Evolutionary Strategies for Multidisciplinary Optimization

Prof. Dr. Thomas Bäck
Managing Director, NuTech Solutions GmbH
CTO, NuTech Solutions, Inc.

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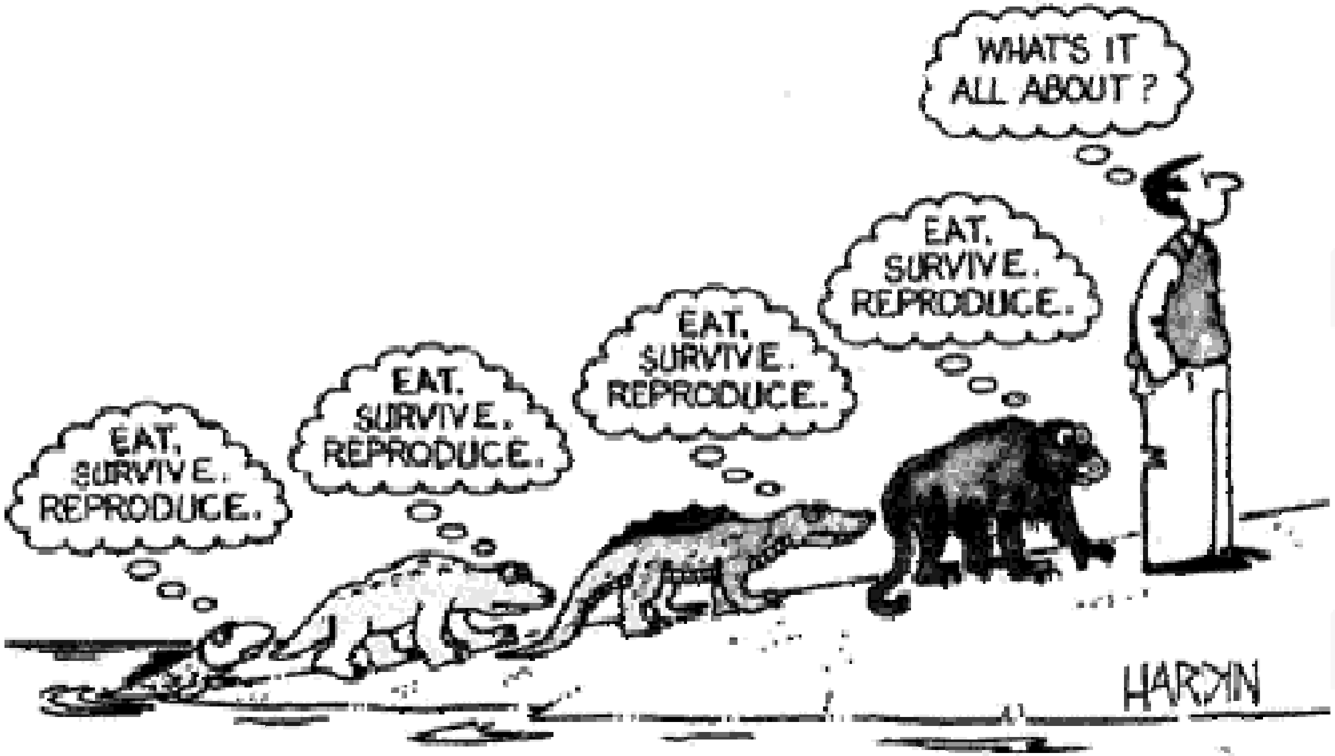
Overview

- Introduction: Optimization and EAs
- Evolutionary Strategies
- Application Examples

Background I

- Biology = Engineering (Daniel Dennett)



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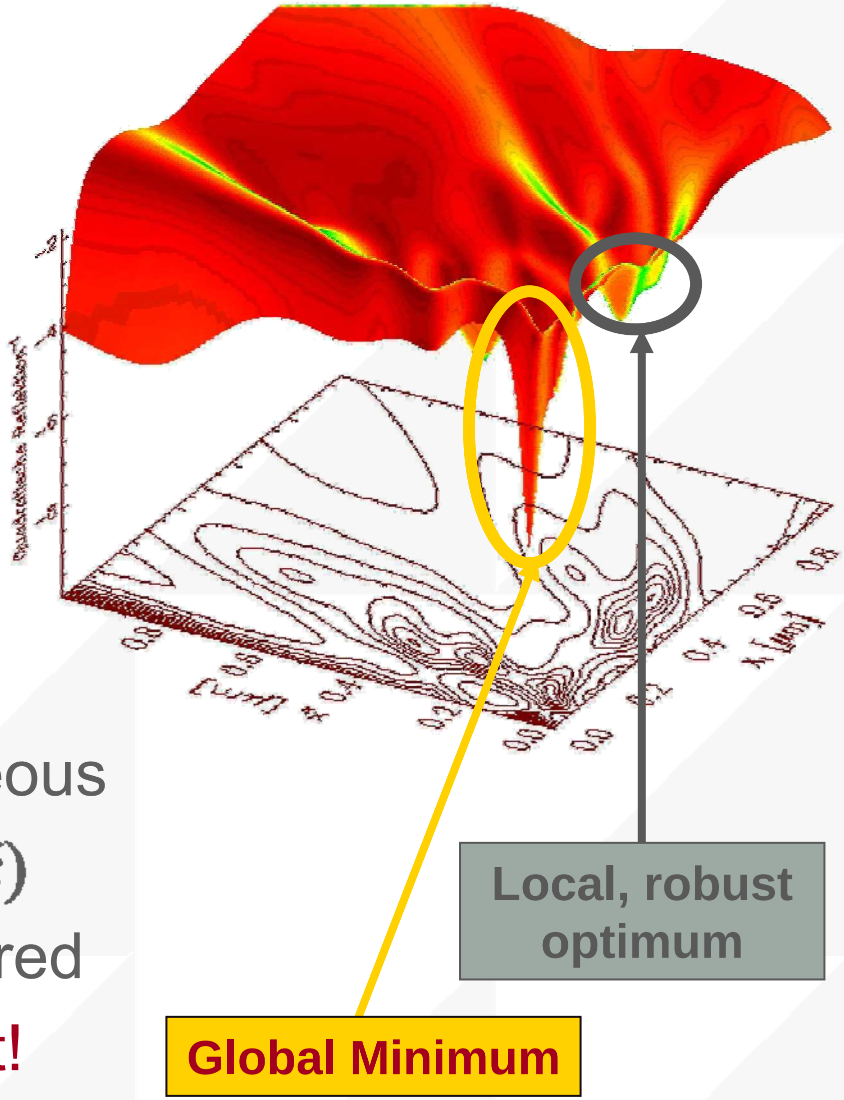
Introduction:

Optimization
Evolutionary Algorithms

Optimization

$$f : M \to \Re, f(\vec{x}) \to \min$$

- f : objective function
  - High-dimensional
  - Non-linear, multimodal
  - Discontinuous, noisy, dynamic
- $M \subseteq M_1 \times M_2 \times \ldots \times M_n$ heterogeneous
- Restrictions possible over $M, f(\vec{x})$
- Good local, robust optimum desired
- Realistic landscapes are like that!



