

# Robustness Evaluations in Virtual Dimensioning of Passive Passenger Safety and Crashworthiness

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## Summary

One of the most important tasks of vehicle development is the steady improvement of passenger passive safety systems. In reality significant result scatter can be observed when performing crash-tests. Cause of this scatter of important vehicle performance variables is scatter of variables concerning the dimensioning of passenger passive safety systems, vehicle structure and testing conditions. A computational robustness evaluation of important result variables can only be obtained by integrating stochastic simulation methods into virtual product design processes [1-2]. Primary result of the robustness evaluations is the calculation of the scatter of performance variables and of the connected probability of achieving safety goals. Secondary result is the investigation of the numerical stability of the models and identification of the input scatter, which is responsible for the output scatter. In this paper the fundamentals of computational robustness evaluation are explained compendiously and the experience gained by the systematical introduction of computational robustness evaluations at BMW AG [3] is discussed. Because of the complexity of finite element-models their robustness has to be examined with great care. Using a robustness evaluation for the front-crash load case of the UL-SAB-study an approach for identification and quantification of numerical noise is presented.

**Keywords:** Robustness evaluation, measures of determination, optiSLang

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# 1 Introduction

In the past deterministic models were used by multi-body or finite element programs for dimensioning of passenger passive safety systems. In reality, however, significant scatter can be observed when performing crash-tests. Cause of this scatter of important vehicle performance variables is scatter of variables concerning the dimensioning of passive safety systems, vehicle structure, the material, the crash-test dummies, the loads and the testing conditions. This results in the necessity to pre-compute not only single values but also to be able to extract information about the scattering of important evaluation criteria.

The necessity of integration of stochastic simulation methods is determined by further trends in virtual product design.

- By increasing optimization, designs can reach their limits and become very sensitive towards scattering
- Because hardware cycles occur later and less often, the influence of scatter, which was still prominent in hardware tests and its influence thereby was at least detected by single tests, has to be taken into account in virtual product design.
- If larger changes in construction are made within very short time (high innovation speed) and more and more complex component systems concur the a priori knowledge (experience) about reliable functionality possibly is very small. Therefore the robustness of the systems has to be determined using virtual models.
- Substantial vehicle concept decisions have to be made in an early stage of development basing on virtual dimensioning. This requires best possible knowledge about the degree of fulfilling the goals (laws, consumer ratings) and respectively a quantitative estimation of the remaining risk.

Robustness evaluations using variation analysis [1] are suitable since it is not the securing of small transgression probabilities that is most important in evaluation of robustness concerning passenger safety. Primary goal of robustness evaluations using variation analysis is the prognosis of a variation range of significant response variables and their evaluation using standards of „robust“ restraint systems. In passive passenger safety limits are defined by the legislator and the vehicle developers set their own limits with a security distance to the statutory limit values. Furthermore it is a goal that vehicles reach an as good evaluation as possible in tests by consumer protection (e.g. Euroncap). These requirements should be met by the majority of the vehicles. However detecting rare transgression probabilities is not the main goal at present. If small probabilities (for example less than 1 in 1000) are to be verified one should use methods from reliability analysis [4-7]. Because the methods of reliability analysis are only feasible in small parameter spaces, robustness evaluations using variation analysis are a necessary preliminary stage for the reduction of parameters.

Secondary goal of robustness evaluations is the identification of correlations between input- and output-scatter and a quantification of the thereby explainable components of the variation of result variables as well as the quantification of the influence of “numerical” noise on the output scatter.

## **2 Computational Robustness Evaluation using Variation Analysis**

For evaluation of robustness all potential input scatter of material, car or test condition are introduced to virtual product design process by using scattering input variables in the numerical models. Using appropriate sampling methods a sample set of  $n$ -possible vehicles and  $n$ -possible crash test conditions are generated and then computed. After the computation the sample set is then evaluated using statistical methods for estimation of variance and correlation. In order to estimate the scatter of the result variables from the sample usually mean value, standard deviation, coefficient of variation and the range of variation (min/max value) are determined for every important response variable. If the detected ranges of variation lie too close to the limit values or even exceed these, one has to ask for the frequency (probability) of exceeding the limits. If overstepping occurs in the calculated support point set, the frequency can be counted. In statistics one would talk about determining the empirical probabilities directly from the histogram. Alternatively distribution functions of the result variables can be assumed and the probabilities can then be computed from the characteristic values of the distribution function.

For significantly scattered result variables or transgression of limits the responsible input scatter is identified using correlation analysis. For this purpose pair wise linear and quadratic correlation coefficients of result and input scatter are computed. The correlation coefficients can obtain values between 0 and 1 (-1) and show the pair wise correlation between a single input scatter and a single output scatter. For identification of mechanisms in which multiple input scattering affects on output scatter the principal components (the eigenvectors of the correlation-matrices) can be evaluated.

In the following it is estimated how much of the result variation can be explained using the identified (linear and quadratic) correlations. This is done by using measures of determination [8]. The determinedness of a result variable regarding the variation of all input scatter describes which percentage of the result variation can be explained by the found correlations to the input variables. If the coefficient of determination of a result variable is high (>90%) the fundamental interrelations can be described using the underlying correlation hypothesis. The smaller the coefficients of determination are the larger the part of the variation of result variables becomes which can not yet be explained by the correlation hypothesis (e.g. linear and quadratic). Typically then non-linear correlations, clustering, “outliers” or a high amount of “numerical noise” exist. This way the measure of determina-

tion therefore provides information on the possible ratio of numerical noise and should be used as an important quality measure for the used modeling. In the robustness evaluations performed so far it could be detected that for coefficients of determination larger than 80% the influence of numerical noise on the performance variables was acceptable.

Choice and complexity of the sampling methods have to be adjusted according to the important statistical measures which are to be estimated. Normally the sampling method is adjusted according to a reliable identification of linear coefficients of correlation. Thereby the number of computations for robustness evaluations of restraint systems results in about 100 to 200 per load case that is to evaluate [8]. Suitable method for this is a Latin-Hypercube-method which fulfills the input distribution function as well as it minimizes the deviation between defined and created input correlations.

### **3 Statistic Description of Input Variables**

Important scattering input variables in virtual dimensioning of restraint systems are e.g. scattering of airbag parameters, scattering in the seat-belt-system and of the seat positions of the crash-test-dummy or scattering of the structural components.

The reliability of the prognosis of the output scatterings is explicitly connected to the closeness to reality of the definition of the input scatterings. In practical applications one can often not assume that all significant input scatterings are captured close to reality at the beginning of stochastic computations. Therefore one will realistically start using relatively rough assumptions concerning the input scatter and the input uncertainties respectively and then improve the knowledge on the significant input scatter step by step.

Input scatter is described using distribution functions. Important distribution function types are e.g. uniform distribution for friction values, normal distribution for mass flow values or log-normal distribution for material strength. If correlations between single scattering input variables exist, they have to be taken into account for the input information using adequate correlation models.

OUTPUT: Streckgrenze vs. OUTPUT: Zugfestigkeit,  $r = 0.660$

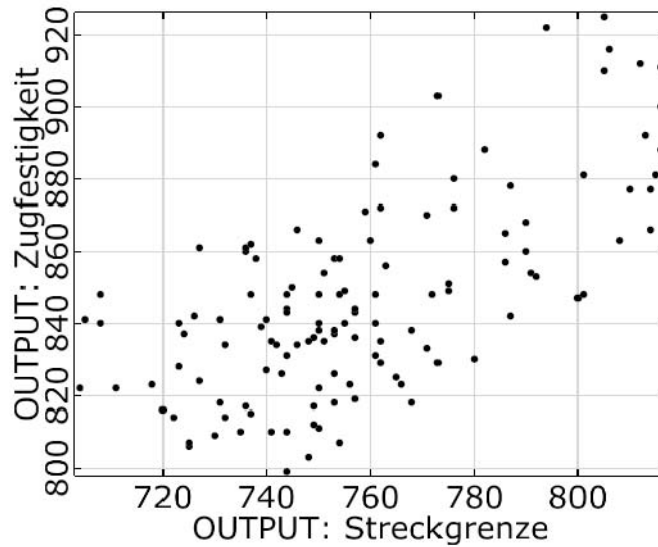


Figure 1: Correlation between the scattering tensile strength and the yield strength of steel

As an example for important interrelations between input scatter the correlation between tensile strength and the yield strength of steel shall be mentioned. In this case one would determine the linear correlation coefficient between both of the scattering input variables for example from available experimental data (shown in figure 1 with a correlation coefficient of 0.66) and consider it as important input information in sampling methods.

