Computation and visualization of statistical measures on FE structures for forming simulations

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Abstract

Input scatters of forming process parameters can lead to significant scattering of the forming results. In order to predict the influence of these scatters on the forming results, stochastic analysis are increasingly employed. Generally, so called robustness evaluations are performed, i.e. sensitivity analyses of input scattering on important result variables. These means prediction can be employed in early stages of the product development process until serial production. However, the biggest potential is expected to lie in a robustness evaluation and consequential definition of actions in a very early development phase.

Based on a deterministic forming simulation, a number of possible realizations of the forming process are computed. The characteristics of the input scattering are described by statistical distribution functions. The robustness of the forming process is analyzed by means of correlation and variation analysis. Based on linear correlation hypotheses and their measures of determination and on variation measures displayed on the FE structures, a first evaluation of the robustness is performed. In the following, statistical measures of linear and quadratic correlation hypotheses and their variations are calculated on local level, i.e. the level of the FE discretisation.

In the present paper, the special demands on reliable correlation coefficients, measures of determination and fractile values regarding the representation on FE meshes are discussed.

Keywords: forming simulation, robustness evaluation, statistical measures

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1 Introduction

For many years, FE methods have been used to evaluate the forming process in industrial applications. In the beginning, the computation times were long, often yielding but unsatisfactory results. Nowadays, thanks to improved material laws and new numerical methods, the simulation is an essential part of the design and ensuring of forming processes. Nevertheless, most investigations are performed assuming deterministic conditions, i.e. all parameters are regarded as fixed. Thus, the simulation results can only describe singular process states. However, real systems are always subjected to scattering of process parameters. Such input scatters can lead to significant scattering of the forming results. Thus, a prognosis of the range of variations proves necessary.

Decisive input scatters might be scattering material properties or sheet thicknesses, variations in the cut blanks ensuing displacements in the moulding press, as well as scattering process forces or friction between forming tool and blanks to form. The quality of the component regarding the scattering of the forming results can be judged by verifying the dimensional accuracy of the blanks to form, as well as the compliance to the admissible thinning and the prescribed surface quality. If the component does not always meet the admissible tolerances because of process variations, the process is judged non robust. The resulting reject rates and necessary post-treatment and control represent a considerable expense factor in the production.
In practice, a trial and error strategy is often employed in order to achieve robustness. However, due to the high pressure of time and cost as well as resulting from a lack of precise knowledge of the effect of scattering input parameters, the area of action is restricted to the scatter reduction of only few process parameters. Thanks to the growing computing capacities and power, stochastic analyses of forming simulations are increasingly employed to predict the influence of process parameter scattering on the forming results. As the knowledge of input scatters usually is limited, it is recommended to start with so called robustness evaluations or sensitivity analyses of input scattering on important result variables. The advantages of this prognosis tool can be exploited in the early phase of the product development process up to the series production. However, the highest potential lies in the robustness evaluation and in the definition of appropriate measures in a very early development phase, as, in this phase, the component design as well as the production process are still accessible to fundamental changes. In the present paper, the current processes as implemented at the BMW group as well as the associated boundaries are described. Hereupon, the strategy to evaluate the process robustness is explained. The reliability of the statistical measures is a crucial criterion to decide whether the results of the stochastic analyses can serve as a base for measure definition. In the following, the conditions for a reliable computation and visualization of appropriate statistical measures are discussed. The implementation of these into the optiSLang post processing is explained and illustrated by an application on a structural component of the three series BMW.

2 Forming simulation process

2.1 Deterministic CAE simulation process

Finite element methods are successfully employed to simulate the forming process in industrial applications. In order to reduce the computation time, the meshes are adaptively refined during the calculation. To make the results comparable therefore it is necessary to map the results of different computation runs onto a uniform mesh.

