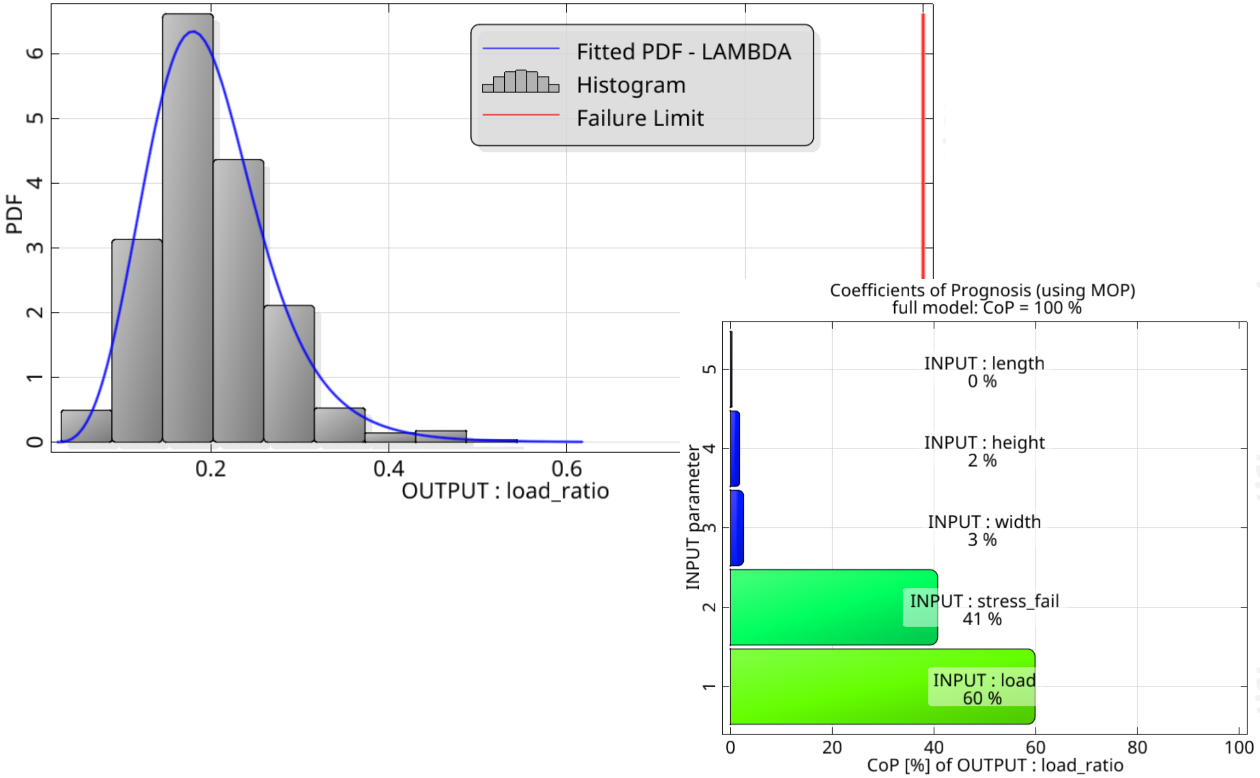


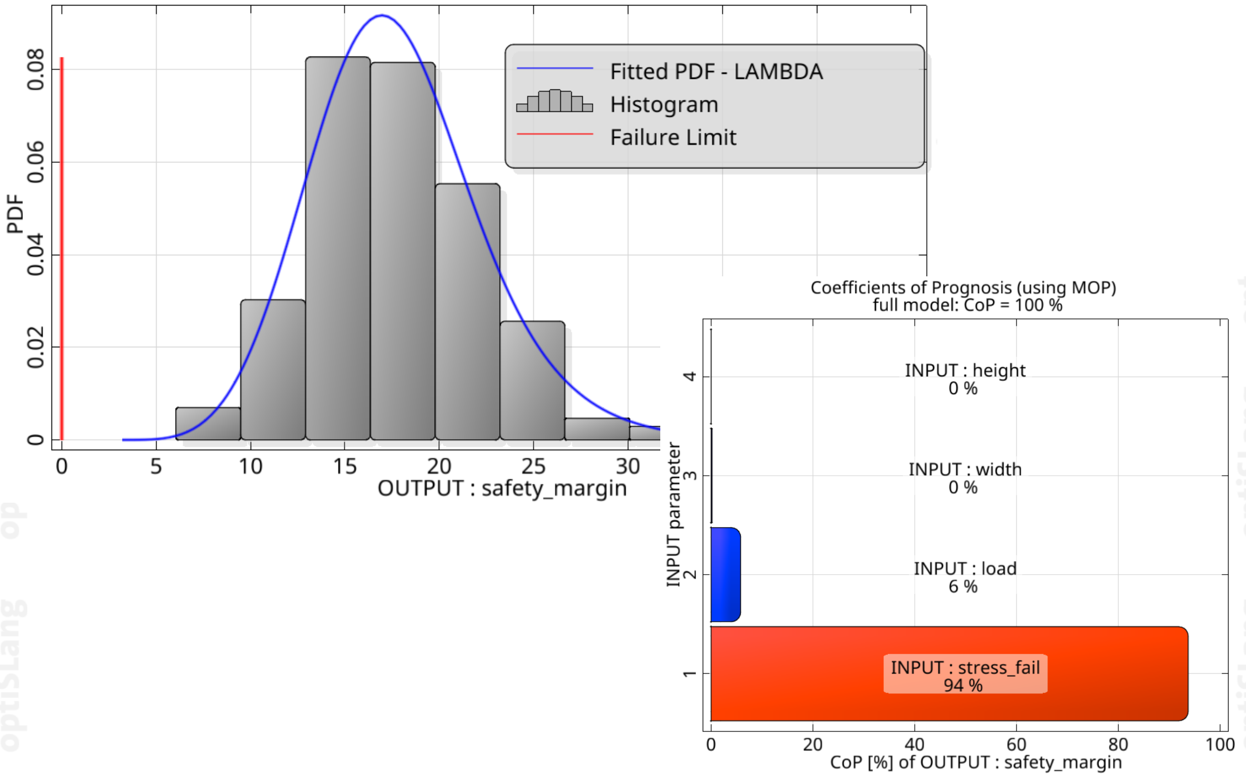


Statistical data			
Mean value:	2.66485	Standard deviation:	0.93736
CoV:	0.351749		
Limit : Failure Limit			
	Lower value = not set	Upper value = 10	Total
P_fit:		2.0222e-05	2.0222e-05
Sigma-Level:		7.82532	

Example: Three-point Bending Beam

- load_ratio = maximum_stress/failure_stress ≤ 1
  safety_margin = failure_stress - maximum_stress ≥ 0
- Load ratio and safety margin consider same failure mechanism but would lead to a different safety assessment with variance-based Robustness evaluation





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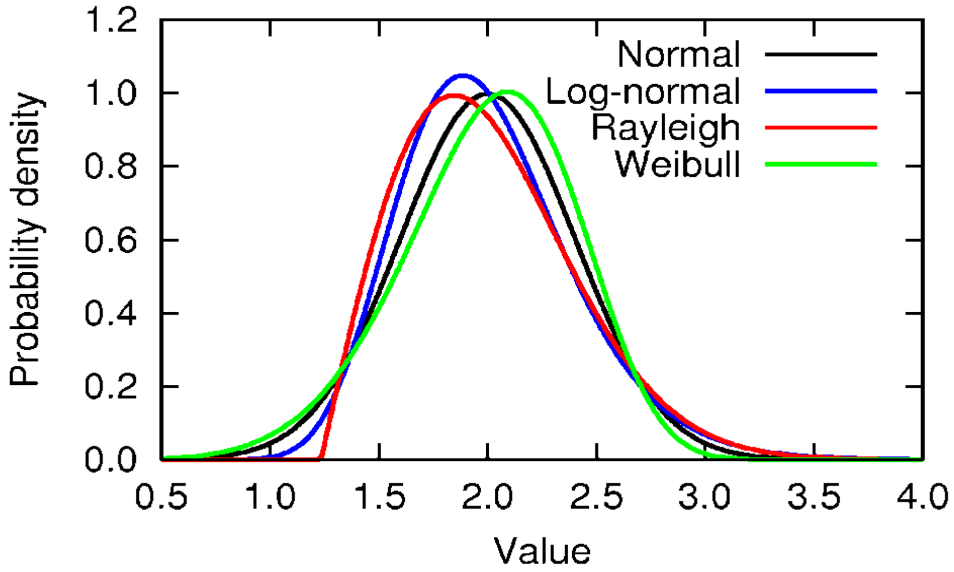
Sigma Level vs. Exceedance Probability

- The sigma level can be used to estimate the probability of exceeding a certain response limit
- Since the distribution type of the response is generally unknown, this estimate may be very inaccurate for small probabilities (sigma levels larger than 3)
- The sigma level deals with single limit values, whereas the failure probability quantifies the event, that any of several limits is exceeded

➔ Reliability analysis should be applied to proof the required safety level



Distribution	Required sigma level (CV=20%) $p_F = 10^{-2}$	$p_F = 10^{-3}$	$p_F = 10^{-6}$
Normal	2.32	3.09	4.75
Log-normal	2.77	4.04	7.57
Rayleigh	2.72	3.76	6.11
Weibull	2.03	2.54	3.49

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Reliability Analysis

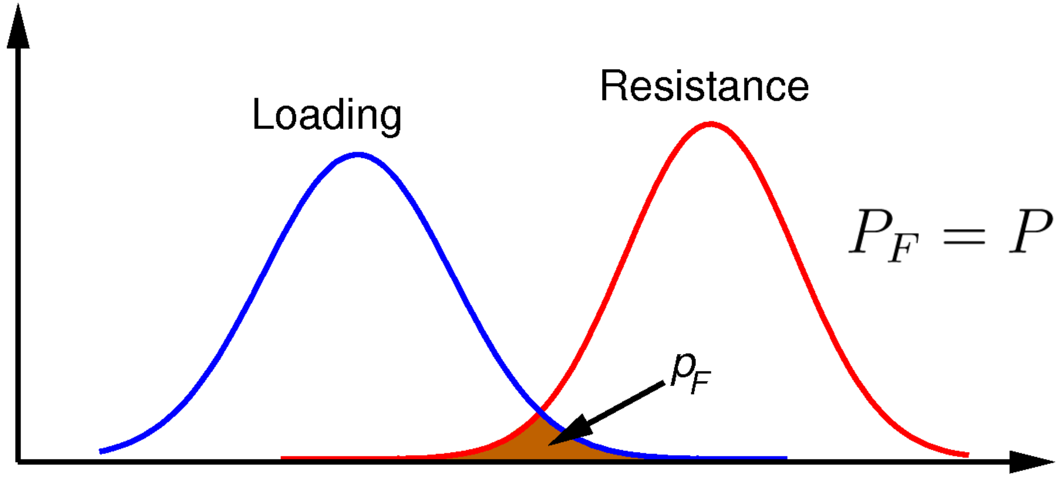
Safety Concept

- Failure occurs if loading S exceeds the resistance R
- Ultimate limit state

$p_F \leq 1.3 \cdot 10^{-6},\ \beta \geq 4.7$

- Serviceability limit state

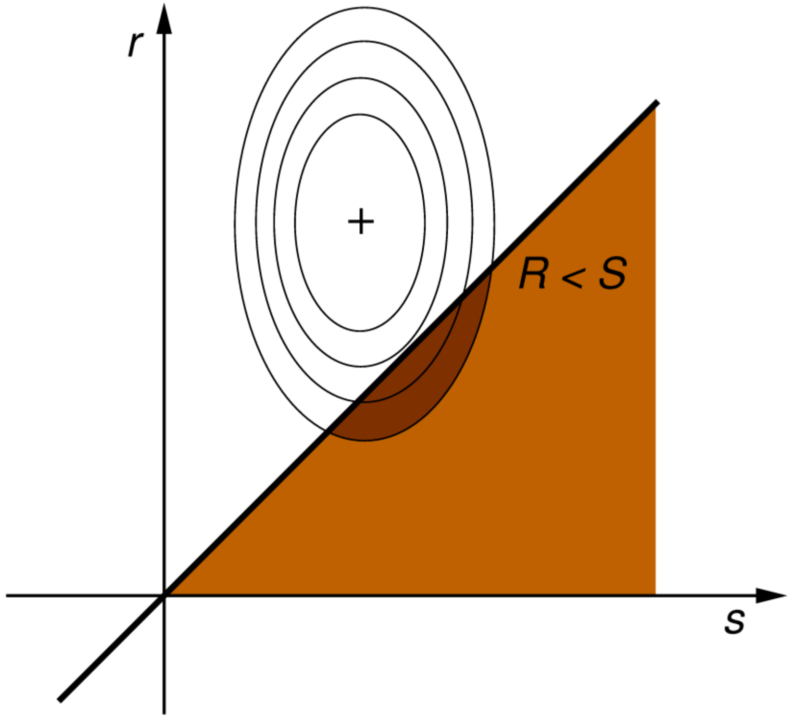
$p_F \leq 1.9 \cdot 10^{-3},\ \beta \geq 2.9$



