

System identification of high-speed railway bridges

Volkmar Zabel^{1*}, Maik Brehm¹

¹ Institute of Structural Mechanics, Bauhaus-University Weimar, Germany

Abstract

With the extension of high-speed railway networks, existing bridges have to be assessed. In many cases a suitable numerical model is required which can represent the basic dynamical properties of the existing system. In general, the correct modeling of an existing bridge is a complicated task. Sometimes additional vibration tests have to be performed to investigate the dynamical properties of the bridge. With the experimentally obtained results a numerical model can be updated to represent an almost realistic behavior. Such models can then be used to assess the bridge with respect to the new demands.

The current paper provides an approach for an optimization-based model updating using vibration data. An example is presented for a high-speed railway bridge at the line Cologne-Brussels. The results are validated by using four different optimization runs.

Keywords: Modal updating, optimization, genetic algorithm, high-speed railway bridge

1 Introduction

European rail companies are extending their high-speed railway networks. These projects include both the construction of new lines and the improvement of existing lines. In the case of an enhancement of an existing line, it is required to proof all bridges with respect to resonance problems, which can occur during train passages. In civil engineering practice this proof is usually performed by numerical analyses. However, the conformance of all requirements can not always be proven by numerical analyses. In these cases experimental investigations are carried out to identify the dynamic structural behavior of the existing bridge.

Zabel et al. (2007) concludes, that the dynamic properties of an existing railway bridge can differ considerably from those of the numerical model. This can be explained by assumptions for the respective numerical models, that do not necessarily coincide with the properties of the existing structure. Even though in many practical applications an experimental proof of the conformance of the requirements is sufficient, there are also situations where an accurate numerical model is desired. This is important, for example, to calculate the structural behavior for several load cases.

To obtain a numerical model that can be used for predictive analyses uncertain system parameters have to be identified based on experimental data. This paper reports about a study

*Contact: Dr.-Ing. Volkmar Zabel, Bauhaus-University Weimar, Institute of Structural Mechanics, Marienstraße 15, D-99423 Weimar, Germany, E-Mail: volkmar.zabel@uni-weimar.de

concerning the identification of several system parameters of a numerical model of a high-speed railway bridge with tracks on ballast that was analyzed within the RFCS research project DETAILS. It is described, how a numerical model of a complex mechanical system consisting of an elastically supported composite bridge deck, a layer of ballast, and the tracks can be updated, such that it meets the modal behavior identified from in-situ tests, utilizing numerical optimization.

2 General aspects of model updating

Model updating is a method to improve the correlation between the numerical model and a realistic structure using measured data (Steenackers and Guillaume (2006)). In the best case, a numerical model can be determined which best fits the measured data. Many errors can effect this procedure. These errors may be an inaccurate model or imprecise and incomplete measurements. Also the updating method itself can be affected by errors.

The first emphasis within a model updating is to avoid or minimize as many errors as possible to obtain realistic results. The second emphasis is to describe the errors in their quality and quantity and implement this information in the updating process. Otherwise the model updating will be significantly influenced by the errors. A very good introduction to model updating is given by Friswell and Mottershead (1995). Basically, three locations can be distinguished, where errors can occur:

- Errors associated with measured data
- Errors associated with numerical model
- Errors associated with updating method

Errors associated with measured data Measurements are usually not perfectly precise. Electronic noise is generated by instruments. Random and systematic errors are introduced by external excitation sources. Systematic errors can occur, for example, due to the additional mass of sensors (normally less relevant for railway bridges) or mistakes in documentations. Signal processing errors may be aliasing and leakage. System identification problems are, for example, the inversion of an ill-conditioned matrix.

Additional errors arise due to an incomplete description of the system behavior by the measured data. Samples are acquired at discrete time instants over finite period, which influence the resolution and range of the frequency. The sensor positions and measurement degree of freedoms are also limited in comparison to a valid numerical model (e.g., finite element model). Friswell and Mottershead (1995)

Errors associated with numerical model In most cases a finite element model will be used as the numerical model to approximate the structure. The most common errors in finite element models are discretization errors and wrong or imprecise boundary conditions. Furthermore, insufficient approximations of physical or geometrical nonlinearities are present.

The definition and choice of the updating parameters (also called design parameters) are also important. Generally, the number of updating parameters should be the smallest meaningful selection of the most sensitive and most uncertain model parameters.

Errors associated with updating methods Depending on the chosen method different errors can occur. Direct methods are mostly affected by numerical errors like ill-conditioned matrices, whereas indirect methods, such as optimization procedures, can have convergence problems in finding the exact or correct minima.

3 Optimization - a suitable model updating technique

Optimization strategies are frequently used tools for model updating. In comparison to other methods, they are flexible and can be used for moderate numerical models. Due to the iterative strategy, many calculations of the problem are needed to obtain an accurate solution. Depending on the optimization method, problems can occur, if local optima are existing.

Very often a high number of uncertain parameters are present in the model. Therefore, it is recommended to use a sensitivity analysis to reduce the number of updating parameters. To avoid the problem of a limited number of updating parameters or to find a local minimum, an evolutionary algorithm is advisable.

3.1 Sensitivity analysis

To identify the most important updating parameters, a sensitivity analysis is recommended. This analysis can include the calculation of the linear and quadratic correlation coefficients between the updating parameters and the output parameters, the coefficient of determination, and the principal component vector. It is recommended to use updating parameters which have a linear correlation coefficient greater than 0.5. The coefficient of determination for one output parameter should be greater than 80%. If a nonlinear correlation is expected, the rank correlation coefficient according to [Spearman \(1904\)](#) should be used to get information about the sensitivity between the design variables and the output variables.

Due to the fact that the correlation coefficients are based on a regression model, they depend on the quality and arrangement of the design and output variables. Clusters and outliers should be avoided to get meaningful results. For example, it has to be ensured that the correct mode will be chosen, when MAC-values ([Allemang and Brown \(1982\)](#)) are used to compare the modes of the numerical model and the real structure.

Further description of the sensitivity analysis is given in [optiSLang \(2006\)](#).

3.2 Optimization method

Basically, three standard optimization methods are distinguished:

- gradient based methods,
- evolutionary algorithms (genetic algorithms, evolution strategies, evolutionary programming), and
- response surface method (RSM) or adaptive response surface method (ARSM).

Gradient based optimization methods are using mainly Quasi-Newton methods, like LBFGS, NLPQL (Nonlinear Programming using a Quadratic or Linear Least-square algorithm) introduced by [Schittkowski \(1985\)](#), or NLPQLP. Line search algorithms ensure the stepwise convergence to the optimum using gradients. According to [optiSLang \(2006\)](#), they are recommended for solving smooth nonlinear optimization problems under the following restrictions:

- The number of design variables is not too large.
- The functions and gradients can be evaluated with sufficiently high precision. Hence, round-off errors of parallel programming with explicite FEM-codes, adaptive procedures, or single precision format limitations have to be avoided.
- The problem is smooth and well-scaled.

