Adaptive Response Surface based FE Model Updating for Operational Modal Analysis of RC Road Bridge

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Abstract

This study attempts to examine the effectiveness of meta-modelling in finite element (FE) model updating problem. Natural frequencies evaluated from eigen analysis of the FE model are approximated individually by meta-models. The updating objective function is defined for residual error between the natural frequencies identified by operational modal analysis and from FE model. A comparative numerical study is performed for updating a reinforced concrete (RC) road bridge FE model. In meta-modelling framework, linear and quadratic polynomial based adaptive response surface method (ARSM) using least square (LS) and moving least square (MLS) regression techniques are presented. Different design of experiment (DOE) schemes (*viz.* Koshal, D–Optimal, Full Factorial and Central Composite Designs) for generating support points and weight functions (*viz.* exponential and regularized) are also considered. Results show the effectiveness of MLS based ARSM that bypasses multiple FE model runs as compared to the conventional gradient based optimization.

Keywords: Response Surface Method, Moving Least Square Technique, Regression Analysis, Finite Element Model Updating, Gradient Based Optimization, Design of Experiment

1. Introduction

Structural parameters in real construction deviate from their characteristic values used in FE model analysis. This is attributed to lack of mimicking the real life conditions such as deformations, joints, boundary conditions, exact material properties among many others. Also, modelling deformations, joints etc. are difficult and cumbersome. Hence, it is computationally appropriate to adjust the properties of FE model to duplicate the response from the actual structure. In this view, updating FE model is an avid domain in damage detection and structural health monitoring [1]. Where both, direct and iterative methods are used for FE model updating [1]. In iterative FE model updating procedure, the physical parameters are updated by minimizing the error between modelled and actual results. This requires to evaluate multiple gradients which are computationally expensive and may cause error or convergence difficulty [2].

In view to overcome the above mentioned shortcomings, meta-modelling by response surface method (RSM) has emerged as an efficient alternative tool for FE model updating [2]. It bypasses the computational effort caused by multiple runs of FE model. Also, it helps in evaluation of the gradient for the model which may be difficult from FE analysis. Ren and Chen [3] applied the RSM for FE model updating of simply supported beam and box-culvert bridge. They optimized weighted residual error between the identified and modelled natural frequencies. It was noted that RSM based FE model updating converged efficiently and faster than the sensitivity based updating methods. Brehm *et al.* [4] presented application of RSM for updating cantilever truss and railway bridge. They also used weights in objective function which is defined for energy based mode

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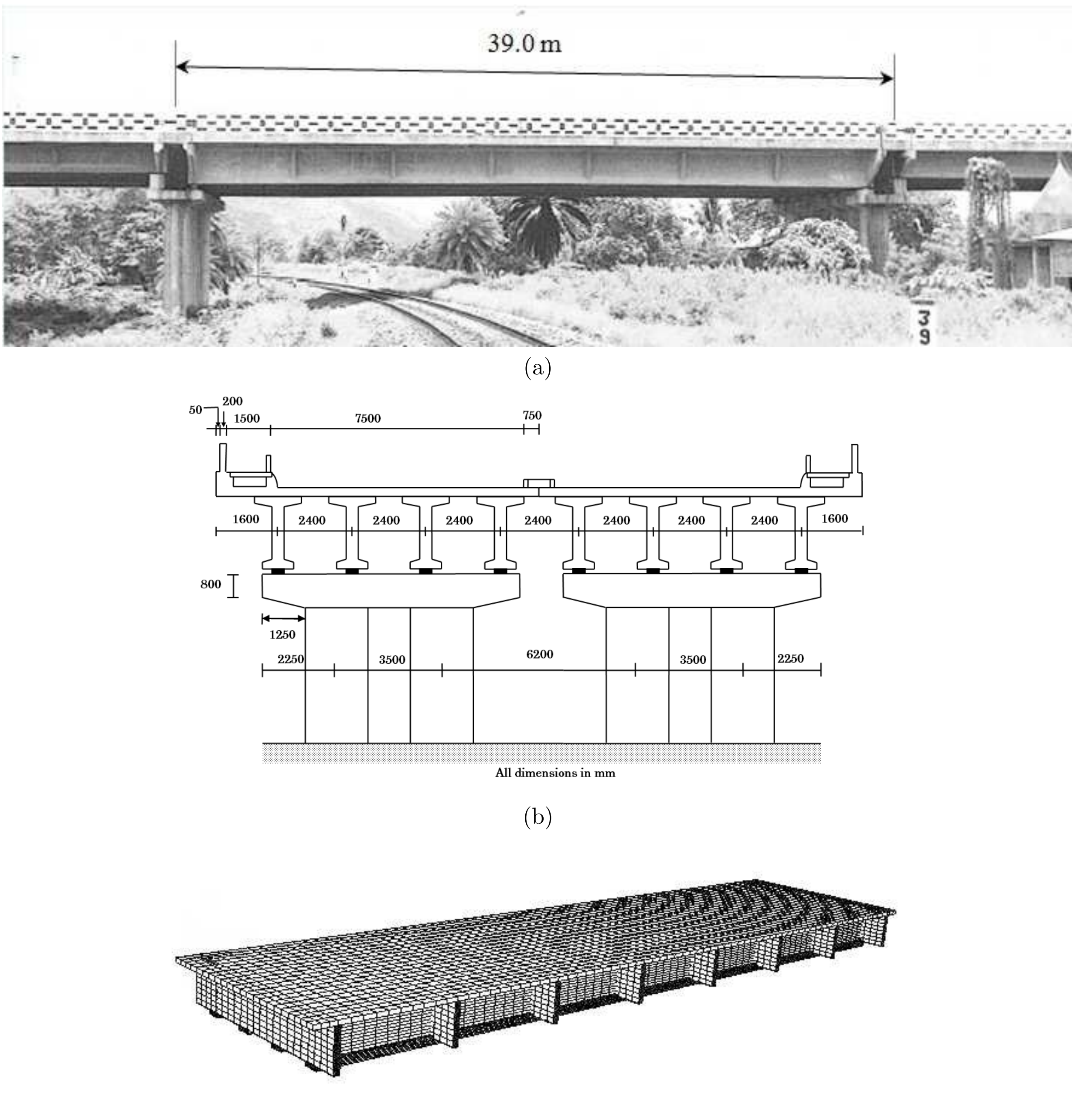