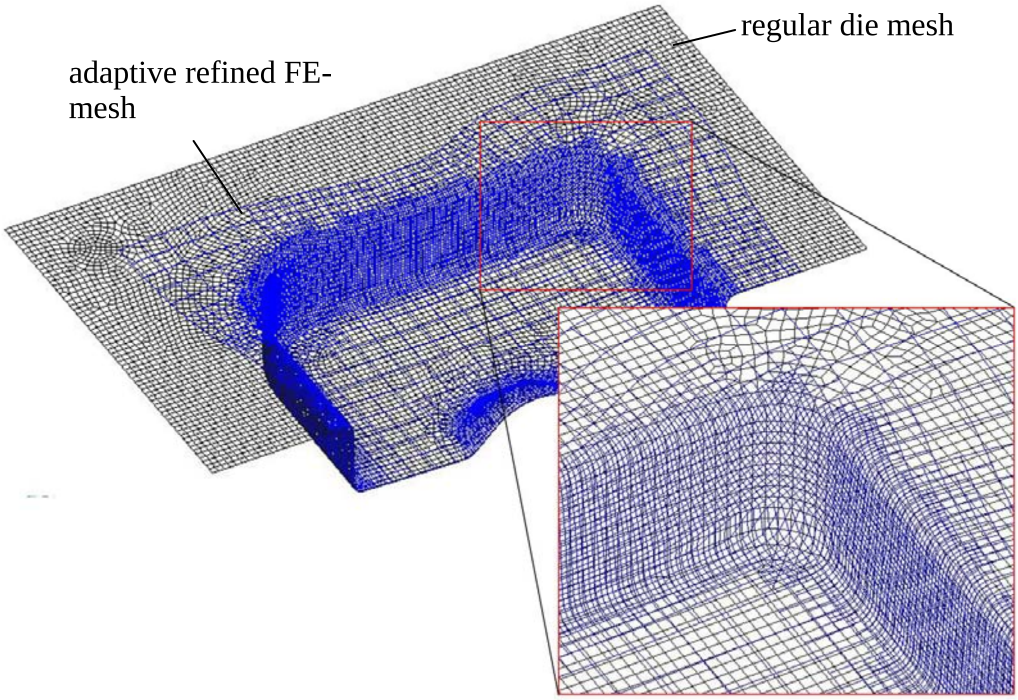

Figure 1: die mesh and FE simulation mesh

To this aim, either the tool mesh of the die or the finite element mesh of a reference simulation can be chosen as a basis. However, it has to be ensured that the stamping draw-in of the reference simulation is minimal in the totality of all simulated configurations. Because of the wide range of available solvers, different finite element solvers are used depending of the application and the available capacity. Therefore, the computation results are transformed into a solver independent internal meta format displaying the physical state variables, e.g. strains and stresses, in a uniform ASCII format.

2.2 Evaluation variables

The objective values of the forming simulation have to be numerically computable and the have to display the regarded quality criterion either in a direct or indirect way. An example for the latter case is stress criteria used to identity fold forming.
Some of the most important objective values are certainly the absolute resulting sheet thickness or the relative thickness reduction, as well as the principal deformation and the principal stresses.

3 Computational robustness evaluation

Robustness evaluations are also known as sensitivity analyses of input scattering on important result variables. Based on a forming simulation with a deterministic set of parameters, e.g. corresponding to the mean values of the scattering parameters, a number of possible realizations of the forming process are computed. The characteristics of the input scattering are described by means of statistical distribution functions, thus defining the probability space of possible realizations. Please note that the statistical measures of the result variables are naturally depending on the quality of the input information about the input scattering. In cases when only coarse assumptions on input scattering are possible, the statistical measures should be judged as nothing more than a trend, taking into account that the measures of small probabilities (e.g. 3 sigma values) are afflicted with very high uncertainties.

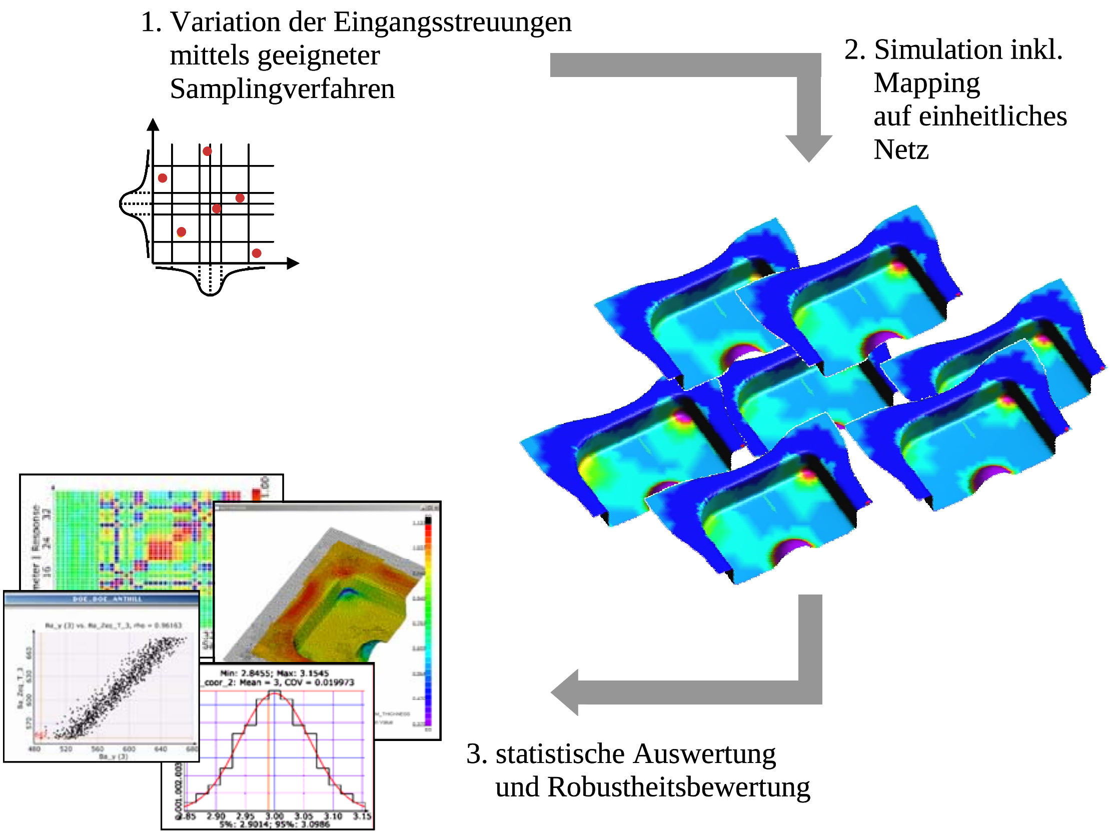

Figure 2: Robustness evaluation procedure

By means of an appropriate sampling strategy, a number of possible samples are generated and calculated. The suggested methodology to generate the samples and to compute the statistical measures ensures an as good estimation as possible of the statistical measures requiring as little computation runs as possible. The sensitivity is estimated by means of statistical measures of variation and correlation analysis and evaluated regarding robustness requirements. Once the most important scatters are identified via robustness evaluations, small probabilities of occurrence can be estimated by means of reliability analysis.

