

# Reliability Analysis within optiSLang 3.1.0

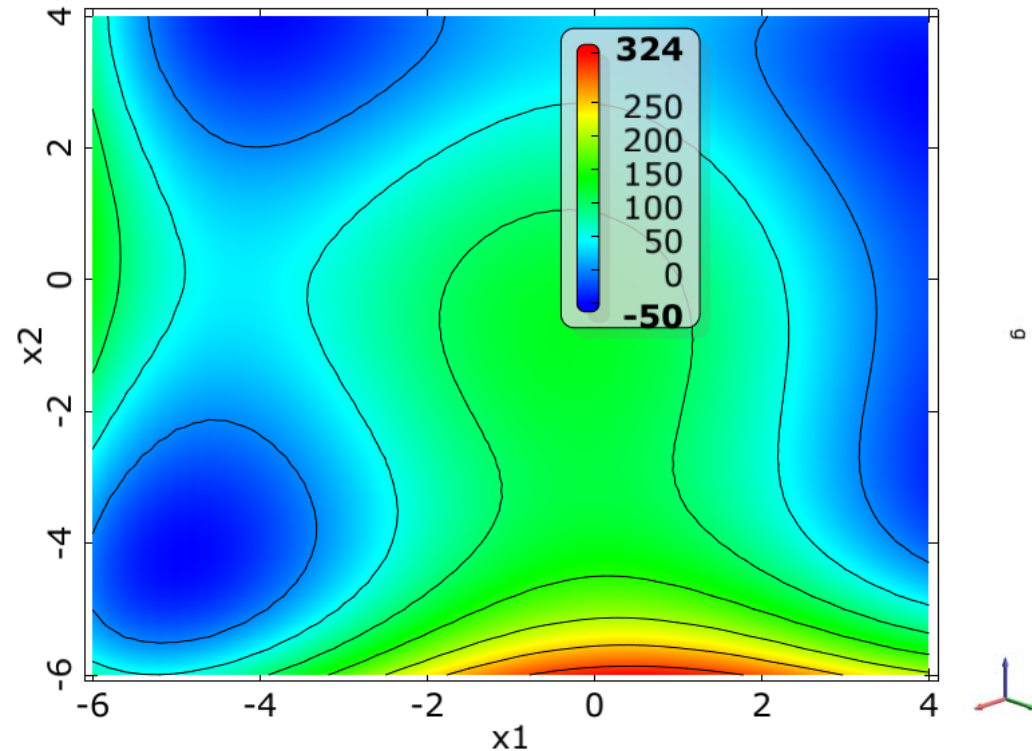
Dirk Roos

**dynardo** – dynamic software and engineering GmbH

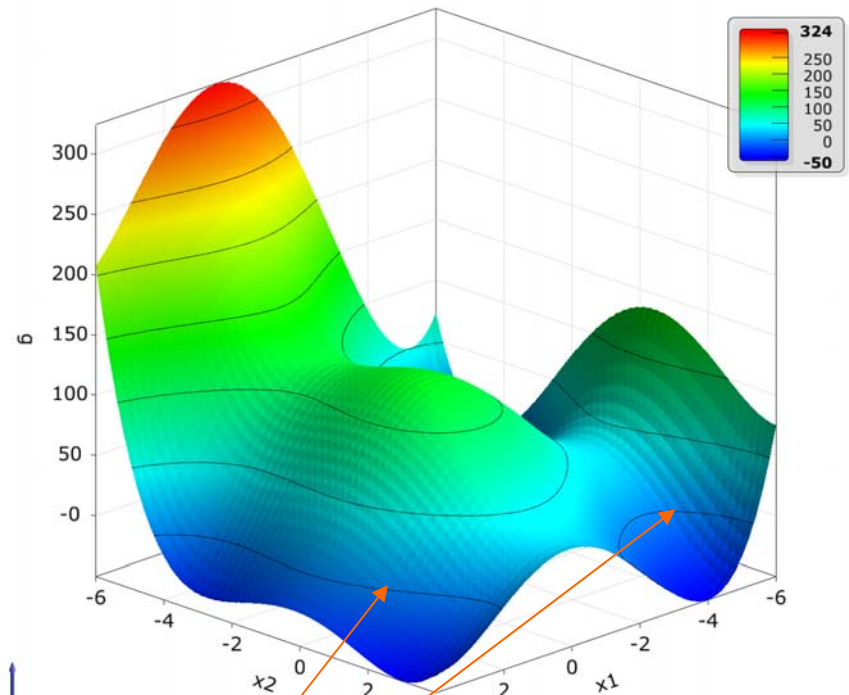


# Reliability Analysis

MLS approximation of  $g$



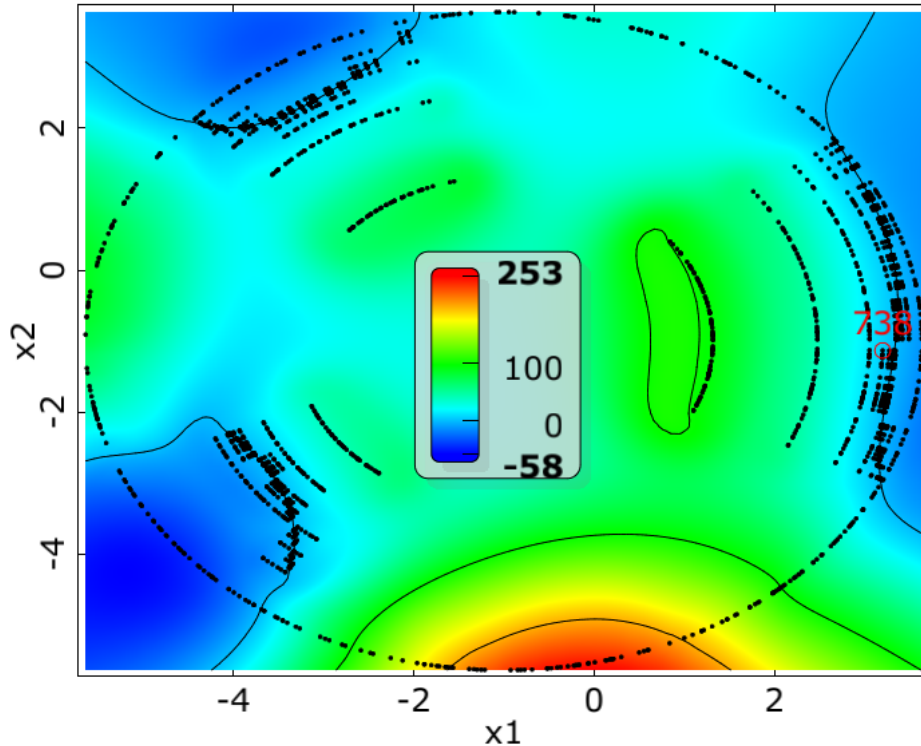
MLS approximation of  $g$



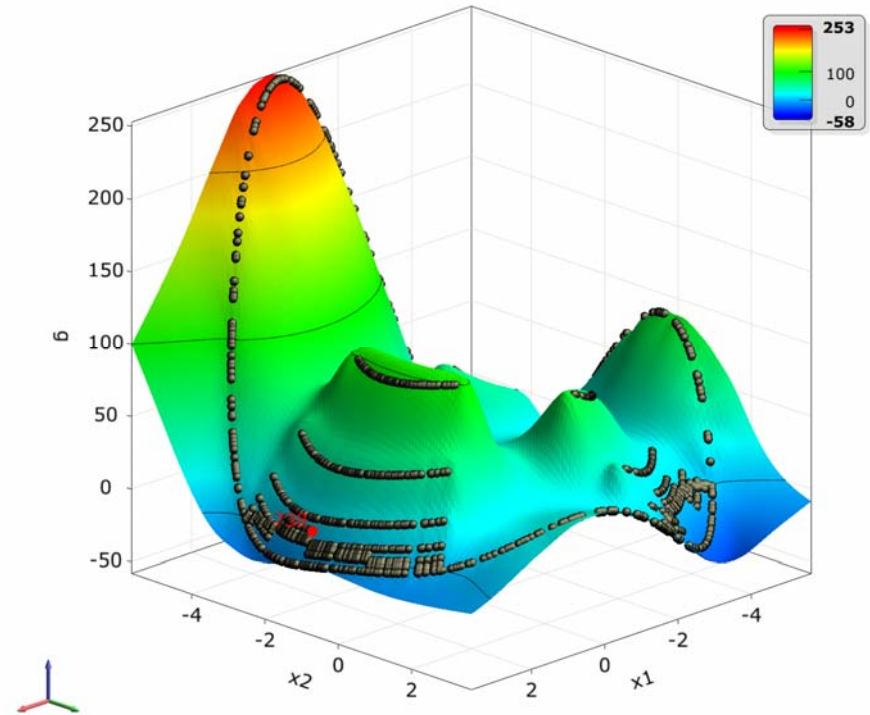
- Himmelblau function
- Nonlinear two dimensional state function  $g(x_1, x_2)$
- Nonlinear limit state function  $g(x_1, x_2) = 0$
- Three separated domains with high failure probability density