Figure 1: View of the RC road bridge (a), the cross-section (b) and the FE Model (c)

assurance criteria. Their RSM used genetic algorithm and gradient based optimization for updating. For linear as well as non-linear problems, Shahidi and Pakzad [5] proposed a unique updating scheme using RSM for each time step. In their time domain FE model updating scheme, a generalized formation of RSM was defined where order and fitting is collectively judged from the individual response surfaces at each time steps. In the view of application of RSM to FE Model updating, Chakraborty and Sen [6] examined the effectiveness of MLS based response surface over LS. In their study, conventional exponential weight function was used for MLS based RSM to update hypothetical examples of 10 bar truss and culvert. They used synthetic data where error was ranging from 10 to 30%.

By far in FE model updating, applications of RSM are subjected to LS regression technique and only recently, studies are using MLS. Thus, a better understanding on MLS based RSM in FE model updating is needed. In this concern, current study examines the different aspects of RSM in updating problems. A numerical study is conducted for iterative FE model updating based on RSM of a real life RC road bridge.

2. Description of RC Road Bridge and FE Model

The structure examined in this study is an existing RC bridge near Indian Institute of Technology Guwahati in North Guwahati. Total length of the RC road bridge is 88.0m with 3 independent spans which are simply supported. The middle span is 39.0m long and the end spans stretch to 24.5m. It allows 2 way traffic flow with 7.5m in each lane. The full view of the bridge is shown in Fig. 1(a). The bridge deck of the main span is resting over 4 prestressed longitudinal beams and 8 diaphragms which act as cross beams. The main span is considered in the study for further investigation. For the initial analysis, a three dimensional (3D) FE modelling is done in ANSYS®. To construct the FE model, SHELL181 and BEAM188 elements [7] are used in the deck and the longitudinal girders, respectively. The material properties are incorporated as per Indian Standard [8]. The corresponding FE model is shown in Fig. 1(b).

3. Parameter Identification and Updating

The middle span of RC road bridge is examined with sensors under traffic flow and the time history responses are recorded for further investigations using signal processing. To solve the identification problem, Hilbert-Huang transformation (HHT) technique [9] is used here. In this approach signal is first decomposed into intrinsic modal function (IMF) by empirical mode decomposition method. After the IMFs are evaluated, Hilbert transform is applied to obtain the instantaneous frequencies and phases. On time averaging of these instantaneous frequencies, one can obtain the natural frequencies of the examined structure (see Mahato *et al.* [10]).