$$P_F = P[R < S]$$

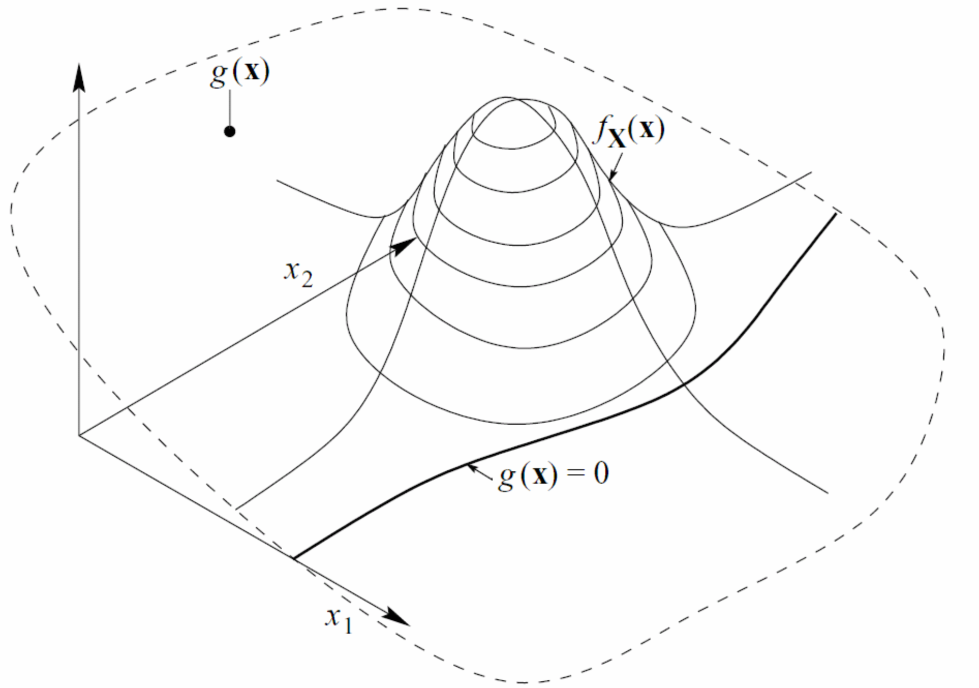
$$p_F = \int\int f_{RS}(r,s) I(r < s)\, dr\, ds$$

$$\beta = -\Phi^{-1}(p_F)$$



Reliability Analysis

- Limit state function $g(\mathbf{x})$ divides the random variable space $\boldsymbol{X}$ into safe domain $g(\mathbf{x}) > 0$ and failure domain $g(\mathbf{x}) \leq 0$
- Multiple failure criteria (limit state functions) are possible
- Failure probability is the probability that at least one failure criterion is violated (at least one limit state function is negative or zero)
- Integration of joint probability density function over failure domain

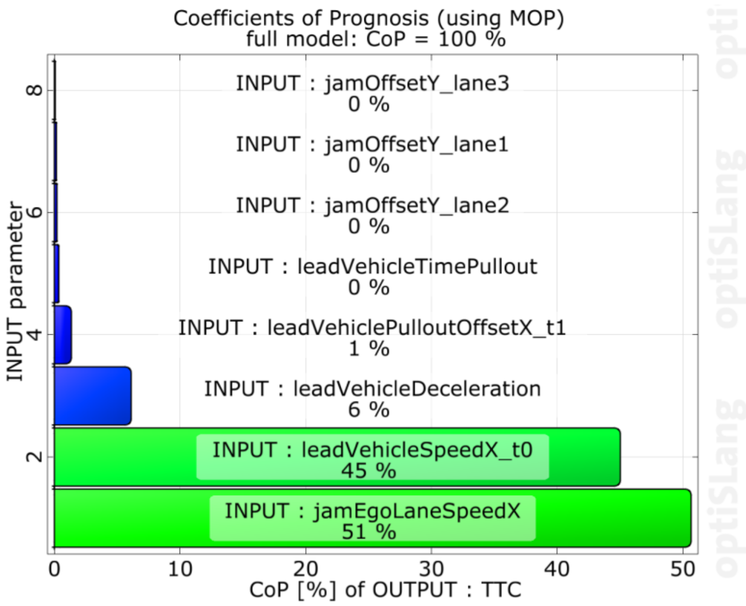


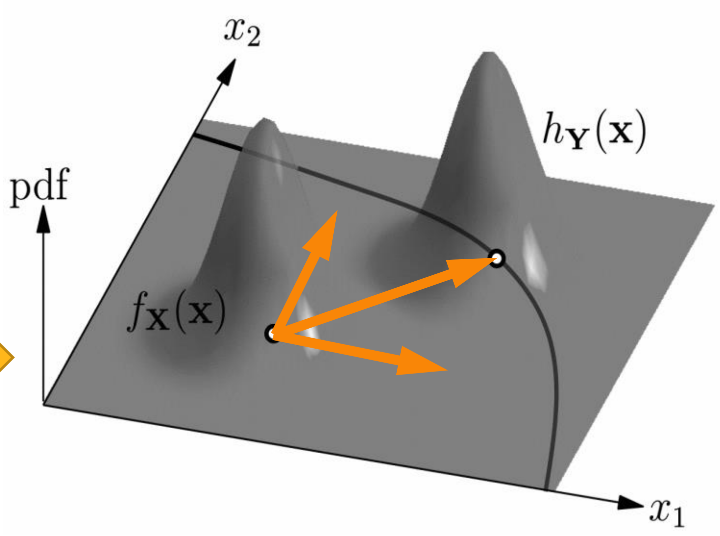
$$P_F = \mathrm{P}[\mathbf{X} : g(\mathbf{X}) \leq 0] = \int \cdots \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I(g(\mathbf{X})) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

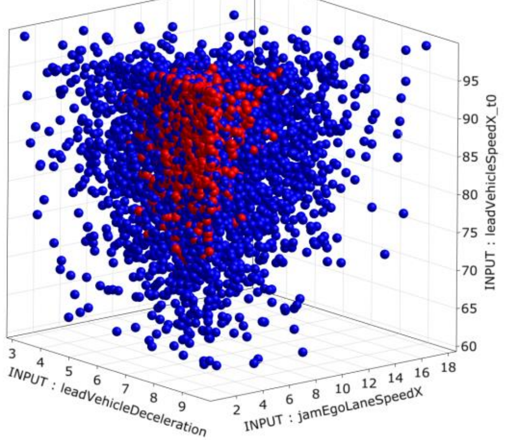
$$I(g(\mathbf{x})) = \begin{cases} 0 \mid g(\mathbf{x}) > 0 \\ 1 \mid g(\mathbf{x}) \leq 0 \end{cases}$$

Reliability Sensitivity Measures

- Correlation and variance-based sensitivity analysis can assess the variable influence only around the mean!
- Sensitivities w.r.t. failure mechanisms are required!



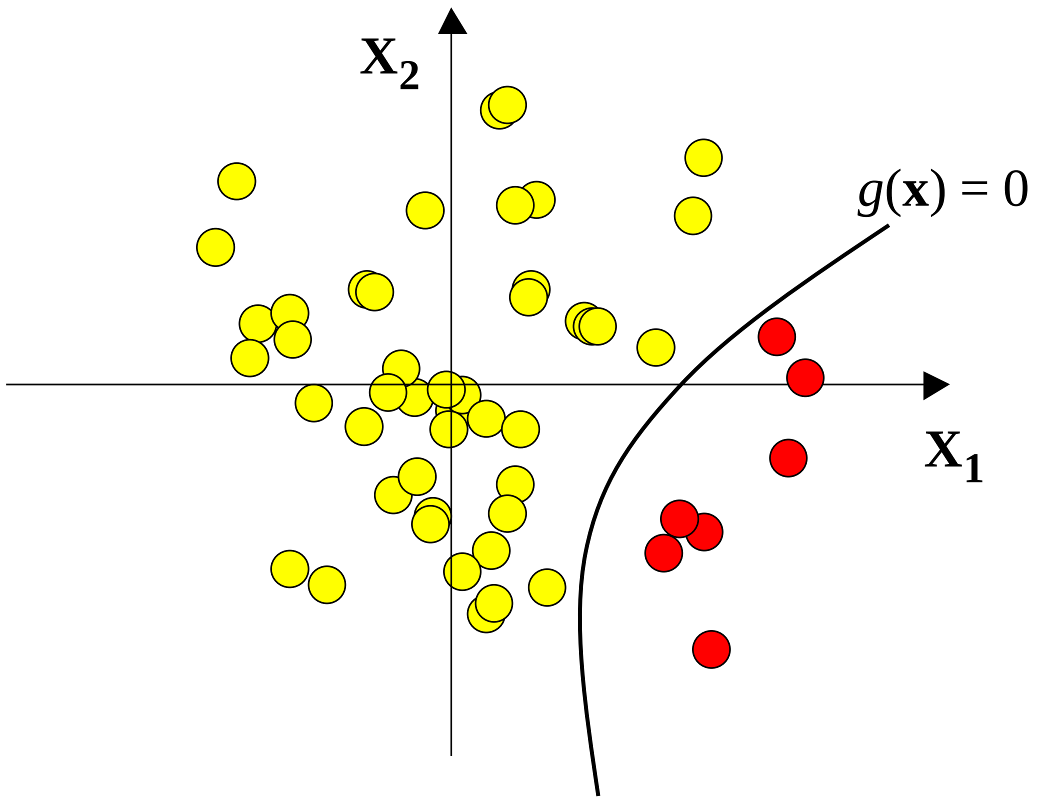




Monte Carlo Simulation

- Robust for arbitrary limit state functions
- Independent of number of random variables
- Huge effort for small failure probabilities

→ Applicable mainly for benchmarking



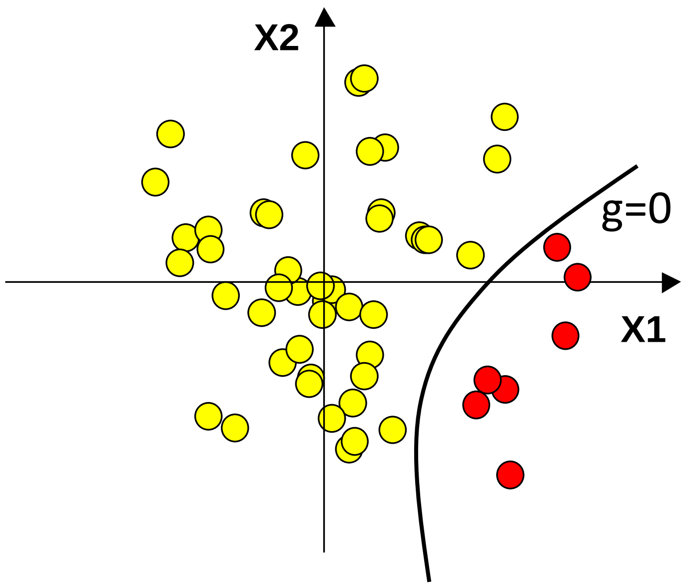
Sigma level	P_F	N for cov(P_F) = 10%
2	2.3E-2	4 400
3	1.3E-3	74 000
4.5	3.4E-6	29 500 000

$$\hat{P}_F = \frac{1}{N}\sum_{i=1}^{N} I\left(g(\mathbf{x}_i)\right), \quad \hat{\sigma}_{P_F} = \sqrt{\frac{\hat{P}_F}{N}}$$