#### 4 Requirements for successful Integration of Robustness Evaluations into the Virtual Product Development Process

For a systematic introduction of stochastic methods of computations the following significant boundary conditions have to be met:

- Numerical model and simulation methods have to possess the ability of prognosis and therefore have to be able to map all significant physical phenomena and compare them to single experimental data. The computational process is to be parameterized concerning the input scatter and has to be processed fully automatically.
- The existing knowledge on input scatter and uncertainties for example in boundary conditions, material values or load characteristics are to be transferred to an appropriate statistical description and have to be integrated in virtual product design as significant input information for stochastic analysis.
- A stochastic method has to be used for robustness evaluations which make sure that the errors within the estimation of the statistical characteristics are small

enough and therefore that the results can be used as reliable foundation of a robustness evaluation.

- For evaluation of the robustness meaningful, reliable statistical characteristics have to be derived. The evaluation process is to be automatized and standardized.

Furthermore one can assume that a consequence introduction of stochastic computation methods can be divided into two phases.

Phase 1: Scatter and uncertainties of input variables are estimated from a few measurements and empirical values:

- Transfer of existing knowledge on input scatter and uncertainties of testing conditions in distribution functions as suitable input for stochastic methods.
- Robustness evaluation of important crash-test load cases, estimation of the variance of important vehicle performance variables, inspection if limit values are exceeded by the variation of the performance variables.
- Inspection of model robustness/stability using coefficients of determination.
- Extraction of most significant correlations between scattering input variables and important performance variables as well as the matching of these mechanisms with expectations and knowledge based on the experiments.

Within and respectively as result of phase 1 the following has to be discussed and arranged:

- At which point in time in the virtual development process robustness evaluations of components, modules or whole vehicles are performed
- For which input scatter the assumptions about the scatter have to be verified
- How scatter of critical performance variables can be reduced or relocated
- Which exceeding probabilities are tolerable for the performance variables

Phase 2: sensitive scattering input variables are known and the assumptions about their scatter are verified:

- With secured knowledge about the input scatter robustness evaluations are performed at predefined milestones of virtual product process
- Assuming that all important input scatter were considered close to reality and that the numerical models show negligible numerical noise then the estimate of the scatter of important input variables is trustworthy.

In the second year of the serial use of stochastic analysis in passenger safety simulation at BMW we currently are in phase 2. The following added value could be obtained concerning dimensioning and increase of the robustness of the restraint systems:

- Development of a better understanding of the transmission mechanisms of input scatter on significant performance variables
- Identification of the significant scattering input parameters and securing of knowledge about their scattering

- Identification of model weaknesses and reduction of numerical noise of significant vehicle performance variables. Thereby increasing the model robustness/stability and of the quality of prognosis of crash-test computations
- Recognizing robustness problems of the restraint systems and in cases of high exceeding of limits with the consequence of redesign of components.

## 4.1 On Numerical Robustness of Crash-Test Computations

The inspection of numerical robustness of numerical models of crash-test computation results from the experience, that the variation of numerical parameters of the approximation method or the variation of demonstrable insignificant physical parameters can lead to large scattering of the result variables or respectively lead to obviously unfeasible results. If n-designs are to be computed and their variation is to be evaluated statistically, the question arises which proportion of the resulting variation can be attributed to problems of the approximation method and the numerical modeling respectively.

The quantitative influence of numerical noise on the result variable can be estimated using coefficients of determination of the robustness evaluation to naturally occurring scatter. If the measure of determination of the robustness evaluation is high, only a small proportion of unexplained variation, which could be caused by numerical noise, is left. In order to use the measure of determination of result variables as a quantitative measure for the numerical model robustness, the proportion of determination of the found correlations has to be estimated with sufficient statistical security. This formulates standards for the sampling method, the number of computations and the statistical algorithms for the evaluation of measures of determination. After very positive experience of evaluating the influence of numerical noise via measures of determination from robustness evaluation this method is for serial production at BMW since 2006 [3]. For “numerically” robust models measures of determination considering linear and quadratic correlations and after elimination of outliers and clustering of over 80% could be determined. If the measures of determination decreased significantly below 80% it was usually an indicator that the corresponding result variable possesses a significant amount of numerical noise. Cause here for were insufficiencies in the result extraction and especially insufficiencies of the numerical models interacting with the approximation methods. After repairing the numerical modeling the measure of determination usually increased to over 80%.

It shall be stated that in theory it is impossible to determine without doubt the proportion of numerical noise.

The subject of bifurcation points surely is to be discussed separately. For the purpose of robust designs one would want to vastly avoid systems with bifurcation points, which can be traversed in multiple ways within the scatter range of input variables and then lead to significantly different system responses. As a matter of principle one would, however, have to be able to find correlations between indicators of bifurcation or results heavily influenced by bifurcation and the input scat-

ter, otherwise the bifurcation occurs at random which implies that we are dealing with a very sensitive dynamic system.

For robust designs the correlations between input variation and output variation should basically be identifiable with high certainty. These correlations also show the possibilities for influencing the result scatter. In order to reduce transgression probabilities it for example is possible to reposition the mean value in the linear case or for quadratic correlations to reduce input scatter or to change the transmission behavior between input and output scatter.

This diagnosis of course excludes systematical errors or the inability to actually map significant physical effects of input scatter on output scatter. The fundamental prognosis ability of the numerical models has to be verified using experimental data.



## 5 Practical Application

### 5.1 Robustness Evaluation for the Load Case Front-Crash FMVSS 208

Since beginning of 2006 computational robustness evaluations using optiSLang [9] are a defined milestone of serial production at BMW AG executed for all relevant load cases for dimensioning of passive safety systems [3]. The procedure is exemplary introduced for the load case FMVSS 208 (front-crash 40 km/h, unbelted, against steep wall). The robustness concerning significant evaluation parameters of driver and passenger was tested.



Figure 2: Simulation Passenger Safety Load Case FMVSS 208

The model was created and computed in MADYMO. A multi-body-formulation was used for most parts of the restraint system and the dummy and a finite-element-formulation was used for the airbag. For the robustness evaluation 200 variants were created in optiSLang using Latin Hypercube Sampling and then computed. Overall 27 physical parameters of the multi-body/finite-element-modeling were varied and 18 dummy result variables were analyzed in the robustness evaluation.

For the definition of the scatter uniform distributions and normal distributions with cut offs at 2 or 3 Sigma Level were used. The following scattering input parameters were considered in robustness evaluation:

- Scattering of the time to fire of airbag and load-limiter
- Scattering of the dummy seat position
- Scattering of mass flow, permeability of the airbag
- Scattering by the load limiter

- Scattering of friction between dummy and airbag, airbag and steering wheel as well as between dummy and seat
- Scattering of impact puls
- Scattering of feet space, foot rest, pedal

The following result variables were examined in the robustness evaluation:

- Head resultant acceleration 3 ms
- Chest resultant acceleration 3 ms
- Pelvis resultant acceleration 3 ms
- HIC15 head injury criterion 15 ms
- Head displacement x
- Pelvis displacement x
- Chest deflection
- Steering column displacement
- Neck compression
- Neck tension
- Neck injury: tension-extension
- Neck injury: tension-flexion
- Neck injury: compression-extension
- Neck injury: compression-flexion
- Distance head – roof (virtual penetration)

Most important result of the robustness evaluation are the predicted intervals of variation for the scatter of the evaluation parameters (figure 3). Even though no limits were exceeded the scatter of single evaluation parameters is high.