Once the investigation of modal parameters (i.e. natural frequencies) from the recorded data is carried out, as specified above, FE model updating problem is solved. Generally, updating is executed for the parameters like material properties, dimensions etc. which plays a vital role in structural behaviour. With the selection of parameters $\mathbf{x}$, updating problem is solved for optimization of residual error ϵ with respect to $\mathbf{x}$ as

$$\begin{aligned} \min_{\mathbf{x}} \quad & \epsilon = \sqrt{\textstyle\sum_{i=1}^{n_f} (\hat{f}_i(\mathbf{x}) - f_i)^2} \\ \text{where,} \quad & \mathbf{x} \in [\mathbf{x}_l, \mathbf{x}_u] \end{aligned} \tag{1}$$

In the above equation, $\hat{f}_i$ and f_i are i^{th} natural frequency identified from operational data of actual structure and FE model, respectively. A total of first n_f natural frequencies are considered for updating in this study where the parameters are bounded in lower and upper limits (i.e. $\mathbf{x}_l$ and $\mathbf{x}_u$, respectively). Furthermore, in the current study RSM based meta-modelling scheme is used for approximating the objective function (i.e. ϵ) for optimization. The methodology of RSM employed here is presented in the following section.

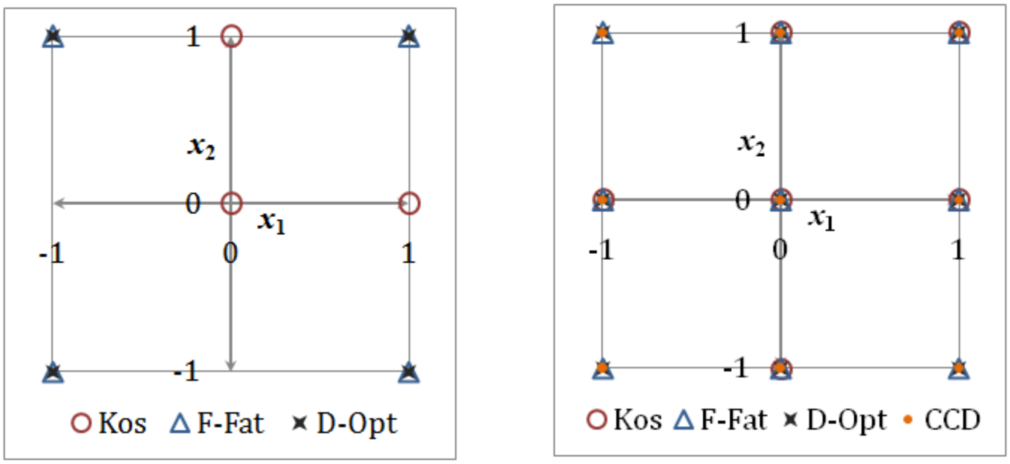
4. Response Surface Meta-modelling

RSM is an approximating polynomial set generally limited to linear or quadratic order based on the accuracy of fit. The polynomial set is estimated by generating a specific pattern of support points following a particular DOE scheme. Original function evaluations at the support points are regressed to obtain coefficients of the polynomial set. In FE model updating purview, RSM is used as approximating surface to the output function of the FE model. Subsequently, optimization algorithm is executed on the response surface to evaluate the minimum point based on the objective function. RSM can be expressed as

$$\mathbf{f} = \mathbf{X}\mathbf{a} + \mathbf{e}_{\mathrm{f}} \tag{2}$$

where, $\mathbf{f}$, $\mathbf{X}$, $\mathbf{a}$ and $\mathbf{e}_{\mathrm{f}}$ are the original function vector, matrix of the RSM polynomial set, corresponding coefficient vector and error vector due to lack of fit. The above generalized expression of RSM can be mathematically represented for linear basis as

$$\mathbf{f} = \alpha_0 + \sum_{i=1}^{n_v} \alpha_i x_i + \mathbf{e}_{\mathrm{f}} \tag{3}$$



(a) (b)

Figure 2: Examples of support points generated by DOE schemes: Koshal Design (Kos), Full Factorial Design (F-Fat), D–Optimal design (D-Opt) and Central Composite Design (CCD) for (a) linear and (b) quadratic approximations