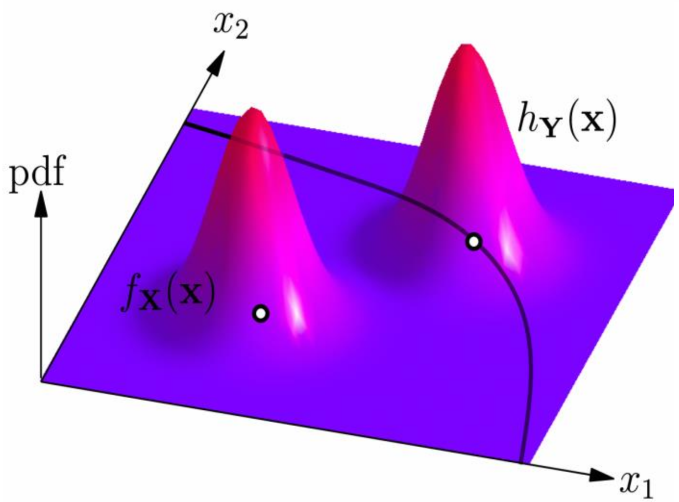
Reliability based Robustness Analysis

- Quantify rare event probabilities with minimum effort and maximum confidence

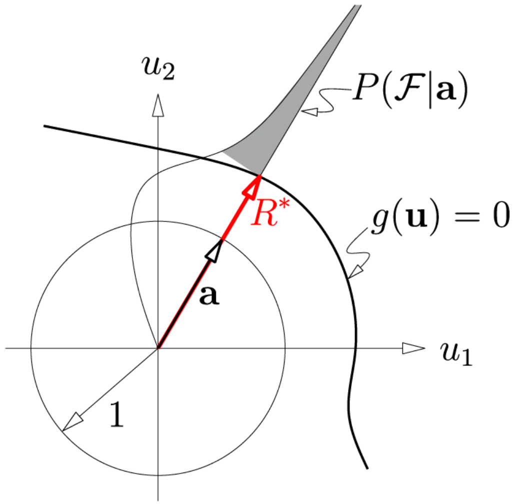
Monte Carlo Sampling



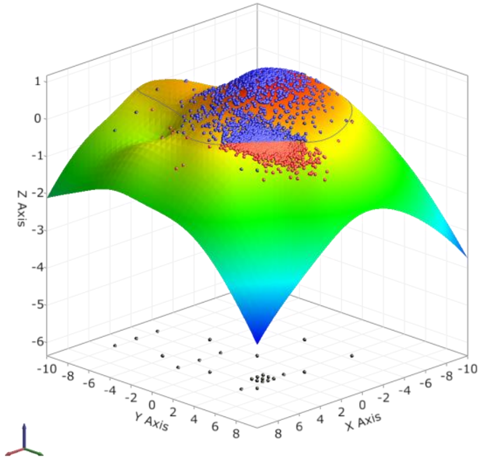
Importance Sampling



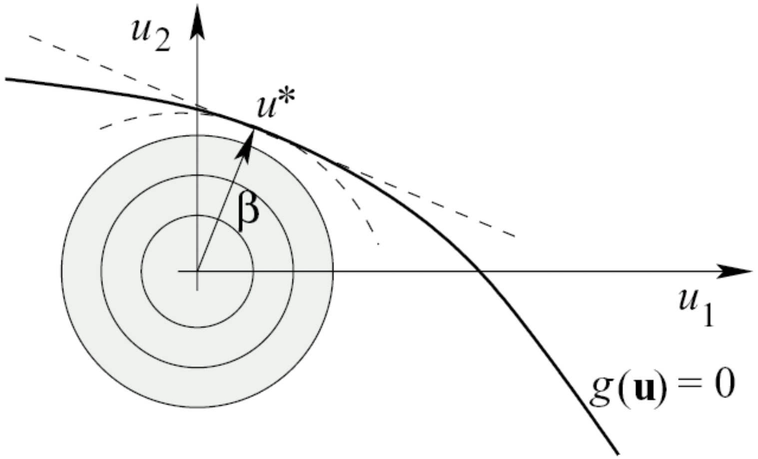
Directional Sampling



Adaptive RSM

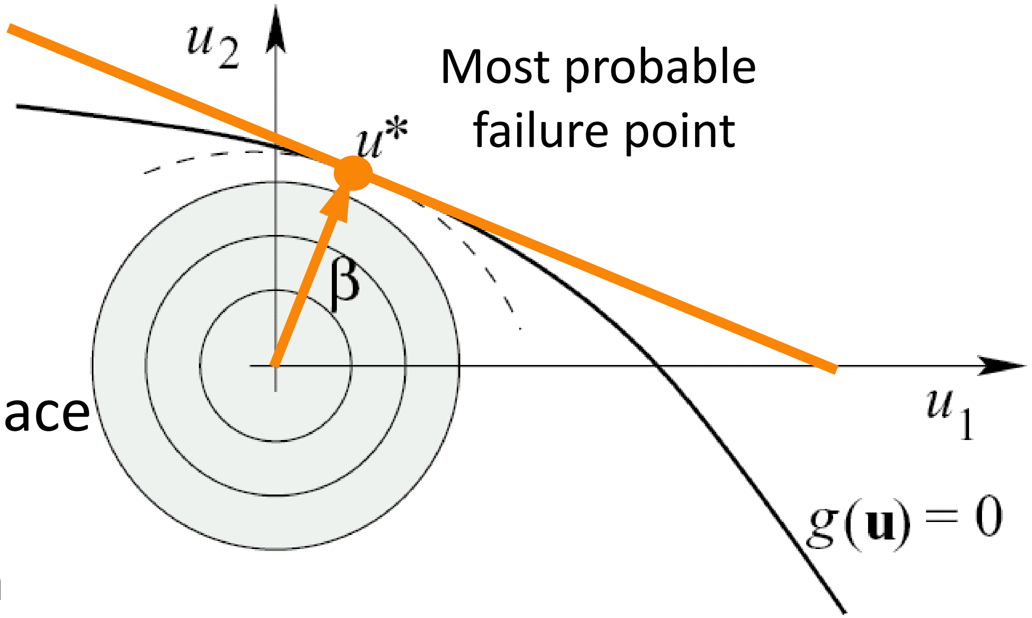


First Order Reliability Method



First Order Reliability Method (FORM)

- Operates in the space of standardized Gaussian variables
- Search for failure point with maximum probability density (*most probable failure point*)
- Equals the point on the limit state surface with minimal distance to origin in standard normal space



- Default algorithm is gradient-based optimization

→ Requires continuous limit state function

$$\mathbf{X} \rightarrow \mathbf{U} \sim \mathcal{N}(0;1) \quad \rho_{i,j\neq i} = 0$$

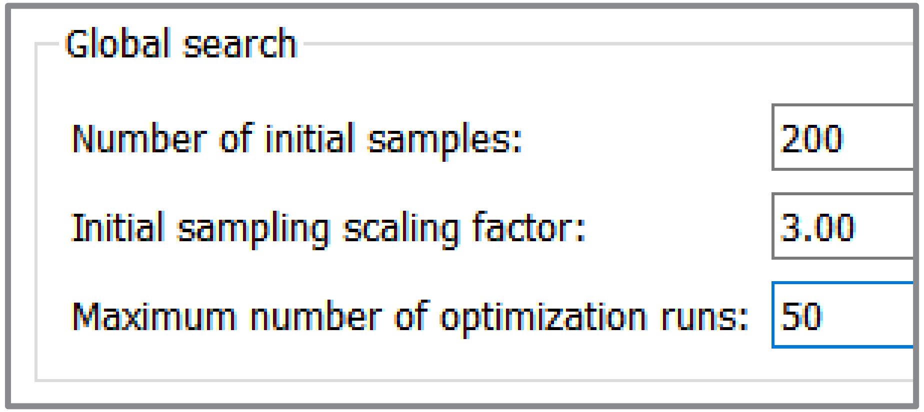
- Probability of failure is calculated after linearization of the limit state function at the design point
- Distance to origin is called reliability index β
- Can be interpreted as generalization of sigma level

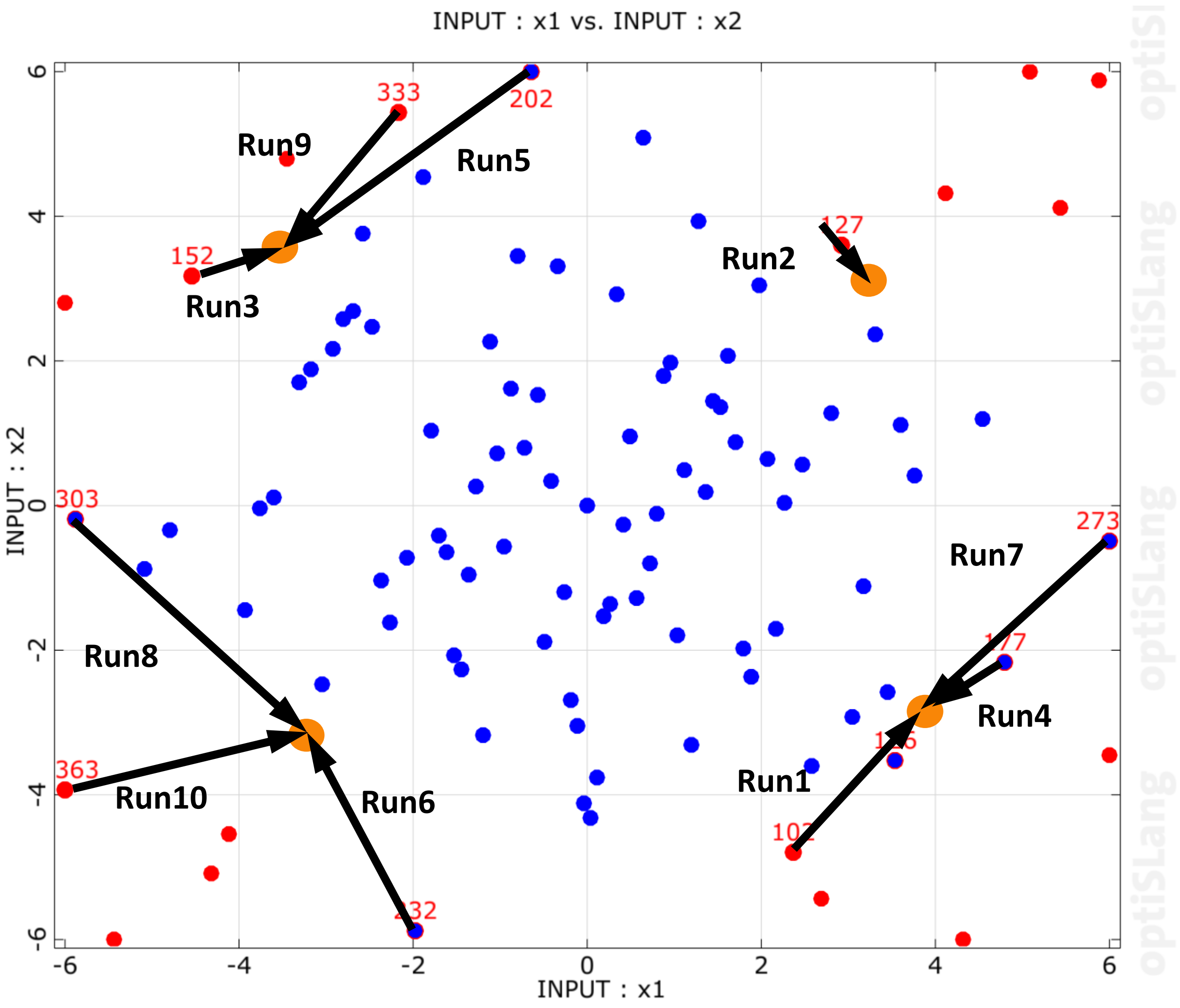
$$\mathbf{u}^* : \frac{1}{2}\mathbf{u}^T\mathbf{u} \rightarrow \min, \quad g(\mathbf{u}) = 0$$

$$P_f = \Phi(-\beta)$$

First Order Reliability Method (FORM)