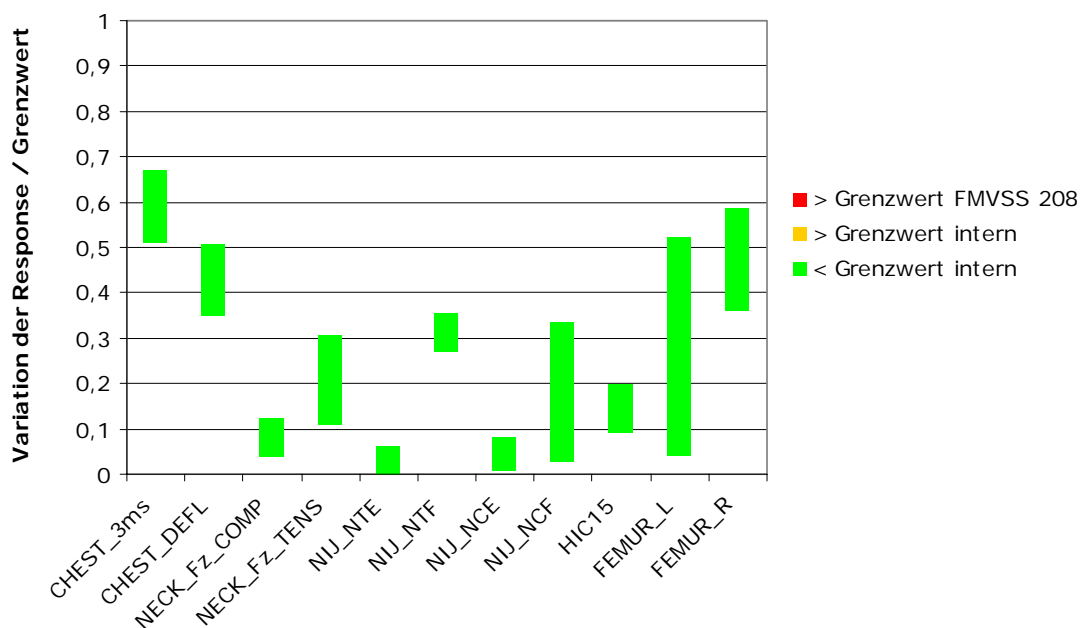


Figure 3: Visualization of the Variation Ranges

Of the 29 sources of input scatter only 9 input variables shows noteworthy correlations to the output variables. As can be seen in the matrix of the linear correlations (figure 4) not for all significant parameters linear correlations (with coefficient of correlation  $> 0.50$ ) to the input scatter could be found. This can be an indicator for a high proportion of numerical noise. Therefore it was investigated if higher measures of determination could be achieved by using quadratic correlations and respectively eliminating of non-linearities (outliers or clustering). However, no correlations, which significantly contribute to the measure of determination, besides linear correlations could be identified. Thereby the determination of the individual result variables strongly varies. For example the maximum force in the femur (figure 5) can be explained with a high determination (90% figure 5), while the variation of the HIC-value can only be explained to less than 50% (figure 6).

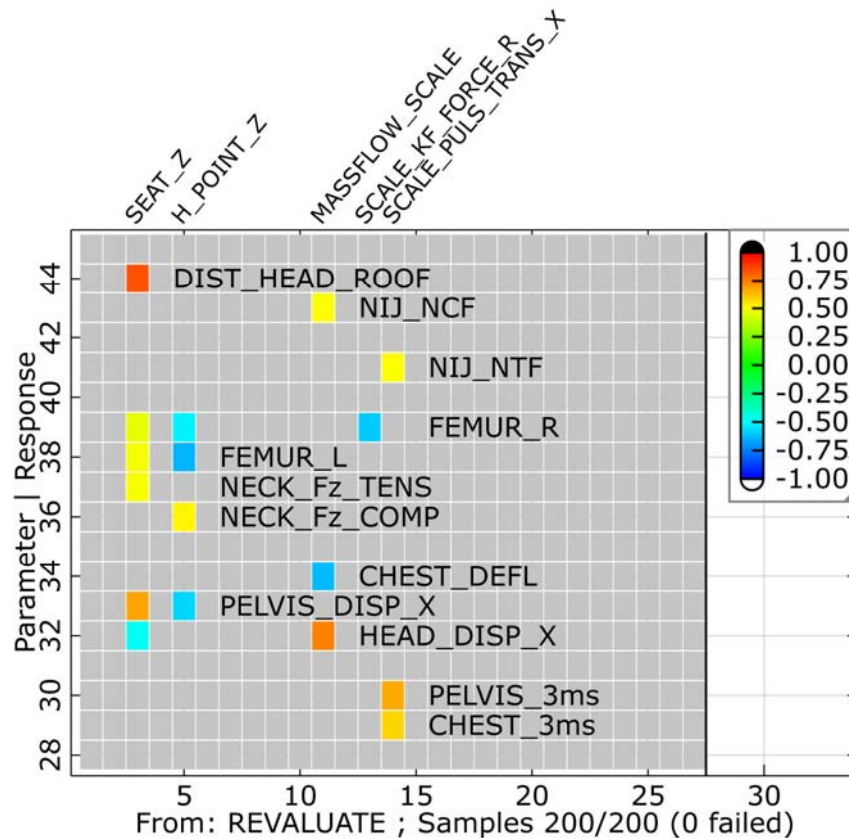


Figure 4: Matrix of Linear Correlations

Therefore a numerical robustness evaluation was performed using the reference model and 5 to 10% of variation of some numerical parameters. Overall 8 numerical parameters, e.g. scaling-factors of the time-steps, the contacts or the “numerical” damping-factors of the multi-body/finite-element-modeling were varied. The scattering of 18 result variables was evaluated.