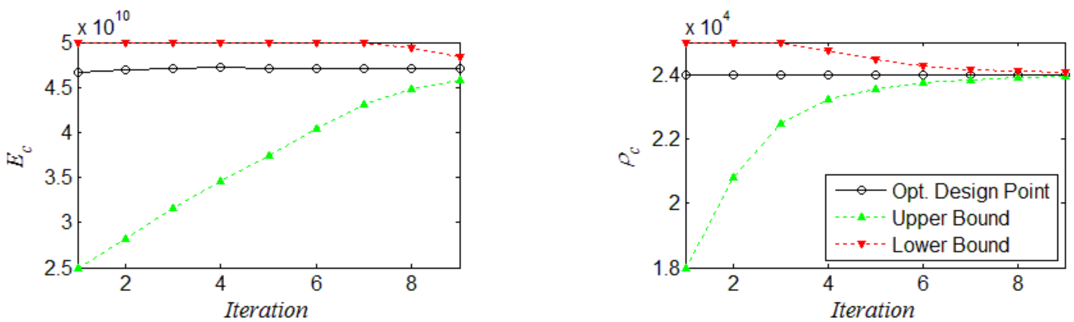


Figure 3: Reduction in bounds of the updating parameter (i.e. E_c and ρ_c) with succeeding iterations of ARSM

and for quadratic basis it is modelled as

$$\mathbf{f} = \alpha_0 + \sum_{i=1}^{n_v} \alpha_i x_i + \sum_{i=1}^{n_v} \beta_i x_i^2 + \sum_{i=1}^{n_v} \sum_{j>i}^{n_v} \beta_{ij} x_i x_j + \mathbf{e}_{\mathrm{f}} \tag{4}$$

where, n_v is number of variables. After the selection of approximation (i.e. linear or quadratic), the unknown coefficients in $\mathbf{a}$ are estimated by LS curve fitting or its modified version of MLS which are discussed below.

4.1. Least Square Technique

This technique is commonly used for regression analysis where it optimizes the square of error between the original function and approximate response. Using Eq. 2, the square of error can be given as

$$e(\mathbf{a}) = \mathbf{e}_{\mathrm{f}}^T \mathbf{e}_{\mathrm{f}} = (\mathbf{f} - \mathbf{X}\mathbf{a})^T \, (\mathbf{f} - \mathbf{X}\mathbf{a}) \tag{5}$$

For minimizing the Eq. 5 with respect to the unknown coefficients $\mathbf{a}$, one can get LS solution of the coefficients for RSM as

$$\mathbf{a} = (\mathbf{X}^T\mathbf{X})^{(-1)} \, (\mathbf{X}^T\mathbf{f}) \tag{6}$$

The solution in the above equation is valid for $(\mathbf{X}^T\mathbf{X})$ being non-singular. In contemporary to LS technique, another regression based technique similar to LS is defined for local approximation of the response surface which is explained in the following section.

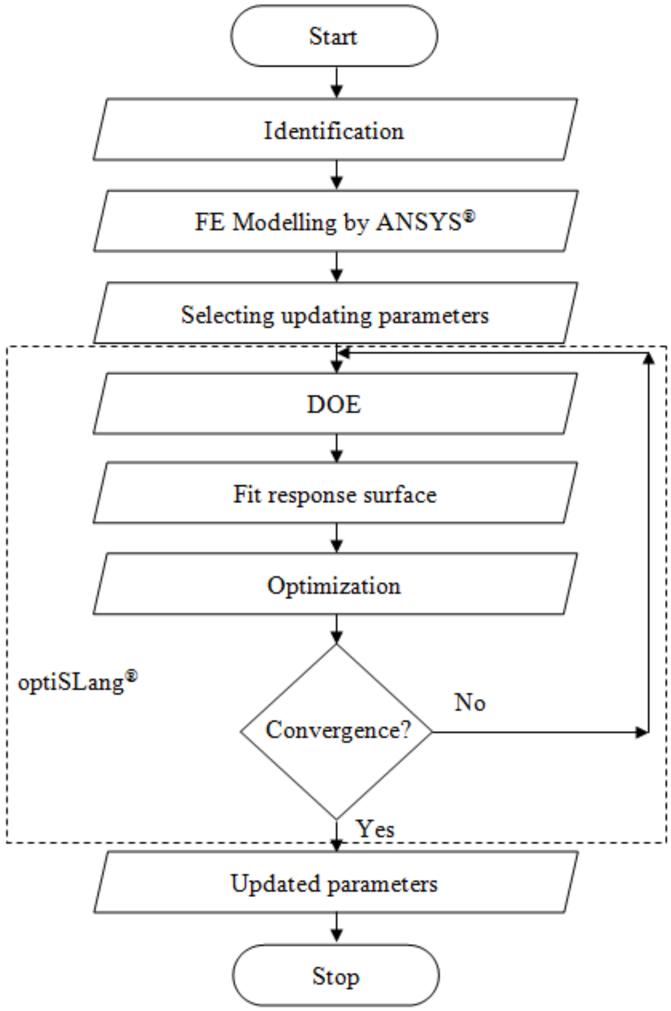


Figure 4: Flowchart of updating scheme followed in this study

4.2. Moving Least Square Technique

A major demerit of LS technique lies in its global fitting of the response surface which results in higher error. Once the coefficients are evaluated, it remains constant for every point in the design domain. This is not in the case of MLS based RSM where the coefficients are function of distance between the points. In MLS technique, the error is also dependent on the distance d of the points by weight functions. Thus, the square of error is modified as