- Multiple design point search is done be NLPQL optimizer with different start points
- This approach detects local optima and thus different failure regions
- Initial presampling generates start points randomly





Calculation of failure probability in FORM

- Calculation of failure probability for multiple failure regions considers the over-lapping of the estimated linearized regions

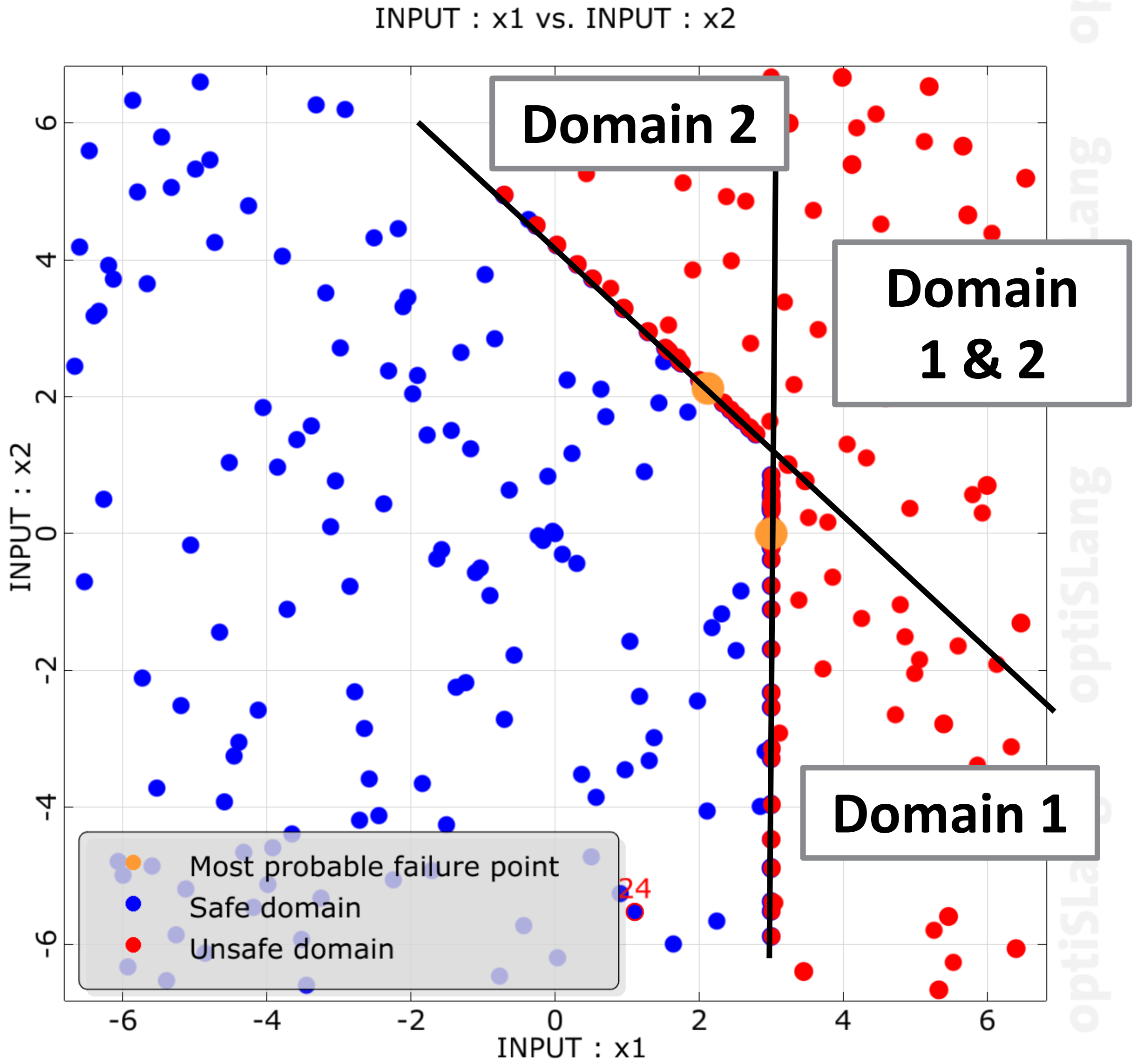
$$\hat{P}_F \leq \sum_{k=1}^{m} \hat{P}_F^k$$

Method : First Order Reliability Method (FORM)

Probability of Failure : 0.00244219
Reliability Index : 2.81456

Most probable failure point(s)

ID :	677	733
Input parameter values		
x1 :	2.12132	3
x2 :	2.12132	1.31347e-07
Reliability index (FORM) :	3	3
Probability of failure (FORM) :	0.0013499	0.0013499



Reliability Sensitivity Measures – First Order Reliability Method

- Sensitivity indices are defined as normalized derivative of failure probability w.r.t. to input parameter variance

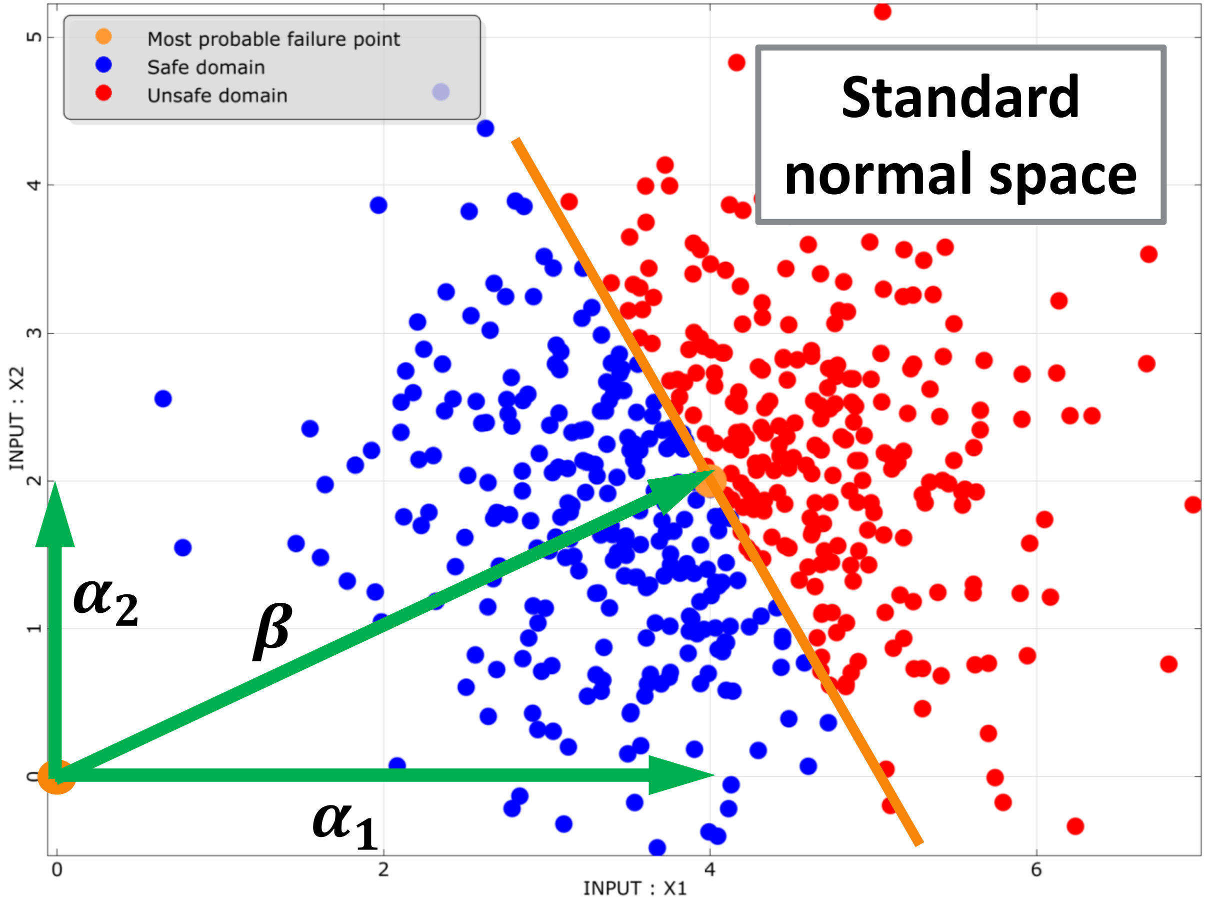
$$S_{P_F}(X_i) = \frac{\partial P_F}{\partial V(X_i)} \left[\sum \frac{\partial P_F}{\partial V(X_k)} \right]^{-1}$$

$$\sum S_{P_F}(X_i) = 1$$

- For FORM with a single failure domain an analytical solution exists

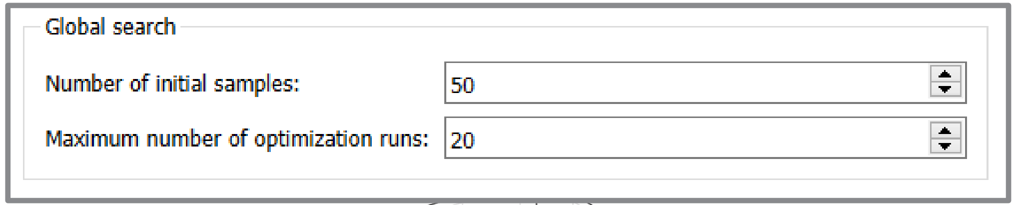
$$S_{P_F}(X_i) = \frac{\alpha_i^2}{\sum \alpha_k^2}$$

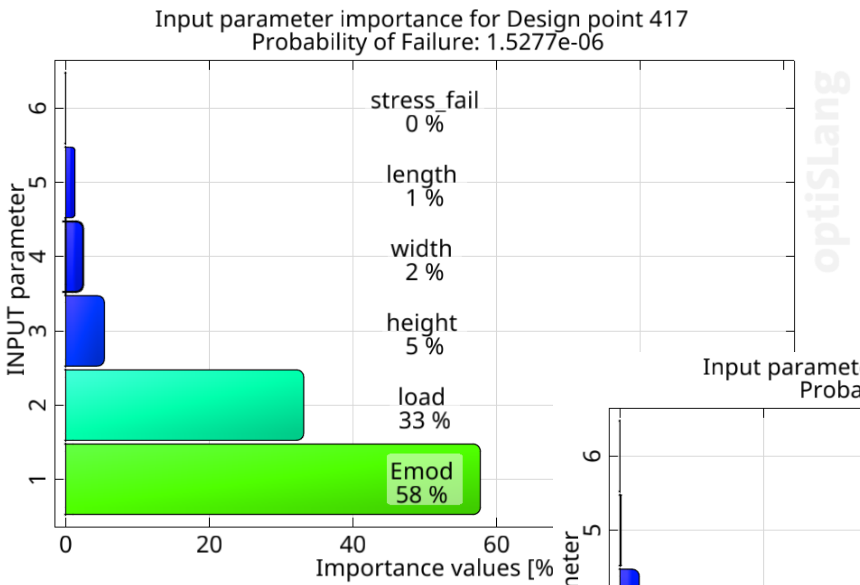
- α_i are the coordinates of the most probable failure point in the standard normal space