# Reliability Analysis

MLS approximation of g



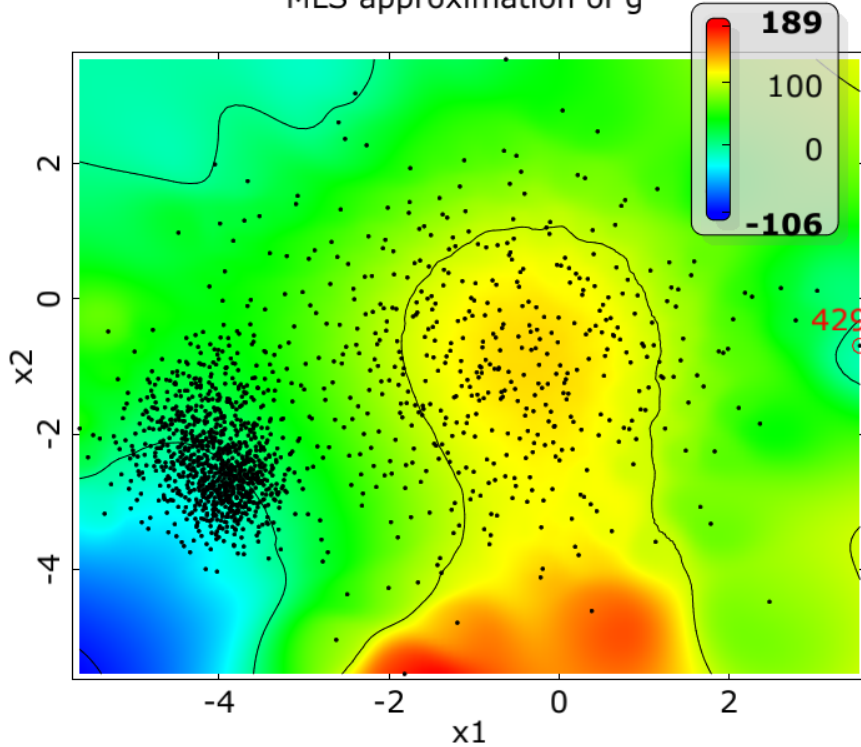
MLS approximation of g



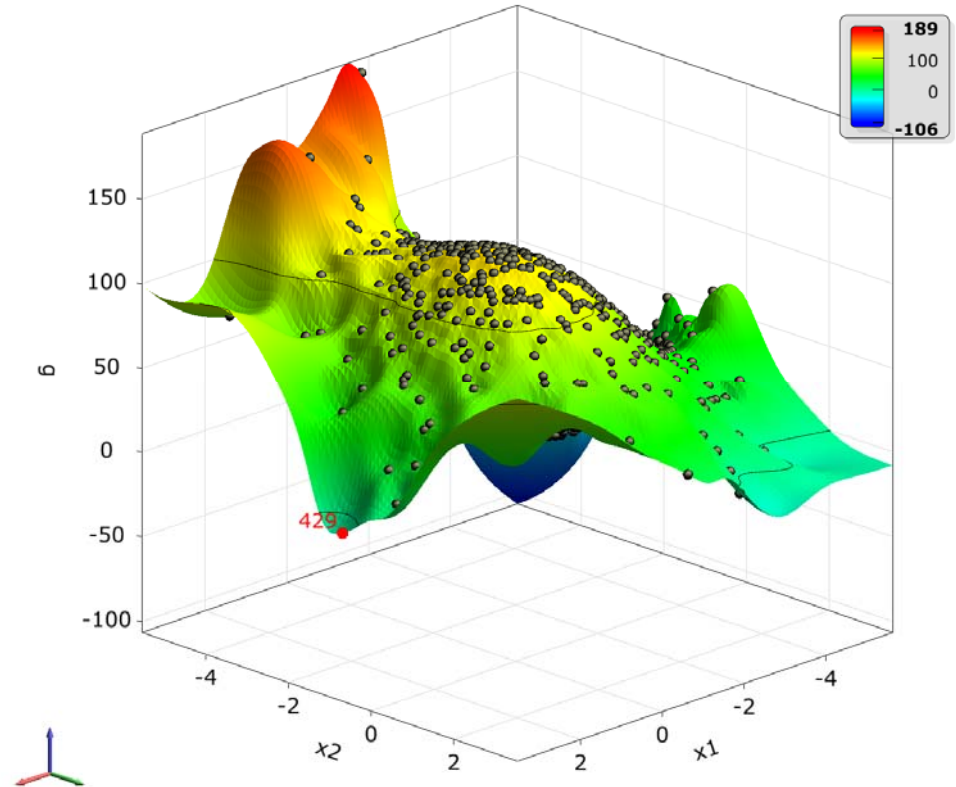
- Directional Sampling
- Design evaluations: 2612
- PF =  $1.99\text{E-}06$  ( $1.99\text{E-}06$ )
- Standard error:  $3.46\text{E-}07$

# Reliability Analysis

MLS approximation of  $g$



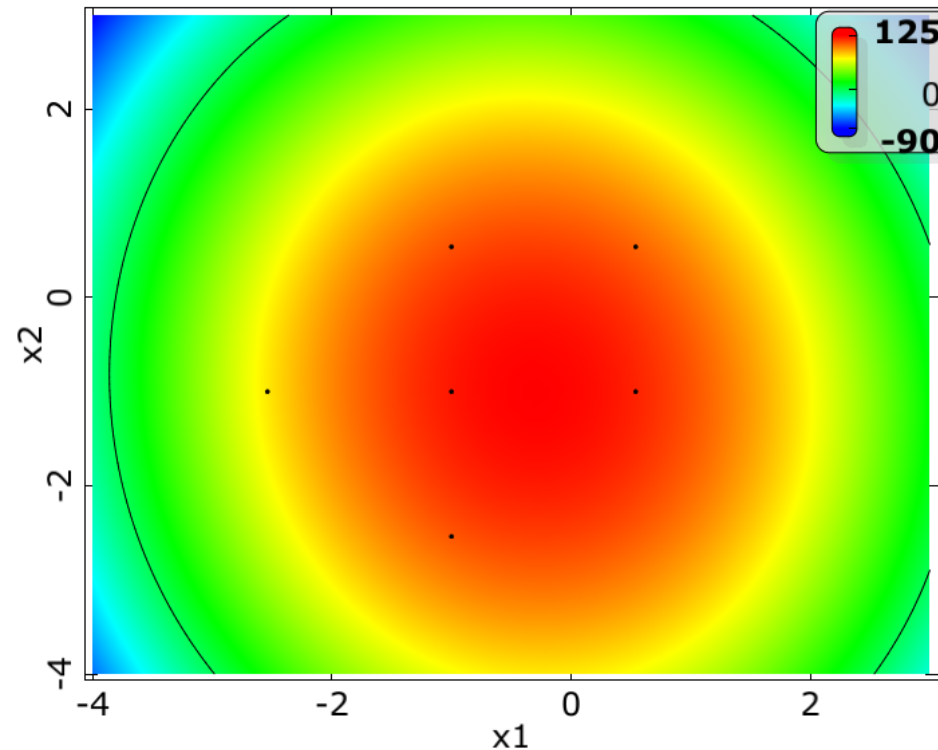
MLS approximation of  $g$



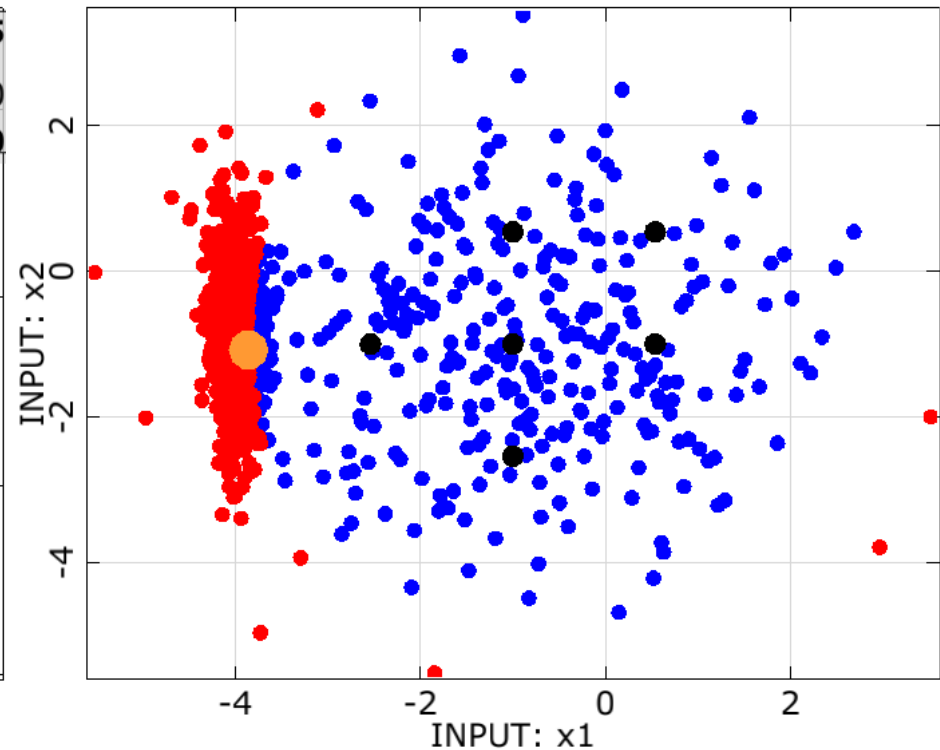
- Adaptive Sampling
- Design evaluations: 1500
- PF =  $1.56E-06$  ( $1.99E-06$ )
- Standard error:  $1.25E-07$