Evolutionary algorithms are stochastic search methods. They imitate natural biological evolution processes like adaptation, selection, and variation. To obtain a better approximation for the solution of the optimization problem, a population of artificial individuals searches the design space of possible solutions based on the Darwin principle ‘survival of the fittest’. The three main classes are genetic algorithms (e.g., [Holland \(1975\)](#)), evolution strategies (e.g., [Rechenberg \(1973\)](#)) and evolutionary programming (e.g., [Fogel et al. \(1966\)](#)). Gradient-based information does not have to be available, which is often difficult for binary or discrete search spaces. Compared to other optimization strategies, these algorithms are more able to find the global minimum if many local minima are present. Unfortunately, significantly more numerical effort has to be invested to obtain the same accuracies. [optiSLang \(2006\)](#) recommends the application of evolutionary algorithms:

- in all cases when gradient based optimization or response surface approximation fails,
- in case of discrete or binary variables dominating the response, and
- in case of a very high number of variables and/or constraints.

Response surface methods generate a response surface with appropriate approximation functions on a suitable set of discrete support points of the objective function. The optimization itself is then performed on the response surface using gradient based or evolutionary algorithms. To approximate the response, several methods like least square or moving least square approximations are possible. The resulting response surface has to be well qualified to represent global trends of the optimization problem. Adaptive response surface methods (e.g., [Etman et al. \(1996\)](#), [Kurtaran et al. \(2002\)](#)) ensure the trends by establishing the response surface adaptively. The extremely fast convergence is the main advantage of this method. It should be applied to reasonably smooth problems with not more than 10 continuous variables.

3.3 Design variables and output variables

For the application of finite element model updating the design variables can be

- material properties,
- geometry data,

- boundary conditions,
- modeling parameters, or
- loads.

For each of those variables, a lower and upper bound have to be defined. If some combinations of design variables are not possible, constraints can be applied as well.

The design variables are linked with the output variables. Hence a numerical model is necessary which can connect the design variables with the output variables in the most realistic way. Obviously, the output variables should be comparable with the features extracted from measured data.

Those features are, for example,

- eigenfrequencies,
- mode shapes, modal deflections, modal curvatures,
- spectra from a Fast Fourier Transformation,
- Frequency Response Functions (FRF),
- Impulse Response Functions (IRF),
- wavelet decompositions,
- wavelet packet energy,
- wavelet packet energy of the modified IRF,
- maximum / minimum of time histories,
- spectrum of time series, and
- rainflows of responses.

However some of the suggested objective functions require the measurement and simulation of the excitation. This is not always possible, for example, for ambient excitation. Due to measurement errors, it is recommended to use a set of global and local criteria to increase the possibility to gain correct values.

3.4 Objective function

A perfect optimization function is sensitive to all design parameters and has a smooth shape. A well-defined global optimum is required to guarantee the uniqueness of the solution. An exhaustive comparison of several objective function by means of an academic example has been performed in [Zabel and Brehm \(2009\)](#).

In the context of stochastic model updating, a certain finite element model should be found, which best represents the measured data. The features extracted from measured data can be collected in a random vector \mathbf{z}_m , which follows a multivariate distribution with the density p_m , the mean value vector $\bar{\mathbf{z}}_m$ and the covariance matrix $\text{Cov}(\mathbf{z}_m, \mathbf{z}_m)$. Analogous a random vector \mathbf{z}_j can be defined, which contains the features extracted from the numerical model in the

optimization step j . The random vector \mathbf{z}_j depends on the vector of design variables $\boldsymbol{\theta}_j$ and is based on a multivariate distribution with the density p_j . The mean value vector is given by $\bar{\mathbf{z}}_j$ and the covariance matrix is denoted by $\text{Cov}(\mathbf{z}_j, \mathbf{z}_j)$.

During the optimization the design parameters should be modified in a way that the objective function I based on a certain distance between the random vectors \mathbf{z}_j and \mathbf{z}_m will be minimized.

$$I(\mathbf{z}_j(\boldsymbol{\theta}_j), \mathbf{z}_m) \longrightarrow \min \quad (1)$$

Depending on the statistical information extracted from the measurement, several objective functions can be applied to solve the optimization problem and to obtain the mean value $\bar{\boldsymbol{\theta}}_j$ and the covariance matrix $\text{Cov}(\boldsymbol{\theta}_j, \boldsymbol{\theta}_j)$. It has to be mentioned that the variation of the design parameters can only be identified, if the variation of the features based on the measurements and the simulation is included in the objective function.

If only the mean value vectors $\bar{\mathbf{z}}_m$ and $\bar{\mathbf{z}}_j$ of the measured and simulated data are available, the weighted Euclidean distance can be used:

$$I_E(\mathbf{z}_j, \mathbf{z}_m) = \sqrt{(\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)^T \mathbf{W} (\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)} \quad (2)$$

The matrix \mathbf{W} is a weighting matrix which can be applied to regularize the function or to adjust the weights of certain features.

The Euclidean distance becomes the Mahalanobis distance ([Mahalanobis \(1936\)](#)), if the inverse of covariance matrix $\text{Cov}(\mathbf{z}_m, \mathbf{z}_m)^{-1}$ of the measured feature vector is used as weighting matrix \mathbf{W} .

$$I_M(\mathbf{z}_j, \mathbf{z}_m) = \sqrt{(\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)^T \text{Cov}(\mathbf{z}_m, \mathbf{z}_m)^{-1} (\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)} \quad (3)$$

If the mean values and the covariance matrix of the measured and simulated values are given, the Kullback-Leibler relative entropy function can be applied. More details can be found in [Kakizawa et al. \(March 1998\)](#). The Kullback-Leibler distance [Kullback \(1978\)](#) is defined by

$$I_{KL}(\mathbf{z}_j, \mathbf{z}_m) = E_{p_j} \left[\ln \frac{p_j(x)}{p_m(x)} \right] = \int_{-\infty}^{\infty} p_j(x) \ln \frac{p_j(x)}{p_m(x)} dx, \quad (4)$$

where $p_j(x)$ and $p_m(x)$ are the densities of the distributions \mathbf{z}_j and \mathbf{z}_m , respectively. E_{p_j} is the expectation value under the density $p_j(x)$. [Kakizawa et al. \(March 1998\)](#) shows that for multivariate normal distributions \mathbf{z}_i and \mathbf{z}_m with dimension N , Equation (4) simplifies to

$$I_{KL}(\mathbf{z}_j, \mathbf{z}_m) = \frac{1}{2} \left(\text{tr}(\text{Cov}(\mathbf{z}_j, \mathbf{z}_j) \text{Cov}(\mathbf{z}_m, \mathbf{z}_m)^{-1}) - \ln \frac{|\text{Cov}(\mathbf{z}_j, \mathbf{z}_j)|}{|\text{Cov}(\mathbf{z}_m, \mathbf{z}_m)|} - N \right) + \frac{1}{2} \left((\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)^T \text{Cov}(\mathbf{z}_m, \mathbf{z}_m)^{-1} (\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j) \right), \quad (5)$$

where $|\cdot|$ denotes the determinant of a matrix and $\text{tr}(\cdot)$ is the trace of a matrix.