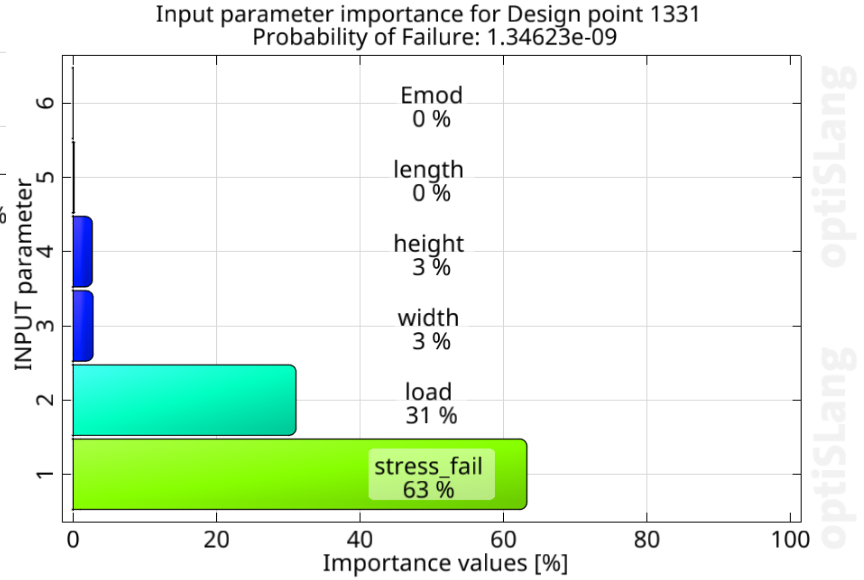


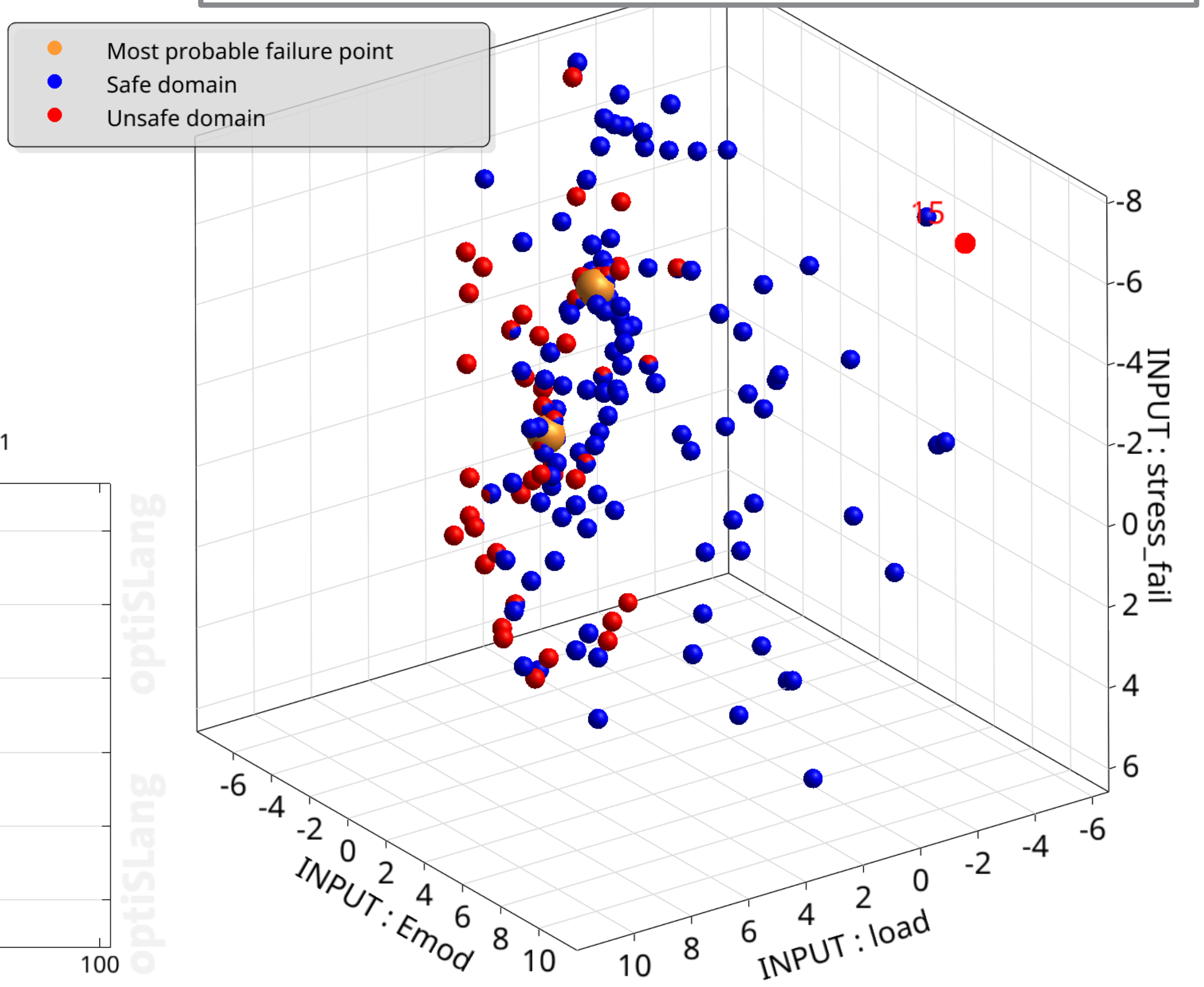
Example: Three-point Bending Beam

- 2 most probable failure points have been found within 20 optimization runs







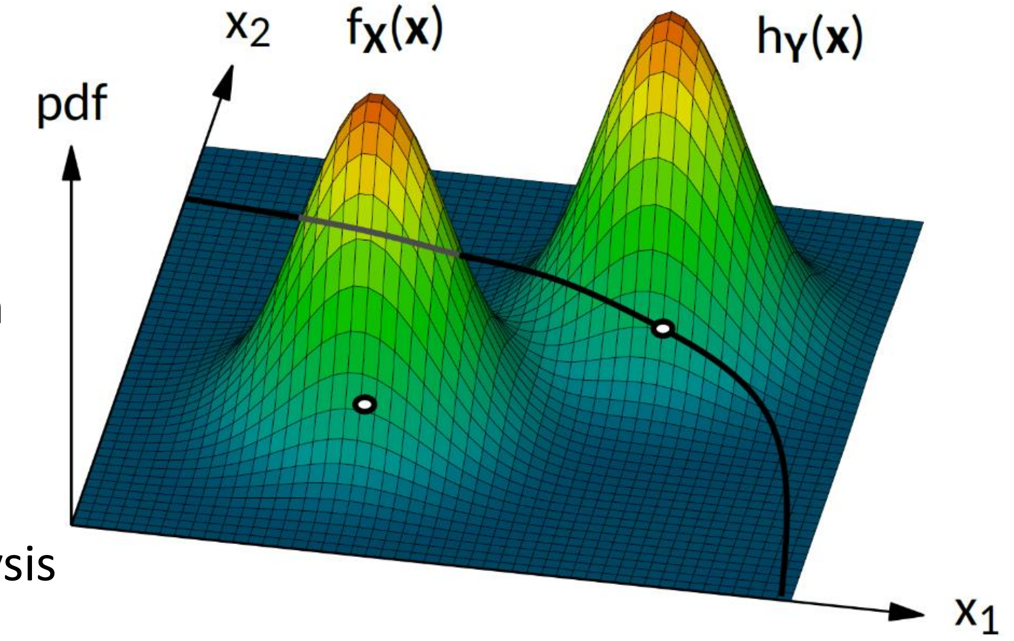


Importance Sampling Approach

- Guide the sampling by making use of information about the failure domain, in order to increase the amount of failure events
- To warrant correct statistics, each sample is weighted by the ratio of original to sampling density

$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{h_{\mathbf{Y}}(\mathbf{x}_i)} I\left(g(\mathbf{x}_i)\right)$$

- Different strategies exist to estimate an "optimal" sampling density:
  - **Adaptive Sampling** detects most dominant failure region
  - **Importance Sampling Using Design Points (ISPUD)** can assess multiple failure regions but requires previous design point search e.g. from First Order Reliability Analysis

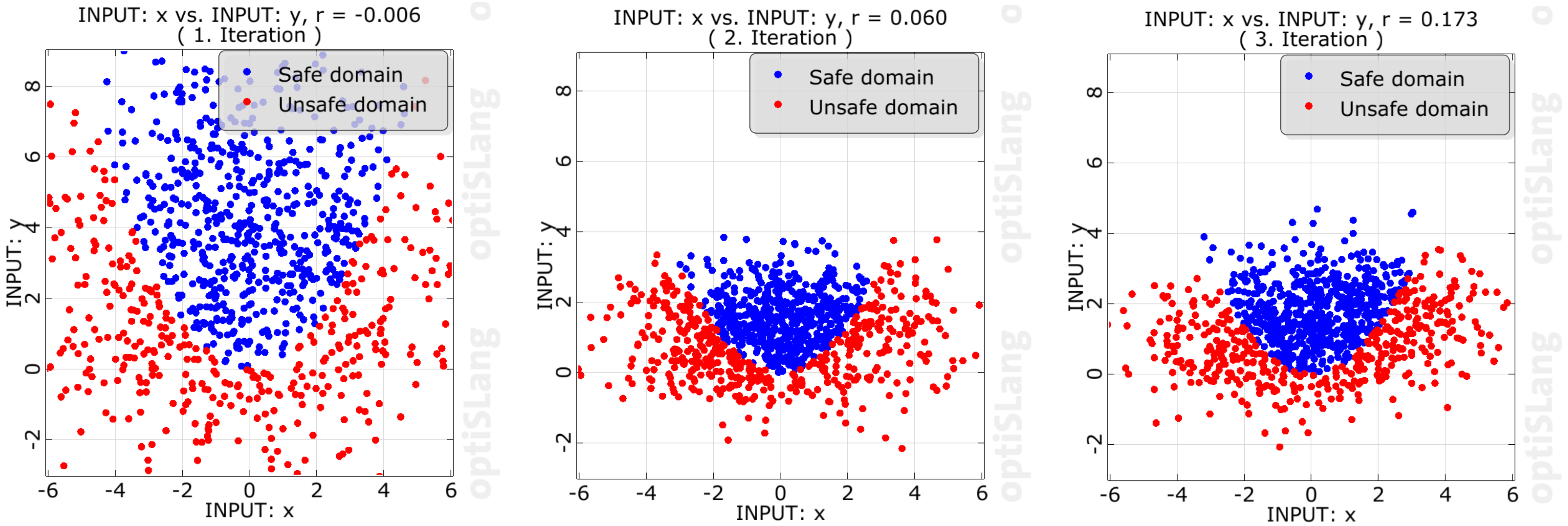


Adaptive Importance Sampling

- Sampling density is defined by mean values vector and covariance matrix of samples in the failure domain
- Search for dominant failure region by 3–5 sampling iterations