3.1 Definition of scattering input parameters

The first step of a robustness evaluation is to assess and model the input scattering. Hereto, the following methods are available:

1. Determining the scatter of the input parameters by means of tests and translating them into appropriate statistical measures, i.e. distribution functions.
2. Assessing the distribution functions based on the relatively coarse knowledge on presumed scatters, generally assuming a uniform distribution and associating upper and lower boundaries of scattering to different levels of probability.

The software used for the robustness evaluation should be capable of taking into account the complete existing knowledge on input information. This means that appropriate distribution functions (normal distribution, cut-off normal distribution, lognormal distribution, Weibull distribution or uniform distribution) should be available, and correlations of single scattering input variables or of spatially correlated random fields need to be considered.
An example shall illustrate this necessity: Generally, in forming simulations, the yield curves of steel are described by the yield strength, tensile strength, and the strain at failure. The yield strength and tensile strength often can be described by a lognormal distribution, while the strain at failure usually follows a normal distribution. But the material variables are significantly correlated. Only by taking into account the complete statistical information (distribution and correlation) in the sampling method, realistic yield curves are computed from a "random" choice of the three correlated scattering input parameters. On the other hand, choosing a normal distribution for all three scattering input variables without taking into account the correlations would result in a large amount of unrealistic yield curves.

Generally, only such input scatters can be taken into account that can be directly or indirectly incorporated into the simulation model. In the forming process, such input scatters are, e.g.:

- Material parameter, e.g. yield strength, tensile strength, n value, R values
- Sheet thickness
- Cut blanks
- Blank position in the tool
- friction value

3.2 Sampling methods

The estimation of the statistical measures from a sample of possible realizations naturally is afflicted with an error. In order keep this error as small as possible while the number of computations is relatively small, Latin Hypercube sampling methods are preferable to Monte Carlo sampling method.
By investigating the linear correlation coefficients, it can be shown that, for the same expected error, Latin Hypercube samplings are ten times more efficient than Monte Carlo samplings. However, the number of computations needed to ensure a

certain error size depends on the total number of scattering input variables plus the number of the result values to estimate. Thus, the probability that the maximum error of single correlation coefficients rises is increasing with an increasing number of result value. Therefore, in many engineering disciplines, typically only a small number of significant result variables are considered in robustness evaluations. In the special case of forming simulations, the necessity arises to visualize the spatially strongly correlated statistical measures on the finite element structure. This results in a very high number of correlation coefficients to estimate. Thus, a special procedure is necessary to even so achieve reliable statistical measures with a relatively small number of calculations (e.g. 100).

3.3 Robustness evaluation by means of statistical measures

The histogram of the result variables contains the statistical measures which together with correlation analysis form the base for identification of noticeable connection between the variation of individual input variables and the variation of individual result variables. Correlations determined by linear and quadratic correlation hypothesis thereby characterize a measure of linear and quadratic connection between parameters. The correlation coefficients in turn form the base of measures of coefficients of determination of individual result variables which are percent wise estimates of the ratio of variance which can be explained by the correlations to all input variables.

The histogram of the result variable contains variation measures, i.e. standard deviation, min/max values or 3-sigma-values, describing the measure of variation. Starting with the linear correlation hypothesis and its measures of determination as well as variation measures displayed on the FE structure a first evaluation of robustness can be made. The found “hot” spots should then be statistically validated locally on element level. Should small measures of determination be found in the area of relevant scatter on the FE structure then continuative statistic evaluations (i.e. quadratic correlations hypothesis or cluster analysis**)** on element level are essential to determine robustness.

3.4 Specific requirements for visualising statistical measures on forming simulations

A visualisation of statistical measures on the FE-mesh considerably facilitates the engineering evaluation since the result values of a forming simulation which are to evaluate are generally spatial correlated values. The statistical measures on the FE structures serve as discussion basis for identification of critical areas and as a basis of comparison for evaluating the quality. In addition this type of representation leads to a high acceptance of the results in the production departements. Therefore it is important to visualize the statistic measures directly on the component and respectively on the corresponding reference mesh and to communicate them in the hardware process. Mean value, variation coefficient, standard deviation and min/max values thereby can be determined in the FE discretisation (node or element wise) and displayed on the FE structure.

As well the computations of the correlation coefficients as the reliable determination of fractile values of small probability levels (i.e. 3-sigma-values as basis of process stability) however have specific requirements.

3.4.1 Projection of result variables into a subspace in order to determine correlation structures

The amount of estimated correlation coefficients and the amount of necessary computations to ensure the desired confidence interval rise significantly, if the correlation coefficient is determined for every point of discretisation or for every finite element.
Concerning forming simulations furthermore it can be assumed that the correlation structures are strongly spatial correlated and that therefore the estimation on discretisation level creates noise on the existing correlation structures. If a single simulation run costs significant CPU, then it can be assumed that the necessary amount of samples to eliminate nameable noise is too costly, too. In the following it is shown that by a projection via stochastic fields a significant reduction of correlations coefficients to estimate and a reduction of noise can be achieved. Thereby a higher accuracy of prognosis can be obtained while using a significantly smaller amount of samples and when using a smaller amount of computations (e.g. 100) small confidence levels of the estimate of the statistical values can be obtained respectively.

For correlation analysis the result variables are projected onto a reduced base in order to reduce the amount of correlations coefficients to estimate (limiting the necessary amount of samples). The base appropriately is assumed as orthogonal over the structure.