# Reliability Analysis

Quadratic regression of g



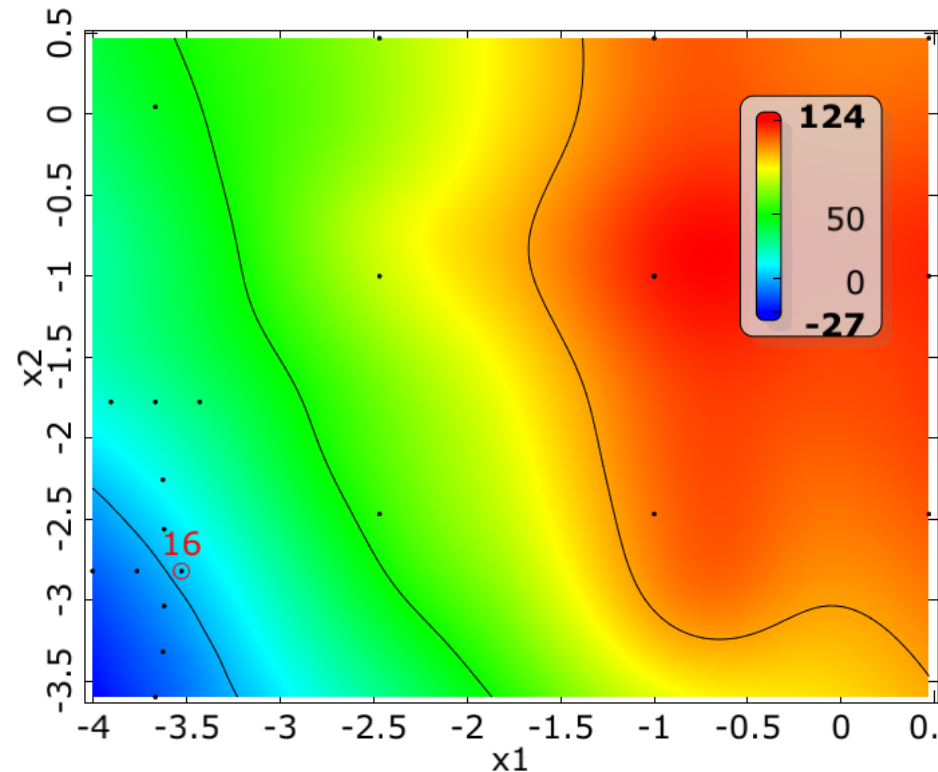
INPUT: x1 vs. INPUT: x2  
( 1. Approximation / All samples )



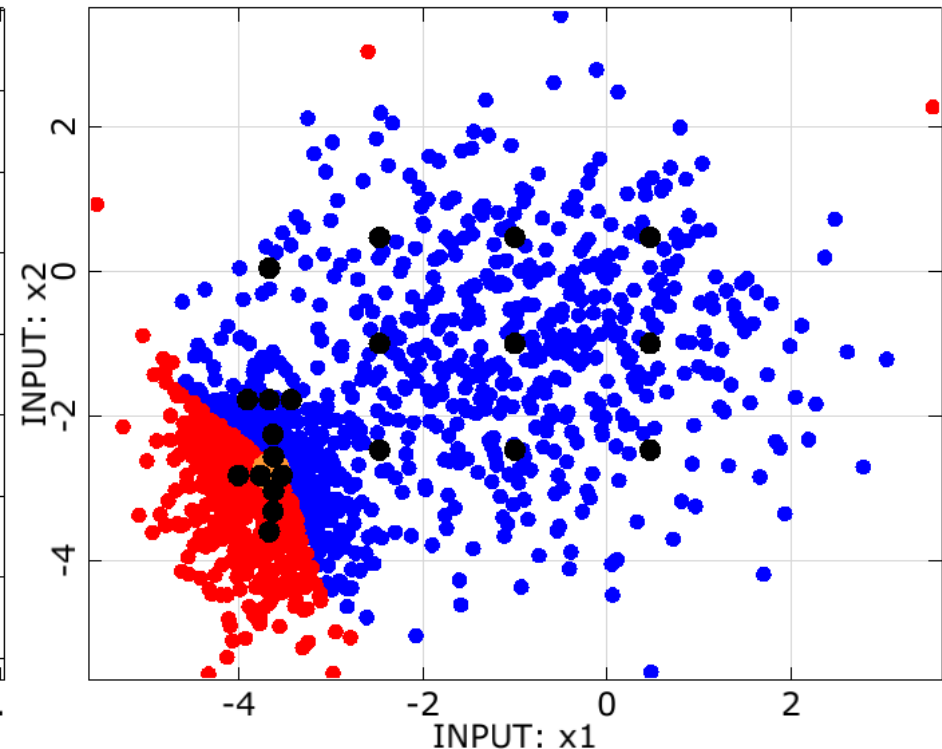
- Classic response surface method
- Design evaluations: 6
- PF = **2.89E-05** (1.99E-06)

# Reliability Analysis

MLS approximation of g

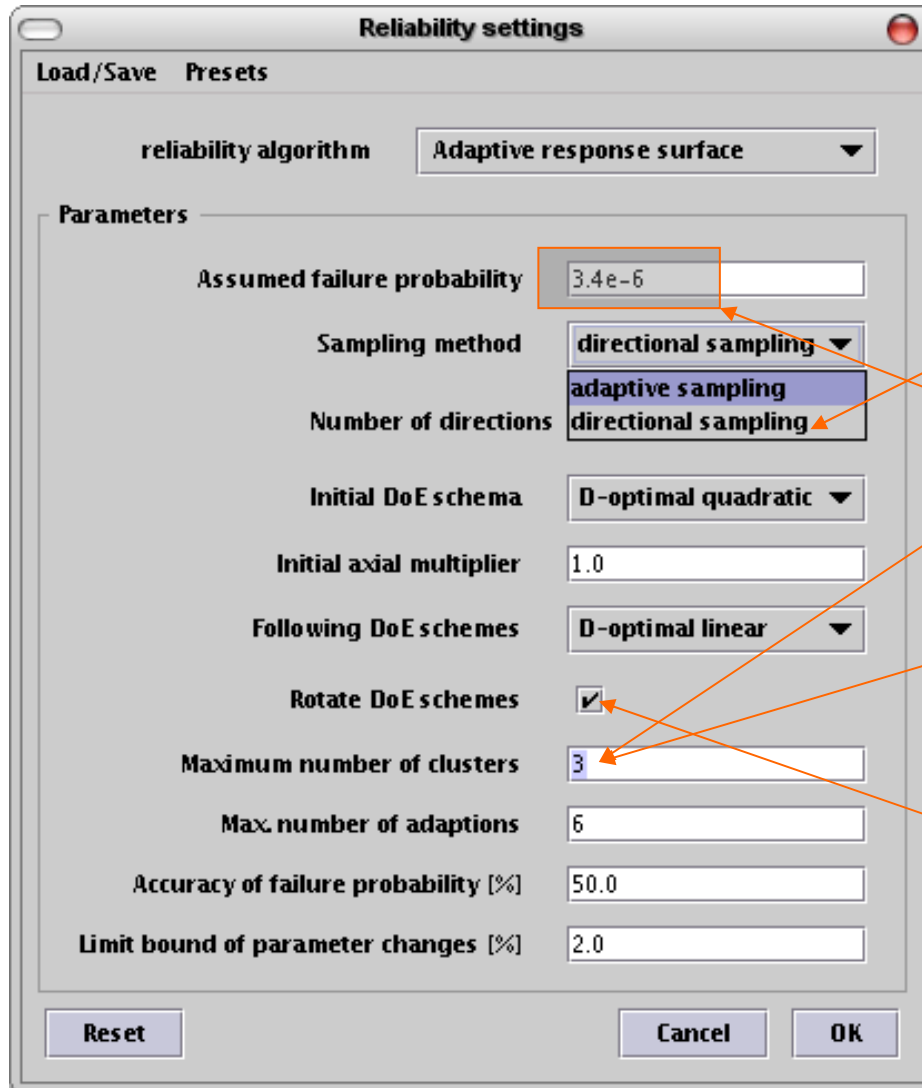


INPUT: x1 vs. INPUT: x2  
( 4. Approximation / All samples )



- Adaptive response surface method
- Adaptive sampling on MLS (**optiSLang** 3.0)
- Design evaluations: 22
- PF = 1.94E-06 (1.99E-06)
- Sigma level dependent
- $n \leq 15$
- Single adaptive DOE
- Supports two unsafe domains only

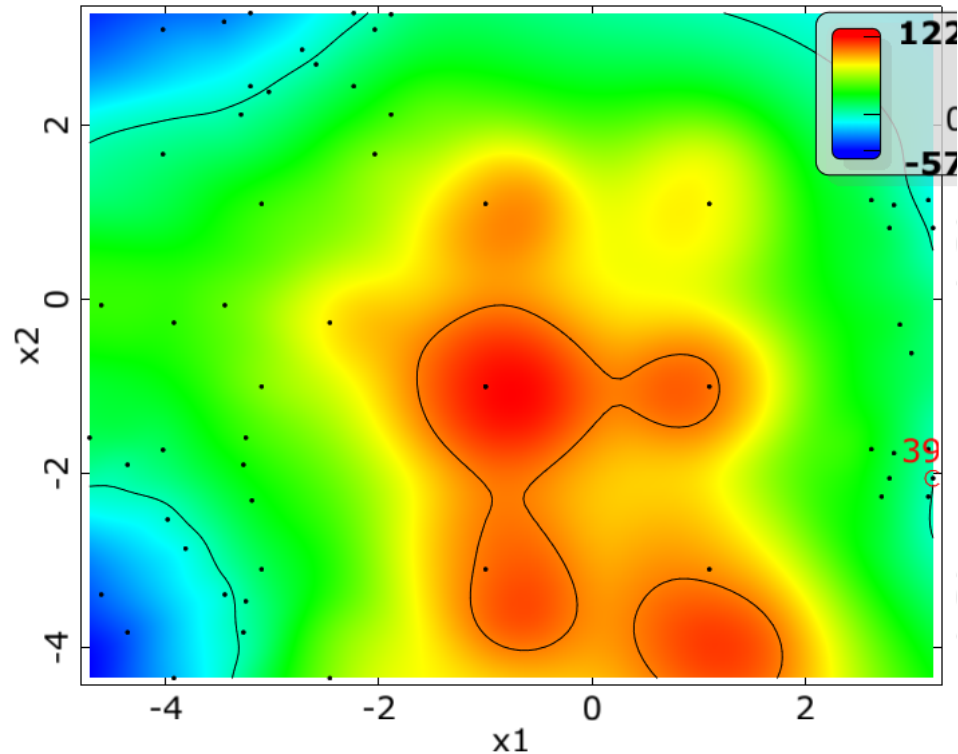
# Adaptive response surface approximation