$$e = \sum_{j=1}^{n_p} w_j e_{\mathrm{f},j}^2 \tag{7}$$

here, w_j and $e_{\mathrm{f},j}$ are weight function and error in response surface fit for j^{th} point, respectively. This modifies the Eq. 5 due to the inclusion of weight matrix $\mathbf{\Theta}$ as

$$e(\mathbf{a}, d) = \mathbf{e}_{\mathrm{f}}^T \mathbf{\Theta}(d)\, \mathbf{e}_{\mathrm{f}} = (\mathbf{f} - \mathbf{X}\mathbf{a})^T \mathbf{\Theta}(d)\, (\mathbf{f} - \mathbf{X}\mathbf{a}) \tag{8}$$

In the above equation, $\mathbf{\Theta}$ is given by

$$\mathbf{\Theta}(d) = \begin{bmatrix} w_1(d) & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & w_j(d) & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & w_{n_p}(d) \end{bmatrix}$$

where, the individual weight functions can be defined using exponential function as

$$w_j(d) = \begin{cases} \exp^{-\left(\frac{d}{D\gamma}\right)^2}, & \text{if } d \leq D \\ 0, & \text{if } d > D \end{cases} \quad (9)$$

The weight function is subjected to a radial distance D which defines its domain of influence over the support points n_p. Additionally, a reducing factor $\gamma = 0.3808$ is used for decaying the shape function. From Eq. 9, it is evident that smaller value of D results in better fit of response surface because of well proportioned shape function. This may result in less number of support points and eventually, reduce error. Also, the weight function is sensitive to D. For higher D, the shape function can have more error. Alternatively, another weight function based on regularized weighted scheme is proposed by Most and Bucher [11] as

$$w_j(d) = \begin{cases} \frac{\bar{w}_j(d)}{\sum_{i=1}^{n_p} \bar{w}_i(d)}, & \text{if } d \leq D \\ 0, & \text{if } d > D \end{cases} \quad (10)$$

The individual weight function $\bar{w}_i(d)$ is given as

$$\bar{w}_i(d) = \frac{\{(\frac{d}{D})^2 + \delta\}^{-2} - (1+\delta)^{-2}}{\delta^{-2} - (1+\delta)^{-2}} \quad (11)$$

where, δ is considered as 10^{-5}. Use of regulated weight function helps in using larger D as the shape function error is relatively less. Hence, the above formulations suggests the overall advantage of MLS technique especially for non-linear surface as it applies local approximations using weight functions as compared to LS based RSM which uses global approximation.

5. Design of Experiment Schemes

The unknown coefficients are determined by supports points generated using a definite DOE scheme. These points are used for evaluating original function using FE model. Thus, strategic choosing of DOE is critical in RSM efficiency as number of FE model runs are dependent but one must not forget the quality of fit. In this study four different DOE schemes are used for evaluating the coefficients of the RSM which are explained below.

5.1. Koshal Design

This scheme uses single parameter at a time for generating the DOE [12]. The sole benefit lies in minimal generation of support points for the coefficient evaluation. This, in turn, effects the quality of response surface fitting. Fig. 2 depicts the Koshal Design for linear and quadratic polynomial basis which generates $n_v + 1$ and $(n_v + 1)(n_v + 2)/2$ support points, respectively.

5.2. Full Factorial Design

Unlike Koshal Design where minimum support points are generated, Full Factorial Design gives all the possible combinations of the support points. This DOE scheme gives p^{n_v} support points. The level of factorial design p is a non-zero integer with 1 for constant basis, 2 for linear basis and so on. For $p = 2$ and 3 (i.e. linear and quadratic basis, respectively) the Full Factorial DOE scheme is shown in Fig. 2. It can be noted that all the DOE schemes, discussed here, are subsets of Full Factorial Design.