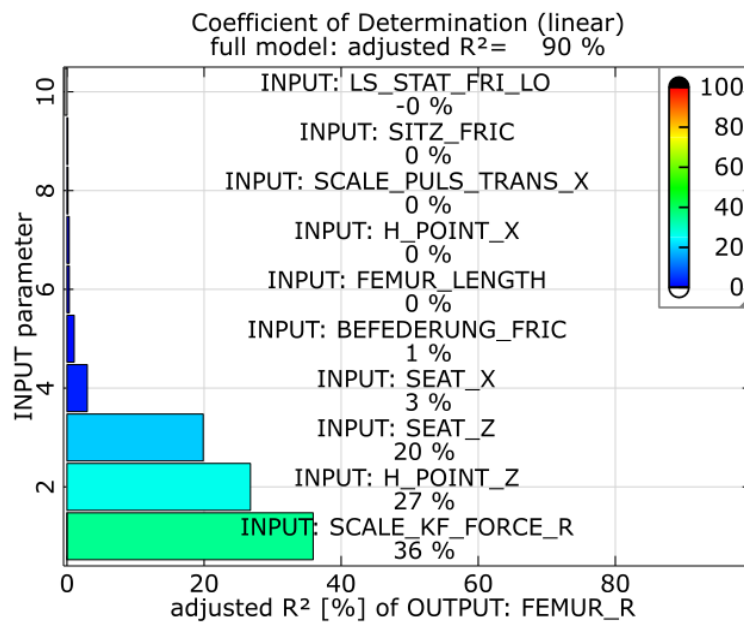


Figure 5: Measure of Determination Femur Force Right

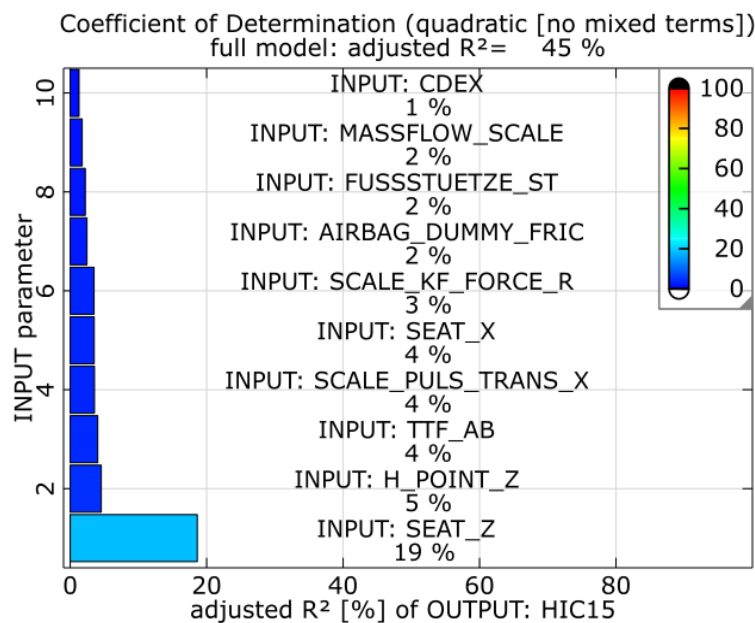


Figure 6: Measure of Determination HIC15

The resulting scatter of the evaluation parameters was compared with the scatter of the physical robustness evaluation (figure 7). As expected the numerical noise of variables with high determination of the physical robustness evaluation, like the femur forces were of negligible proportion. For the evaluation parameter HIC15 as expected significant scatter occurred caused by the variation of numerical parameters. The large scattering of the chest-values in comparison to the physical robustness evaluation are also critical in this model. Although these evaluation parameters show measures of determination of about 80% in the physical robust-

ness evaluation their scatter caused by the variation of numerical parameters is very high. As can be seen in this example, one can not assume that the measure of numerical noise related to the variation interval, can be linearly obtained from the measures of determination.

If noteworthy variations occur within the numerical robustness evaluations, one can assume that the prognosis of scatter of the physical robustness evaluation tends to be too high. No noticeable correlations (linear or quadratic) of single input variations of numerical parameters concerning the observed scattering of the evaluation parameters. Thereby the cause for the numerical noise could not be directly identified from correlation analysis. But of course by checking designs with minimal and maximal performance values often sources of numerical problems can be identified.

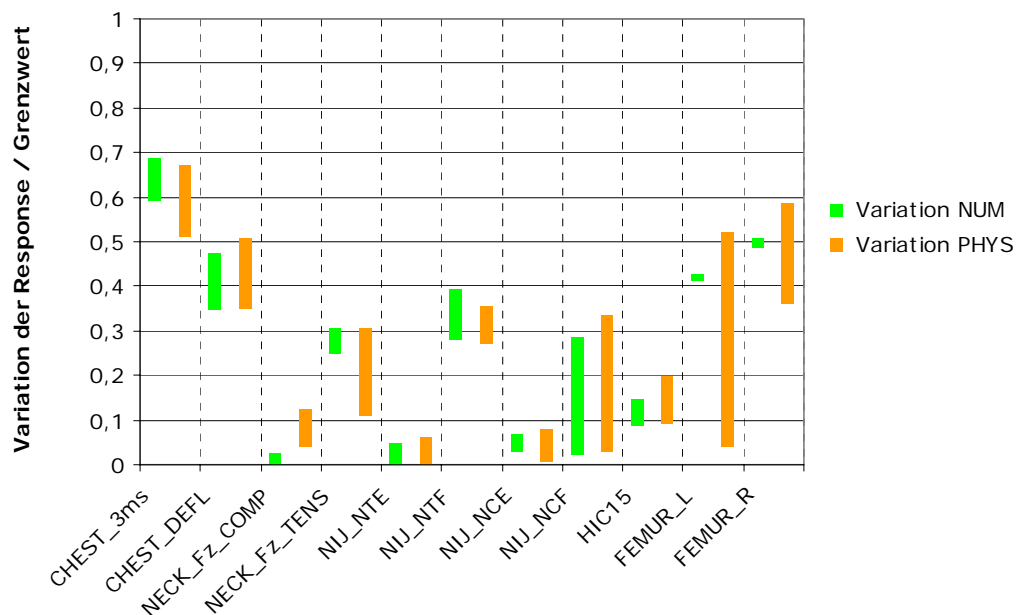


Figure 7: Comparison of the Variation Intervals of physical and numerical robustness evaluation

The robustness evaluation in the early stage of vehicle development indeed showed that the evaluation parameters including the consideration of input scatterings lie below target limit values. At the same time it was shown that the multi-body/finite-element-model shows a high amount of numerical noise for this load case, which leads to a high amount of uncertainty within the prognosis of deterministic results (single values) or of stochastic values (variation ranges). Therefore until the next milestone the models are reworked with the goal to reduce the numerical noise.

## 5.2 Robustness Evaluation of a front-crash load-case of the ULSAB Car Body

On request of the FAT working group 27 of the German automobile industry a front-crash load-case of the ULSAB car body with 14 km/h against a rigid wall (figure 1) was evaluated concerning robustness. The goal of the study was to showcase the possibilities of computational robustness evaluations in crashworthiness. LSDYNA was used for FEM computing. optiSLang [9] was used for the process automation and for the robustness evaluation. Evaluation parameters of the robustness study were energy, forces and deformation of the main crash boxes as well as the relative displacement of the front wall. Input scatter were sheet thickness and yield stress of overall 36 sheet metal components in the front end, the coefficient of friction as well as the test boundary conditions barrier impact speed and barrier impact angle.

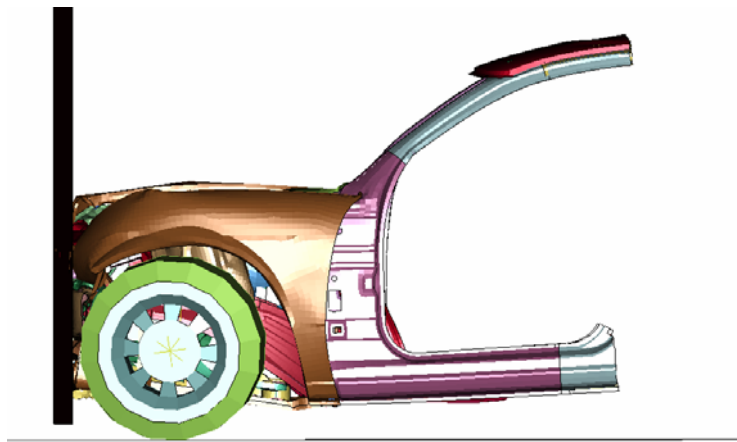


Figure 8: Front-Crash ULSAB Car Body, Side View

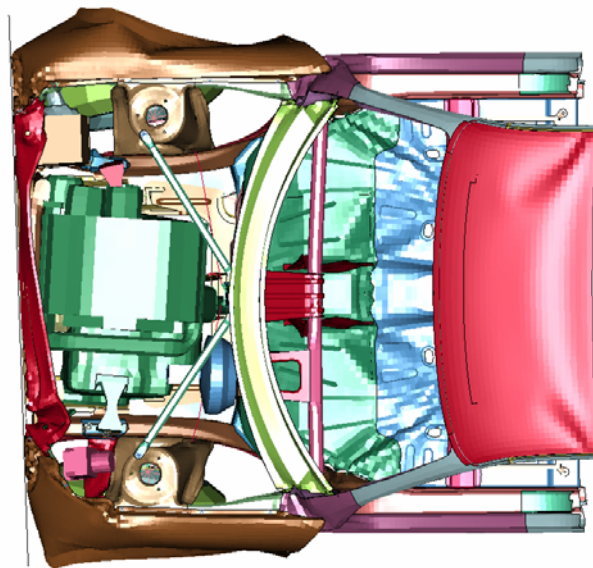


Figure 9: Front-Crash ULSAB Car Body, Top View

Normal distribution was assumed for the scattering value sheet metal thickness and a lognormal distribution for the scattering value tensile strength and yield strength. For the scattering of the test boundary conditions a cut off normal distribution was used and for the coefficient of friction a uniform distribution was used. For the robustness evaluation 169 variants of the 84 overall input scatterings were created using Latin Hypercube Sampling. During the evaluation of the variation intervals significantly too large scatter could be detected concerning nodal intrusion values (figure 10/11).