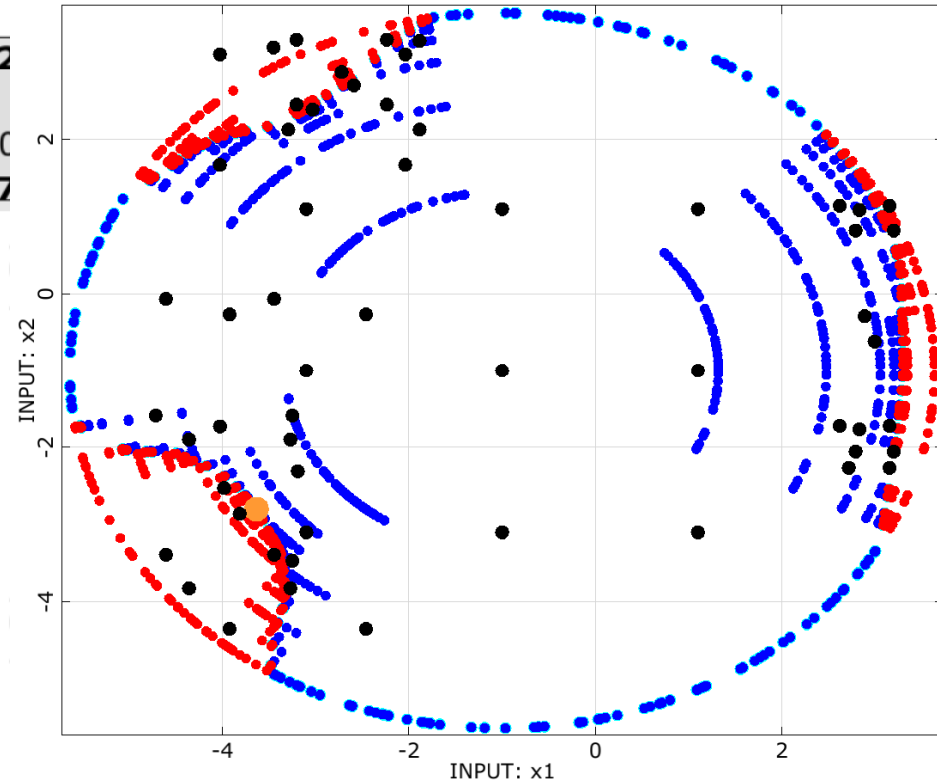
- Sampling methods on the MLS approximation:
  - Adaptive Sampling
  - Directional Sampling
    - supports more than two failure domains
    - and sigma level independent
- Cluster analysis to detect number of failure domains with high failure probability
- Rotatable adaptive designs of experiments to improve the approximation accuracy

# Reliability Analysis

MLS approximation of  $g$



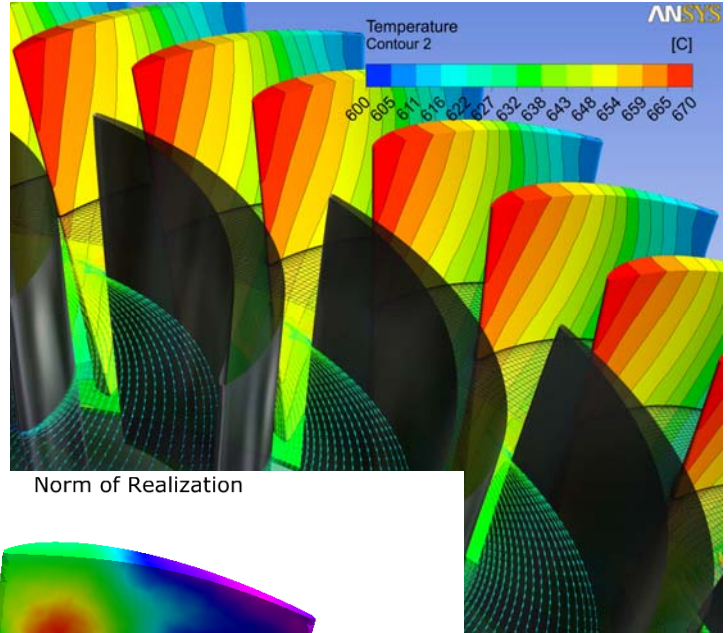
INPUT:  $x_1$  vs. INPUT:  $x_2$   
( 5. Approximation )



- Adaptive response surface method
- Directional sampling on MLS ([optiSLang 3.1](#))
- Design evaluations: 58
- PF =  $1.67E-06$  ( $1.99E-06$ )

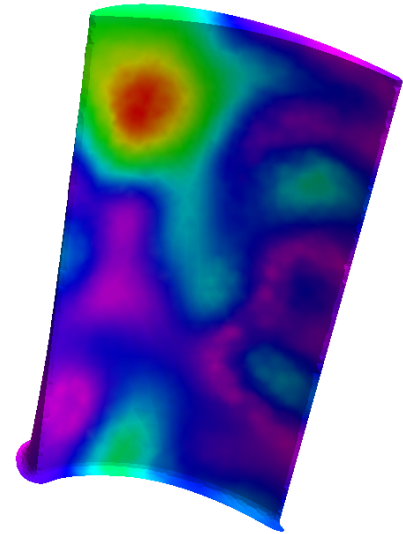
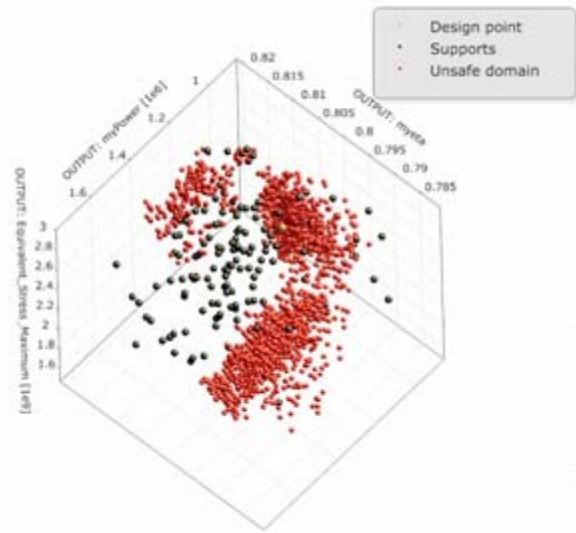


# Reliability Analysis with Manufacturing Tolerances



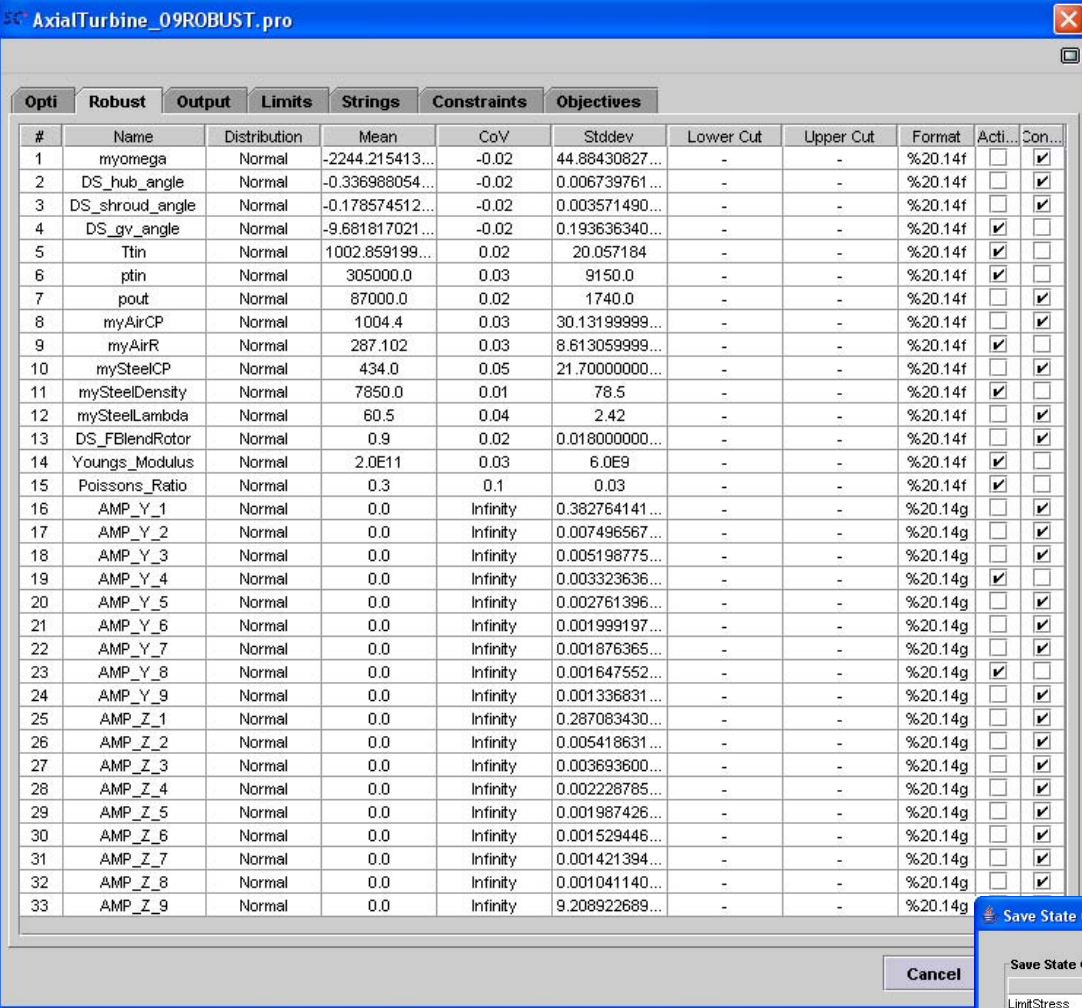
Norm of Realization

OUTPUT: myeta vs. OUTPUT: myPower vs. OUTPUT: Equivalent\_Stress\_Maximum (2. Approximation)