5.3. D–Optimal Design

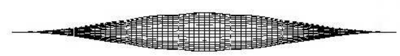
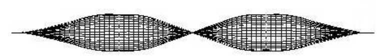
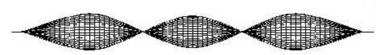
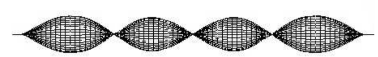
In this design scheme, variance of the error is minimized by choosing a subset of Full Factorial Design. This eventually helps in reducing the support points without much reduction in the fitting quality for larger number of parameters. The optimal criteria to improve the fit of response surface is given by maximizing $|\mathbf{X}^T\mathbf{X}|/n_p^{n_v}$ [13], here the number support points required are defined by the user. In this study, 1.5 times of support points generated as per Koshal Design is used. Thus, D–Optimal Design is same as Full Factorial Design for number of parameters ≤ 2 as shown in Fig. 2. But further increase in number of parameter have a significant decrease in number of support points.

5.4. Central Composite Design

Central Composite Design scheme has radial as well as box corner points along the center point. A total of $1 + 2n_v + 2^{n_v}$ support points are generated. In this study, *faced* Central Composite Design is used where the radial points are situated at the middle of the adjacent box points as shown in Fig. 2 (b). Alike Full Factorial Design, this design generates less number of support points (but more than D–Optimal Design) with adequate fitting accuracy.

It is evident from the Fig 2, number of points generated by Koshal Design $\leq$ D–Optimal Design $\leq$ Central Composite Design $\leq$ Full Factorial Design. Also, one can notice that for 2 parameters (i.e. $n_v = 2$) in linear approximation, D–Optimal and Full Factorial Designs have same points. Moreover,

Table 1: Natural frequencies and corresponding mode shapes from FE model of RC road bridge

Sl. No.	Identified Natural Frequencies (Hz)	Updated Natural Frequencies (Hz)	Updated Mode Shapes
Mode 1	3.04	3.06	
Mode 2	7.74	7.61	
Mode 3	13.81	13.39	
Mode 4	19.72	19.86	
Mode 5	-	26.68	
Mode 6	-	27.99	
Mode 7	-	28.73	
Mode 8	-	30.72	
Mode 9	-	33.48	
Mode 10	-	34.07	

for quadratic approximation, D–Optimal, Central Composite and Full Factorial Designs generates identical points.

6. Numerical Study

Initially, investigation of the recorded data using operational modal analysis is performed, as per § 3, to identify the natural frequencies of the RC road bridge. Detailed discussion on the identification is beyond the scope of this study. However, one may refer to Mahato *et al.* [10] for this purpose. Identified first four natural frequencies are reported in Table 1. FE modelling of the bridge is done on ANSYS® platform using the parametric design language (APDL). The objective function for updating problem constitutes of the residual error ϵ of the first four natural frequencies (i.e. n_f in Eq. 1 is equal to 4) from the identification and FE model. The updating parameters selected in this study are elastic modulus (E_c) and density (ρ_c) of concrete. These updating parameters are bounded between $[2.5 \times 10^{10},\ 5.0 \times 10^{10}]$ and $[1.8 \times 10^{4},\ 2.5 \times 10^{4}]$, respectively.

In this study, optimization of the residual error for updating FE model is done using optiSLang® [14]. For performing ARSM, 100% initial range is chosen for spreading support points over the entire domain. Additionally, steady convergence is ensured by setting the zooming factor as 80%. This factor helps in reducing the domain of new support points at each iteration from the preceding iteration. Reduction in extreme values of the support points can be expressed for E_c and ρ_c in Fig. 3 with succeeding iterations. Here, the lower and upper bound presents the extreme values

Table 2: Minimized objective function, number of iterations and FE model runs from direct gradient optimization and different ARSM schemes

Optimization Method	Basis	Regression	Weight Type	DOE	Iterations	Number of Runs	Minimized Objective Function
Direct Gradient	-	-	-	-	7	38 (38)	2.1688
ARSM	Linear	LS	-	D-Opt	11	67 (60)	2.1316
			-	Kos	9	46 (30)	2.1319
	Quadratic		-	CCD	6	67 (56)	2.1317
			-	Kos	6	49 (38)	2.1315
ARSM	Linear	MLS	Exponential	D-Opt	11	67 (62)	2.1317
				Kos	9	46 (30)	2.1319
	Quadratic			CCD	2	23 (20)	2.1320
				Kos	6	49 (38)	2.1315
ARSM	Linear	MLS	Regularized	D-Opt	5	31 (29)	2.1318
				Kos	9	46 (30)	2.1319
	Quadratic			CCD	2	23 (20)	2.1320
				Kos	6	49 (38)	2.1312

Note: Here (.) gives the actual number of FE model runs for the complete iterations.