An appropriate base results from spectral decomposition of the covariance function of probability fields. Assuming that a structural result $H(x)$ is spatial correlated stochastic distributed the mean value function then is defined by

$$H(x) = E[H(x)] \qquad (1)$$

E refers to the expectation operator and the mean value over the ensemble of all realizations (fig. 3) respectively. The covariance function is defined by

$$C_{HH}(x, y) = E\left[\{H(x) - \overline{H}(x)\}\{H(y) - \overline{H}(y)\}\right] \qquad (2)$$

The covariance function shows the correlation between the values of the stochastic field at different positions x and y. In most instances of FE-applications the stochastic field $H(x)$ is a priori discretized which leads to

$$H_i = H(x)\ ;\ i = 1 \ldots N \qquad (3)$$

A spectral representation of this discrete stochastic field is given by

$$H_i = \sum_{k=1}^{N} \phi_k(x_i) c_k = \sum_{k=1}^{N} \phi_{ik} c_k \qquad (4)$$

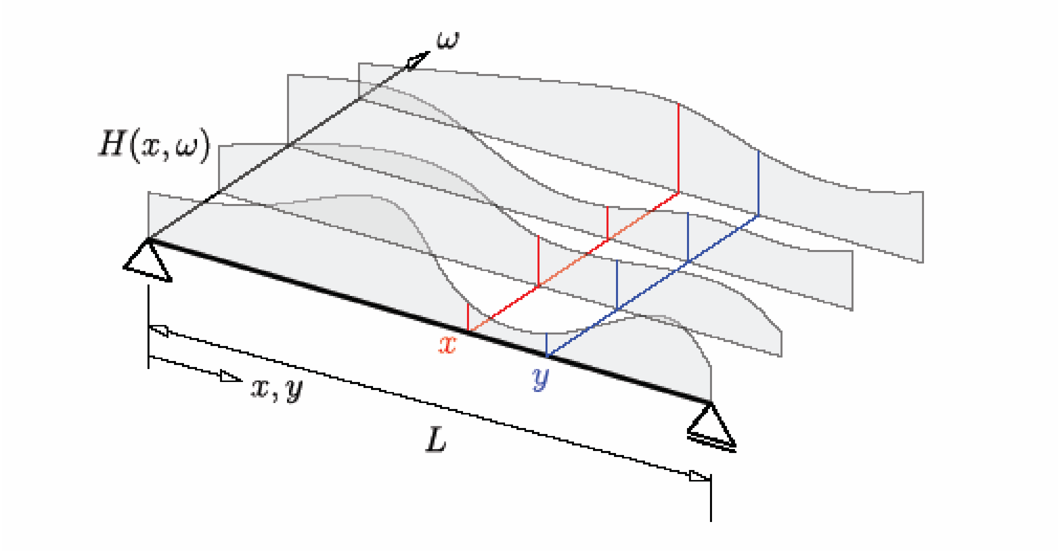

Figure 3: ensemble of realisation of one-dimensional probability fields of structural result H (the covariance matrix describes the values of the probability fields at any positions x and y)

This is the index notation of a matrix-vector multiplication:

$$H = \Phi c \qquad (5)$$

The orthogonality condition for the columns of Î¦

$$\Phi^T \Phi = I \qquad (6)$$

are automatically fulfilled if these columns Ï†k are solutions to the following Eigenvalue problem:

$$C_{HH\phi_k} = \sigma_{c_k}^2 \phi_k \;\; ; \; k = 1..N \qquad (7)$$

In the following examples it is assumed that an isotropic exponential covariance function is used to generate the orthogonal base Ï†k . Therefore only a number m << N of base vectors actually are computed and used. Per result value the covariance matrix of the projected value as well as the matrix of the correlation coefficient corresponding to the input scatter is computed. In the subspace also coefficients of determination are computed from the correlation between result variables and input parameters assuming a linear correlation hypothesis. After determining the correlation in the subspace the results are projected back onto the FE-mesh.

3.4.2 Determination of reliable fractile values

Fractile values with small probabilities (in this case 3-sigma-values usually this means a value with a failure probability of 0.0013) generally can be estimated from the sample set when doing computational robustness evaluations or calculated by assuming distribution functions from mean value and standard deviation. The calculation of the fractile values via distribution functions is strongly recommended because when doing robustness evaluation it can be assumed that there are too few realisations for a reliable estimation of small probabilities from the sample set. The determination of fractile values from mean value and standard deviation is however tightly bound to the assumption of a certain distribution function generally the normal distribution. This is a hypothesis for which's validation often no traceable justification is given. It therefore is recommended that non-normality of scatter in the result variables is eliminated by transforming to normal distribution. After determining the fractile values in normalized space they are transformed back. By doing so the distribution hypothesis to determine fractile values is assured and is not subject to scattering. Thereby especially implausible jumps of the 3-sigma-values of adjacent discretion points can be avoided resulting from various distribution hypothesis. An appropriate transformation is given in form of a quadratic function

$$x = a + bu + cu^2 \tag{8}$$

Whereas the coefficients are computed by regression over the 20% highest values (superior fractile value) of the sample and respectively the 20% lowest values (inferior fractile value) of the sample (cp. Fig4). The wanted„ 3σ " value results from