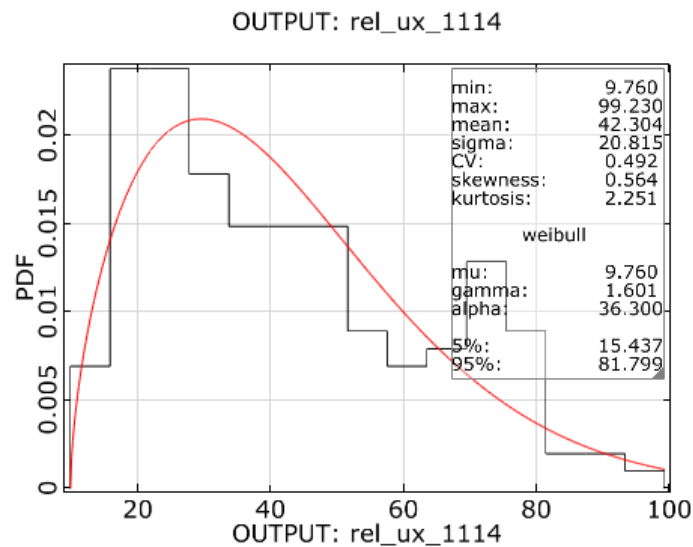


Figure 10: Histogram of the Intrusion at Node 1114

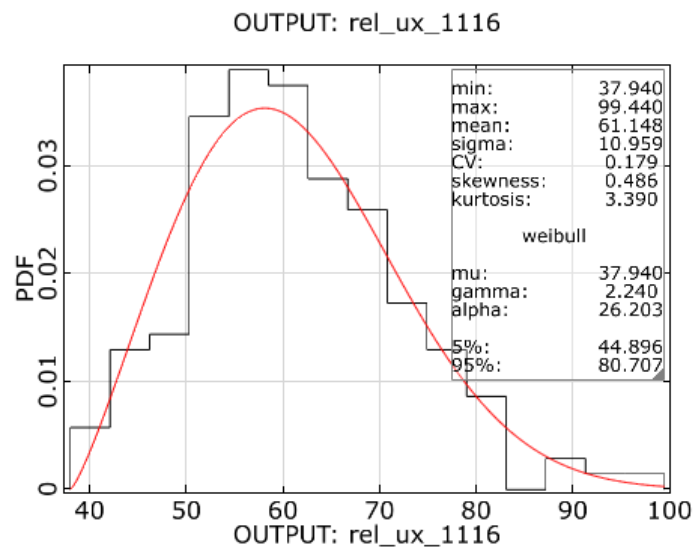


Figure 11: Histogram of the Intrusion at Node 1116

Using correlation analysis and evaluation of the coefficients of determination the reasons for the scatter of the result variables were investigated. While high meas-



ures of determination of  $> 90\%$  were calculated for some evaluation parameters, like for the maximum force in the crash box (figure 12), the measures of determination of the front wall intrusion considering linear and quadratic correlations were very small, lying in the rang of about 50% to 60% (figure 13). This leads to the question whether the high proportion of inexplicable intrusion is caused by non-linearity of the crash analysis or if it is caused by numerical problems.

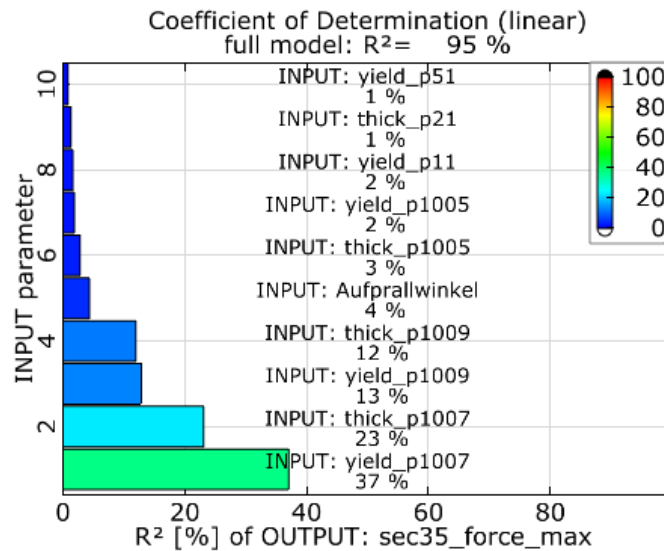


Figure 12: Measures of Determination of the Maximum Forces in the Crash Box

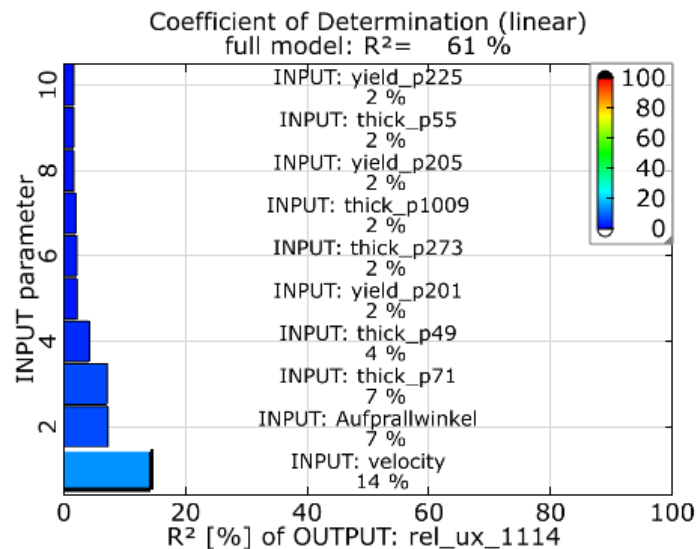


Figure 13: Measure of Determination of Intrusion at Node 1114

In order to determine the significance of the statistical measures the parameter space was reduced to those 15 variables, which had shown significant linear or quadratic correlations in the 84-dimensional response space and a second robustness evaluation was performed. Essentially those variables were the sheet thickness and yield strength of crash box, further sheet metal component in the load



transfer path as well as scattering of the test boundary conditions. In the 15-dimensional space 100 variants were generated and evaluated using Latin Hypercube Sampling.