Another statistical distance has been proposed by [Bhattacharyya \(1943\)](#)

$$I_B(\mathbf{z}_j, \mathbf{z}_m) = -\ln E_{p_j} \left[\left(\frac{p_m(x)}{p_j(x)} \right)^{\frac{1}{2}} \right] = -\ln \int_{-\infty}^{\infty} p_j(x) \left(\frac{p_m(x)}{p_j(x)} \right)^{\frac{1}{2}} dx, \quad (6)$$

which simplifies to

$$I_B(\mathbf{z}_j, \mathbf{z}_m) = \frac{1}{2} \left(\ln \frac{|\frac{1}{2}(\text{Cov}(\mathbf{z}_j, \mathbf{z}_j) + \text{Cov}(\mathbf{z}_m, \mathbf{z}_m))|}{|\text{Cov}(\mathbf{z}_m, \mathbf{z}_m)|} - \frac{1}{2} \ln \frac{|\text{Cov}(\mathbf{z}_j, \mathbf{z}_j)|}{|\text{Cov}(\mathbf{z}_m, \mathbf{z}_m)|} \right) + \frac{1}{8} \left((\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)^T \left(\frac{\text{Cov}(\mathbf{z}_m, \mathbf{z}_m) + \text{Cov}(\mathbf{z}_j, \mathbf{z}_j)}{2} \right)^{-1} (\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j) \right) \quad (7)$$

in case of multivariate normal distributions \mathbf{z}_i and \mathbf{z}_m (Kailath (1967)). The Bhattacharyya distance is equivalent to the Chernoff distance (Renyi (1961))

$$I_C(\mathbf{z}_j, \mathbf{z}_m) = -\ln E_{p_j} \left[\left(\frac{p_j(x)}{p_m(x)} \right)^\alpha \right] \quad (8)$$

for $\alpha = 0.5$.

Khodaparast and Mottershead (2008) proposed a distance, where the weighted Euclidean norm and the Frobenius norm of the covariance matrices are combined.

$$I_{EF}(\mathbf{z}_j, \mathbf{z}_m) = \|\text{Cov}(\mathbf{z}_m, \mathbf{z}_m) - \text{Cov}(\mathbf{z}_j, \mathbf{z}_j)\|_F + \frac{1}{2} \left((\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j)^T \mathbf{W} (\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j) \right) \quad (9)$$

The matrix \mathbf{W} is a weighting matrix similar to Equation (2).

4 High-speed railway bridge Erfthal

4.1 Finite element model of the bridge

To keep the number of degree-of-freedom as small as possible the finite element model has been modeled with shell, beam, and spring elements.

Concrete slab, support beams and ballast Shell elements with four nodes are used for the concrete slab. The two concrete slabs are made of concrete of type B25(C20/25) and B35(C30/37). In Schneider (1998) the Young's modulus is denoted by $2.9 \cdot 10^{10}$ and $3.2 \cdot 10^{10}$ N/m², respectively. The coefficient of variation 0.15 is given in Faber (2000). The density is defined to be 2400 kg/m³ and the Poisson ratio is 0.2. The support beams have the same material behavior as the concrete slab.

The ballast has a density between 1700 and 1900 kg/m³. The stiffness of the ballast is neglected in conventional considerations. However, in the current model, the mass and the stiffness will be added to the concrete slab.

The connections between the two superstructures and between the superstructures and the soil, both established by the ballast, are realized by springs (3 translational and 2 rotational degree-of-freedom).

HEM1000 steel beams The embedded HEM1000-beams are modeled with 2-node beam elements. The geometry parameters of the section are given in Schneider (1998) and assumed to be deterministic.

According to Schneider (1998), the Young's modulus, Poisson ratio, and density of steel are $2.1 \cdot 10^{11}$ N/m², 0.3, and 7850 kg/m³, respectively. The coefficient of variation of the Young's modulus is set to 0.03 (Faber (2000)).

Table 1: Static stiffness of the elastomeric bearings with a shear modulus of $1 \cdot 10^6 \text{ N/m}^2$

Stiffness	Bearing type 1 400x500	Bearing type 2 450x550
k_{u_x}	$3.9000 \cdot 10^6 \text{ N/m}$	$4.8490 \cdot 10^6 \text{ N/m}$
k_{u_y}	$3.9000 \cdot 10^6 \text{ N/m}$	$4.8490 \cdot 10^6 \text{ N/m}$
k_{u_z}	$7.1044 \cdot 10^8 \text{ N/m}$	$1.0769 \cdot 10^9 \text{ N/m}$
k_{r_x}	$1.5407 \cdot 10^7 \text{ Nm/rad}$	$2.8551 \cdot 10^7 \text{ Nm/rad}$
k_{r_y}	$7.5819 \cdot 10^6 \text{ Nm/rad}$	$1.5147 \cdot 10^7 \text{ Nm/rad}$
k_{r_z}	$1.0187 \cdot 10^5 \text{ Nm/rad}$	$1.5813 \cdot 10^5 \text{ Nm/rad}$

Elastomeric bearings The elastomeric bearings are represented by springs with 3 translational and 3 rotational degree-of-freedom. The stiffness has been adopted by [DINEN1337-3 \(2005\)](#). Thus, the static stiffness of elastomeric bearing is given in Table 1.

According to [DINEN1337-3 \(2005\)](#) the shear modulus of the bearing and, consequently, the stiffness depends on the fabrication, temperature, and aging of the material. The possible range for the shear modulus G after the fabrication at $(23 \pm 5)^\circ\text{C}$ is $0.6 \cdot 10^6 \text{ N/m}^2$ and $1.35 \cdot 10^6 \text{ N/m}^2$. At a temperature of $(-25 \pm 2)^\circ\text{C}$ the shear modulus should not be higher than 3 times the shear modulus at fabrication temperature. During aging, the shear modulus can increase by $0.15 \cdot 10^6 \text{ N/m}^2$. In addition, the dynamic shear modulus can be 1.25 to 3 times higher than the static shear modulus. Finally, the dynamic shear modulus during the measurement at a temperature of around 15°C can vary approximately between $0.75 \cdot 10^6 \text{ N/m}^2$ and $6.3 \cdot 10^6 \text{ N/m}^2$ with a mean value of about $2.5 \cdot 10^6 \text{ N/m}^2$.

Rail, pads, and sleepers For this kind of structure, the rail has an important influence on the structure's behavior. Hence, the rail with sleepers and pads are modeled. The sleepers and rails are 2-node beam elements connected with springs (rail pads). The connection to the slab is given by springs between the sleepers and the slab.

A collection of the vertical dynamic stiffness of pads is given in [Knothe \(2001\)](#). Thereby, the values differ between $0.6 \cdot 10^8 \text{ N/m}$ and $7.7 \cdot 10^8 \text{ N/m}$.

The material parameters of rail steel are similar to those of structural steel. The sleepers (type B75) and rail (UIC60) are described for example in [Matthwes \(2003\)](#). The concrete of the sleeper is of type B60 (C50/60) and has a static Young's modulus of about $3.7 \cdot 10^{10}$ ([Schneider \(1998\)](#)). [Faber \(2000\)](#) suggests a coefficient of variation of about 0.15.