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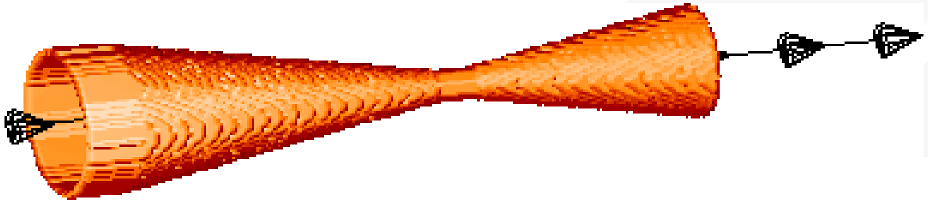
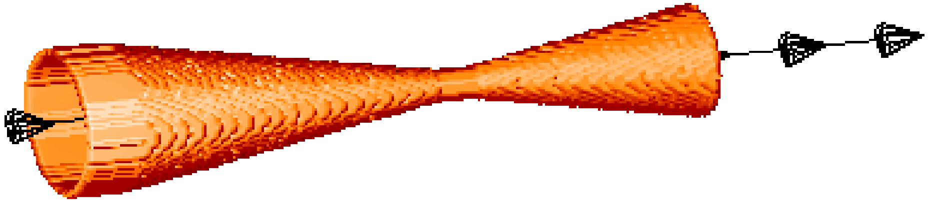
Optimization Creating Innovation

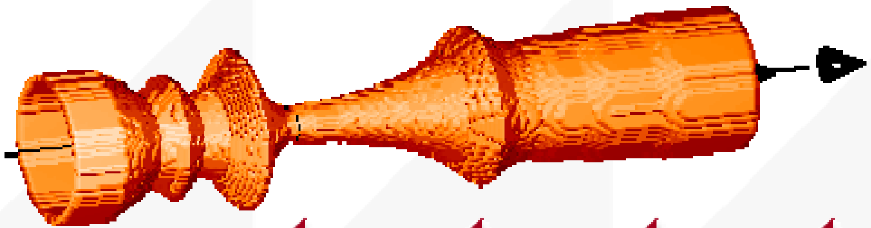
- Illustrative Example: Optimize Efficiency
  - Initial:



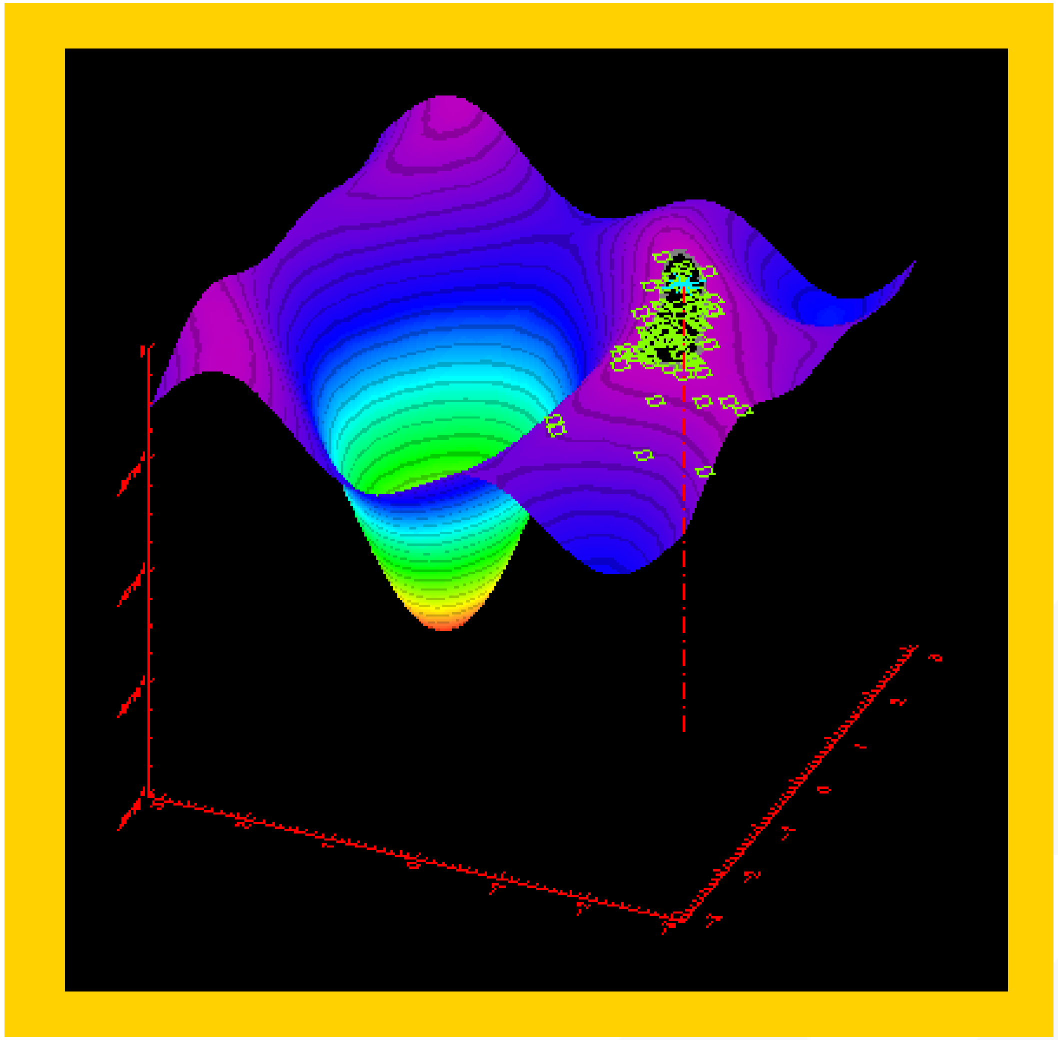
  - Evolution:



- 32% Improvement in Efficiency !



Dynamic Optimization



- Dynamic Function
- 30-dimensional
- 3D-Projection

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Iterative Optimization Methods

- General description:

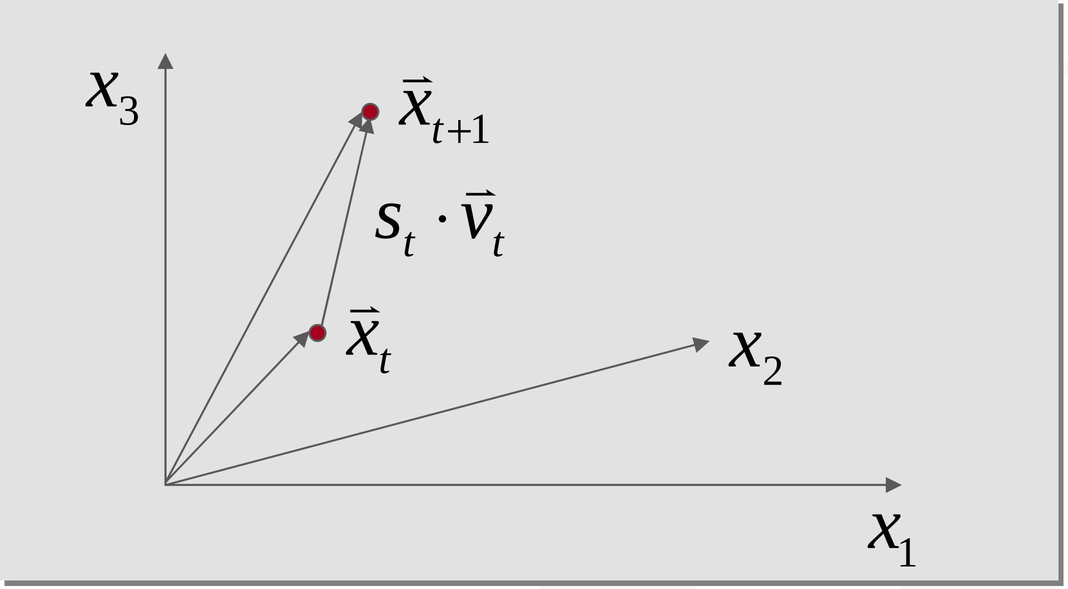
$$\vec{x}_{t+1} = \vec{x}_t + s_t \cdot \vec{v}_t$$