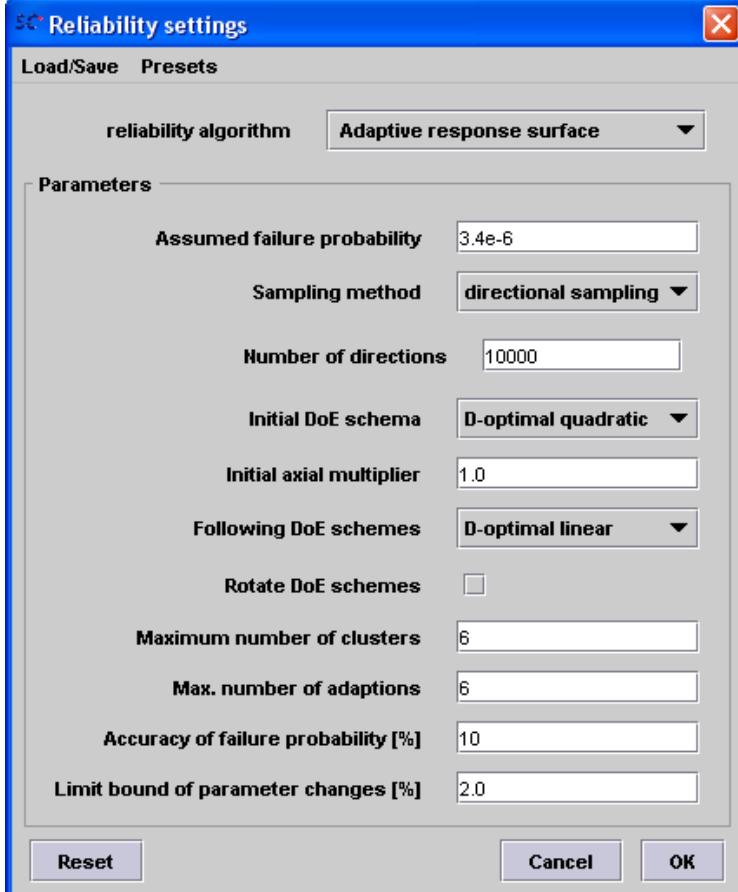


apfsLang apfsLang apfsLang apfsLang

# Example: Axial Turbine Blade



#	Name	Distribution	Mean	CoV	Stddev	Lower Cut	Upper Cut	Format	Acti...	Con...
1	myomega	Normal	-2244.215413...	-0.02	44.88430827...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
2	DS_hub_angle	Normal	-0.336988054...	-0.02	0.006739761...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
3	DS_shroud_angle	Normal	-0.178574512...	-0.02	0.003571490...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
4	DS_gv_angle	Normal	-9.681817021...	-0.02	0.193636340...	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
5	Ttin	Normal	1002.859199...	0.02	20.057184	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
6	ptin	Normal	305000.0	0.03	9150.0	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
7	pout	Normal	87000.0	0.02	1740.0	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
8	myAirCP	Normal	1004.4	0.03	30.13199999...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
9	myAirR	Normal	287.102	0.03	8.613059999...	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
10	mySteelCP	Normal	434.0	0.05	21.70000000...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
11	mySteelDensity	Normal	7850.0	0.01	78.5	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
12	mySteelLambda	Normal	60.5	0.04	2.42	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
13	DS_FBlendRotor	Normal	0.9	0.02	0.018000000...	-	-	%20.14f	<input type="checkbox"/>	<input checked="" type="checkbox"/>
14	Youngs_Modulus	Normal	2.0E11	0.03	6.0E9	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
15	Poissons_Ratio	Normal	0.3	0.1	0.03	-	-	%20.14f	<input checked="" type="checkbox"/>	<input type="checkbox"/>
16	AMP_Y_1	Normal	0.0	Infinity	0.382764141...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
17	AMP_Y_2	Normal	0.0	Infinity	0.007496567...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
18	AMP_Y_3	Normal	0.0	Infinity	0.005198775...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
19	AMP_Y_4	Normal	0.0	Infinity	0.003323636...	-	-	%20.14g	<input checked="" type="checkbox"/>	<input type="checkbox"/>
20	AMP_Y_5	Normal	0.0	Infinity	0.002761396...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
21	AMP_Y_6	Normal	0.0	Infinity	0.001999197...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
22	AMP_Y_7	Normal	0.0	Infinity	0.001876365...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
23	AMP_Y_8	Normal	0.0	Infinity	0.001647552...	-	-	%20.14g	<input checked="" type="checkbox"/>	<input type="checkbox"/>
24	AMP_Y_9	Normal	0.0	Infinity	0.001336831...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
25	AMP_Z_1	Normal	0.0	Infinity	0.287083430...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
26	AMP_Z_2	Normal	0.0	Infinity	0.005418631...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
27	AMP_Z_3	Normal	0.0	Infinity	0.003693600...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
28	AMP_Z_4	Normal	0.0	Infinity	0.002228785...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
29	AMP_Z_5	Normal	0.0	Infinity	0.001987426...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
30	AMP_Z_6	Normal	0.0	Infinity	0.001529446...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
31	AMP_Z_7	Normal	0.0	Infinity	0.001421394...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
32	AMP_Z_8	Normal	0.0	Infinity	0.001041140...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>
33	AMP_Z_9	Normal	0.0	Infinity	9.208922689...	-	-	%20.14g	<input type="checkbox"/>	<input checked="" type="checkbox"/>



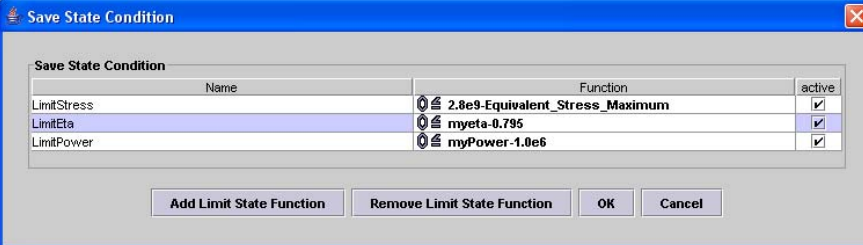
**reliability algorithm** Adaptive response surface

**Parameters**

- Assumed failure probability: 3.4e-6
- Sampling method: directional sampling
- Number of directions: 10000
- Initial DoE schema: D-optimal quadratic
- Initial axial multiplier: 1.0
- Following DoE schemes: D-optimal linear
- Rotate DoE schemes:
- Maximum number of clusters: 6
- Max. number of adaptations: 6
- Accuracy of failure probability [%]: 10
- Limit bound of parameter changes [%]: 2.0

