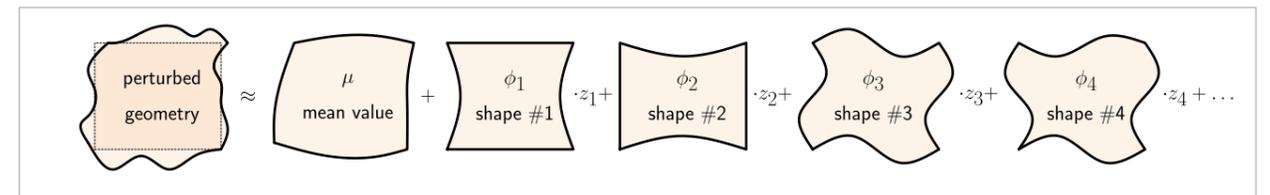Fig 1: Representation of a random field (e.g. a geometric perturbation from a CAD model) based on mean value and a linear combination of deterministic scatter shapes and associated random scaling factors (random field amplitudes) z_i.

RANDOM FIELDS AND FIELD META MODELS – CORRELATION ANALYSIS IN TIME AND SPACE

The extended capabilities of Statistic on Structures can process and analyse spatially or temporally distributed product parameters for a realistic simulation and evaluation of the design performance.

Introduction

Progress in measurement technology makes it possible to capture accurate data of real component geometries. This also includes the recording of spatial distribution of material or signal-based properties, such as dynamic processes in time, frequency fields or load-displacement curves. Computing-based manufacturing technologies (Industry 4.0) provide such spatially or temporally distributed product parameters in large quantities. To use this data (Big Data) for a realistic simulation and evaluation of design performance, extended methods of correlation analysis are required.

Random fields and field meta models, which Dynardo has been developing intensively in recent years, are suitable for the modeling of correlations in time and space. Using random fields, parametric models for arbitrary geometries and signals will automatically be generated and can be directly used for the process of optimization, model calibration or robustness evaluation. For example, the generation of random imperfections will be possible with the help of random fields. Field meta models can be used, for instance, to approximate the result of an FEM solution including spatial and temporal distribution. Thus, the accuracy of the meta

modeling and the sensitivity analysis of distributed values will be increased while simplifying the effort of processing such parameters.

This article gives a very brief overview on the methodology and application areas of random fields and field meta models.

Methodology

Typically, the variation of a parameter can be described by a representative value (e.g. statistical mean), the magnitude of variation (e.g. statistical variance) and even more detailed by its statistical distribution type. This can be applied directly to distributed quantities, where one can describe the variation by the statistical parameters - for each point - in time or space. Further and most importantly, the correlation among all individual points in time/space need to be specified. In practice, however, one can hardly specify the whole covariance matrix of a spatial or temporal field. However, it is possible to approximate the covariance matrix through a truncated series expansion as shown in Figure 1. This series represents the correlation structure of a field in

sparse memory layout. It is defined by "scatter shapes" and associated "amplitudes". The scatter shapes are typically determined by analysis of a test set. The amplitudes are a sub space representing the total field variation in terms of a few scalar parameters.

The methodology ensures that a minimum number of parameters yields a certain accuracy in describing spatial or temporal variation patterns. Hence, **it can be used to identify an optimal parameterization of a variation in space or time based on analysis of measured or virtually created data**.

An extension of the random field parameterization is field meta modeling. Therein, one does not represent the field variations in terms of a new parameter space, but in terms of the original input parameters. The field meta models are nonlinear functions that are chosen to approximate the average field response in an optimal way through variations of the input parameters as illustrated in Figure 2. Therefore, these surrogates are called field meta model of optimal prognosis (F-MOP). They also allow to compute sensitivity indices based on prognosis quality (field coefficient of prognosis – F-CoP).
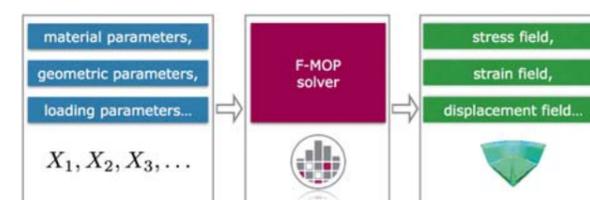
Figure 3. Based on the analysis of a design of experiments, a random field model is established. Only 7 parameters can be used to explain nearly 80% of the whole variance observed in the DoE, while already 11 parameters represent more than 90% of the variance. The scatter shapes are wavelets as seen in Figure 4. This result can be confirmed to be an optimal parameterization by analytical solutions.
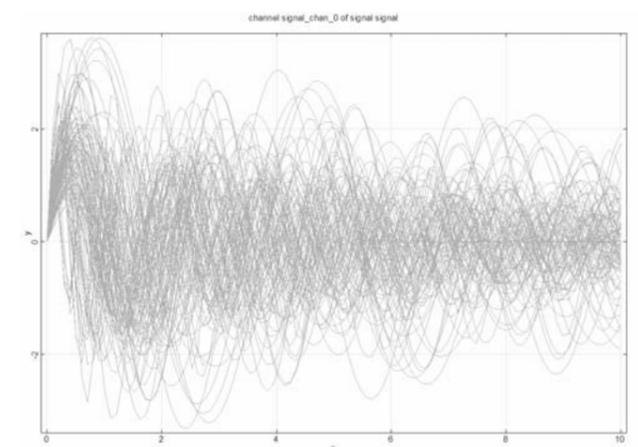