New Point

Actual Point

Directional vector

Step size (scalar)



- At every Iteration:
  - Choose direction
  - Determine step size
- Direction:
  - Gradient
  - Random
- Step size:
  - 1-dim. optimization
  - Random
  - Self-adaptive

The Fundamental Challenge

- Global convergence with probability one:

$$\lim_{t \to \infty} \Pr(\vec{x}^* \in P(t)) = 1$$

- General, but for practical purposes useless
- Convergence velocity:

$$\varphi = E(f_{\max}(P(t+1)) - f_{\max}(P(t)))$$

- Local analysis only, specific (convex) functions

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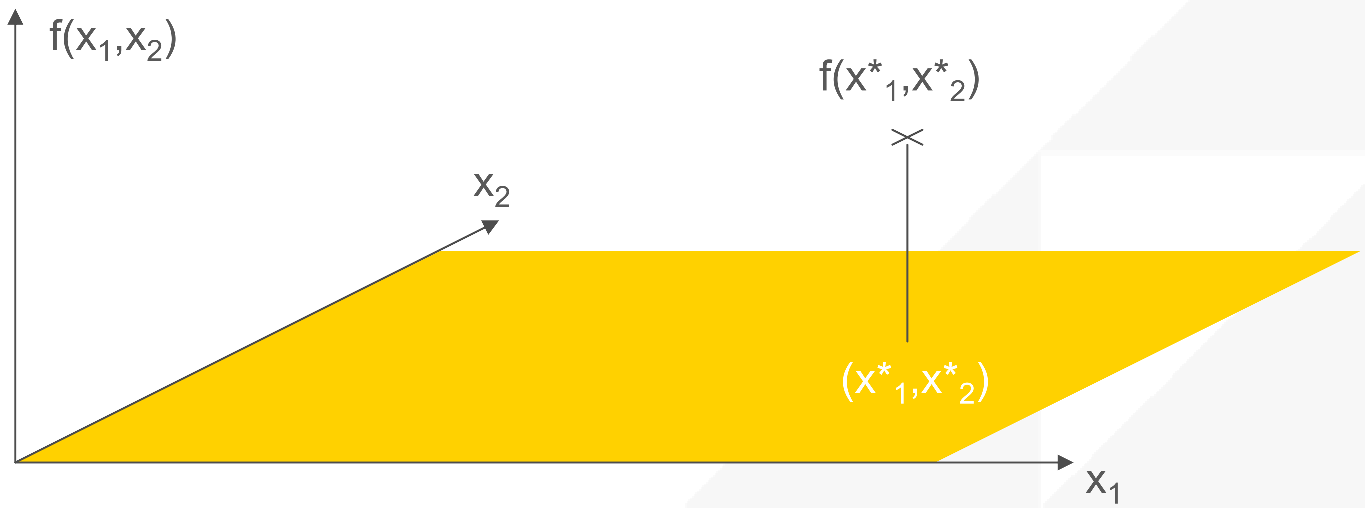
An Infinite Number of Pathological Cases !



- NFL-Theorem:
  - All optimization algorithms perform equally well iff performance is averaged over all possible optimization problems.
- Fortunately: We are not Interested in „all possible problems“

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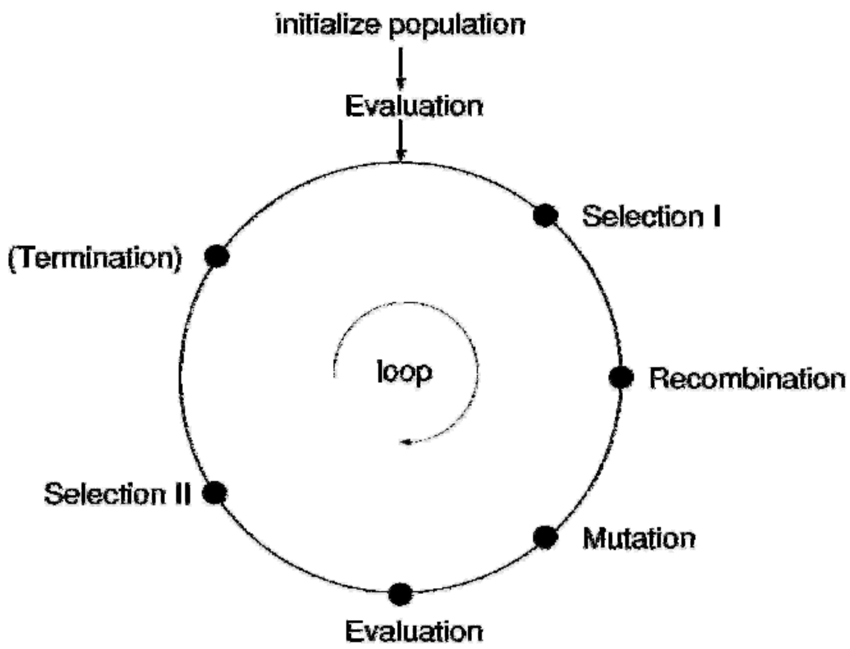
Evolution Strategies

Generalized Evolutionary Algorithm

```
t := 0;
initialize(P(t));
evaluate(P(t));
while not terminate do
    P'(t) := mating_selection(P(t));
    P''(t) := variation(P'(t));
    evaluate(P''(t));
    P(t+1) := environmental_selection(P''(t) ∪ Q);
    t := t+1;
od
```



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Evolution Strategy – Basics

- Mostly real-valued search space $\mathbb{R}^n$
  - also mixed-integer, discrete spaces
- Emphasis on mutation
  - n-dimensional normal distribution
  - expectation zero
- Different recombination operators
- Deterministic selection
  - (μ, λ)-selection: Deterioration possible
  - $(\mu+\lambda)$-selection: Only accepts improvements
- $\lambda >> \mu$, i.e.: Creation of offspring surplus
- Self-adaptation of strategy parameters.