$$\xi = x(3) = a + 3b + 9c \tag{9}$$

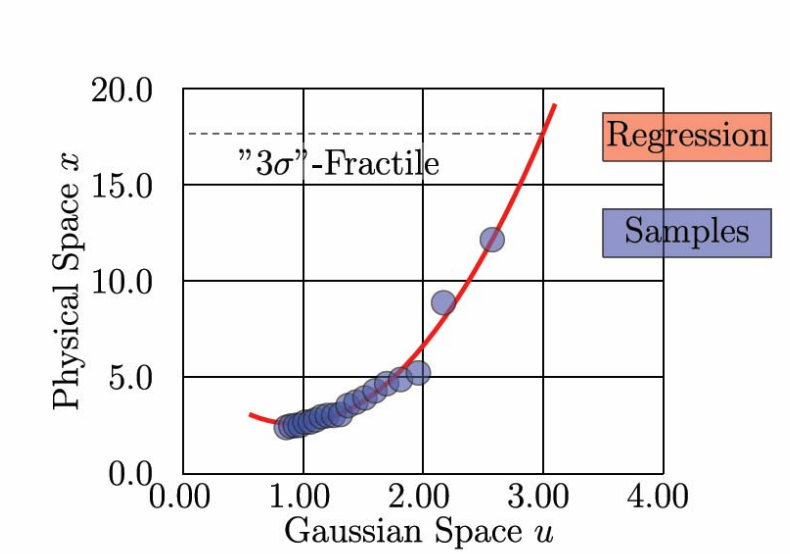

Figure 4: Determination of fractile values by means of transformation on normal distribution

4 Example – Rear extension sideframe, three series BMW

For demonstration purposes the possibilities of the optiSLang post processing are demonstrated using a car body part, the extension of the rear extension sideframe.

4.1 Description of components and robustness task

The component is manufactured in several maintenance sequences, i.e. using a deep draw operation and several following operations. During the last following operation amongst others the flange is set, as shown in the following figure. Occasionally difficulties arose concerning this matter during tool try-out, so the process could not be called robust. The process only could be stabilised by complex changes of the geometry.

Figure 5: rear extension sideframe, three series BMW

The blank is cut from the usable waste of the blank of a different, larger component. That way the comparatively small component is dependent on the tolerance limits of the large coil. Thus the assumption of the manufacture was that the size of the cut blanks as well as the connected variation in length between the gauge pin were the reason for the process not being robust. This assumption should be reviewed by doing sensitivity- and robustness analysis using optiSLang.

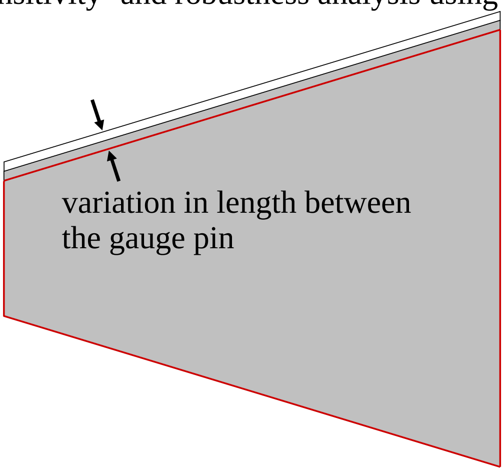

Figure 6: Blank

For this drawing simulations were reviewed, because it was know from real measurements that the thinning of the critical area underlies measurable fluctuations during this process step. During the following operations a significant amplification of the thinning arises which can ultimately lead to material failure. The reason for this risk of cracks in the manufacturing process lies in the pre damage of the material by the draw operation.

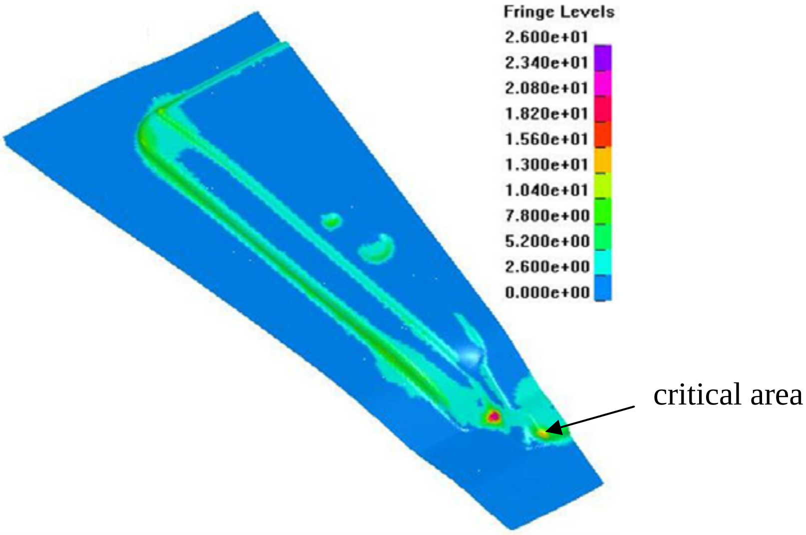

Figure 7: Drawing simulation

4.2 Results of the robustness evaluation using OptiSLang

Besides considering the cut blanks and respectively the position of the blank also the variation of the sheet thickness, the yield stress und R-Values (R90, R45, R0) were considered as input scatter. Since no real distribution function of the parameters was known a uniform distribution of the values with upper and lower boundary of assumed scatter was adopted. As the estimation of the input scatter was relatively raw, in the following mainly correlation coefficients are evaluated and using those sensitivities considering the suspected causes of the crack problem are performed.

Overall 100 computations (optiSLang Latin Hypercube Sampling) were made using the solver LS-Dyna, which in each case were mapped on a standardized mesh and converted to an internal meta format. The statistical measures then were determined and visualized using the aforementioned optiSLang post processing. In Figure 8 the result value thickness reduction is displayed element wise as minimal value and respectively maximum value. Thereby blue stands for large and red for a small thinning value.