An isometric view of the finite element model generated with [SLang \(2007\)](#) is shown in

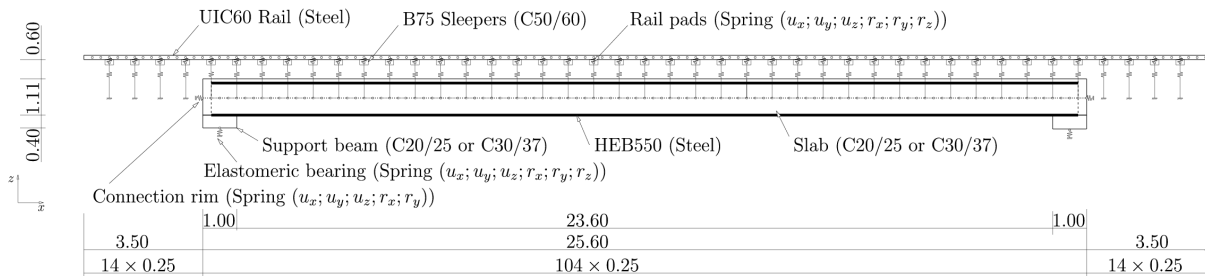


Figure 1: Longitudinal section of the finite element model

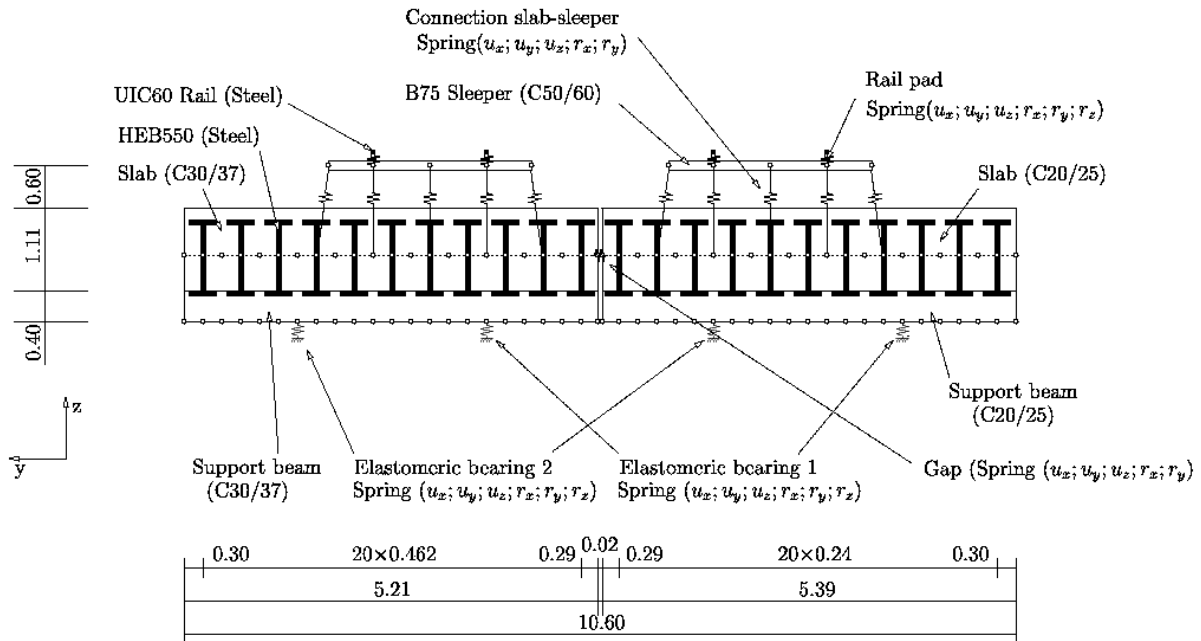


Figure 2: Cross section of the finite element model

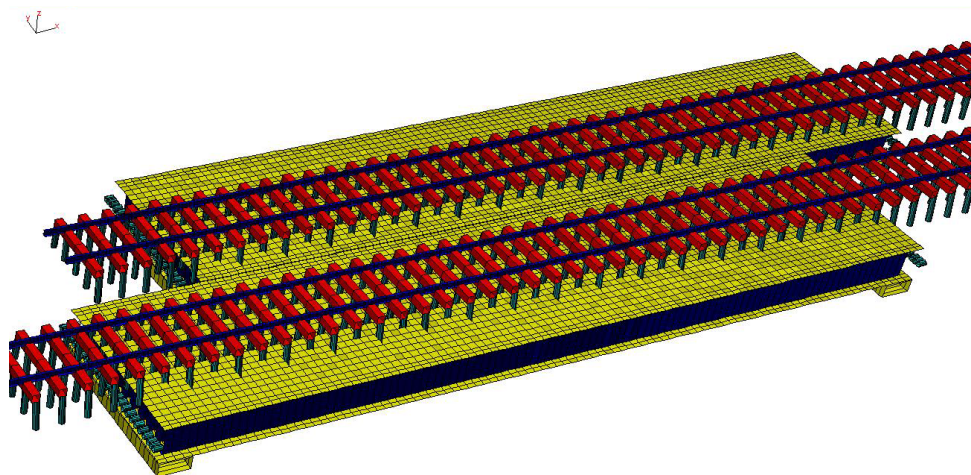


Figure 3: Isometric view of the finite element model

Figure 3. More detailed information of the model can be extracted from Figures 1 and 2.

4.2 Sensitivity analysis

Since the uncertainty of the identification of a sensible parameter set by a numerical optimization grows and the probability to find the global optimum decreases with an increasing number of design variables, a reduced set of the 36 updating parameters has to be selected. This was the motivation to perform a sensitivity analysis. The linear and quadratic correlation coefficients are presented in Figures 4 and 5, respectively. By excluding all coefficients between -0.2 and 0.2 the important parameters can be extracted. The most important updating parameter is the vertical stiffness of the gap. The remaining 14 updating parameters are indicated in Table 3.

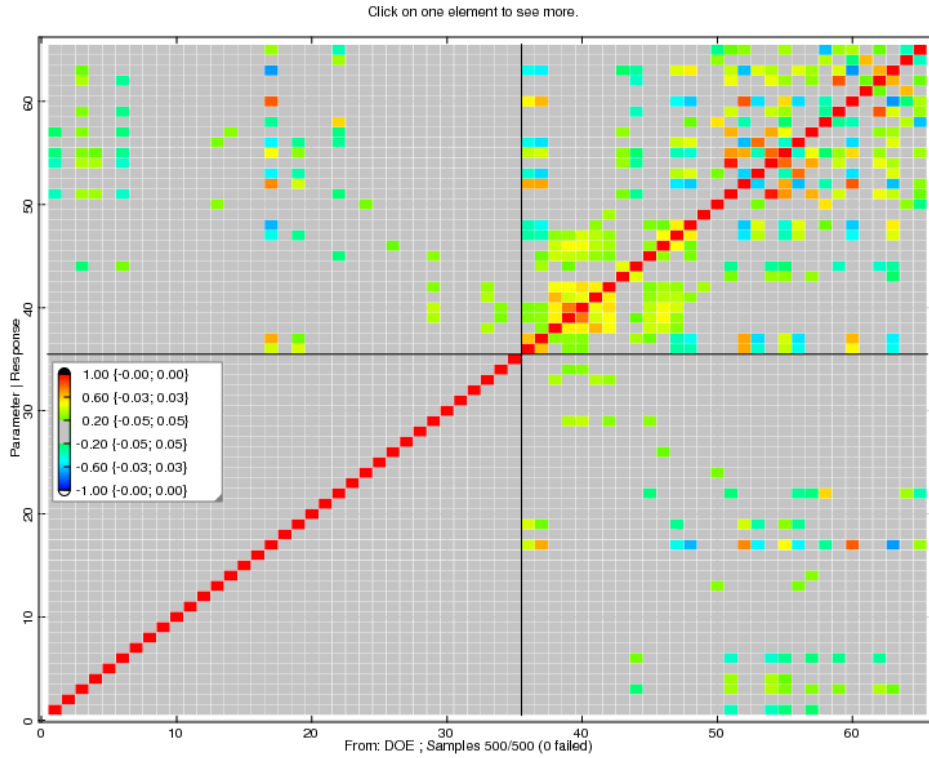


Figure 4: Linear correlation matrix of input and output variables using 500 latin hypercube samples