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Representation of search points

- Self-adaptive ES with single step size:
  - One σ controls mutation for all x_i
  - Mutation: N(0, σ)

$$\vec{a} = ((x_1, \ldots, x_n), \sigma)$$

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Evolution Strategy:

Algorithms
Mutation

Operators: Mutation – one σ

- Self-adaptive ES with one step size:
  - One σ controls mutation for all x_i
  - Mutation: N(0, σ)

$$\vec{a} = ((x_1, \ldots, x_n), \sigma)$$

$$\vec{a}' = ((x_1', \ldots, x_n'), \sigma')$$

$$\sigma' = \sigma \cdot \exp(\tau_0 \cdot N(0,1))$$

$$x_i' = x_i + \sigma' \cdot N_i(0,1)$$

Individual before mutation

Individual after mutation

1.: Mutation of step sizes

2.: Mutation of objective variables

Here the new σ‘ is used!

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Operators: Mutation – one σ

- Thereby τ_0 is the so-called learning rate
  - Affects the speed of the σ-Adaptation
  - τ_0 bigger: faster but more imprecise
  - τ_0 smaller: slower but more precise
  - How to choose τ_0?
  - According to recommendation of Schwefel*:

$$\tau_0 = \frac{1}{\sqrt{n}}$$

*H.-P. Schwefel: Evolution and Optimum Seeking, Wiley, NY, 1995.

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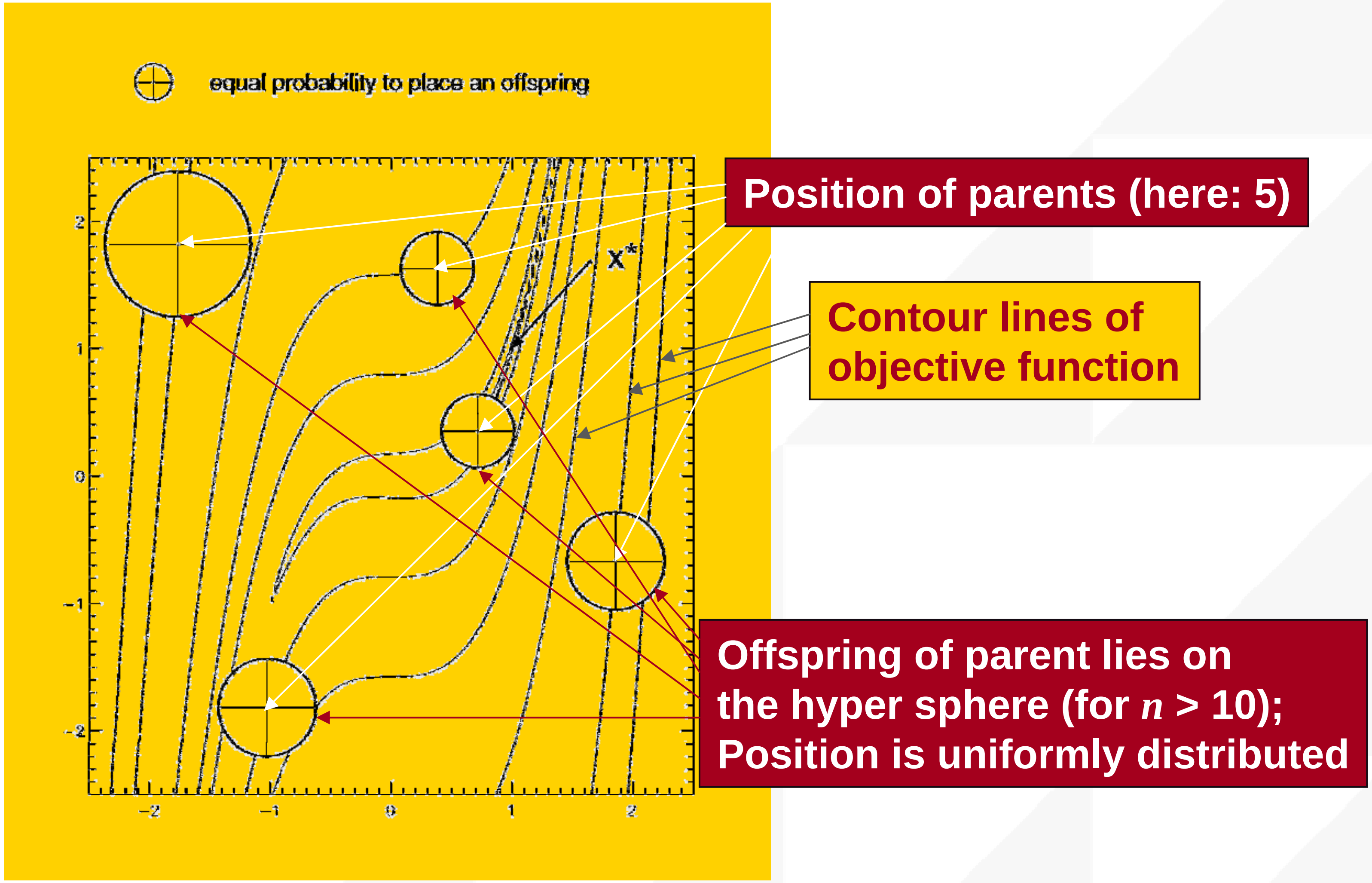
Operators: Mutation – one σ



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Evolution Strategy

Algorithms
Selection

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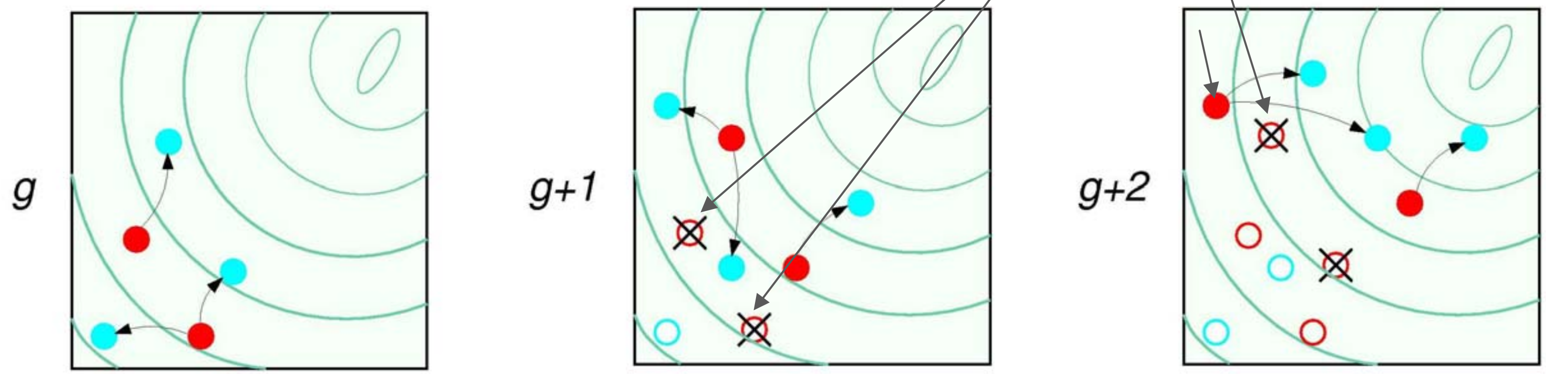
Operators: Selection

Parents don‘t survive …

Parents don‘t survive!