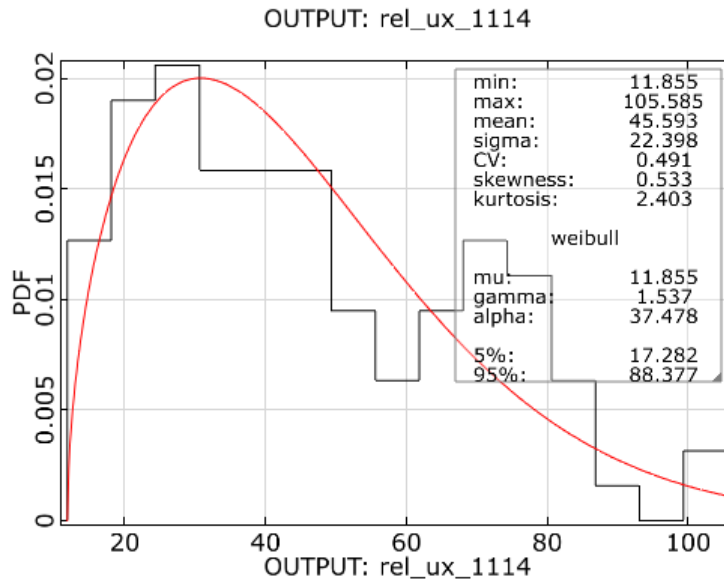


Figure 14: Histogram of the Intrusion at Node 1114

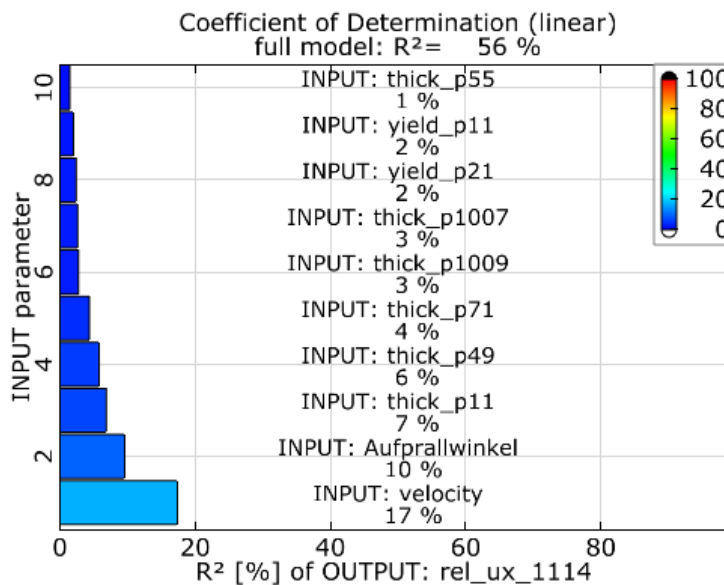


Figure 15: Measure of Determination of the Relative Displacement of the Front wall Node 1114

Fortunately the variation prognosis (figure 14) as well as the measure of determination (figure 15) turned out to be very stable. Thereby it could be shown that the variables that were preliminary selected as of no importance indeed had no significant influence on the result scattering and that the determined statistical measures are trustworthy. However, still only about 50% of the result variation could

be described by linear and quadratic correlation. In order to further investigate the cause for the unexplained variation components of the front wall intrusion the statistic measures of the 100 computations on the FE-structure were investigated using the post-processor Statistics\_on\_Structure (Sos) [10].

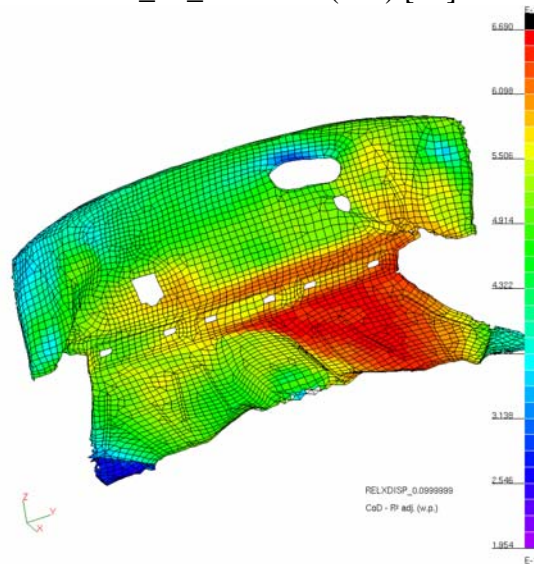


Figure 16: Measure of Determination of the Relative Displacement of the Front Wall

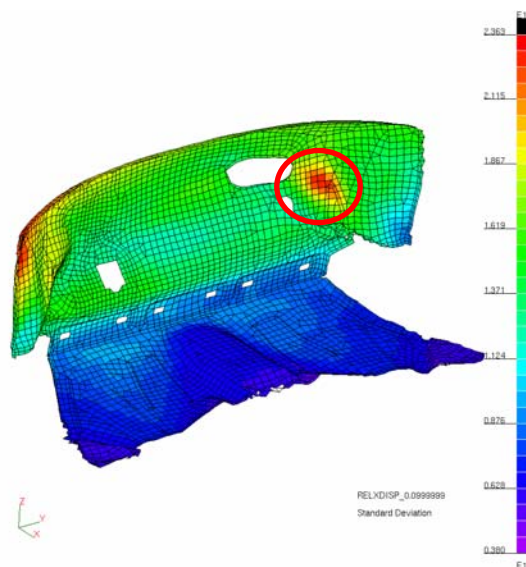


Figure 17: Standard Deviation of the Relative Displacement of the Front Wall

The evaluation of the measures of determination (figure 16), standard deviation and correlation relationships show the largest scatter in the interconnection between the crash box in the front wall (figure 17). The comparison of load cases with minimal (figure 17) and maximal (figure 18) relative displacement at this point showed, that the crash box buckles during displacements and one could have

reasoned that the low determination of the relative displacement could have been associated to this bifurcation problem of the buckling crash box.

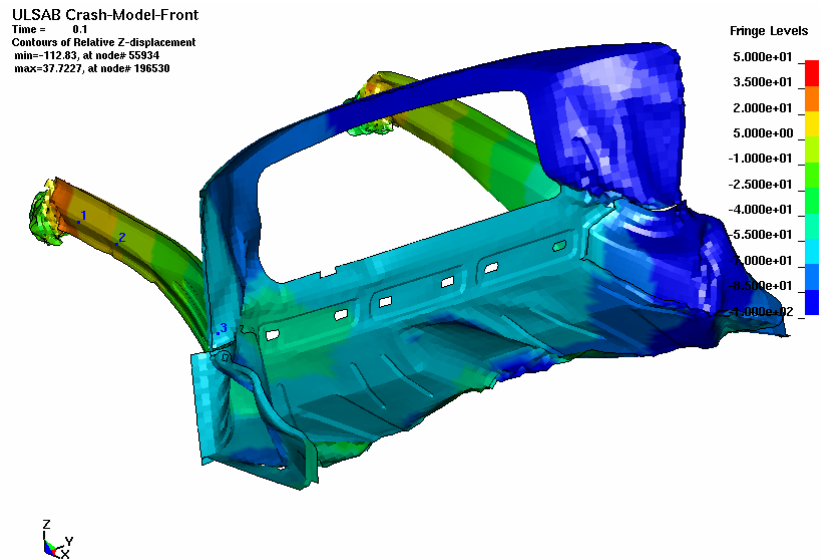


Figure 18: Design with Minimal Front Wall Intrusion

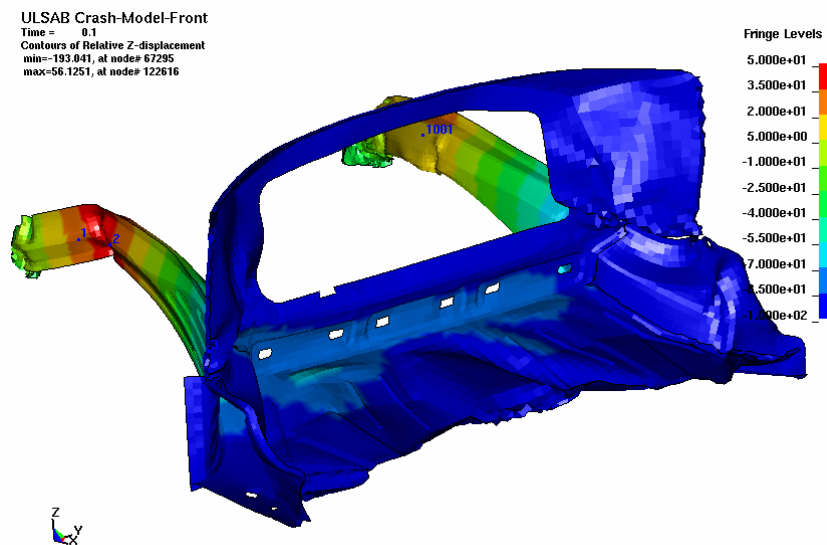


Figure 19: Design with Maximal Front Wall Intrusion

Since the buckling of the crash box comes along with a strong vertical (z) displacement correlations to that indicator of the buckling were looked for. However, the measure of determination of the vertical displacement only averages out to about 50% (figure 20). The remaining 50% of the variation also can not be explained using quadratic correlation analysis or visual examination for possible clustering.