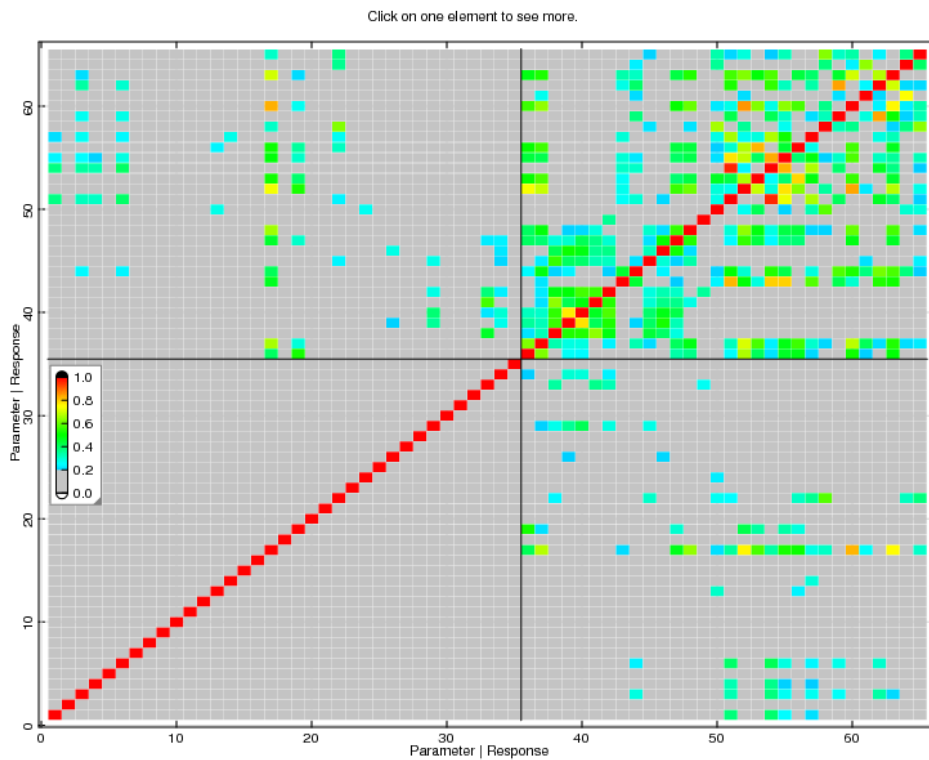


Figure 5: Quadratic correlation matrix of input and output variables using 500 latin hypercube samples

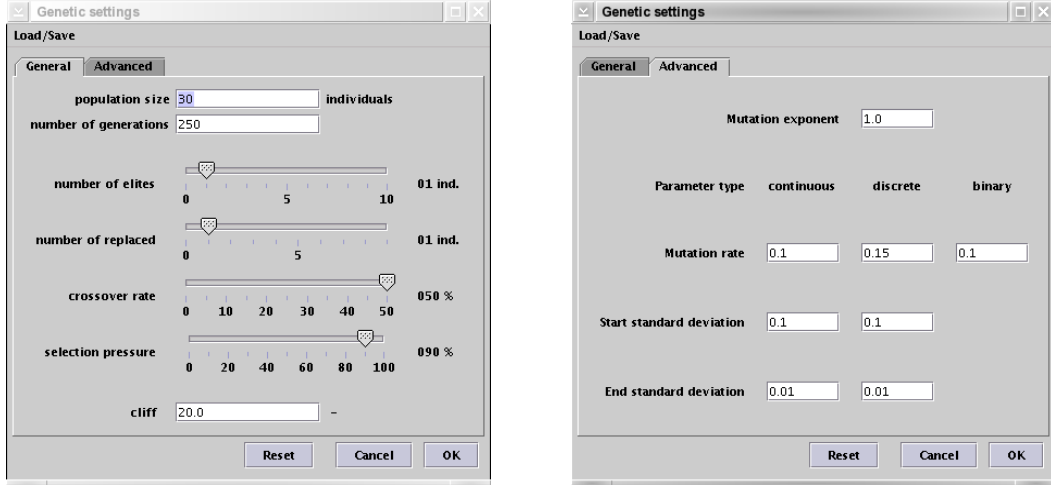


Figure 6: Parameters of the genetic algorithm

4.3 Optimization

By using the 14 most important updating parameters (bold fonts in Table 3), the model updating by means of optimization will be undertaken. The genetic algorithm provided by the software *optiSLang* (2006) was used. Figure 6 shows the chosen parameters of the algorithm.

By the previously performed system identification, mean values μ and standard deviation σ of the frequencies f and the modal vectors Φ of the first seven modes can be extracted. To consider the mean values and variances of the frequencies, an objective function similar to Equation (3) was used. By scaling the residuals of the frequencies, MAC-values and selected items of the modal displacements, the objective function can be written as

$$I = 1.0I_1 + 1.0I_2 + 0.01I_3 \quad (10)$$

with

$$I_1 = \sum_{i=1}^7 \frac{\|\mu^{f_i^m} - \mu^{f_i^j}\|}{\sigma^{f_i^m} S} \quad \text{with} \quad S = \sum_{i=1}^7 \frac{\mu^{f_i^m}}{7\sigma^{f_i^m}} \quad (11)$$

$$I_2 = \sum_{i=1}^7 (1 - \text{MAC}(\Phi_i^m, \Phi_i^j)) \quad (12)$$

$$I_3 = \sum_{i=1}^2 \sum_h \|\Phi_i^m[j] - \Phi_i^j[h]\| \quad \text{with} \quad h = 1, 11, 12, 22, 23, 33, 34, 44 \quad (13)$$

whereas the superscripts m and j denote the values extracted from the measurement and simulation. The weightings are set according to engineering judgment. The selection of h corresponds to the modal deflections at specific measurement points in the vicinity of the bearings.

4.4 Results of model updating

The optimization was performed four times with always the same initial settings of the optimization algorithm. The final values of the weighted objective functions are always very similar. However, the very best result (run 4) is presented in Tables 2 and 3. In general, all frequencies

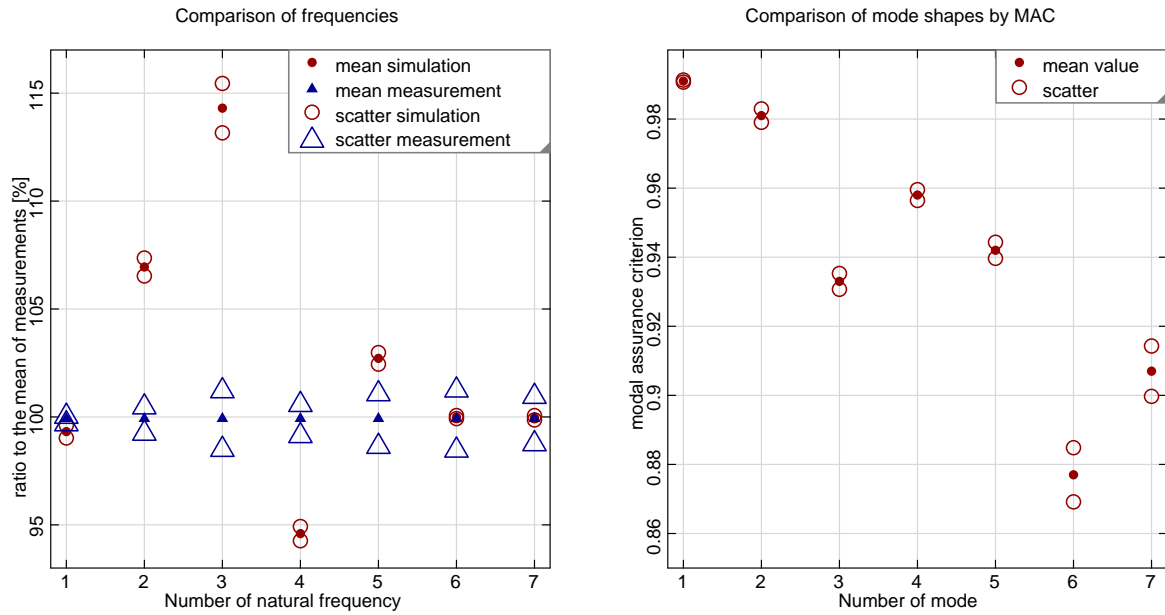


Figure 7: Comparison of obtained output values.