Fig. 3: Observed variations of a dynamic process in a design of experiments
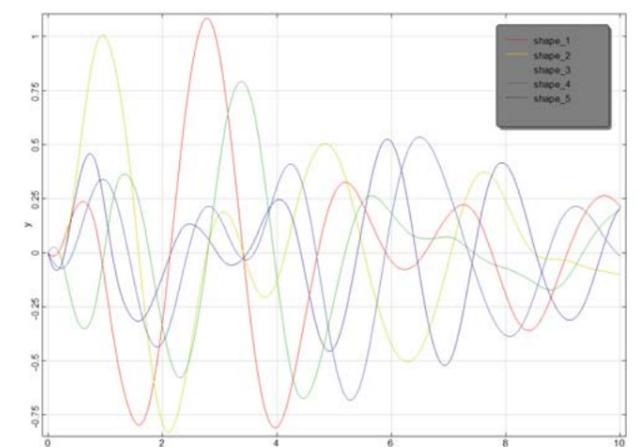

Fig 2: Approximation of FEM solutions by field meta models replacing the original CAE solver

Correlation analysis of signals

Signals are data being distributed in a one-dimensional space. Signals are, for example, dynamic processes (time, frequency), load-displacement curves, transient external loading and boundary conditions, etc.

Case 1: Parameterization of a dynamic process

This example represents a simple dynamic process defined by a damped harmonic excitation in time. Although the mechanical model is very simple, the task of finding a good parameterization is a challenge: The response changes its magnitude, its damping ratio and its frequency as illustrated in

Fig 4: Identified scatter shapes describing variations of a dynamic process

Case 2: Modeling ad-hoc variations of signals

Sometimes, one wants to study the performance of a numerical model given some variations of input signals. If measurements are available, the random field parameterization can be used to create an accurate statistical model for the

observed signal variations. Typically, however, no or only a few measurements are existing during the design process.

In this case, signal variations were modeled by a "synthetic random field". Therein, the correlation is assumed using some autocorrelation model between the individual points in time. The distribution of mean and standard deviation can be either estimated from a few measurements or assumed as well. The results are typically sinus-like functions in 1D similar to the Fourier series, as being illustrated in Figure 5. Therein, a synthetic random field model is created with zero mean and constant unit standard deviation.
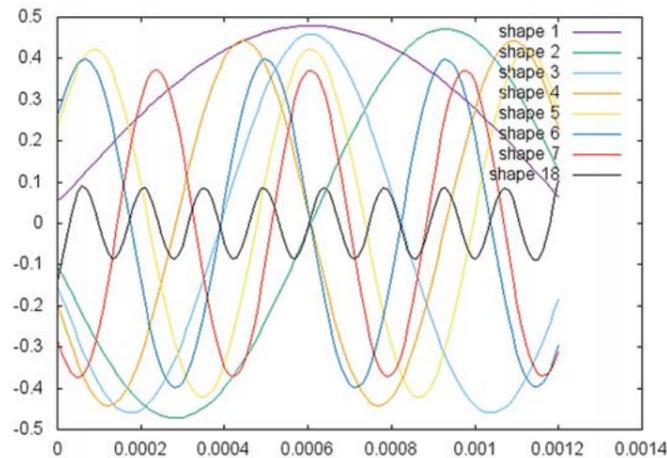

Fig 5: Scatter shapes of a synthetic random field model for mean=0 and std-dev=1 at all points

Case 3: Sensitivity analysis of a signal

The field meta models can be used to perform an extended sensitivity analysis based on signal data. They allow to identify which input parameter is responsible for the response variation for each individual point in time. It is recommended to perform a sensitivity analysis based on field meta models (in favor of the scalar MOP) if

- The location of hot spots is not yet known
- The location of hot spots is varying
- Hot spots do not exist since one is interested in a distributed effect (for example the shape of the signal in an interval)
- The accuracy of a scalar MOP is too small, but the reason is unknown.