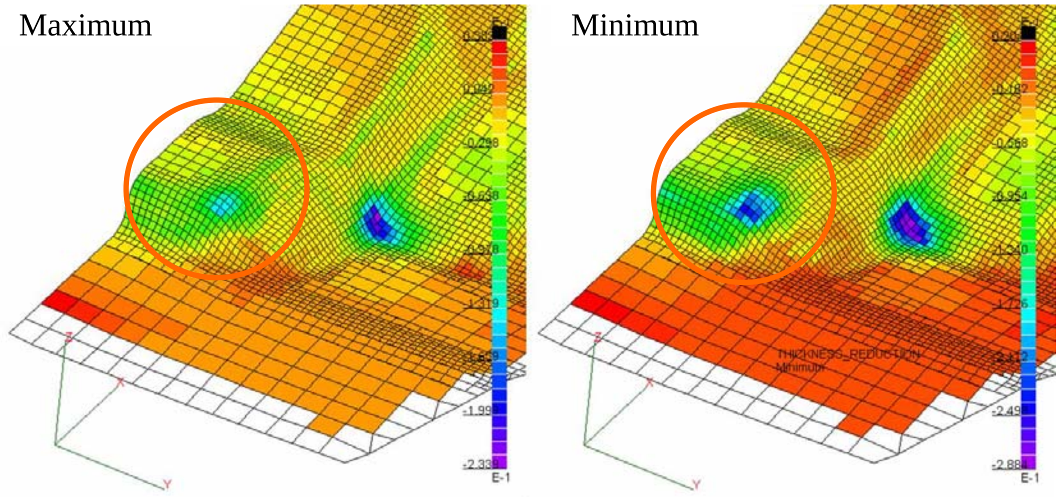

Figure 8: Min/Max-values thickness reduction

In the following figure the mean deviation of the result variable thickness reduction is shown. One can recognize that the thickness reduction varies strongest in the area considered critical.

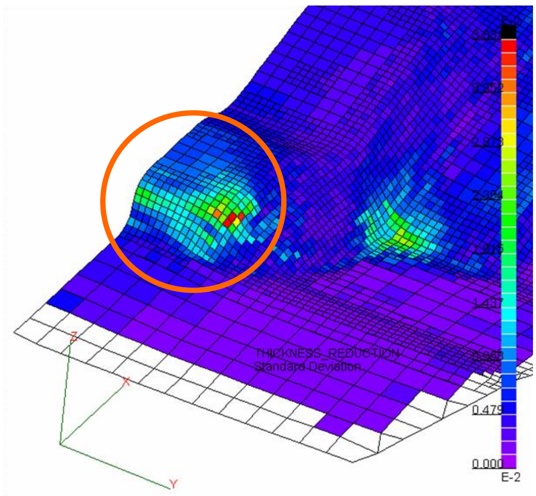

Figure 9: Standard deviation - thinning

The measure of determination of the examined result value can now also be mapped on the FE-mesh and gives information about the amount of variation of this result variable over all linear correlations to the input variables that can be explained. A larger measure of determination shows that linear correlation analysis is sufficient to identify the important input parameters. A smaller measure of determination shows that nonlinear correlations are important or that numerical noise of the CAE-computation affects the result variable significantly. Therefore the measure of determination may also give some a measure of quality for the numeric CAE-process. In the present case the measure of determination of linear correlations is large (>80%) in the region of interest. The most important coherences can be explained this way using linear correlation coefficients.

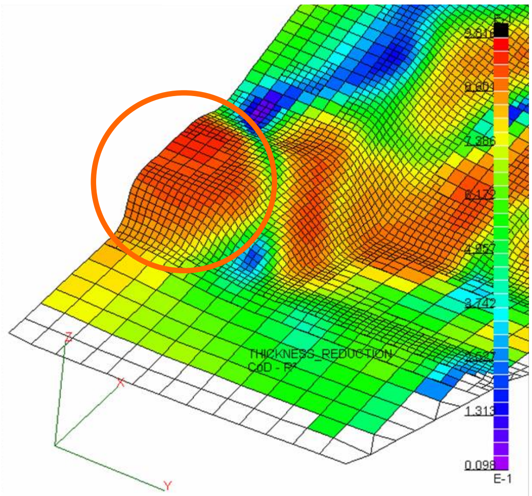

Figure 10: linear measure of determination – thinning

Using plots of the correlation values of the single scattering input variable it can be detected which input scatter determines the result variable the most. In Figure 11 the correlation values of the R-values and the position Y are visualised for thinning. It can be clearly seen that the variation of the sheet thickness is dominated by the R-values. The position Y does not show any noteworthy correlation in this area.

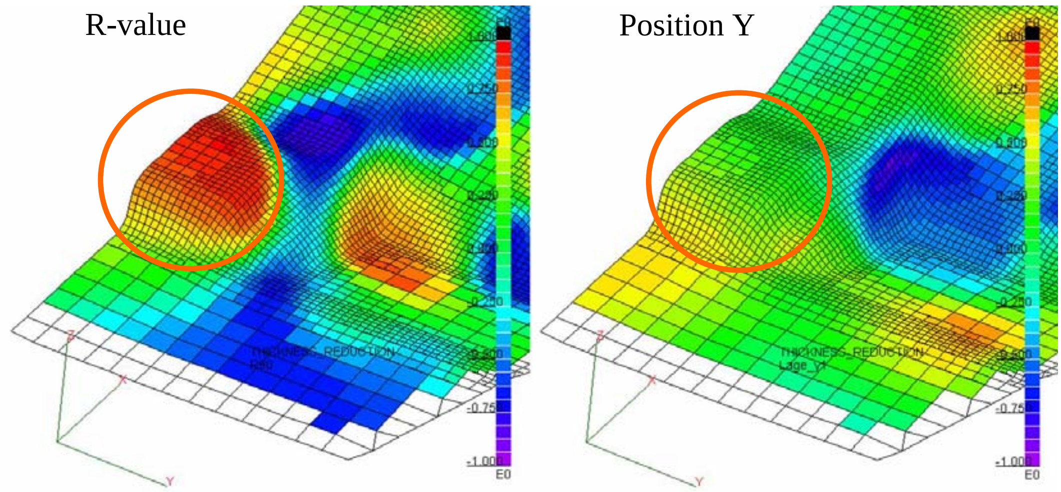

Figure 11: Correlation to Thickness Reduction

For further detailing of the statistical measure now histogram, measures of determination and anthill-plots can be displayed element wise. For this purpose the element with the greatest thinning in the area of interest is chosen.