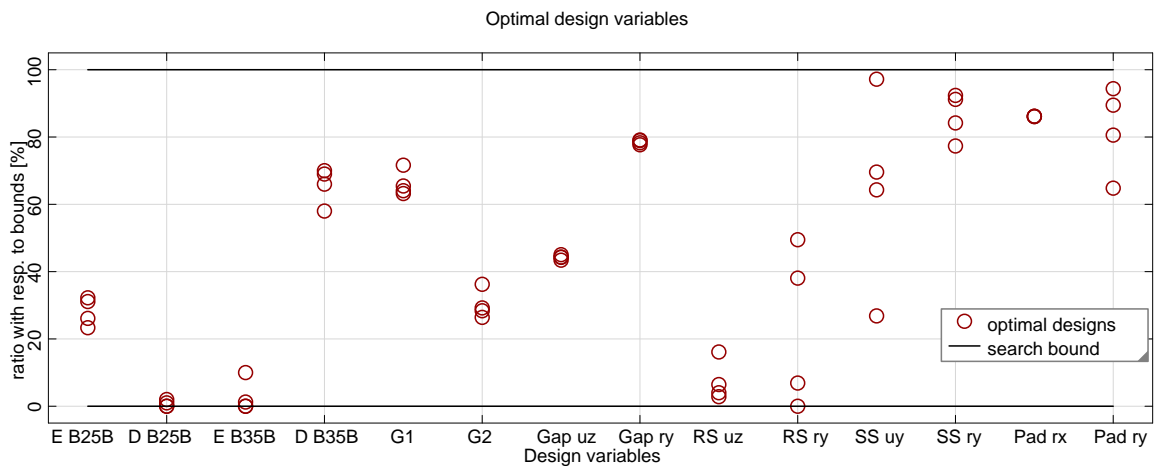


Figure 8: Comparison of optimal design variables from four different optimization runs. The stiffness of the springs are given as logarithmic values.

are close to the measured values and the MAC values are close to 1. Only the frequency of the third mode has a deviation of 13.1% with respect to the experimentally determined frequencies.

It can be observed, that the coefficients of variation (cov) of the measured frequencies correlate with the deviation between the frequencies obtained from simulation and measurement. For example, the frequency with the highest coefficient of variation has the largest error. This can be explained to a certain extent by the introduction of weighting factors in the objective function (Equation (11)), which depend on the coefficient of variation (cov).

The values of the output parameters of all four optimization runs are given in Table 4. The low coefficient of variation (cov) of the output parameters show the good quality of the optimization algorithm. The coefficient of variation of the frequencies of the measurement (Table 2) and of the simulation (Table 4) are close to each other. Figure 7 summarizes the mean values and the standard deviations of the frequencies obtained from the optimization runs with

respect to the mean values of the experimental modal analysis. The mean values and standard deviations of the modal assurance criterion is visualized in Figure 7 on the right hand side.

The optimized updating parameters of each optimization run are collected in Table 5. The coefficient of variation (cov) indicates the sensitivity of the parameter with respect to the objective function. In some cases the parameters reach the defined boundaries. However, enlarged design spaces for these parameters are physically not meaningful. The results of all four optimization runs are presented in Figure 8, where values of the design variables are given as ratio to their boundaries. It can be observed that the stiffness of the springs associated with the gap between the superstructures can be identified with high accuracy, whereas, for example, the stiffness between the slab and the sleepers is not sensitive enough to gain reliable results.

5 Conclusion

The application of optimization strategies to model updating problems is possible. However, the success depends on several factors, like a smooth objective function which is sensitive to all unknown design parameters. Even though many numerical tools are available, a full automatic model updating is not possible. The engineer's experience and judgment is essential for reasonable model updating results.

For the given example of the high-speed railway bridge Erfthal, the modeling of the finite element model was important. In the present case, the subsequent sensitivity analysis was informative enough to select a suitable number of design parameters for the optimization. Due to the high number of 14 design variables, a genetic algorithm has been used. To verify the obtained optimal solution, four independent optimization runs have been performed. The gained modal parameters are similar for all runs and close to the modal parameters extracted from the experimental modal analysis. The variation of the design variables with respect to the allowed range of the parameters presents an information about the final sensitivity and reliability of the optimal values of the design variables.

Further research is concentrated on the investigation of the influence of uncertain measured data within the model updating procedure.

Acknowledgement

The research work presented in this paper has been supported by the European Research Fund for Coal and Steel (RFCS) through the project DETAILS (DEsign for opTimal life cycle costs (LCC) of high-speed rAILway bridges by enhanced monitoring Systems).

Table 2: Comparison of the natural frequencies and mode shapes of the measurement and simulation

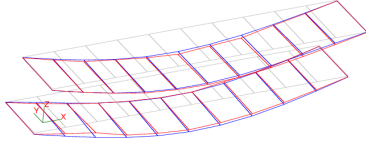
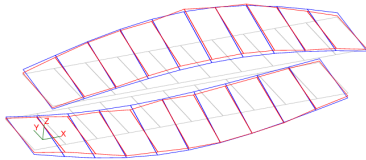
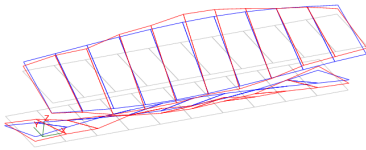
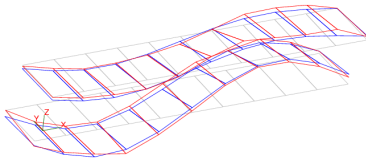
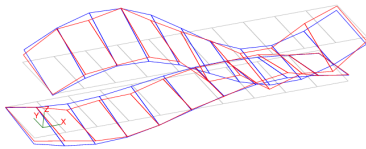
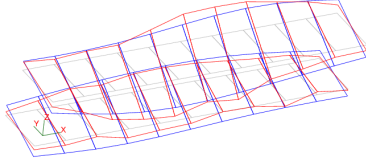
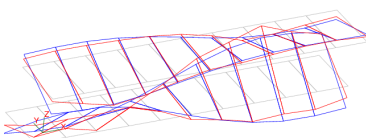
Mode	Natural Frequency		Error [%]	MAC	Mode shape Measurement (blue) Simulation (red)
	Measurements mean[Hz] (cov)	Simulation [Hz]			
1	3.68 (0.00174)	3.655	-1.1	0.991	
2	5.24 (0.00610)	5.604	6.3	0.981	
3	9.36 (0.01358)	10.699	13.1	0.933	
4	13.17 (0.00721)	12.458	-5.4	0.958	
5	13.71 (0.01219)	14.082	2.5	0.942	
6	15.09 (0.01392)	15.089	0.1	0.877	
7	20.98 (0.01097)	20.972	0.1	0.907	