The result of a F-MOP is the visualization of the Field-CoP describing the distribution of the CoP along the time axis. By that one can identify at which locations of the signal the meta model is able to predict the variations well (or not), and at which locations of the signal input parameters are of large or small influence onto the signal variation.

The following example is presented by Römelsberger et al [1]. Therein an optimization on a meta model was attempted in order to optimize the design of a dual band WiFi

antenna. The response of interest were the minima of the "return loss" in frequency domain. A scalar MOP obtained an accuracy of CoP=71% only. Using a field meta model for the return loss spectrum, however, one can identify the intervals in frequency domain, which are responsible for the low accuracy of the meta model. The signal responses of a design of experiments are illustrated in Figure 6. The corresponding F-CoP values are shown in Figure 7. The upper curve is the F-CoP of the whole model defining the model accuracy for the respective frequency value. For the right minimum the accuracy is sufficiently large, but the meta model cannot well predict the variations in the center and for the left minimum. The center is, however, not relevant for the optimization. The left minimum is, however, the reason why the scalar MOP cannot approximate the minimum value well: The position of the minimum is varying and, further, it is a true singularity being dominated by round-off errors in the CAE process. The other curves in Figure 7 denote the sensitivity of the individual input parameters which is varying in the frequency domain.

The advantage of a Field-MOP is that it captures automatically all support points of a signal. Alternatively, one needs to manually define few hot spots for which a MOP is evaluated, but still misses the shape of the signal in the interval between the selected locations.
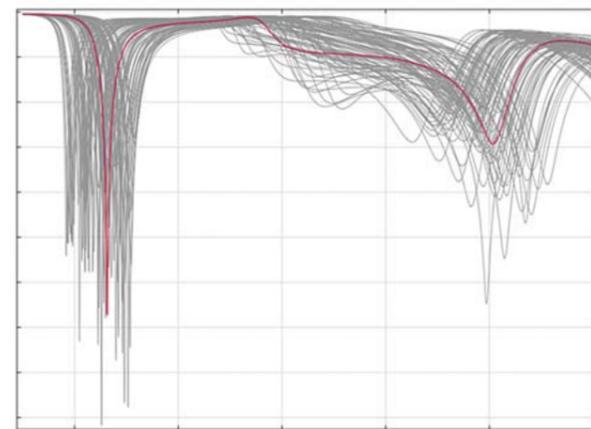

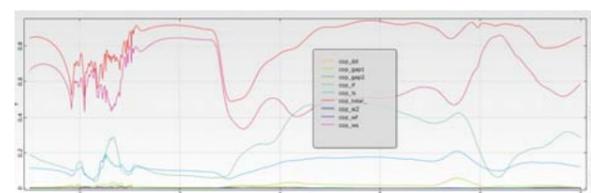Fig. 6: Response signals of the "return loss" in frequency domain of a design of experiments

Fig. 7: F-CoP of the "return loss" in frequency domain

Case 4: Pre-Optimization on a signal meta model

A sensitivity analysis is an important part of the design process, typically being implemented before the actual optimiza-

tion takes place. If the meta model is accurate enough, it can be used to "pre-optimize" the design without extra CAE solver run. The identified optimum of the meta model can later be used as a start point of the direct optimization.