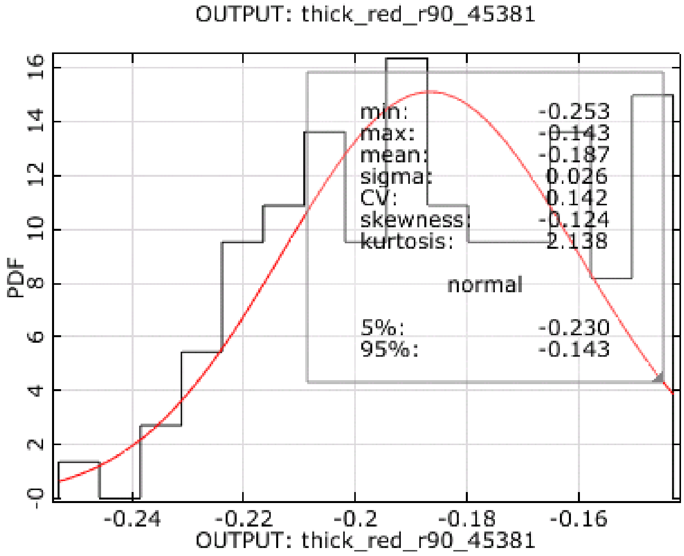

Figure 12: Histogram of the element with the highest thinning in the „hot spot“

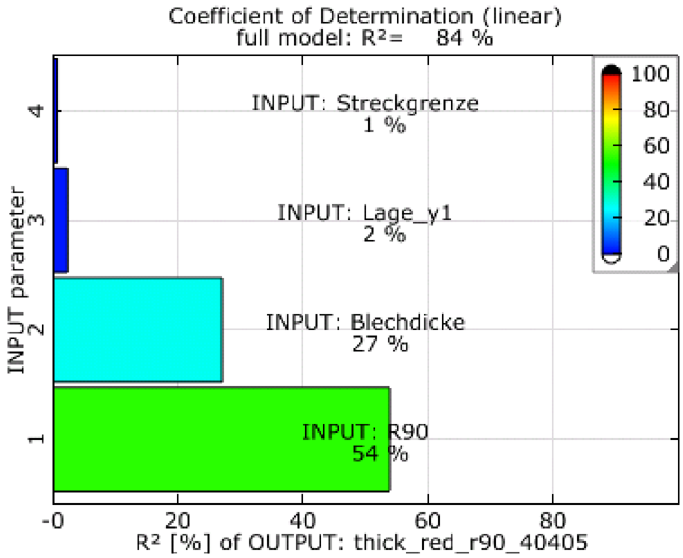

Figure 13: Measure of determination of linear correlation

The local measure of determination of linear correlations (figure 13 ⇨ 84 %) affirms the results on the FE-mesh. Taking into account quadratic correlations the measure of determination rises to 91% (figure 14). The distinct dominance of the material characteristic R-value is reflected in the Anthill-Plot regarding thinning (figure 15), the linear correlation, which according to figure 13 constitutes to 54% of the variation in thinning can be clearly seen (orange line). The scatter of all the other process parameters furthermore causes the scatter around this linear correlation.