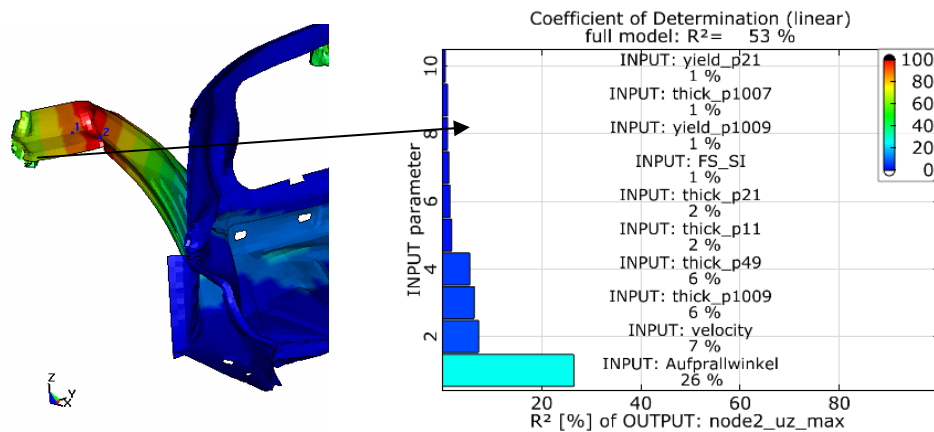


Figure 20: Measure of Determination as Indicator of the Stability Problem

Therefore it can be assumed that 50% of the buckling crash box is caused by “numerical noise”. For further investigation of the causes of the chaotic activation of the buckling the input scatter was decreased even further and robustness evaluations only concerning the input scatter of the testing boundary conditions velocity and impact angle were performed (robustness evaluation 3). Furthermore the input scatter of impact velocity and impact angel were reduced by 90% in the fourth robustness evaluation, in order to verify if the robustness of the structure is dependent on the amount of the input scattering. Using the Latin Hypercube Sampling 36 variants were created respectively and subsequently computed.

Intrusion = relative X-Displacement <b>Node 1114</b>	Robustness 1 84 scattering parameters	Robustness 2 15 scattering parameters	Robustness 3 2 scattering parameters	Robustness 4 2 scattering parameters very little scatter
Mean Value	42.5	44.5	52	53
Variation Interval Max-Min	89.5	93.7	63	68
Coefficient of Determination $R^2/\text{adjusted}R^2$	61/23	56/47	43/35	

Table 1: Statistical Measures of the Relative Displacement in the Node 1114

As can be seen in table 1, the variation interval of the relative displacement is only reduced by 30% even when reducing the input scatter to two scattering variables and a large amount of output scatter remains even when the input of the two variables is reduced to 10%. This again leads to the conclusion that either the connected “physical” correlation is relatively independent of the input scatter and therefore the structural response is very instable or that numerical problems cause the scatter in the response behavior.

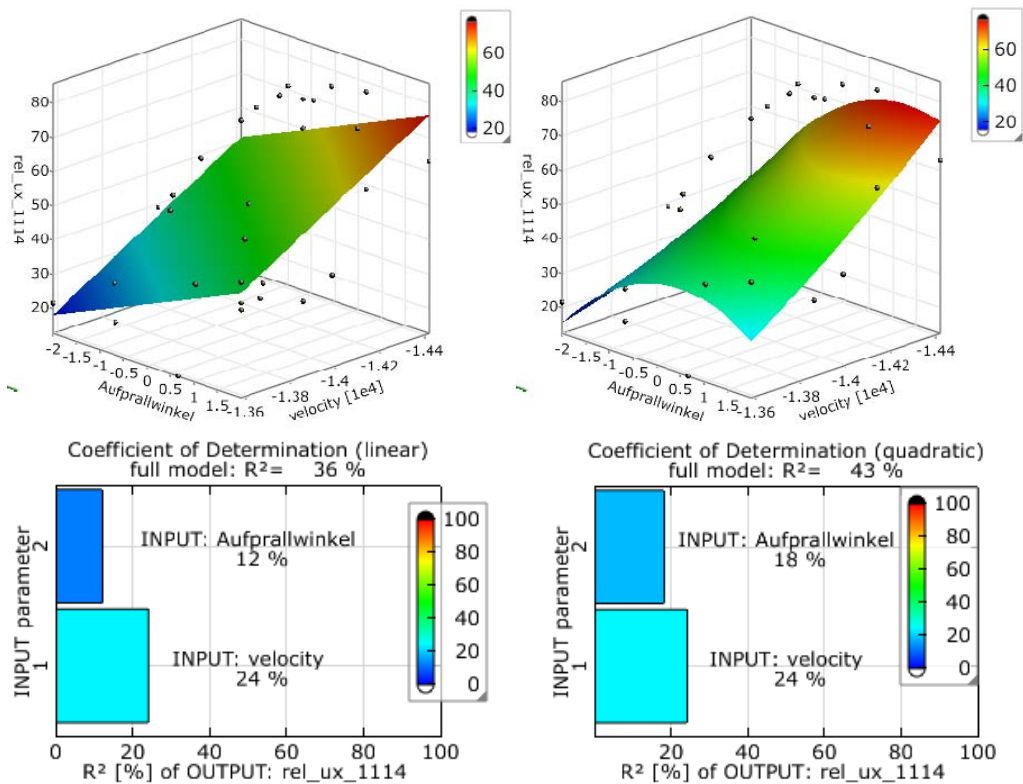


Figure 21: Visualization using Linear Regression Model Linear, Quadratic Regression Model as well as the Corresponding Measures of Determination

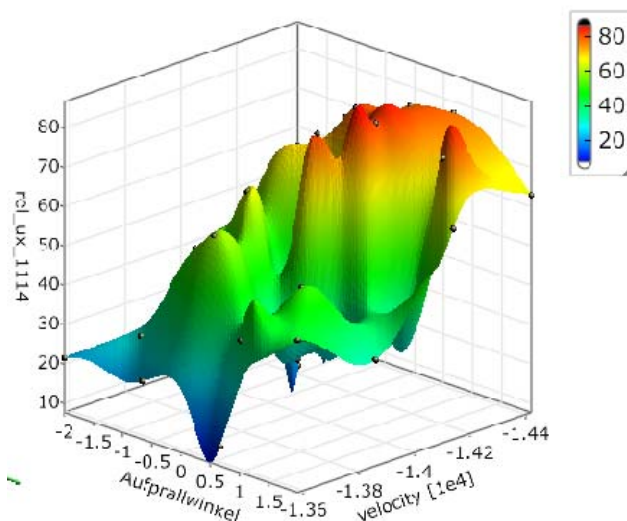


Figure 22: Visualization of the scatter using Moving Least Square Regression

The visualization of the scatter in figure 21/22 shows, that there is no traceable physical correlation above quadratic correlation hypothesis. In the following a

“numerical” robustness evaluation only concerning the time step of the explicit time step integration was performed. The 10 computations in turn showed about the variation space of the robustness evaluations 3 and 4.