Table 3: Results of the optimization

#	Parameter	Unity	initial value	lower bound	upper bound	optimum
1	Young's m. B25 + Ballast	N/m ²	2.90 10 ¹⁰	2.70 10 ¹⁰	4.50 10 ¹⁰	3.28 10 ¹⁰
2	Poisson Ratio B25	-	2.00 10 ⁻¹	1.80 10 ⁻¹	2.20 10 ⁻¹	
3	Density B25 + Ballast	kg/m ³	3.95 10 ³	3.00 10 ³	4.00 10 ³	3.00 10 ³
4	Young's m. B35 + Ballast	N/m ²	3.20 10 ¹⁰	2.90 10 ¹⁰	4.50 10 ¹⁰	3.06 10 ¹⁰
5	Poisson Ratio B35	-	2.00 10 ⁻¹	1.80 10 ⁻¹	2.20 10 ⁻¹	
6	Density B35 + Ballast	kg/m ³	3.95 10 ³	3.00 10 ³	4.00 10 ³	3.66 10 ³
7	Young's m. HEM1000	N/m ²	2.10 10 ¹¹	2.00 10 ¹¹	2.30 10 ¹¹	
8	Poisson Ratio HEM1000	-	3.00 10 ⁻¹	2.50 10 ⁻¹	3.50 10 ⁻¹	
9	Density HEM1000	kg/m ³	7.85 10 ³	7.70 10 ³	8.00 10 ³	
10	Young's modulus sleeper	N/m ²	3.70 10 ¹⁰	3.00 10 ¹⁰	5.00 10 ¹⁰	
11	Poisson Ratio sleeper	-	2.00 10 ⁻¹	2.00 10 ⁻¹	3.00 10 ⁻¹	
12	Density sleeper	kg/m ³	2.20 10 ³	2.20 10 ³	3.00 10 ³	
13	Shear m. Elastomer 1	N/m ²	1.50 10 ⁶	9.40 10 ⁵	4.50 10 ⁶	3.22 10 ⁶
14	Shear m. Elastomer 2	N/m ²	1.00 10 ⁶	9.40 10 ⁵	4.50 10 ⁶	1.88 10 ⁶
15	Ballast Gap u_x	N/m	3.00 10 ⁷	3.00 10 ⁵	3.00 10 ¹¹	
16	Ballast Gap u_y	N/m	5.00 10 ⁸	5.00 10 ⁵	5.00 10 ¹¹	
17	Ballast Gap u_z	N/m	3.00 10 ⁶	3.00 10 ⁵	9.49 10 ⁷	4.01 10 ⁶
18	Ballast Gap r_x	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ⁵	
19	Ballast Gap r_y	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ¹⁰	9.83 10 ⁷
20	Ballast Rim-Soil u_x	N/m	3.00 10 ⁷	3.00 10 ⁴	3.00 10 ¹¹	
21	Ballast Rim-Soil u_y	N/m	5.00 10 ⁸	5.00 10 ⁴	5.00 10 ¹¹	
22	Ballast Rim-Soil u_z	N/m	3.00 10 ⁶	3.00 10 ⁶	3.00 10 ¹¹	4.76 10 ⁶
23	Ballast Rim-Soil r_x	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ⁶	
24	Ballast Rim-Soil r_y	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ¹⁰	2.67 10 ⁴
25	slab-sleeper u_x	N/m	5.00 10 ⁶	5.00 10 ⁴	5.00 10 ¹¹	
26	slab-sleeper u_y	N/m	5.00 10 ⁶	1.58 10 ⁵	5.00 10 ¹¹	2.39 10 ⁹
27	slab-sleeper u_z	N/m	5.00 10 ⁷	5.00 10 ⁵	5.00 10 ¹¹	
28	slab-sleeper r_x	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ⁵	
29	slab-sleeper r_y	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ⁵	1.24 10 ⁴
30	slab-sleeper r_z	Nm/rad	1.00 10 ¹	1.00 10 ¹	1.00 10 ⁵	
31	Rail Pad u_x	N/m	1.00 10 ⁸	1.00 10 ⁵	1.00 10 ¹⁰	
32	Rail Pad u_y	N/m	1.00 10 ⁸	1.00 10 ⁵	1.00 10 ¹⁰	
33	Rail Pad u_z	N/m	1.00 10 ⁸	5.01 10 ⁶	1.58 10 ⁹	
34	Rail Pad r_x	Nm/rad	1.00 10 ⁵	1.00 10 ¹	1.00 10 ⁵	2.80 10 ⁴
35	Rail Pad r_y	Nm/rad	1.00 10 ⁵	1.00 10 ¹	1.00 10 ⁵	5.95 10 ⁴
36	Rail Pad r_z	Nm/rad	1.00 10 ⁵	1.00 10 ¹	1.00 10 ⁵	

Table 4: Comparison of the natural frequencies and MAC values of the four optimization runs

output parameter	Simulation						Measurement	
	run 1	run 2	run 3	run 4	mean	cov	mean	cov
1. frequency	3.660	3.656	3.663	3.639	3.655	0.00294	3.68	0.00174
2. frequency	5.620	5.621	5.604	5.571	5.604	0.00415	5.24	0.00610
3. frequency	10.680	10.872	10.658	10.584	10.699	0.01144	9.36	0.01358
4. frequency	12.450	12.412	12.511	12.460	12.458	0.00329	13.17	0.00721
5. frequency	14.130	14.054	14.094	14.050	14.082	0.00267	13.71	0.01219
6. frequency	15.090	15.088	15.076	15.103	15.089	0.00071	15.09	0.01392
7. frequency	20.970	20.959	20.957	21.002	20.972	0.00100	20.98	0.01097
1. MAC	0.990	0.991	0.991	0.991	0.991	0.00030	0.991	-
2. MAC	0.981	0.984	0.980	0.979	0.981	0.00194	0.981	-
3. MAC	0.934	0.936	0.933	0.931	0.933	0.00229	0.933	-
4. MAC	0.957	0.956	0.959	0.959	0.958	0.00156	0.958	-
5. MAC	0.940	0.945	0.942	0.942	0.942	0.00233	0.942	-
6. MAC	0.882	0.884	0.871	0.871	0.877	0.00784	0.877	-
7. MAC	0.910	0.916	0.903	0.902	0.907	0.00729	0.907	-

Table 5: Results of the different optimization runs with same initial configuration