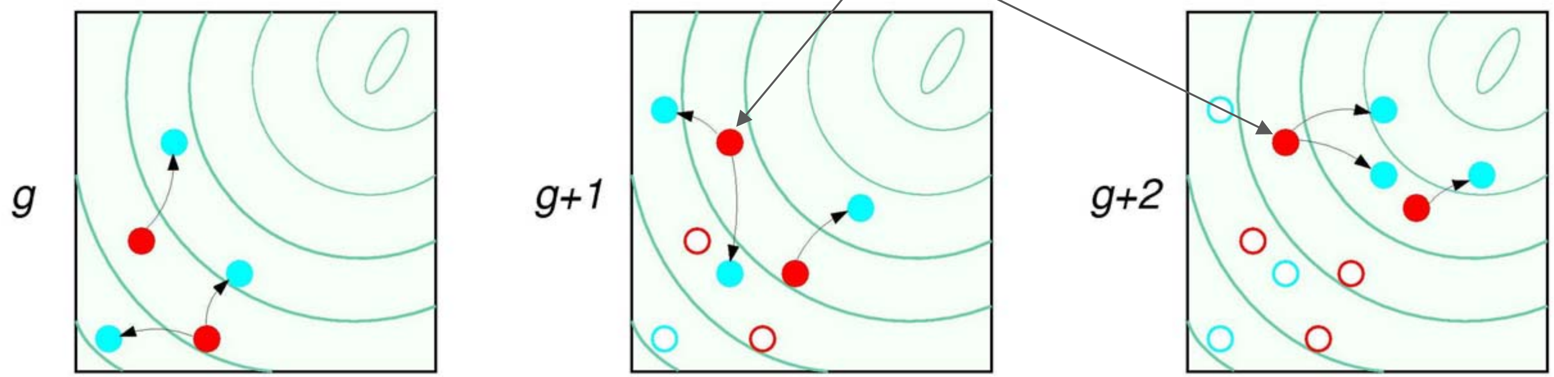
… but a worse offspring.

- Example: (2,3)-Selection



- Example: (2+3)-Selection

… now this offspring survives.



Operators: Selection

Exception!

- Possible occurrences of selection
  - (1+1)-ES: One parent, one offspring, 1/5-Rule
  - (1,λ)-ES: One Parent, λ offspring
    - Example: (1,10)-Strategy
    - One step size / n self-adaptive step sizes
    - Mutative step size control
    - Derandomized strategy
  - (μ,λ)-ES: μ > 1 parents, $\lambda > \mu$ offspring
    - Example: (2,15)-Strategy
    - Includes recombination
    - Can overcome local optima
  - (μ+λ)-strategies: elitist strategies

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Evolution Strategy:

Self adaptation of step sizes

Self-adaptation

- No deterministic step size control!
- Rather: Evolution of step sizes
  - Biology: Repair enzymes, mutator-genes
- Why should this work at all?
  - Indirect coupling: step sizes – progress
  - Good step sizes improve individuals
  - Bad ones make them worse
  - This yields an indirect step size selection

Self-adaptation: Example

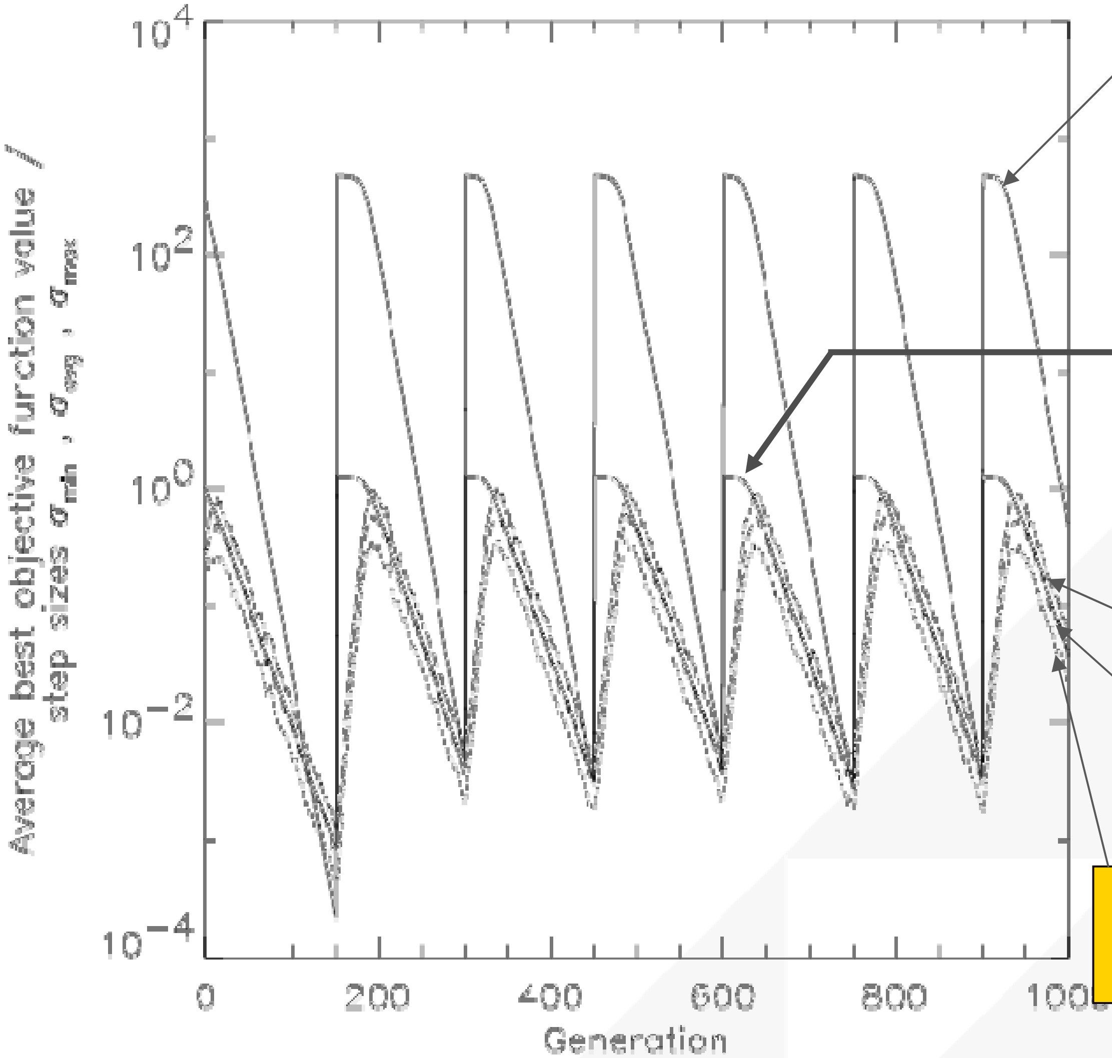
- How can we test this at all?
- Need to know optimal step size …
  - Only for very simple, convex objective functions
  - Here: Sphere model