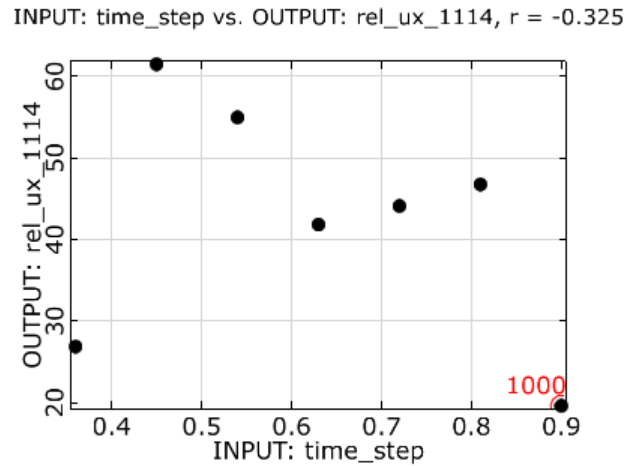


Figure 23: Anthill Plot of the Variation of the Critical Time Step Scaling concerning the Displacement of the Node 1114

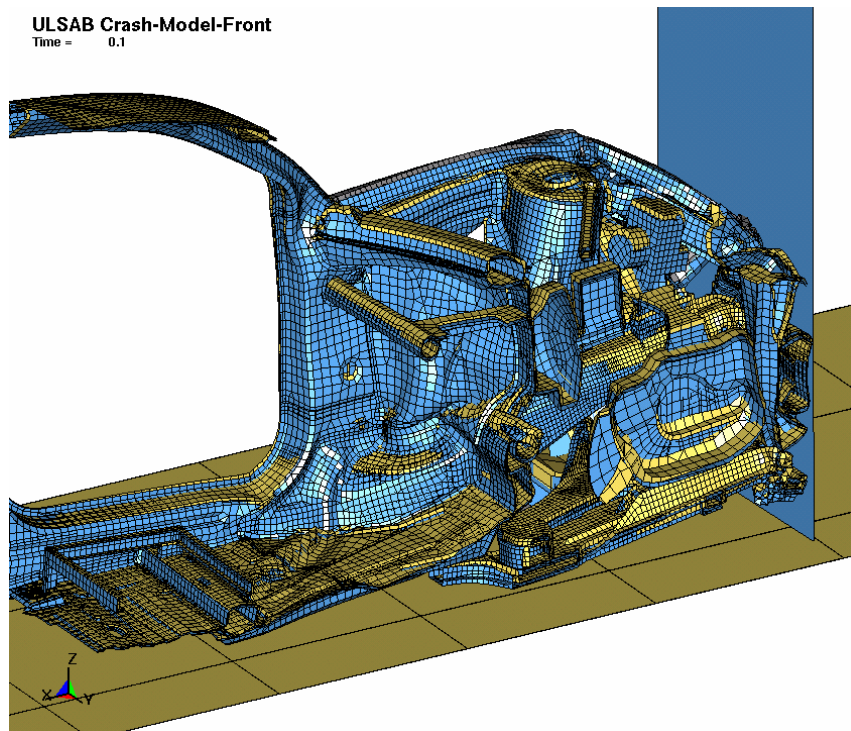


Figure 24: Comparison of two Computations with differing Time Steps

In the following further analysis for identifying the problem were performed and an insufficient meshing of some parts of the crash box supporting structure was diagnosed. This causes “contact locking” of an element cluster during the crash



test simulation while folding this support structure, which is connected to an random impulse, which leads to buckling of the crash box in a random way.

Thereby this benchmark example could demonstrate within different sub spaces of the robustness problem in exemplary manner that robustness evaluations can provide reliable statistical measures for the quantitative estimation of the influence of “numerical noise” on result variables. In practical applications it therefore would be advisable after the first robustness evaluation with small measures of determination searching for the cause of numerical problems by comparing single computation runs and using projection of statistical measures on the FE-structure.

## **6 Outlook and Conclusion**

A new systematic approach was developed, for determining the robustness of important performance parameters of crash test computation qualitatively and quantitatively. Primary result of the robustness evaluation is the estimation of the scatter of important result variables. Furthermore sensitive scattering input variables can be identified as well as the determination of result variables can be examined. Assumptions concerning activated nonlinear correlations (clustering/outliers/bifurcation) caused by input scatter can be verified.

Using measures of determination the quantitative influence of numerical noise on the variation of result variables can be estimated and thereby an important contribution to the reliability of prognosis and quality of the crash test computations can be given.

The breakthrough in practical application and the acceptance of stochastic analysis for robustness evaluations was achieved by supplying linear and quadratic correlations and the corresponding measures of determination as well as by projection of statistical measures on the finite element structure.

The quantitative estimation of the measures of determination and the securing of large measures of determination are not only meaningful in robustness evaluations of final designs. If crash test are integral part of multi-disciplinary optimization tasks [11] the measures of determination should also be secured for result values. Here measures of determination in the design space of optimization can be used as quality criteria for the applicability of results in constraints or objective functions [12].

## Literature

- [1] Will, J.: Robustheitsbewertungen in der virtuellen Produktentwicklung; Proceedings 24. CAD-FEM Users Meeting 2006, Fellbach, Chapter 1., Germany
- [2] Will, J.; Möller, J-St.; Bauer, E.: Robustheitsbewertungen des Fahrkomfortverhaltens an Gesamtfahrzeugmodellen mittels stochastischer Analyse, VDI-Berichte Nr.1846, 2004, S.505-527
- [3] Will, J.; Baldauf, H.: Integration rechnerischer Robustheitsbewertungen in die virtuelle Auslegung passiver Fahrzeugsicherheit bei der BMW AG, VDI-Berichte Nr. 1976, Berechnung und Simulation im Fahrzeugbau, 2006 Seite 851-873
- [4] Bucher, C.: Adaptive sampling-an iterative fast Monte Carlo procedure. Structural Safety, 5(2):119–126, 1988.
- [5] Bayer; V.; Bucher, C.: Importance sampling for first passage problems of nonlinear structures. Probabilistic Engineering Mechanics, 14:27–32, 1999.
- [6] Macke, M.; Bucher, C.: Importance sampling for randomly excited dynamical systems. Journal of Sound and Vibration, (268):269–290, 2003.
- [7] Roos, D.; Adam, U.; Bayer, V.: Design Reliability Analysis; Proceedings Weimarer Optimierungs- und Stochastiktage 3.0, 2006, Weimar, Germany
- [8] Will, J.; Bucher, C.: Statistische Maße für rechnerische Robustheitsbewertungen CAE-gestützter Berechnungsmodelle, Proceedings Weimarer Optimierungs- und Stochastiktage 3.0, 2006, Weimar, Germany
- [9] optiSLang - the Optimizing Structural Language Version 2.1, DYNARDO, Weimar, 2006, [www.dynardo.de](http://www.dynardo.de)
- [10] Will, J.; Bucher, C.; Ganser, M.; Grossenbacher, K.: Berechnung und Visualisierung statistischer Maße auf FE-Strukturen für Umformsimulationen; Proceedings Weimarer Optimierungs- und Stochastiktage 2.0, 2005, Weimar, Germany
- [11] Duddeck, F.: Multidisziplinäre Optimierung im Produktentwicklungsprozess der Automobilindustrie; Proceedings Weimarer Optimierung- und Stochastiktage 2.0, 2005, Weimar, Germany
- [12] Blum, S.; Will, J.: Combining Robustness Evaluation with Current Automotive MDO Application, Proceedings Weimarer Optimierungs- und Stochastiktage 3.0, 2006, Weimar, Germany