Buttons: Reset, Cancel, OK

- **n = 9 relevant random parameters**

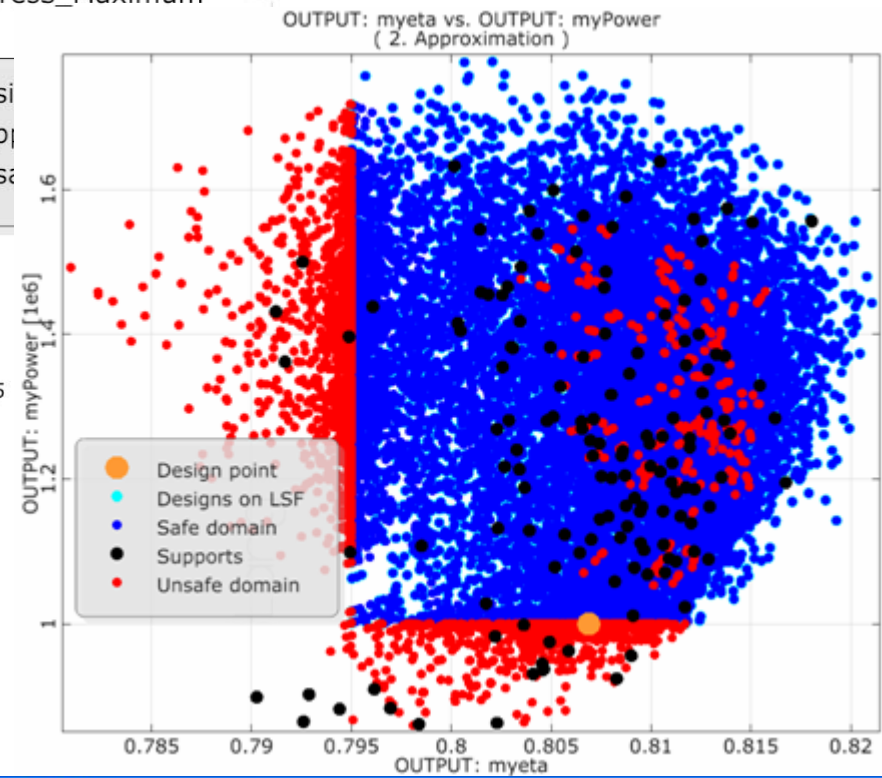
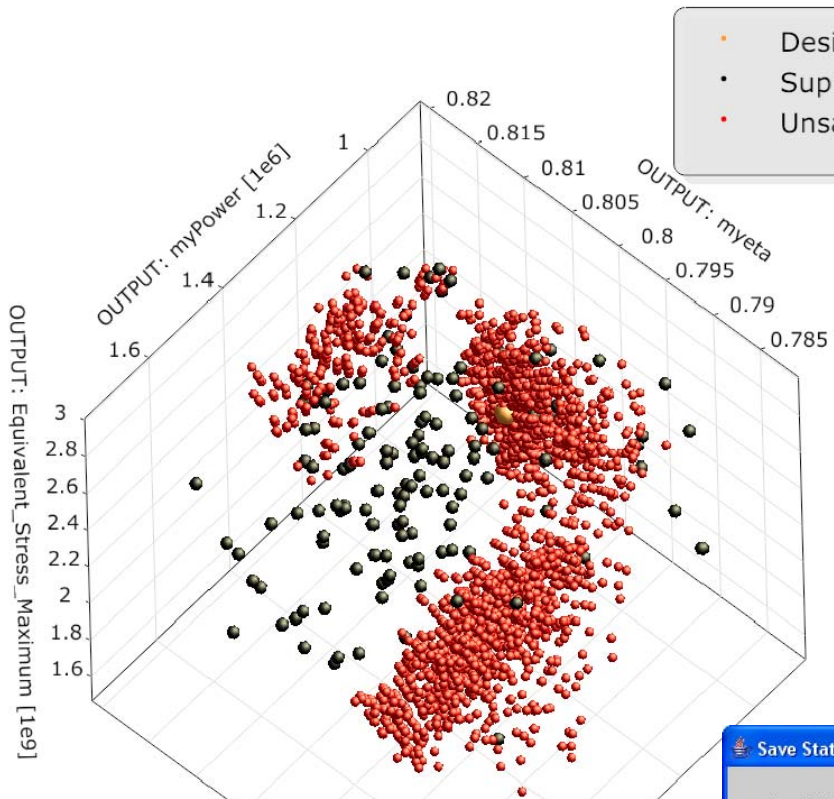


Name	Function	active
LimitStress	$0 \leq 2.8e9 \cdot \text{Equivalent\_Stress\_Maximum}$	<input checked="" type="checkbox"/>
LimitEta	$0 \leq \text{myeta} - 0.795$	<input checked="" type="checkbox"/>
LimitPower	$0 \leq \text{myPower} - 1.0e6$	<input checked="" type="checkbox"/>

Buttons: Add Limit State Function, Remove Limit State Function, OK, Cancel

# Example: Axial Turbine Blade

OUTPUT: myeta vs. OUTPUT: myPower vs. OUTPUT: Equivalent\_Stress\_Maximum ( 2. Approximation )



Save State Condition

Name	Function	active
LimitStress	2.8e9-Equivalent_Stress_Maximum	<input checked="" type="checkbox"/>
LimitEta	myeta-0.795	<input checked="" type="checkbox"/>
LimitPower	myPower-1.0e6	<input checked="" type="checkbox"/>

Add Limit State Function Remove Limit State Function OK Cancel

- 3 clustered areas with high failure probability

# Reliability Analysis

- $n = 9$  random parameters
- $N = 141$  design evaluations
- $P_f = 2.4e-7 < 3.4e-6$  (4.5 sigma)
- Six Sigma Design

## Method : Directional Sampling on Adaptive Response Surfaces (ARSM-DS)

Selected data : 2. Approximation

Number of designs : 141 (7 failed)

Complete directions : 10000 / 10000

Number of samples :

Total : 19590

Safe domain : 15143

Unsafe domain : 4447

Failure strings : 0

Unsuccessful : 0

Probability of failure :  $2.45e-07$  (  $2.45e-07$  )

Standard deviation error :  $4.771e-08$  (  $4.771e-08$  )

Most probable failure point:

myomega : -2217.04008841

DS\_gv\_angle : -9.99561718295

Ttin : 960.730885684

ptin : 266629.901461

pout : 90929.4317189

myAirCP : 1023.31803947

myAirR : 279.505216069

Youngs\_Modulus : 202109405683

Poissons\_Ratio : 0.285449914136

Distance median - design point (beta) : 5.623

Probability of failure (FORM):  $9.372e-09$

- Calculation of the MPP or design point
- Corresponding FORM result

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# RDO - Examples

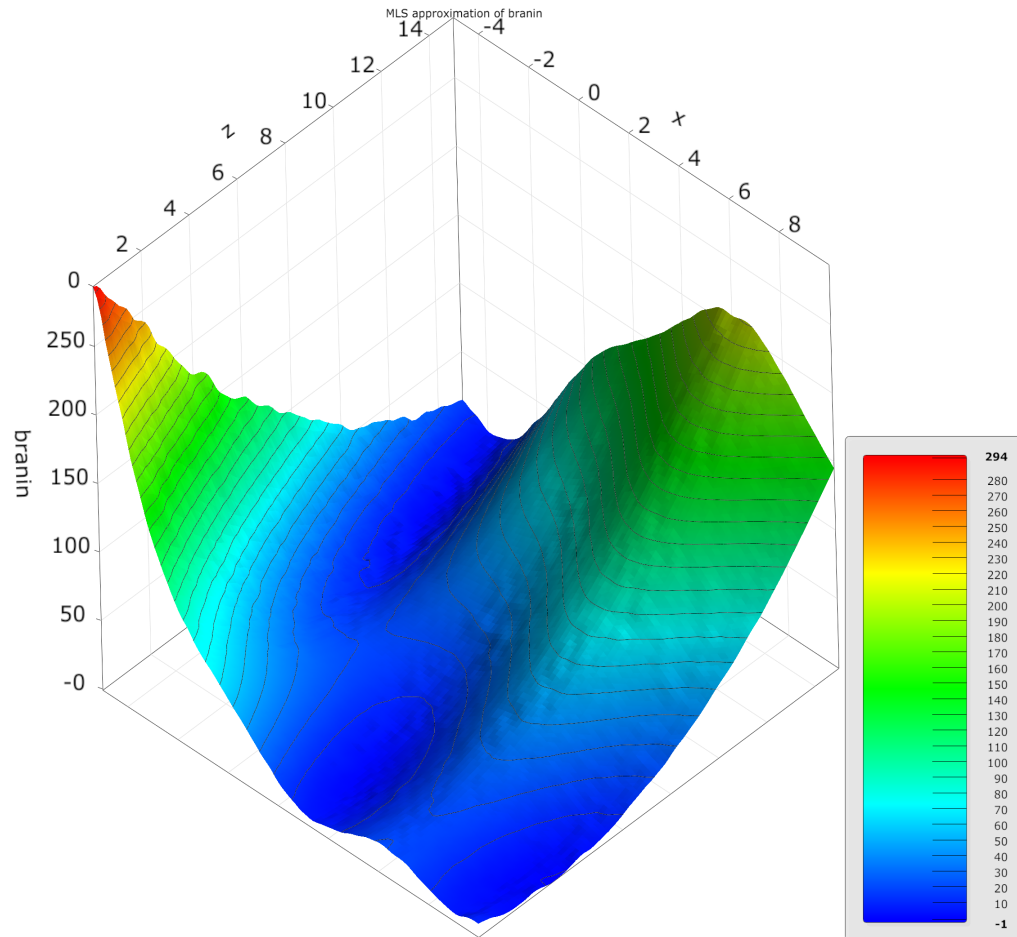
# Overview

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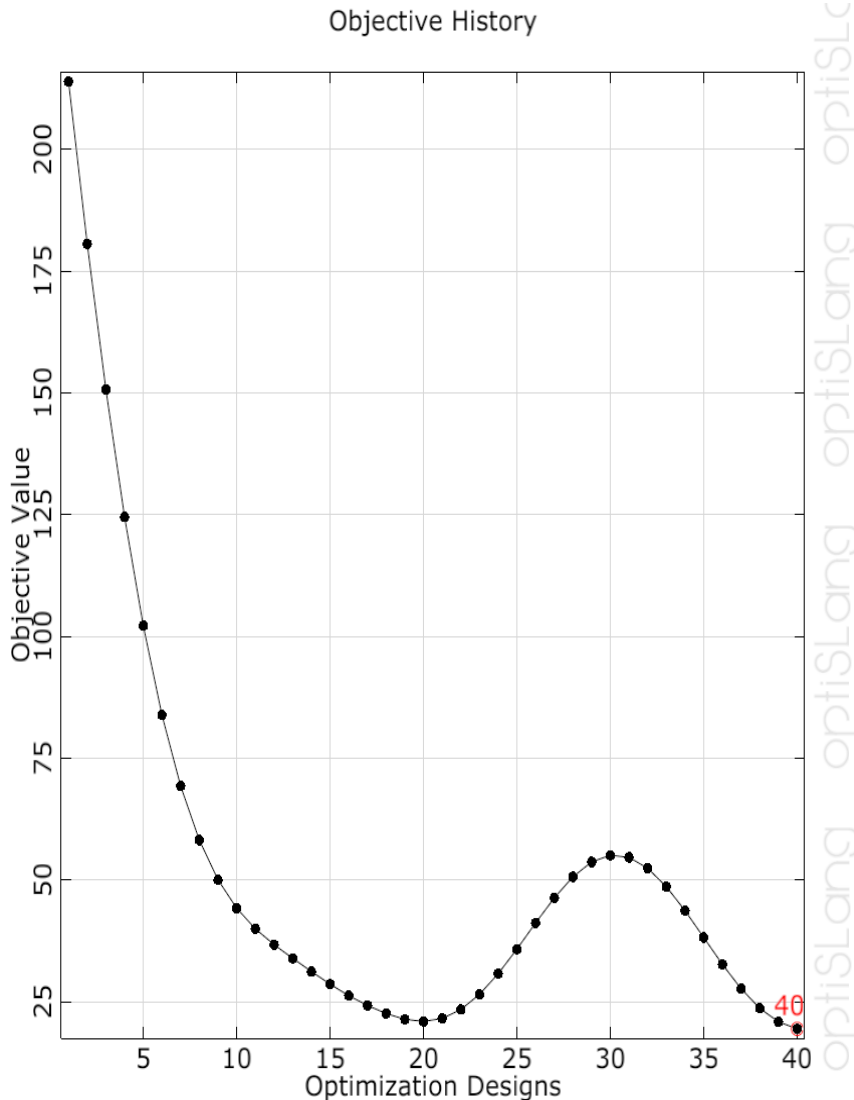
- Braninfunction
- Beam under dynamic load
- New highdimensional Example