of the parameter in the domain of support points generated for the current iteration and the design point specifies the optimized point in that iteration. Moreover, one extra center point along the support points is generated as per the DOE and inclusion of the support points from past iterations are used in regression for better fitting of response surface. Linear scale transformation of the parameter in [0, 10] is performed to reduce the ill-conditioning of the matrices in the regression analysis. Thus, the radial distance D is defined as 14.14. The convergence criteria of the ARSM for stopping the iterations loop is defined as 0.01% of variation in the parameters and objective function. As discussed, ARSM using LS and MLS regression techniques with linear as well as quadratic polynomial basis is executed with varying DOEs. A schematic flowchart of the complete updating procedure is presented in Fig. 4. DOE schemes D–Optimal and Full Factorial have identical support points, for two parameters (i.e. $n_v = 2$) under linear approximation. For quadratic approximation D–Optimal, Full Factorial and Central Composite Designs have identical support points. Hence, all these identical cases as per the approximation basis show similar results. A comparative study is performed for ARSM based on LS and MLS with exponential and regularized weight for varying polynomial basis and DOE.

In this study, the response surfaces are fitted individually for all the four natural frequencies as shown in Fig. 5 then the objective function (see Fig. 6) is optimized by gradient method. Fig. 5 shows a close fit of response surface and support points for natural frequencies f_1 to f_4 for regularized weighted MLS based ARSM and same was noticed for the other cases. One can notice nearly linear relation between the natural frequencies and updating parameters with higher sensitivity towards E_c as compared to ρ_c. On other hand, Fig. 6 shows nonlinear relation of the residual error ϵ with respect to the updating parameters. The outcome of updating problem solved for different cases is presented in Table 2. It is observed that the convergence was achieved for all the ARSM cases and the difference in minimized objective functions is insignificant. Results show that LS and MLS by exponential weight function have similar number of iterations, FE runs and minimized objective function value for linear approximation. But for quadratic approximation, Central Composite Design converged faster in MLS based ARSM by dropping the number of iterations from 6 to 2 and subsequently, reducing the FE runs from 67 to 23. Thus, reduction of 65% in FE runs is noticed. Sim-

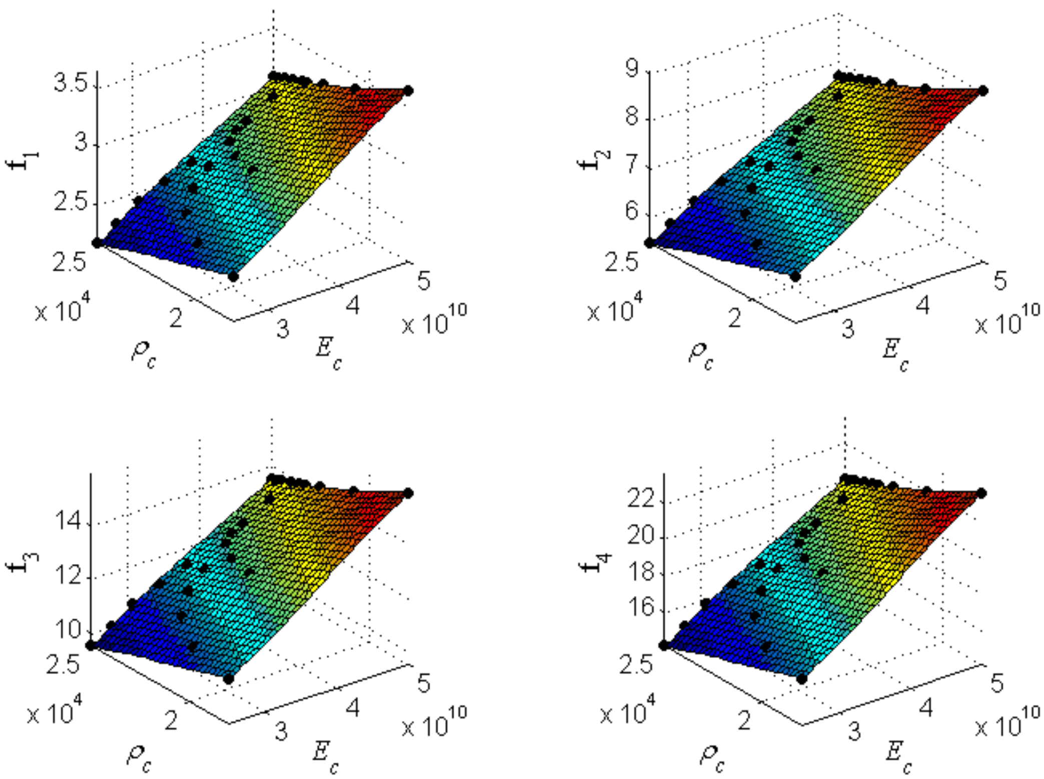


Figure 5: Response surfaces constructed for all the n_f natural frequencies with support points (•)