#	Parameter	run 1	run 2	run 3	run 4	mean	cov
1	Young's m. B25 + Ballast	$3.17 \cdot 10^{10}$	$3.12 \cdot 10^{10}$	$3.26 \cdot 10^{10}$	$3.28 \cdot 10^{10}$	$3.21 \cdot 10^{10}$	0.0229
3	Density B25 + Ballast	$3.01 \cdot 10^3$	$3.02 \cdot 10^3$	$3.00 \cdot 10^3$	$3.00 \cdot 10^3$	$3.01 \cdot 10^3$	0.0039
4	Young's m. B35 + Ballast	$2.90 \cdot 10^{10}$	$2.90 \cdot 10^{10}$	$2.92 \cdot 10^{10}$	$3.06 \cdot 10^{10}$	$2.94 \cdot 10^{10}$	0.0256
6	Density B35 + Ballast	$3.70 \cdot 10^3$	$3.69 \cdot 10^3$	$3.58 \cdot 10^3$	$3.66 \cdot 10^3$	$3.66 \cdot 10^3$	0.0149
13	Shear m. Elast. 1	$3.49 \cdot 10^6$	$3.27 \cdot 10^6$	$3.19 \cdot 10^6$	$3.22 \cdot 10^6$	$3.29 \cdot 10^6$	0.0415
14	Shear m. Elast. 2	$2.23 \cdot 10^6$	$1.98 \cdot 10^6$	$1.95 \cdot 10^6$	$1.88 \cdot 10^6$	$2.01 \cdot 10^6$	0.0760
17	Ballast Gap u_z	$3.83 \cdot 10^6$	$3.65 \cdot 10^6$	$3.85 \cdot 10^6$	$4.01 \cdot 10^6$	$3.84 \cdot 10^6$	0.0386
19	Ballast Gap r_y	$1.27 \cdot 10^8$	$1.32 \cdot 10^8$	$1.11 \cdot 10^8$	$9.83 \cdot 10^7$	$1.17 \cdot 10^8$	0.1326
22	Ballast Rim-Soil u_z	$4.16 \cdot 10^6$	$1.92 \cdot 10^7$	$6.30 \cdot 10^6$	$4.76 \cdot 10^6$	$8.61 \cdot 10^6$	0.8282
24	Ballast Rim-Soil r_y	$1.00 \cdot 10^1$	$4.17 \cdot 10^1$	$2.84 \cdot 10^5$	$2.67 \cdot 10^4$	$7.77 \cdot 10^4$	1.7780
26	slab-sleeper u_y	$3.28 \cdot 10^{11}$	$8.78 \cdot 10^6$	$5.27 \cdot 10^9$	$2.39 \cdot 10^9$	$8.39 \cdot 10^{10}$	1.9393
29	slab-sleeper r_y	$4.44 \cdot 10^4$	$2.33 \cdot 10^4$	$4.95 \cdot 10^4$	$1.24 \cdot 10^4$	$3.24 \cdot 10^4$	0.5403
34	Rail Pad r_x	$2.78 \cdot 10^4$	$2.78 \cdot 10^4$	$2.78 \cdot 10^4$	$2.80 \cdot 10^4$	$2.78 \cdot 10^4$	0.0028
35	Rail Pad r_y	$1.67 \cdot 10^4$	$3.79 \cdot 10^4$	$3.91 \cdot 10^3$	$5.95 \cdot 10^4$	$2.95 \cdot 10^4$	0.8275

References

- ALLEMANG, R. J. ; BROWN, D.L.: A Correlation Coefficient for Modal Vector Analysis. In: *Proceedings of the 1st International Modal Analysis Conference (IMAC I)*. Orlando, Florida, USA, 1982, S. 110–116
- BHATTACHARYYA, A.: On a measure of divergence between two statistical populations defined by probability distributions. In: *Bulletin of the Calcutta Mathematical Society* 35 (1943), S. 99–109
- DINEN1337-3. *DIN EN 1337-3:2005, Lager im Bauwesen – Teil 3: Elastomerlager*. DIN Deutsches Institut für Normung. 2005
- ETMAN, J. ; ADRIAENS, J. ; SLAGMAAT, M. van ; SCHOOF, A.: Crashworthiness design optimization using multipoint sequential linear programming. In: *Structural Optimization* 12 (1996), S. 222–228
- FABER, Michael et al.: JCSS Probabilistic Model Code / Joint Committee on Structural Safety. 2000. – Technical Report
- FOGEL, L. J. ; OWENS, A. J. ; WALSH, M. J.: *Artificial intelligence through simulated evolution*. New York : John Wiley & Sons, 1966
- FRISWELL, M.I. ; MOTTERSHEAD, J.E.: *Finite Element Model Updating in Structural Dynamics*. Kluwer Academic Publishers, 1995
- HOLLAND, J. H. *Adaptation in natural and artificial systems*. 1975
- KAILATH, Thomas: The Divergence and the Bhattacharyya Distance Measures in Signal Selection. In: *IEEE transactions on communication technology* 15 (1967), S. 52–60
- KAKIZAWA, Y. ; SHUMWAY, R. H. ; TANIGUCHI, M.: Discrimination and Clustering for Multivariate Time Series. In: *Journal of the American Statistical Association* 93 (March 1998), no. 441, S. 328–340
- KHODAPARAST, H. H. ; MOTTERSHEAD, J. E.: Efficient Methods in Stochastic Model Updating. In: SAS, P. (Hrsg.) ; BERGEN, B. (Hrsg.): *ISMA International Conference on Noise and Vibration Engineering*. Leuven, Belgium, 15 - 17 September 2008, S. 1855 – 1869
- KNOTHE, Klaus: *Gleisdynamik*. Ernst & Sohn, 2001
- KULLBACK, Salomon: *Information Theory and Statistics*. Gloucester MA, 1978
- KURTARAN, H. ; ESKANDARIAN, A. ; MARZOUGUI, D. ; BEDWI, N.: Crashworthiness design optimization using successive response surface approximations. In: *Computational Mechanics* 29 (2002), S. 409–421
- MAHALANOBIS, P. C.: On the generalised distance in statistics. In: *Proceedings of the National Institute of Sciences of India* 2 (1936), no. 1, S. 49–55
- MATTHWES, Volker: *Bahnbau*. 6. Wiesbaden : Teubner, 2003

- OPTISLANG: – *the optimizing Structural Language, Version 2.1*. Weimar : dynardo GmbH, 2006
- RECHENBERG, I.: *Evolutionstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution*. Stuttgart : Frommann-Holzboog, 1973
- RENYI, Alfred: On Measures of Entropy and Information. In: *Proceedings of 4th Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley : University of California Press, 1961, S. 547–561
- SCHITTKOWSKI, K.: NLPQL: A fortran subroutine solving constrained nonlinear programming problems. In: *Annals of Operations Research* 5 (1985), S. 485–500
- SCHNEIDER, Klaus-Jürgen (Hrsg.): *Bautabellen für Ingenieure*. 13. Düsseldorf : Werner Verlag, 1998
- SLANG: – *the Structural Language, Version 5.1.0*. Weimar : Institute of Structural Mechanics, Bauhaus-University Weimar, 2007
- SPEARMAN, C.: The proof and measurement of association between two things. In: *American Journal of Psychology* 15 (1904), S. 72–101
- STEENACKERS, G. ; GUILLAUME, P.: Finite element model updating taking into account the uncertainty on the modal parameters estimates. In: *Journal of Sound and Vibration* 296 (2006), S. 919–934
- ZABEL, Volkmar ; BREHM, Maik: Stochastic Model Updating methods – a comparative study. In: *Accepted paper of IMAC XXVII A Conference and Exposition on Structural Dynamics*. Orlando, Florida, USA, 9 – 12 February 2009
- ZABEL, Volkmar ; BREHM, Maik ; BUCHER, Christian: Climatic influences on the dynamics of railway bridges with steel girders embedded in concrete. In: *International Conference on Experimental Vibration Analysis for Civil Engineering Structures (EVACES)*. Porto, Portugal, 24-26 October 2007