$\vec{x}^*$ **: Optimum**

$$f(\vec{x}) = \sum_{i=1}^{n} (x_i - x_i^*)^2$$

  - Dynamic sphere model
    - Optimum locations changes occasionally

Self-adaptation: Example



Objective function value

According to theory ff optimal step sizes

$$\sigma_{opt} = c_{\mu,\lambda} \frac{R}{n}$$

Largest …

average …

… and smallest step size measured in the population

Self-adaptation

- Self-adaptation of one step size
  - Perfect adaptation
  - Learning time for back adaptation proportional n
  - Proofs only for convex functions
- Individual step sizes
  - Experiments by Schwefel
- Correlated mutations
  - Adaptation much slower

Mixed-Integer Evolution Strategies

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Mixed-Integer Evolution Strategy

- Generalized optimization problem:

$$f(r_1, \ldots, r_{n_r}, z_1, \ldots, z_{n_z}, d_1, \ldots, d_{n_d}) \rightarrow min$$

subject to:

$$r_i \in [r_i^{min}, r_i^{max}] \subset \mathbb{R}, \ \ i = 1, \ldots, n_r$$

$$z_i \in [z_i^{min}, z_i^{max}] \subset \mathbb{Z}, \ \ i = 1, \ldots, n_z$$

$$d_i \in D_i = \{d_{i,1}, \ldots, d_{i,|D_i|}\}, i = 1, \ldots, n_d$$

Mixed-Integer ES: Mutation

Learning rates (global)

Learning rates (global)

Geometrical distribution

Mutation Probabilities

$$
\begin{array}{l}
\textbf{for } i = 1, \ldots, n_r \textbf{ do} \\
\quad s_i' \leftarrow s_i \exp(\tau_g \mathrm{N}_g + \tau_l \mathrm{N}(0,1)) \\
\quad r_i' = r_i + N(0, s_i') \\
\textbf{end for} \\
\textbf{for } i = 1, \ldots, n_z \textbf{ do} \\
\quad q_i' \leftarrow q_i \exp(\tau_g \mathrm{N}_g + \tau_l \mathrm{N}(0,1)) \\
\quad z_i' \leftarrow z_i + \mathrm{G}(0, q_i') \\
\textbf{end for} \\
p_i' := 1/[1 + \frac{1-p_i}{p_i} * \exp(-\tau_l * \mathrm{N}(0,1))] \\
\textbf{for } i \in \{1, \ldots, n_d\} \textbf{ do} \\
\quad \textbf{if } \mathrm{U}(0,1) < p_i' \textbf{ then} \\
\quad\quad d_i' \leftarrow \text{uniformly randomly value from } D_i \\
\quad \textbf{end if} \\
\textbf{end for}
\end{array}
$$

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Literature

- H.-P. Schwefel: *Evolution and Optimum Seeking*, Wiley, NY, 1995.
- I. Rechenberg: *Evolutionsstrategie 94*, frommann-holzboog, Stuttgart, 1994.
- Th. Bäck: *Evolutionary Algorithms in Theory and Practice*, Oxford University Press, NY, 1996.
- Th. Bäck, D.B. Fogel, Z. Michalewicz (Hrsg.): *Handbook of Evolutionary Computation, Vols. 1,2*, Institute of Physics Publishing, 2000.

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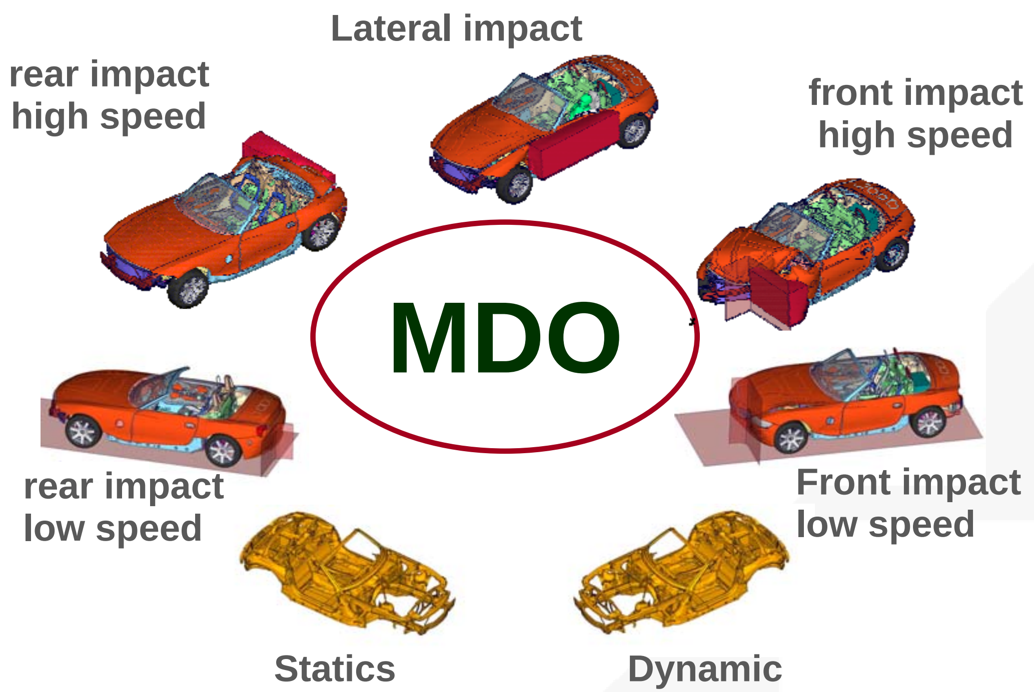
Multidisciplinary Optimization (MDO)



- Different disciplines involved (crash cases, NVH, statics, …).
- Very demanding simulation (multiple CPUs, many hours).
- Large number of parameters and constraints.
- Very few evaluations (shots) possible (often < 300).

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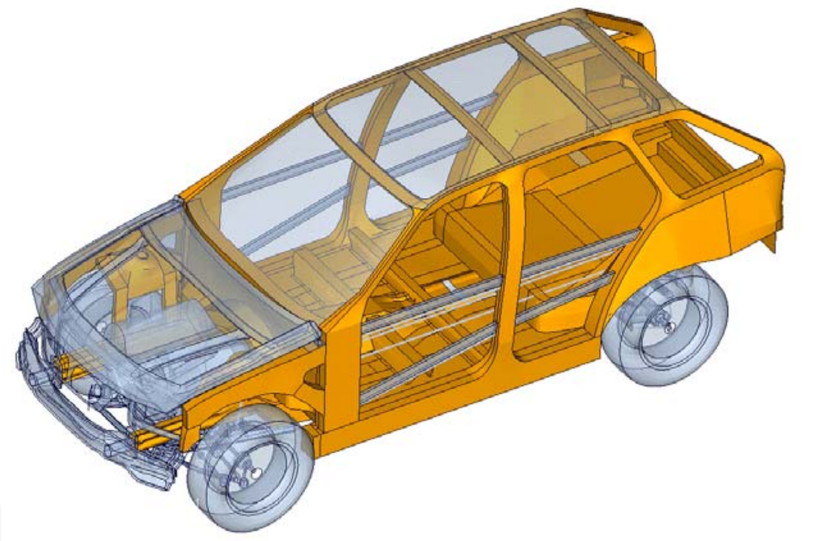
Example 1