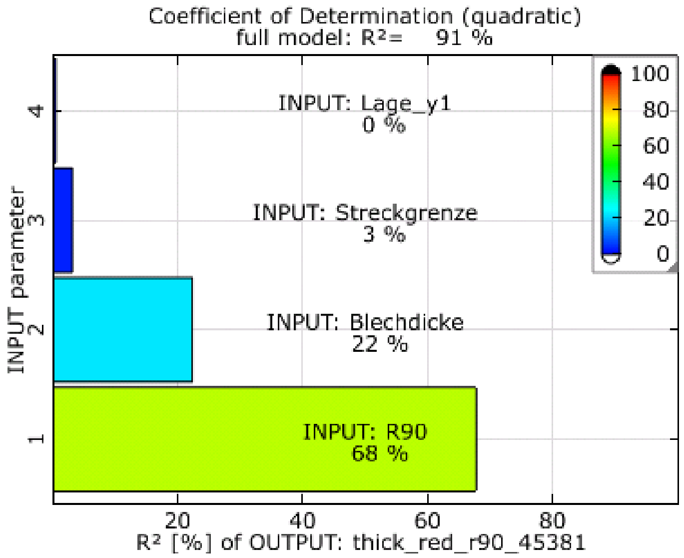

Figure 14: Measure of determination of linear and quadratic correlation

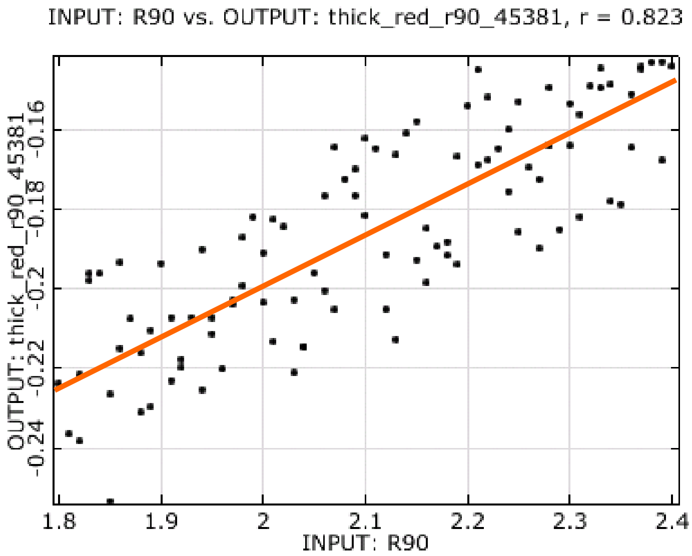

Figure 15: Anthill plot thickness reduction over R90, element 45381

4.3 Interpretation of the results

Using the visualisation it becomes apparent that the area of the component viewed as critical shows a large scatter of the result variable “Thickness Reduction”. The suspected influence of the size of the blank has very small correlation coefficients and measures of determination in this area and therefore can not be identified as the main cause. The greatest influence on the scatter of the thinning results from the assumed scatter of the R-value, followed by the scatter of the sheet thickness. Scatter of the yield stress and the position of the blank in contrast do not noteworthy affect the thinning in the area of interest.

Using the visualization of statistical measures the sphere of influence of different input parameters can quickly be identified. On local level these insights can be verified using further statistical measures. This allows a reliable identification of decisive input scatter on scatter in the forming process. At this point it shall be pointed out that the above results apply under the assumption of scatter as described in chapter 4.2. When adding further scatter the statistical measures can all change significantly if these scatter have significant influence on the scatter of the result variables.

5 Outlook

The procedure shall be further automatized and made accessible by the regular design process. Here fore, the optiSLang post processing module shall be enhanced by a graphical interface to the meta format.

The number of stochastic fields used for the projection of the result variables is crucial for the usability of the linear correlations and measures of determination on the finite element mesh.
As can be seen in 17, 100 stochastic fields are sufficient to determine the correlations in the areas of maximum variation more reliably than without projection (figure 16). Using 200 stochastic fields, only correlations in areas of little variation change compared to the projection with 100 stochastic fields. This „error“ of the projection might be tolerable, as the correlation relationships are of no interest in these areas. Nevertheless, this points out the necessity to verify correlation relationships on element level.

In view of the automatization, methodological enhancements might prove necessary to calculate and evaluate a spatial quality measure of the projection. This might serve as a criterion for the choice of the number of stochastic fields.

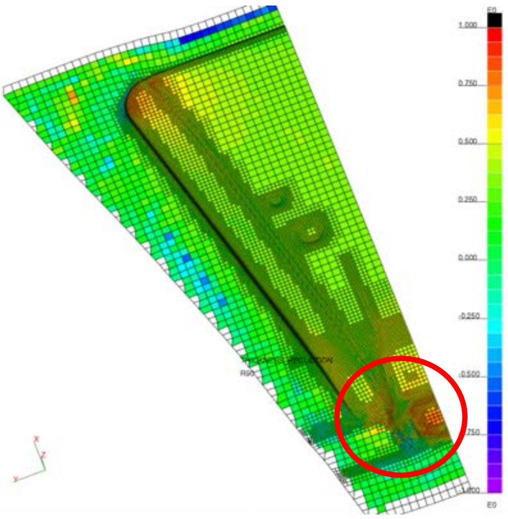

figure 16: projected linear correlations. without projection

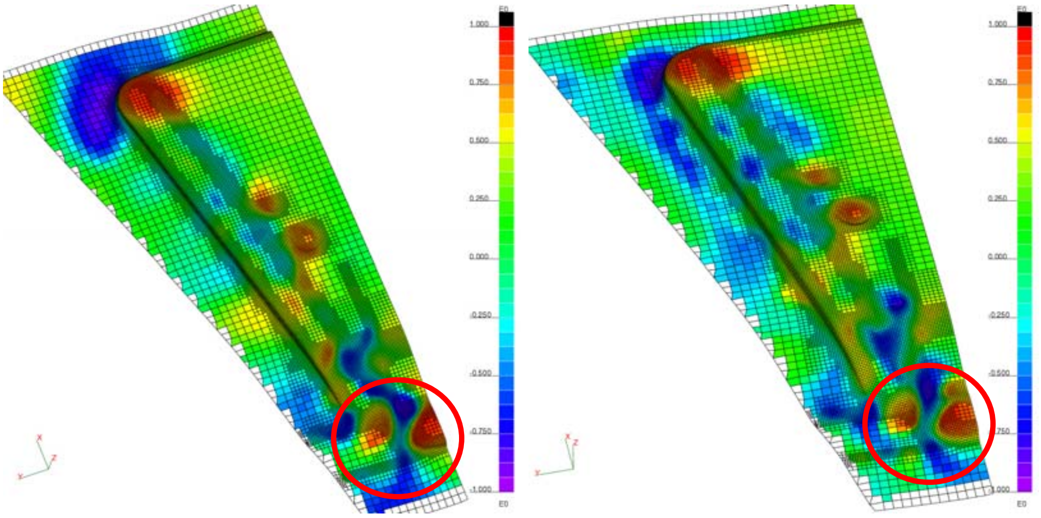

figure 17: projected linear correlations. left with 100, bottom right with 200 stochastic fields projected

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