# Braninfunktion

$$f(x, z) = \left( z - \frac{5.1}{4\pi^2} x^2 + \frac{5}{\pi} x - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \right) \cos(x) + 10$$



# Braninfunction



deterministic parameter:

$$-5 \leq x \leq 10$$

stochastic parameter:

$$\sigma_z = 2$$

$$\mu_z = 2$$

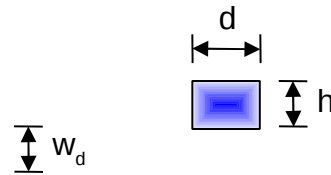
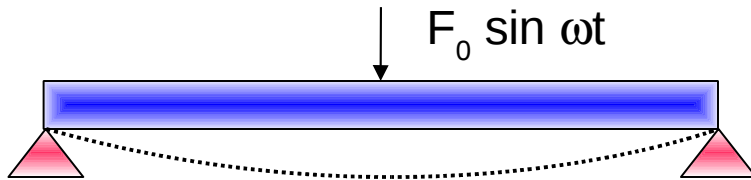
minimize:

$$\delta_{qbranin}(x) = \mu(f(x)) + \sigma(f(x))$$

- ARSM / 25LHS
- SR:  $1.0\sigma$  / 5%
- Minimum found at  $x = 10$
- 47 runs



# Example - simple beam



Optimization parameter:

$d[0..1]$

$h[0..1]$

Stochastic parameter:

$F_0[\mu=2e5, \sigma=2e4]$

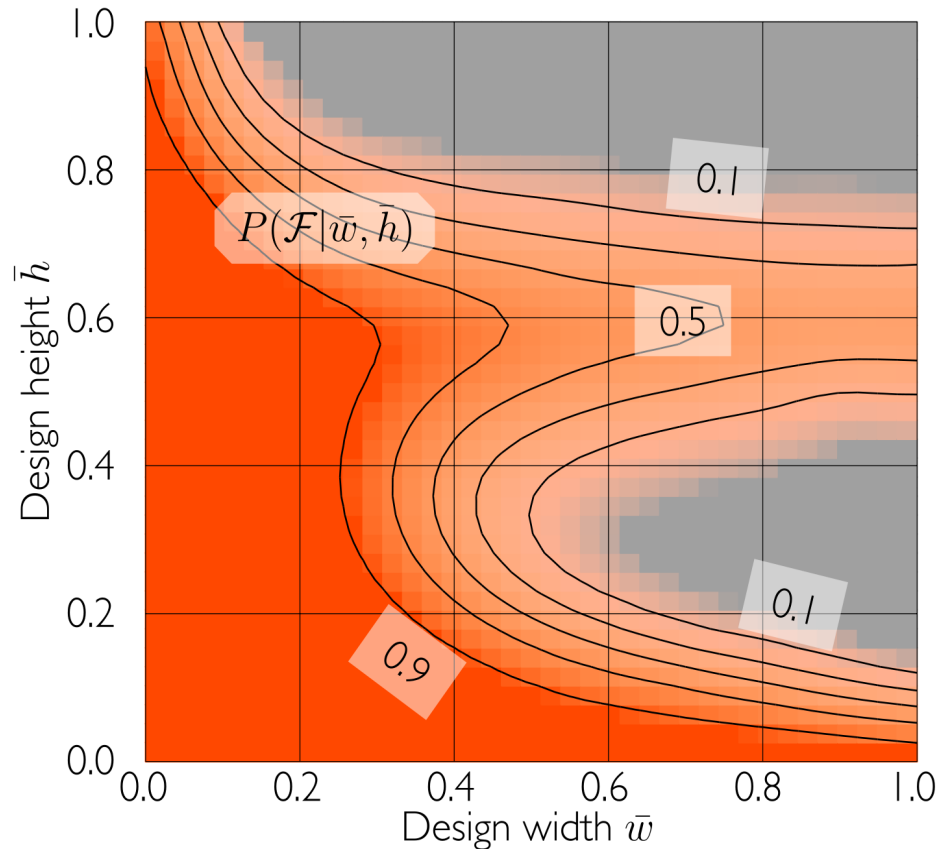
$\omega[\mu=60, \sigma=6]$

Objective:

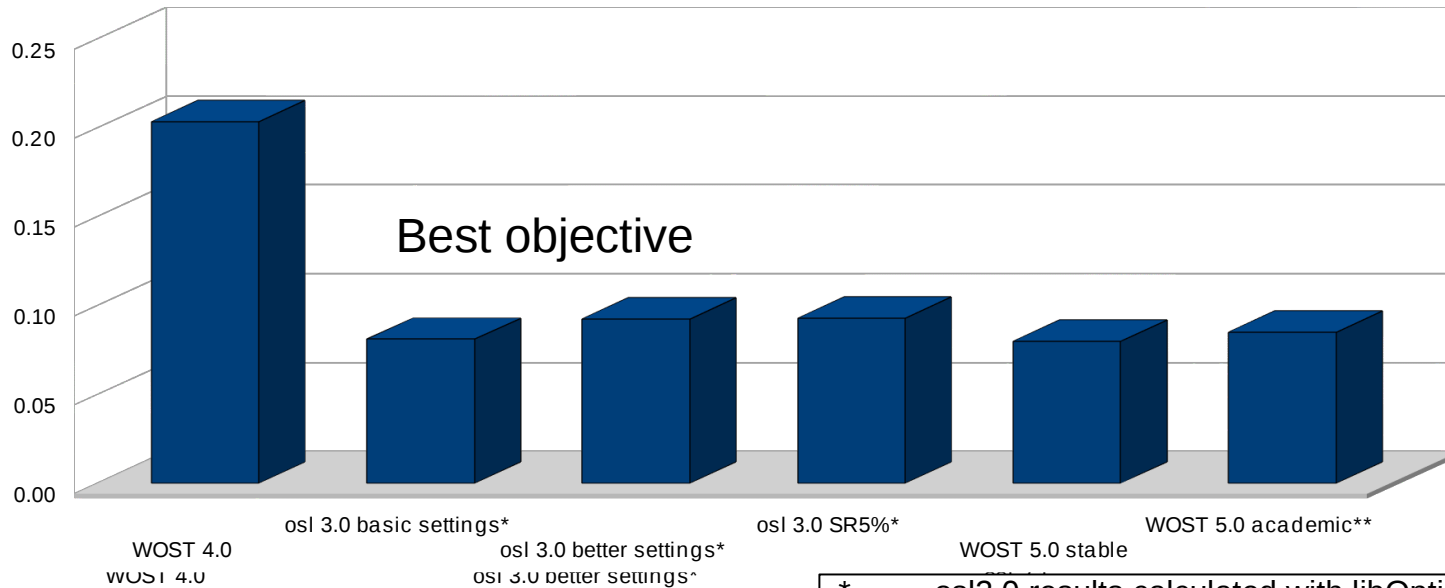
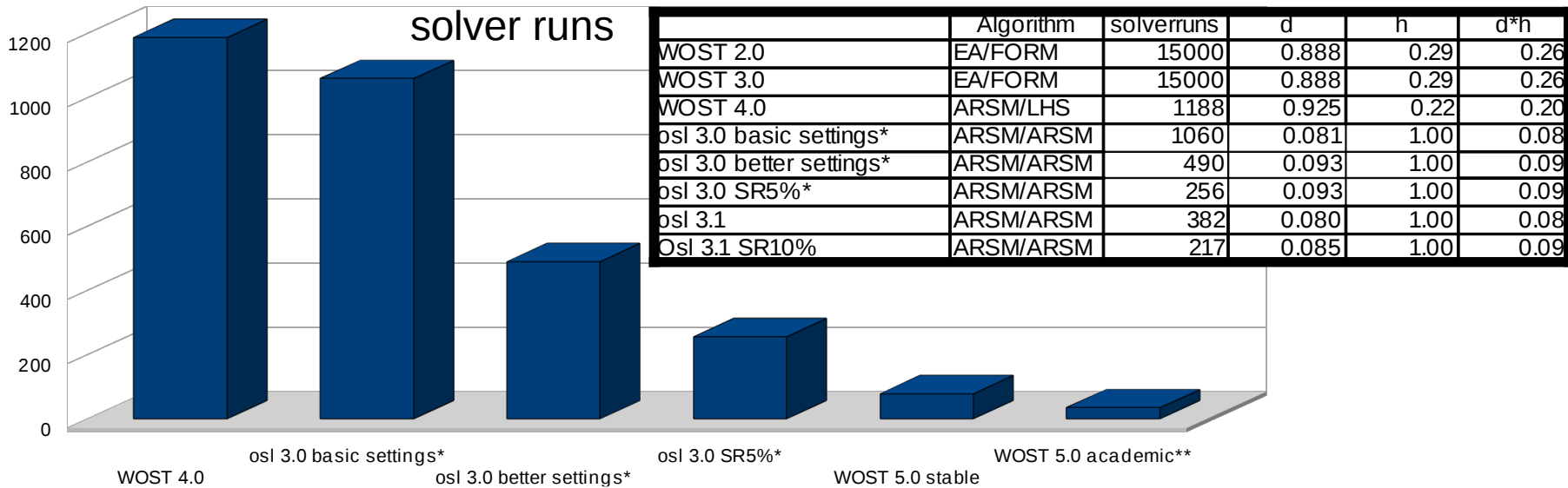
$\min(d \cdot h)$