Courtesy of

MDO Crash / Statics / Dynamics

- Minimization of body mass
- Finite element mesh
  - Crash ~ 130.000 elements
  - NVH ~ 90.000 elements
- Independent parameters: Thickness of each unit: 109
- Constraints: 18



Algorithm	Avg. reduction (kg)	Max. reduction (kg)	Min. reduction (kg)
Best so far	-6.6	-8.3	-3.3
NuTech ES	**-9.0**	**-13.4**	**-6.3**

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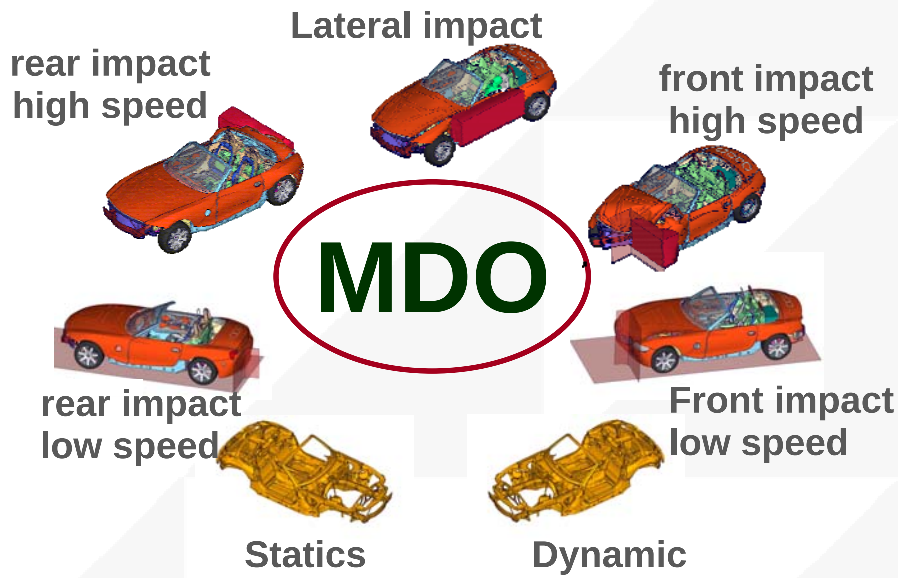
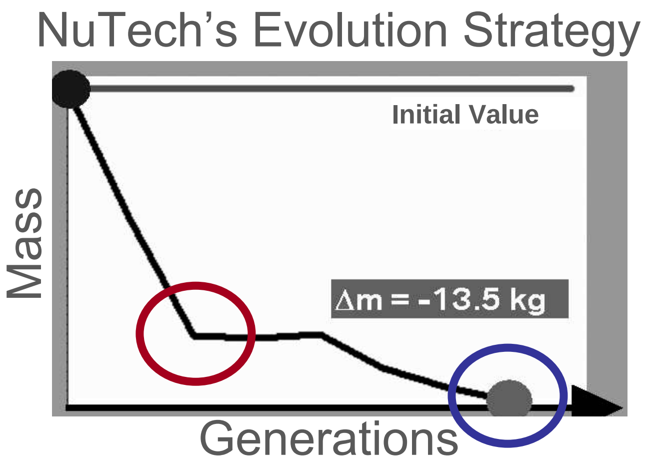
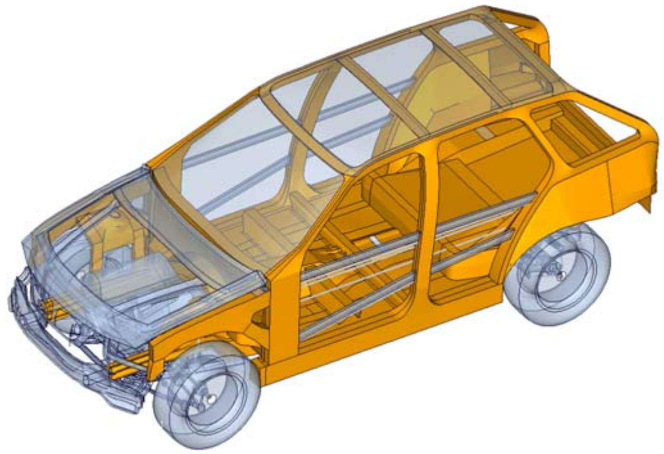
MDO Production Runs (I)

- Minimization of body mass
- Finite element mesh
  - Crash ~ 1.000.000 elements
  - NVH ~ 300.000 elements
- Independent parameters:
  - Thickness of each unit: 136
- Constraints: 47, resulting from various load cases
- 180 (10 x 18) shots ~ 12 days
- No statistical evaluation due to problem complexity



MDO Production Runs (II)





- 13,5 kg weight reduction by NuTech's ES.
- Beats best so far method significantly.
- Typically faster convergence velocity of ES.
- Reduction of development time from 5 to 2 weeks allows for process integration.
- Still potential for further improvement after 180 shots.

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Example 2

Courtesy of

Audi

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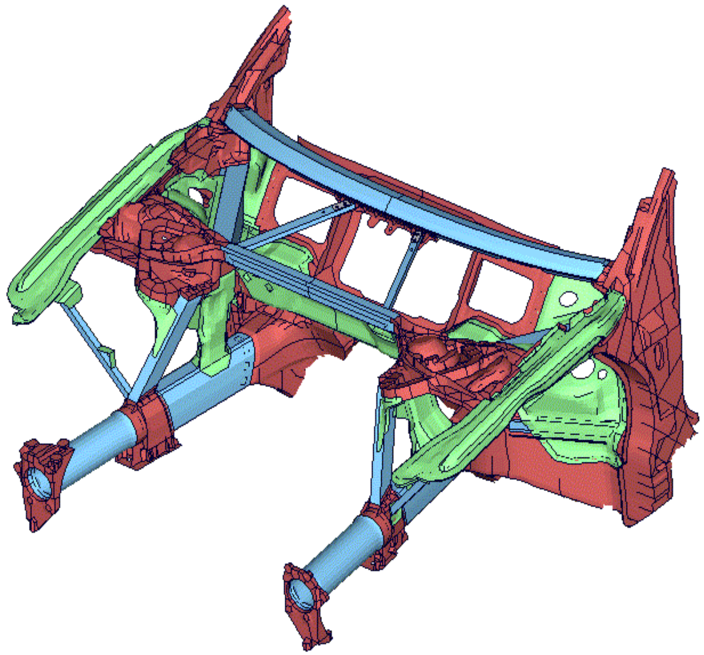
MDO ASF® Front Optimization

- Pre-optimized Space-Frame-Concept – improvement possible?
- Goal: Minimization of structural weight
- Degrees of freedom:
  - Wall thicknesses of the semi-finished products sheet & profile
  - Material characteristic profile
- Limitation of design space:
  - Semi-finished products technology
  - Technique for joining parts



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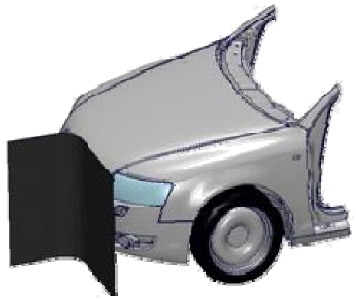
MDO ASF® Disciplines