→ Applicable for non-smooth and even discontinuous limit state functions

→ Limited to small to medium number of random variables



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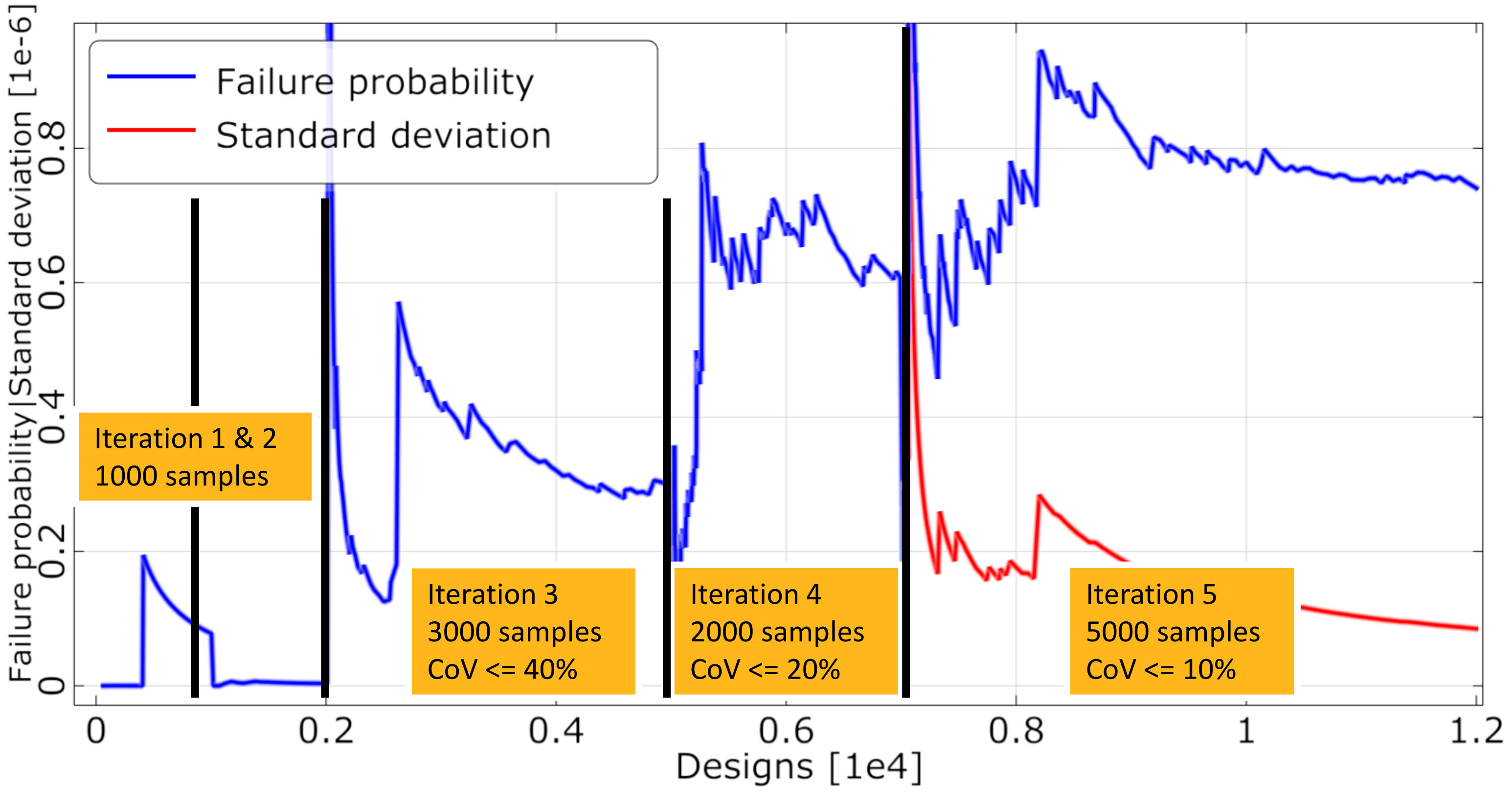
Automatic Sample Size & Error Estimator

- Error estimator of Importance Sampling enables automatic adjustment of sample number for Adaptive Sampling and ISPUD
- Iterations and number of samples are automatically adjusted to reach required accuracy of failure probability



Method : Adaptive Sampling (AS)

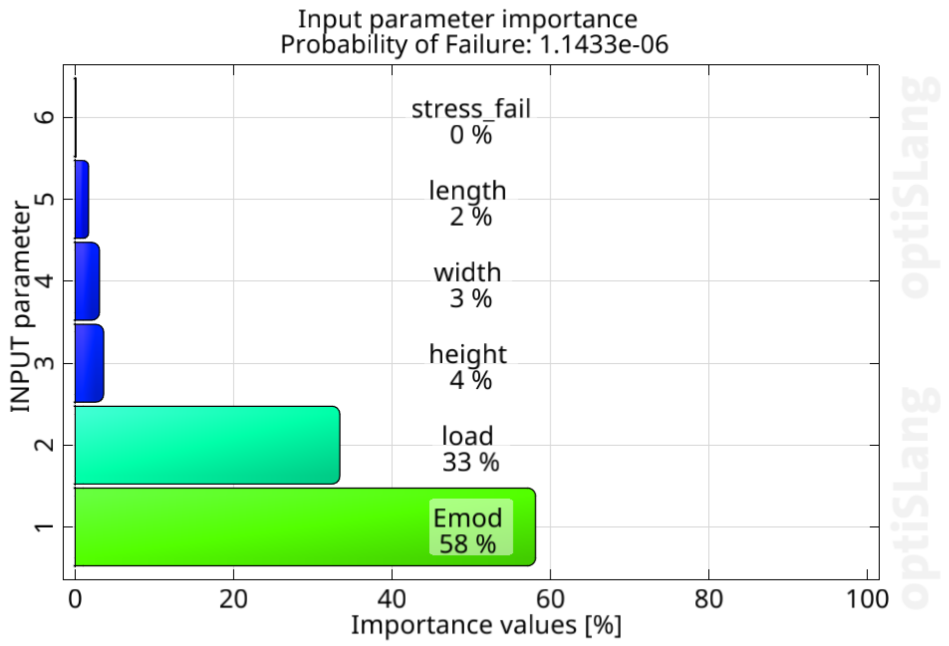
Complete iterations :	5 / 5
Selected data :	All designs
Probability of Failure :	8.86918e-6
Standard deviation error :	9.99696e-7
Reliability Index :	4.29161

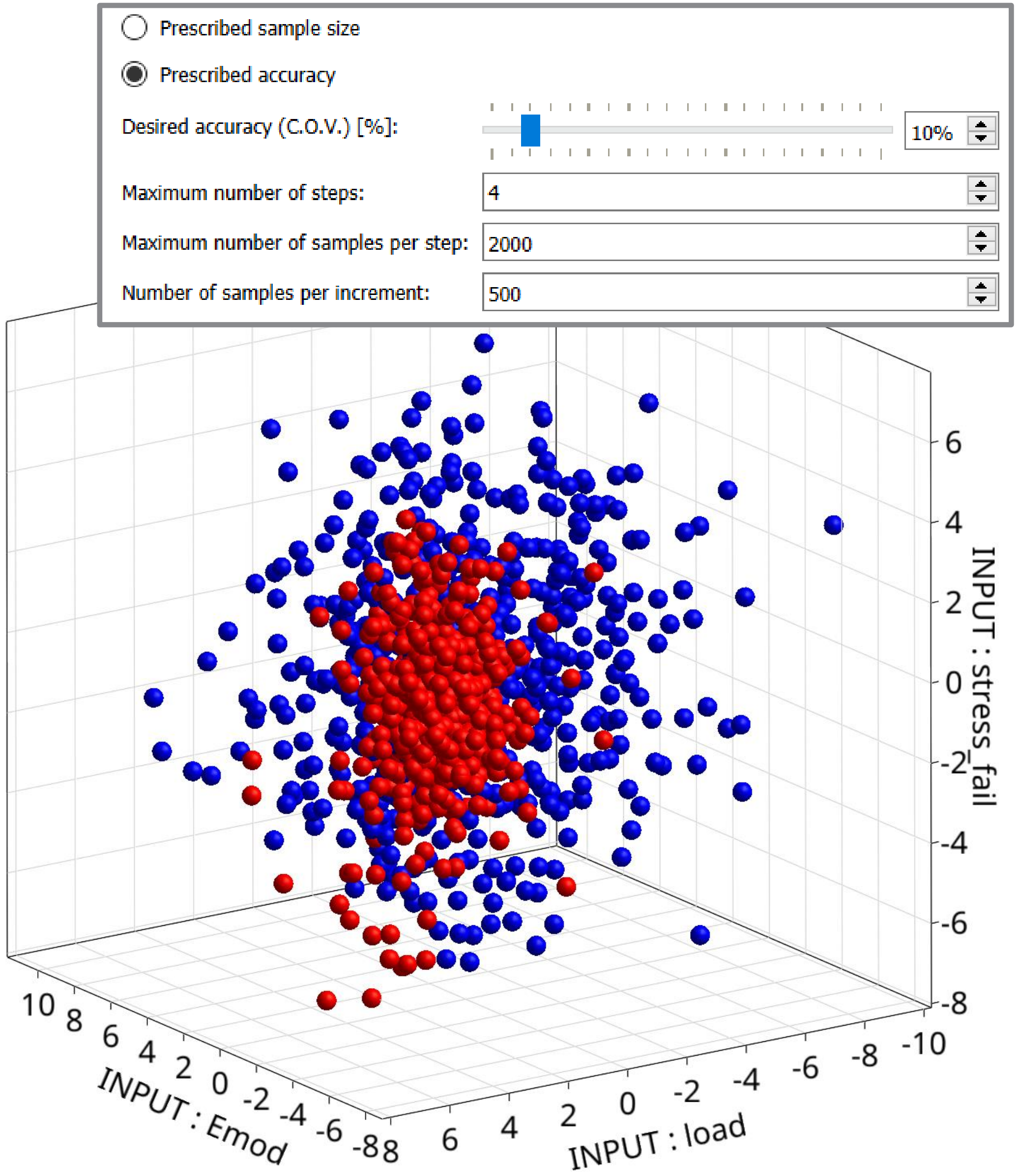
Number of designs

Total :	18000
Safe domain :	12060
Unsafe domain :	5940
Failure strings :	0
Failed :	0

Example: Three-point Bending Beam

- Only a single failure region can be detected with Adaptive Sampling
- Automatic sample size required 1500 samples to reach 10% accuracy





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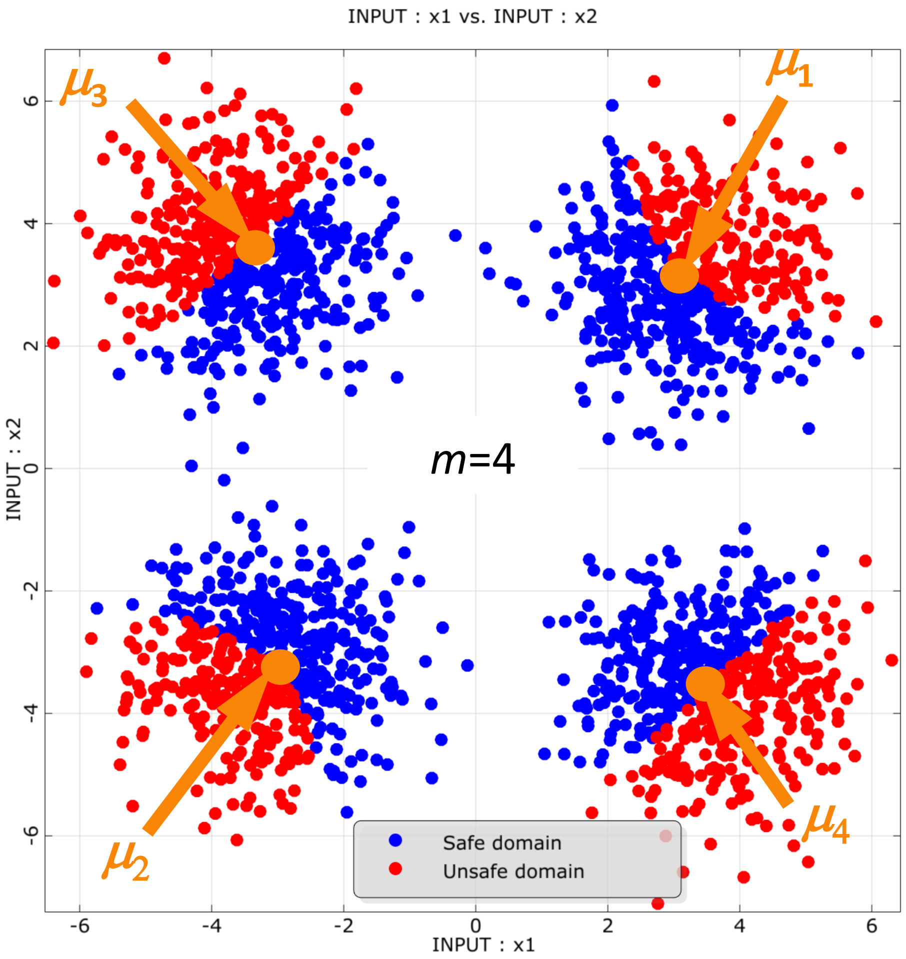
Importance Sampling Using multiple Design Points