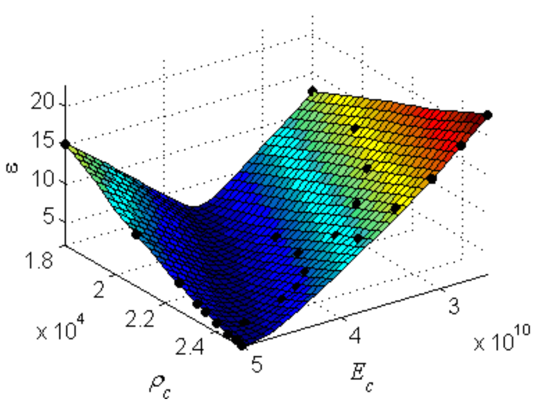


Figure 6: Objective function with respect to the updating parameters along with support points generated in ARSM as shown in Fig. 5

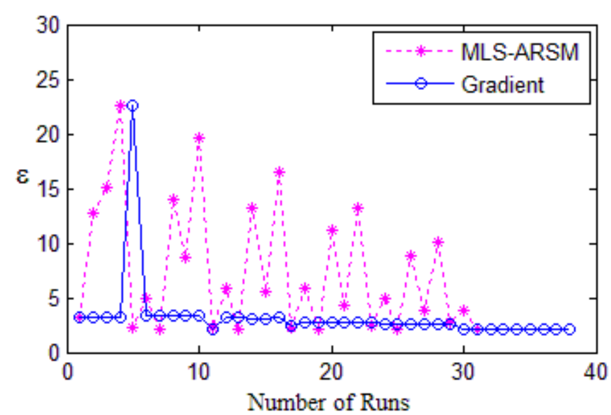


Figure 7: Convergence of the objective function with MLS based ARSM and gradient based optimization

ilarly for MLS with regularized weight function, D–Optimal design also shows faster convergence with 5 iterations and 31 FE runs as compared to 11 iterations and 67 FE runs for both LS and MLS using exponential weight function based ARSM. In all the six cases of Koshal DOE (see Table 2), one can notice no change in number of iterations and support points. This is plausibly due to the fact that this DOE scheme generates minimal support points which is minimal for coefficient evaluation and subsequently, the effect of weight function in MLS is not significant. Thus for similar polynomial basis, the minimized objective functions are almost identical at each iterations. The increase in support points, as in the case of D–Optimal Design (also, Full Factorial and Central Composite Designs) shows a significant improvement in MLS based ARSM, especially with regularized weights. Moreover, direct gradient optimization without employing ARSM is also performed for elucidating the effectiveness of meta-modelling. It is also observed that ARSM formed by LS converges slower than the direct gradient optimization. Similar conclusions can be seen for Koshal Design based support points too. But quadratic polynomial approximation using exponential and regularized weight functions show speedy convergence for Central Composite Design (also, D–Optimal and Full Factorial Designs). Additionally, linear approximation based on regularized weight function shows better efficiency than direct gradient optimization. A convergence comparison of the both is presented in Fig. 7. In this study, the duplicate FE runs are avoided and thus, reducing actual FE runs (represented in first brackets in Table 2). The above numerical study and discussion justifies the use of MLS based response surface meta-modelling in FE model updating.

7. Conclusions

This study deals with application of ARSM in FE model updating of RC road bridge. Response surface methodology surrogates the physically time consuming FE model analysis of multivariate input-output relations. Additionally, these surrogating methodology helps in considering unsmooth, nonlinear or discontinuous functions with efficiency. A comparative numerical study is performed using LS based RSM and MLS based RSM with exponential and regularized weight functions for linear and quadratic approximations. Also, efficiency of different DOE schemes are studied in this framework. Following are the conclusions from this study

- Generally, it is noted that MLS based RSM are better than LS based RSM. Especially, regularized weighted MLS based RSM performs better than exponentially weighted MLS based RSM.
- Selection of DOE scheme for support points is critical. Koshal Design shows no difference in FE runs and optimized results with LS and MLS based RSM. Whereas D–Optimal, Full Factorial and Central Composite Designs show appreciable difference in performance of meta-models with MLS based RSM.
- As compared to gradient method based optimization for FE model updating, MLS based RSM converged faster except for exponential weighted linear polynomial basis and Koshal Design.

Application of ARSM in FE model updating can be studied further in details with more DOE schemes and weight functions by using more number of updating parameters.

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