Damage according
to insurance classification,
Component Model,
2 CPUs

Global dynamic stiffness,
Trimmed Body,
1 CPU

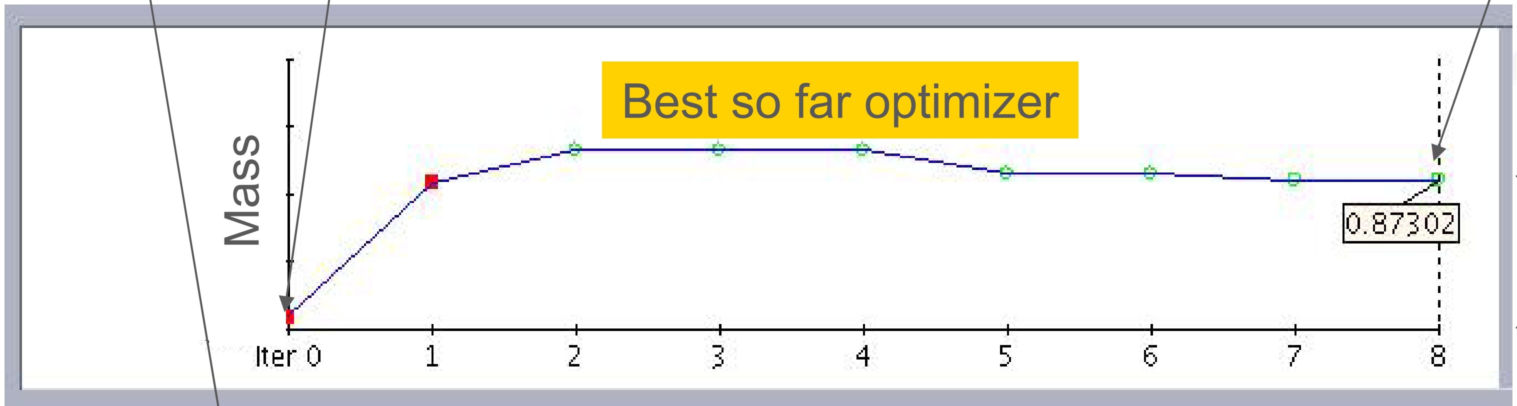
Front Crash (EURO NCAP),
Complete Body
4 CPUs

Resources per Design: 7 CPUs, approx. 23h

MDO Run Comparison

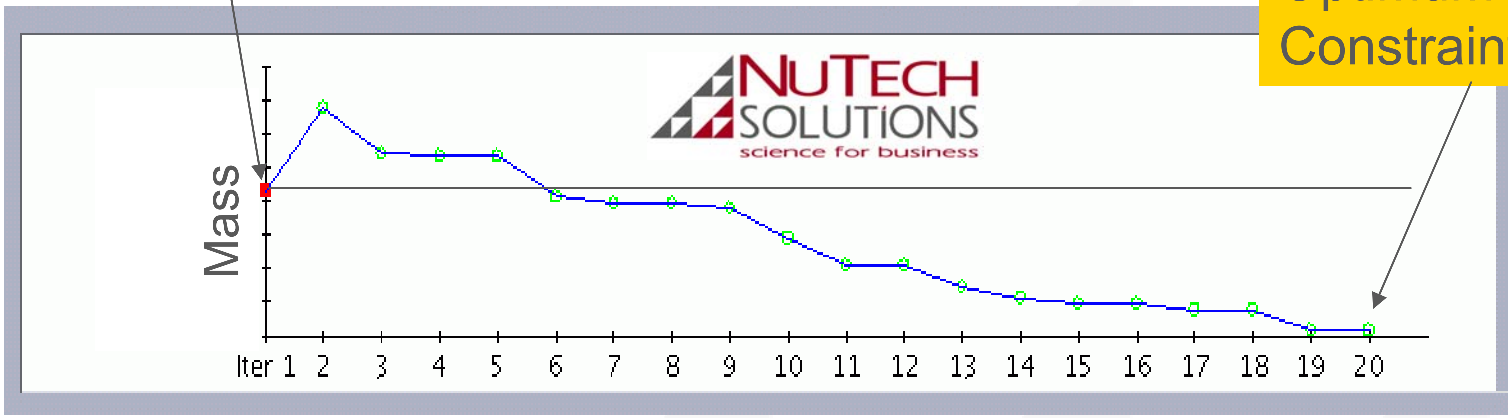
Initial design, constraints violated

Optimum (exp. 924), Constraints satisfied



Increased weight!

Optimum (exp. 376), Constraints satisfied



Decreased weight!

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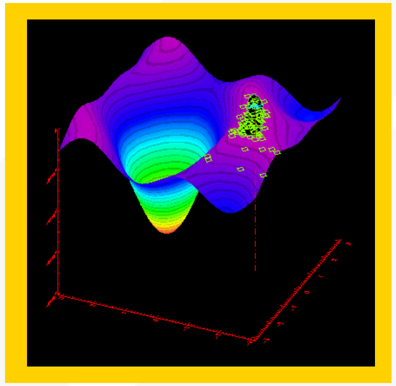
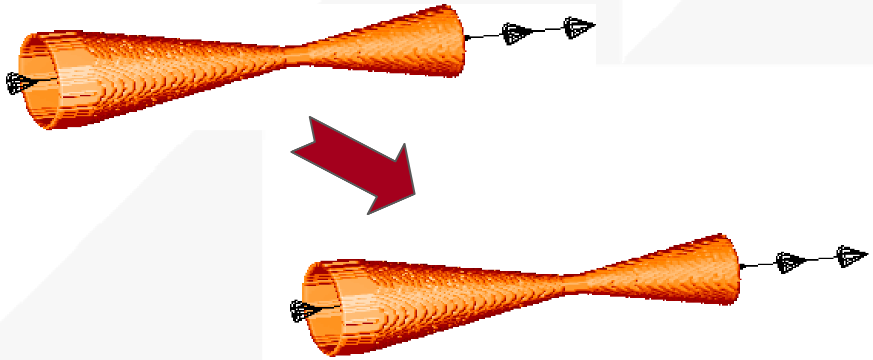
Concluding Remarks

- ES are very useful for MDO tasks
  - High dimensionality
  - Few evaluations (shots) possible
  - Mixed-integer tasks
  - MCDM tasks





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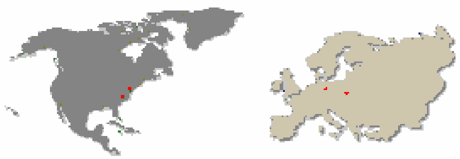
Corporate Headquarters:

Charlotte, NC
121 West Trade Street
Interstate Tower, Suite 1900
Charlotte, NC 28202

Phone: (704) 943-5400
Fax: (704) 943-5401



Dortmund, Germany
Martin-Schmeisser-Weg 15
44227 Dortmund
Germany
Phone: +49-231-725-4630

Warsaw, Poland
Łucka 11, lok.508
00-842 Warszawa
Poland
Phone: (+4822) 656-33-99
Fax: (+4822) 656-31-21

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