- Joint failure probability and error estimator for multiple sampling densities

$$\hat{P}_F = \frac{1}{N}\sum_{i=1}^{N} w(\mathbf{x}_i) I\left(g(\mathbf{x}_i)\right)$$

$$\hat{\sigma}_F^2 = \frac{1}{N^2}\sum_{i=1}^{N} w^2(\mathbf{x}_i) I\left(g(\mathbf{x}_i)\right) - \frac{\hat{P}_F^2}{N} \qquad w(\mathbf{x}_i) = \frac{f_{orig}(\mathbf{x}_i)}{f_{mod}(\mathbf{x}_i)}$$



- Original joint probability density in standard Gaussian space

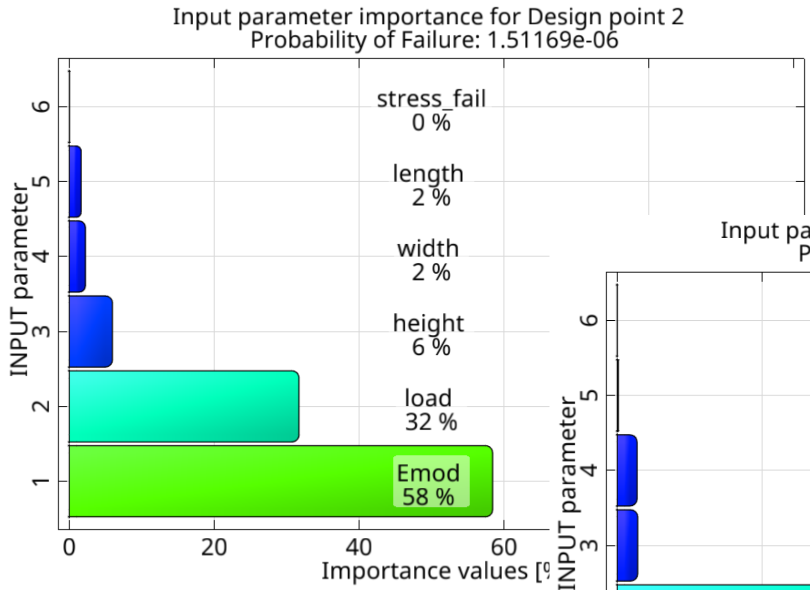
$$f_{orig}(\mathbf{x}_i) = \exp\left(-\frac{\|\mathbf{x}_i\|^2}{2}\right)$$

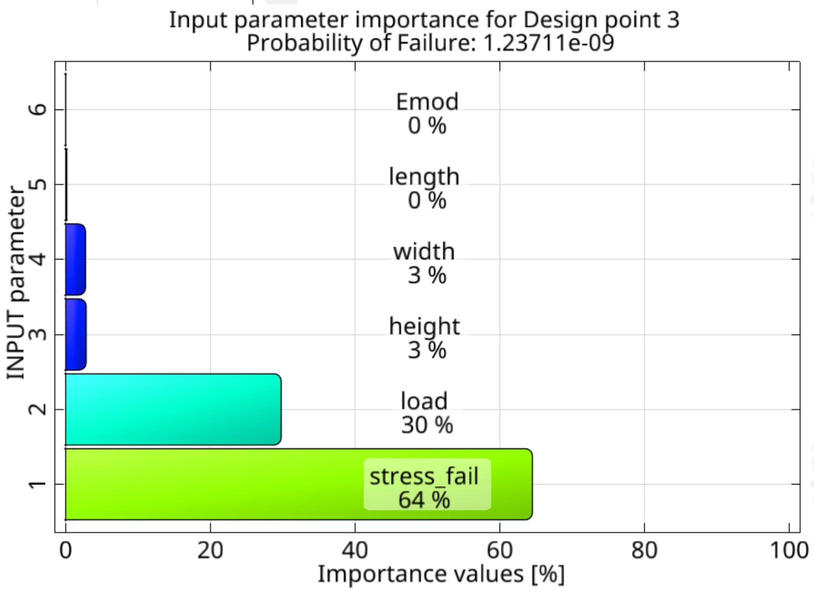
- Modified global sampling density as sum of *m* individual local densities

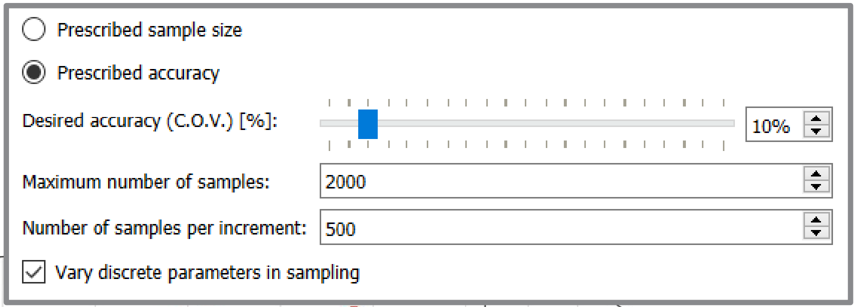
$$f_{mod}(\mathbf{x}_i) = \frac{1}{m}\sum_{k=0}^{m} \exp\left(-\frac{\|\mathbf{x}_i - \boldsymbol{\mu}_k\|^2}{2}\right)$$

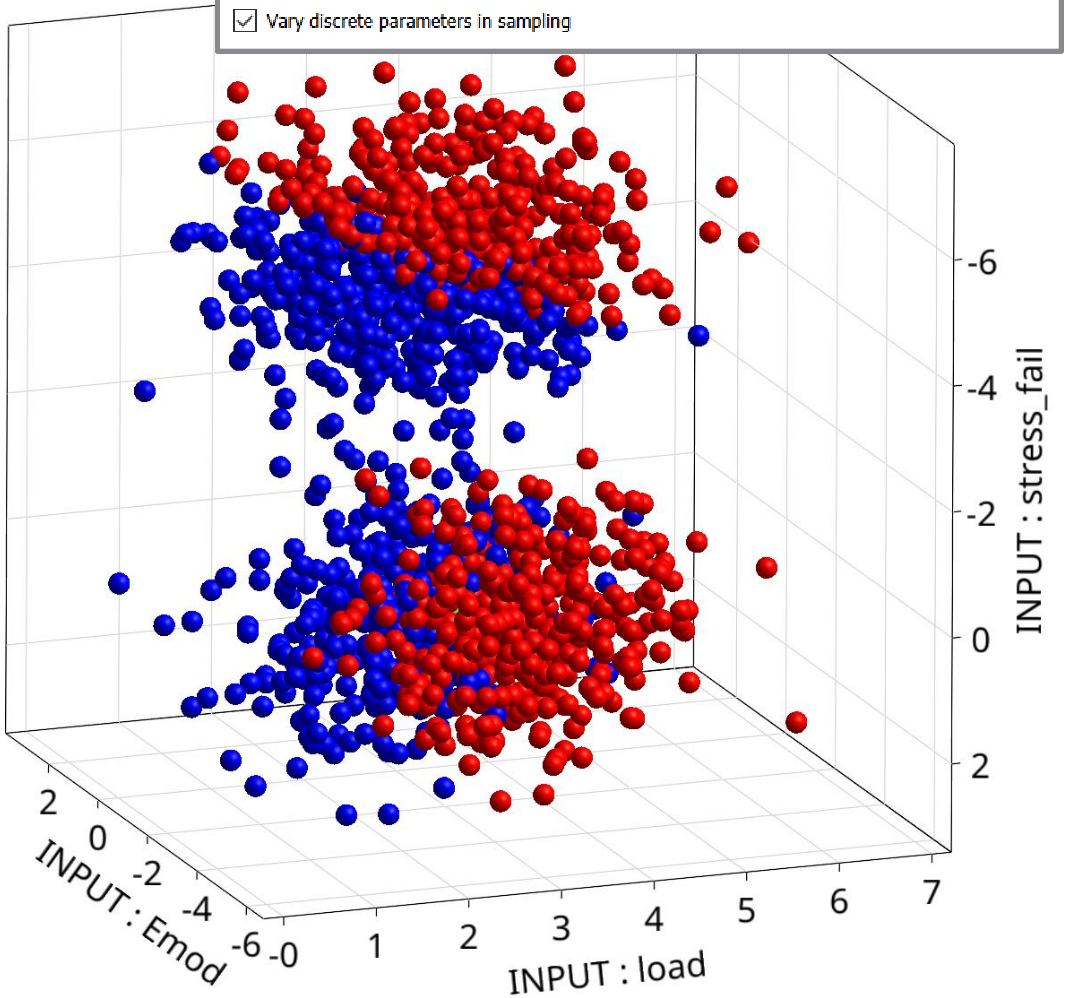
Example: Three-point Bending Beam

- Both failure regions could be proven by the ISPUD
- Automatic sample size required 1500 samples to reach 10% accuracy









Best practice

Overview on Reliability Algorithms

Approach	Non-linearity	Failure domains	Parameters	No. solver runs
Robustness Sampling + Distribution fit	continuous, monotonic	one dominant (for each KPI)	many	100-500 (Pf $\geq 10^{-3}$)
Monte Carlo Simulation	arbitrary	arbitrary	many	10^5 (Pf $\approx 10^{-3}$) 10^8 (Pf $\approx 10^{-6}$)
Directional Sampling	arbitrary	arbitrary	<= 10	1000-5000
Adaptive Response Surface Method	continuous	few dominant	<= 20	200-500
Adaptive Importance Sampling	arbitrary	one dominant	<= 20	1000-5000
FORM + ISPUD	continuous	few dominant	<= 50	2000-10000

- Advanced reliability methods reduce numerical costs by a factor of at least **1000**
- Verification using a second method is recommended

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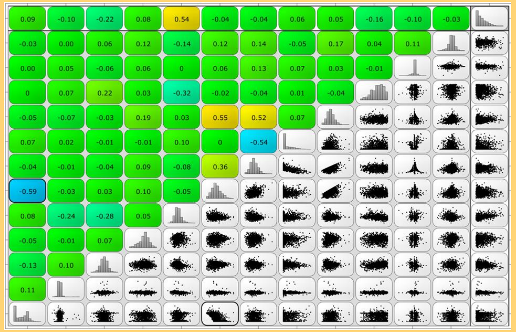
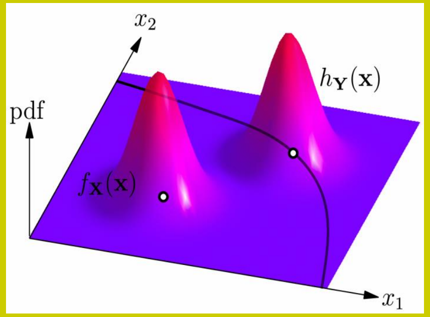
Requirements to a Successful Scenario-based Reliability Analysis

1. Qualified knowledge of uncertain parameters of the analyzed logical scenarios including their interdependencies and constraints
2. Suitable probabilistic model representing the observed uncertainty scatter by marginal distribution functions and dependencies e.g. by the Nataf correlation model



3. Suitable simulation model, which should cover all important effects and phenomena of the investigated scenario with sufficient accuracy
4. Parametric simulation model should be valid for each possible input parameter combination within the statistical assumptions

5. Qualified reliability analysis methods which should provide reliable estimates of statistical errors
6. Due to assumptions in different methods, we recommend to verify the estimates of failure probability for a certain scenario by a second reliability method



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Scenario Based Safety Assessment of Automated Driver Assistance Systems using Reliability Analysis