Constraint:

$P_f(w_d > 0.005) < 0.01$



# Results - a comparison



\* osl3.0 results calculated with libOptiSLang++  
 \*\* not stable yet  
 \*\*\* exact number of solverruns  $20 \cdot 50 \cdot N_{FORM}$

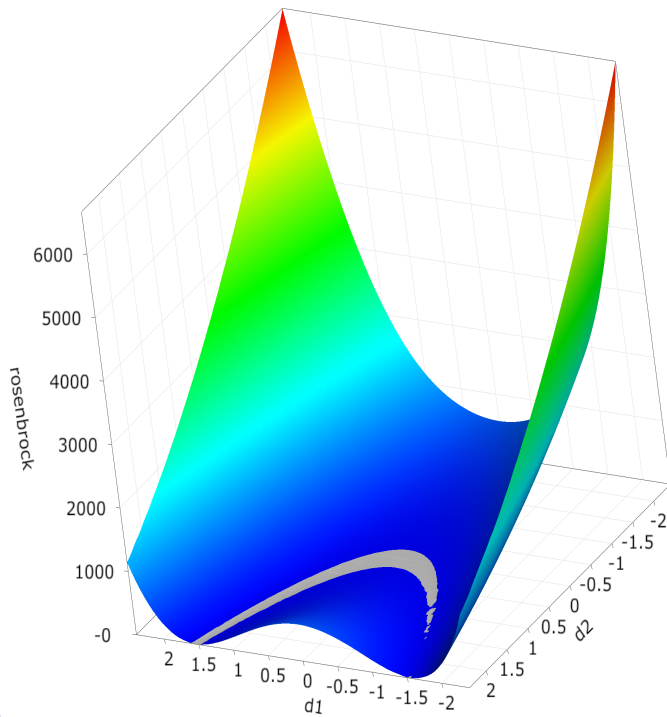
# Highdimensional Example

Combination of Rosenbrock-function and Hyperplane

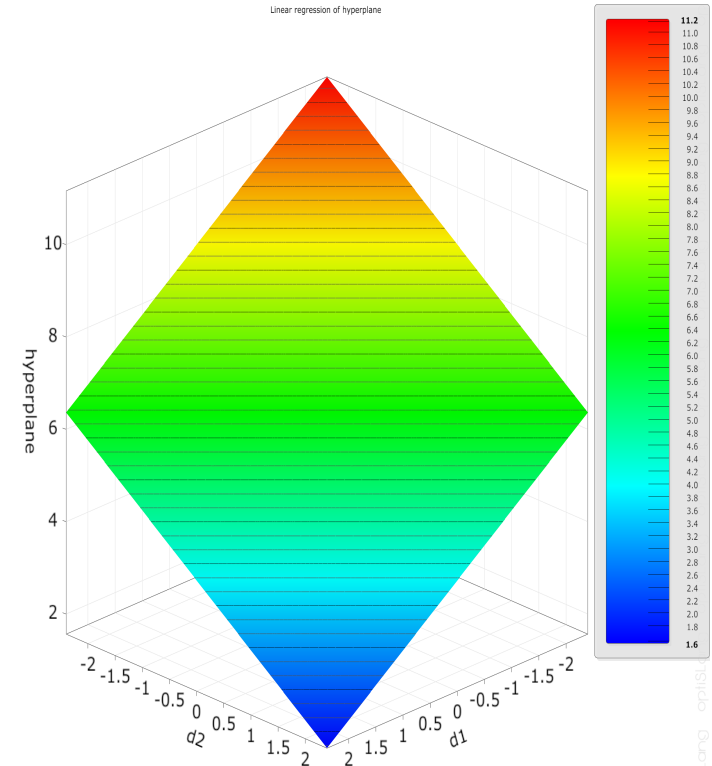
$$\text{rosenB}(d_0, \dots, d_n) = \sum_{i=0}^{n-1} [(1 - x_i)^2 + 100(x_{i+1} - x_i^2)^2]$$

$$\text{hyperP}(\beta, p_0, \dots, p_m) = \beta\sqrt{m} - \sum_{i=0}^m p_i$$

MLS approximation of rosenbrock



Linear regression of hyperplane



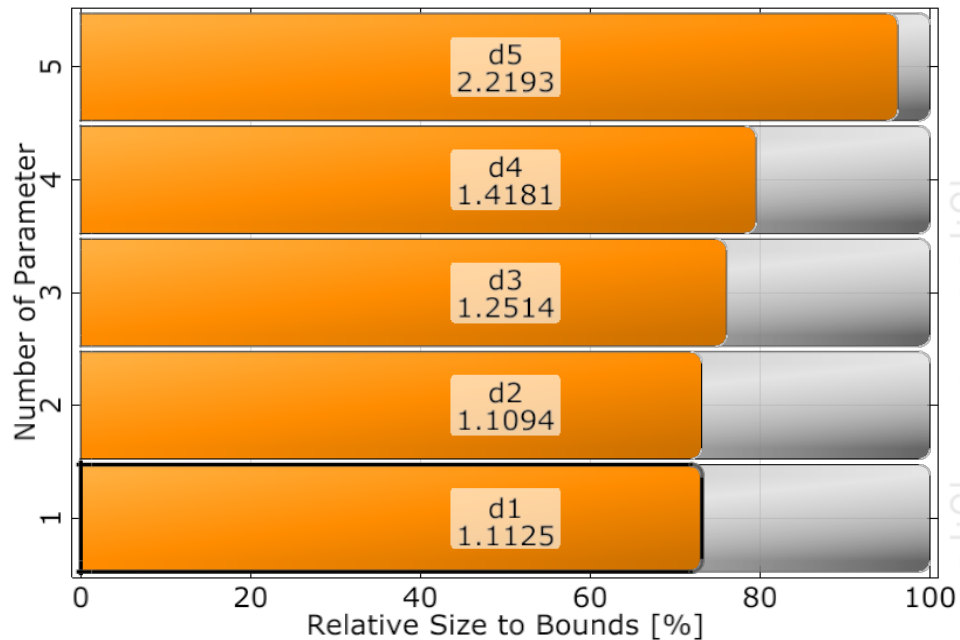
# Highdimensional Example

$$\min(\text{rosenB}(d_0, \dots, d_n))$$

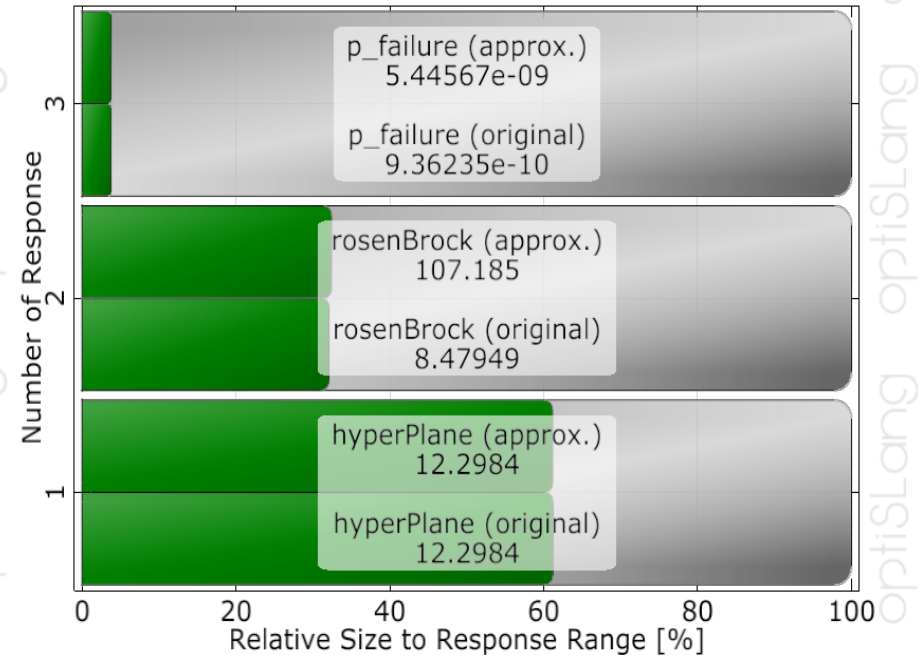
$$-2.4 \leq x_i \leq 2.4$$

$$P[\text{hyperP}(\beta, p_0, \dots, p_m) + \sqrt{n} * (d_0) < 0] \leq 3.4E - 6$$

Design Number: 3515



RESPONSE DATA: (Design Number: 3515)



# Outlook

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- Better decision criteria for SR
  - Easy and understandable parameters
  - Apply to total space
  - Find and use best metamodel
- 
- Braninfunction: 17 designs
  - Simple beam: 81 designs
  - Rosenbrock/hyperplane: less than 500 designs

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# RDO - Examples