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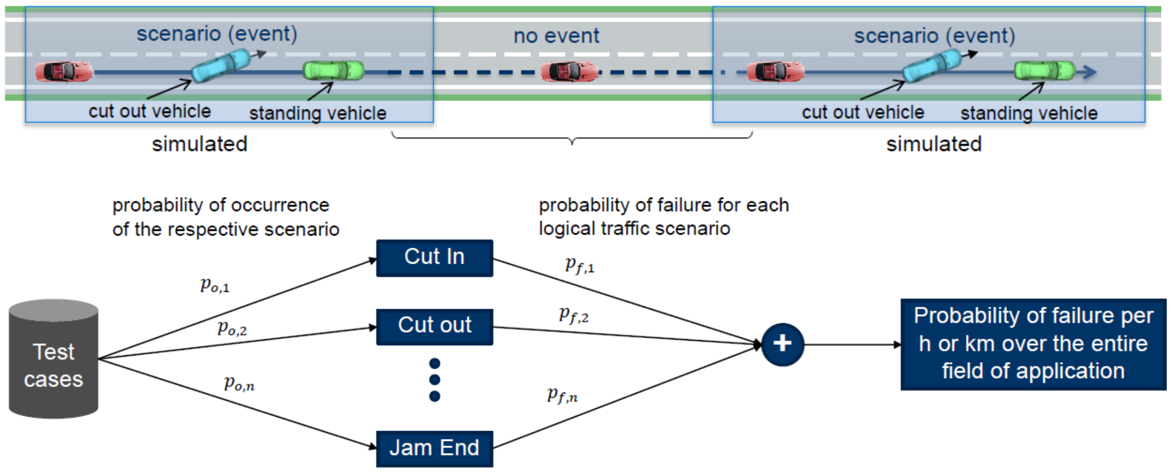
Scenario-Based Evaluation/ Risk Quantification

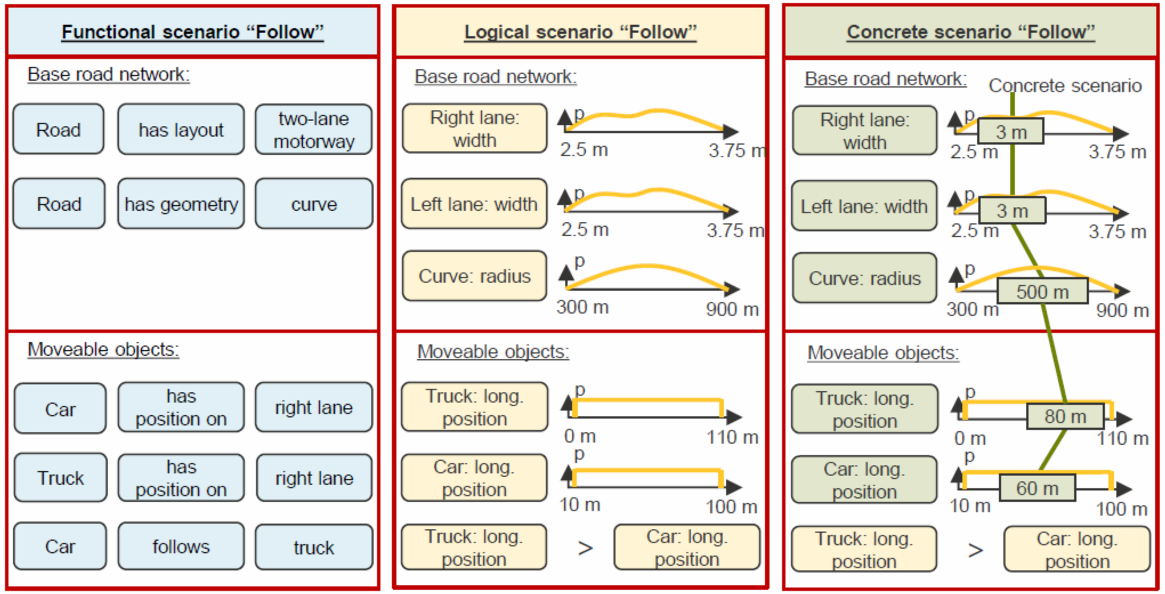
Challenge:
Required mileage needed to proof the probability of failure of the system is impossible to reach on real road

Solution:
Performs robustness evaluation and reliability analysis for parameterized driving scenarios in a way that is **much more efficient than Monto-Carlo Simulation**.
1/ Logical scenario: Reliability analysis on probability of failure by decision trees and handbooks
2/ Concrete scenario: Reliability analysis (e.g. Adaptive Importance Sampling) to obtain probability of failure by simulated concrete scenarios

Benefits:
Only « interesting » concrete scenarios are simulated

$$P(\text{crash}/\text{km}) = P(\text{crash} \mid \text{scenario}_1)P(\text{scenario}_1/\text{km}) + \cdots + P(\text{crash} \mid \text{scenario}_{rest})P(\text{scenario}_{rest}/\text{km})$$

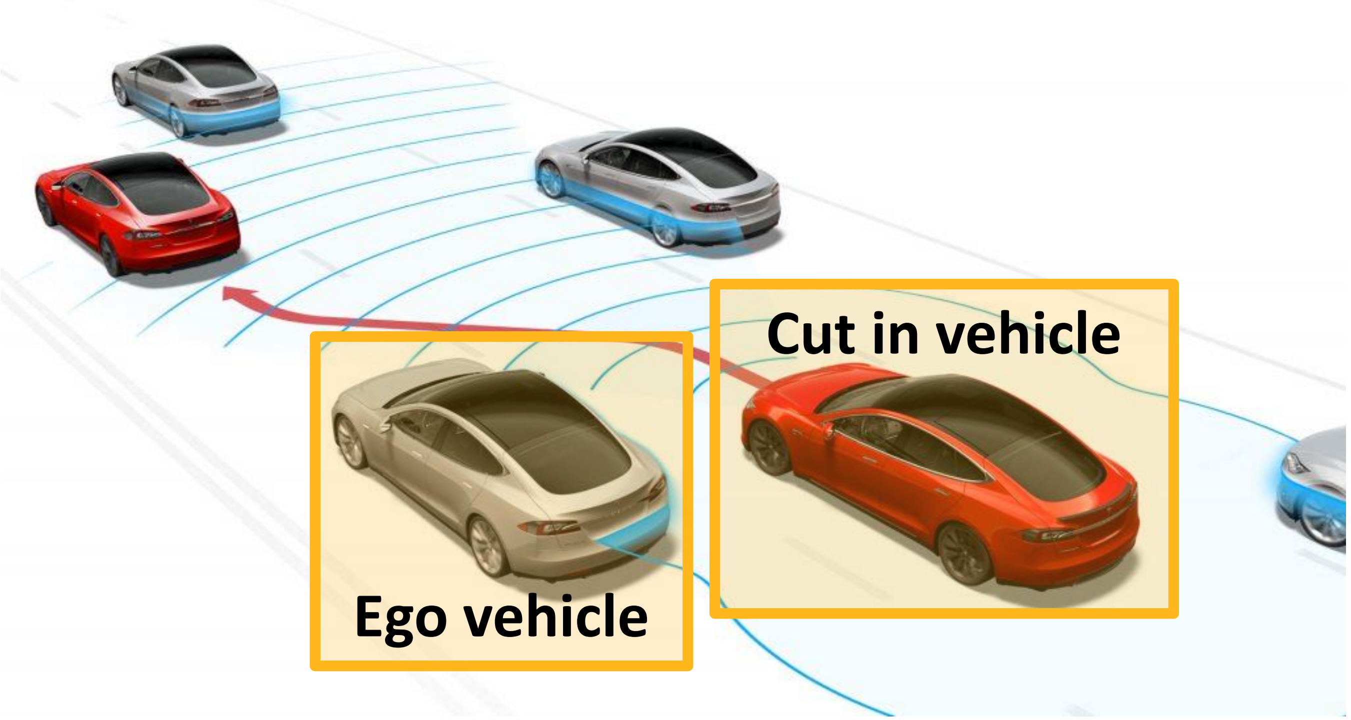


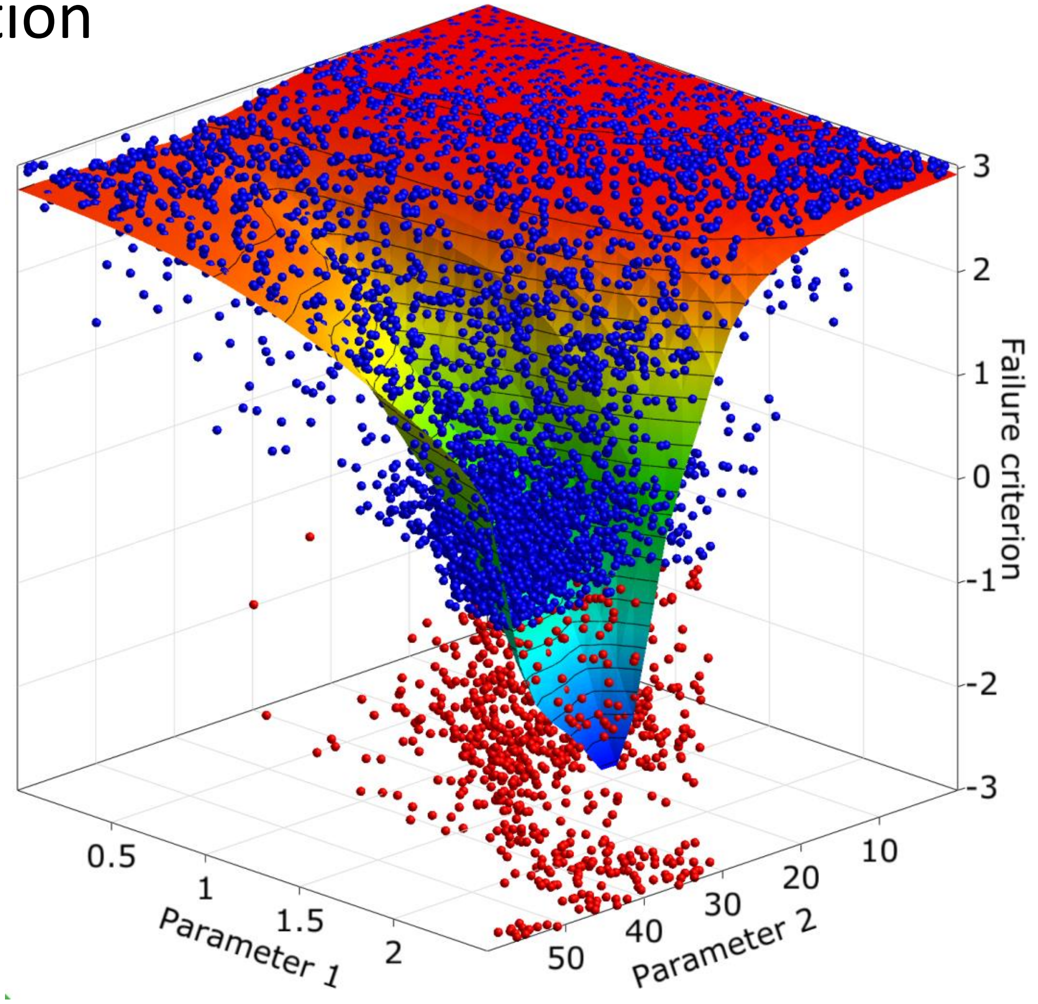


Source picture and formula: http://www.pegasusprojekt.de

Customer Application: Safety Assessment of ADAS Systems

- 5000 data points of a cut-in scenario simulation
- 10 input parameters, 75 response values
- Scalar failure criterion combining several failure mechanism





Customer Workflow for Safety Assessment of Single Scenario

1. **Sensitivity analysis with Adaptive Surrogate** within +/- six sigma ranges of scattering inputs by using local refinement considering critical failure modes
2. **First Order Reliability Analysis** by searching for important failure regions on meta-model using only the important inputs
3. **Importance sampling** in important failure regions with real solver runs and all inputs parameters