Field meta models are recommended if the shape of the signal is of interest. This is the case in many applications of model calibration, e.g. the identification of material parameters based on measured load-displacement curves. The basic idea is that one approximates the whole load-displacement curve by a field meta model and then minimizes the error norm between the approximated signal and the reference curve.
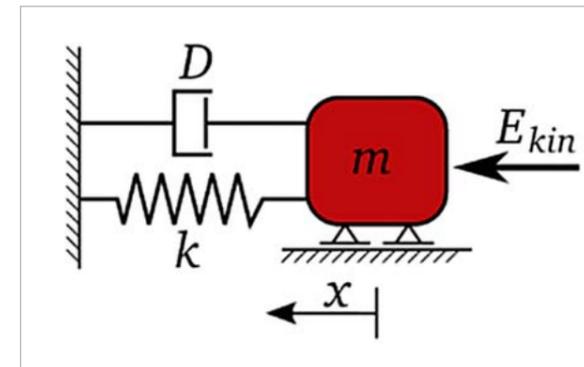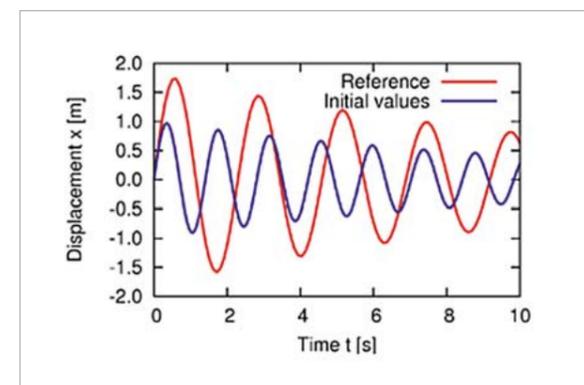

Fig 8: SDOF oscillator

Fig 9: Response of a SDOF oscillator model (blue) and reference signal (red)

As an example, assume that the dynamic process of example 1 is the result of a numerical oscillator model as illustrated in Figure 8. The task is to identify the material parameters of a given measured reference signal as shown in Figure 9. In terms of scalar parameters one would need to identify the locations of the local minima and maxima and then minimize the position error with respect to the same extrema in the reference signal. The identification of these hot spots, however, needs manual programming and, thus, is prone to errors. Using a field meta model for the whole signal, one only needs to minimize the error norm between both signals. The result is shown in Figure 10 being very close to the true optimum.
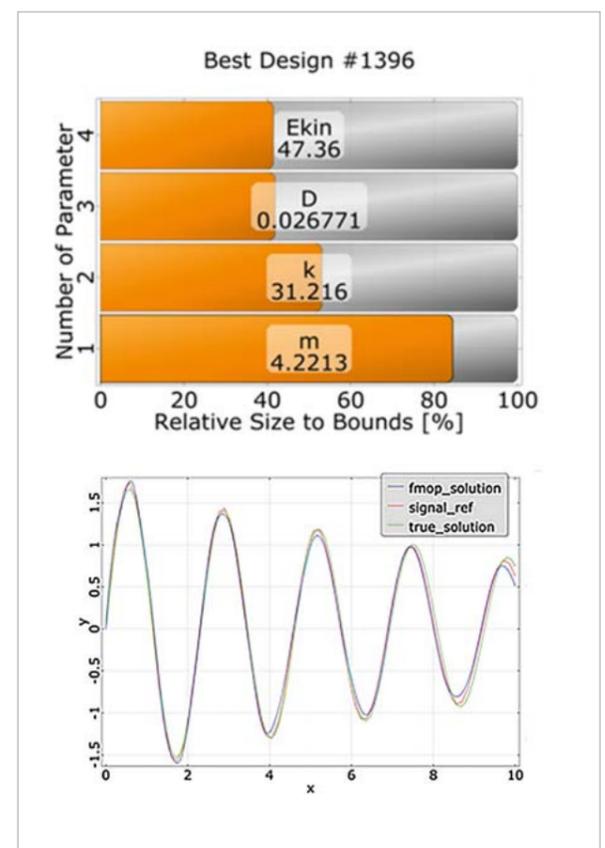

Fig 10: Solution of a pre-optimization of a model calibration

Correlation analysis in space

The methodology and work flows for modeling of signal variations can also be applied to spatial variations in 3D. Typical quantities are geometric variations (e.g. random imperfections due to manufacturing) like boundary coordinates or shell thickness, state variables (e.g. stresses, displacements) used to evaluate structural performance, damage (e.g. distribution of plastic strains), material properties (e.g. surface friction properties, fiber angles) etc.

Case 1: Parameterization of geometric imperfections based on measurements

Nowadays, detailed laser scans of manufactured products are often done in order to ensure the dimensional accuracy at critical locations. The same information can be reused, for example, to create a realistic statistical model of the geometric imperfections in order to virtually create random geometries, e.g. for a robustness analysis.

The following example is presented by Nunes et al [2]. Therein, the robustness of an automotive brake system regarding brake squeal noise was analyzed. The squeal noise frequencies predicted by the numerical model did not match the observed frequencies in the hardware experiments. Engineers suspected the geometric imperfections of the brake pad surfaces. To model these variations, laser scans of brake pad sur-
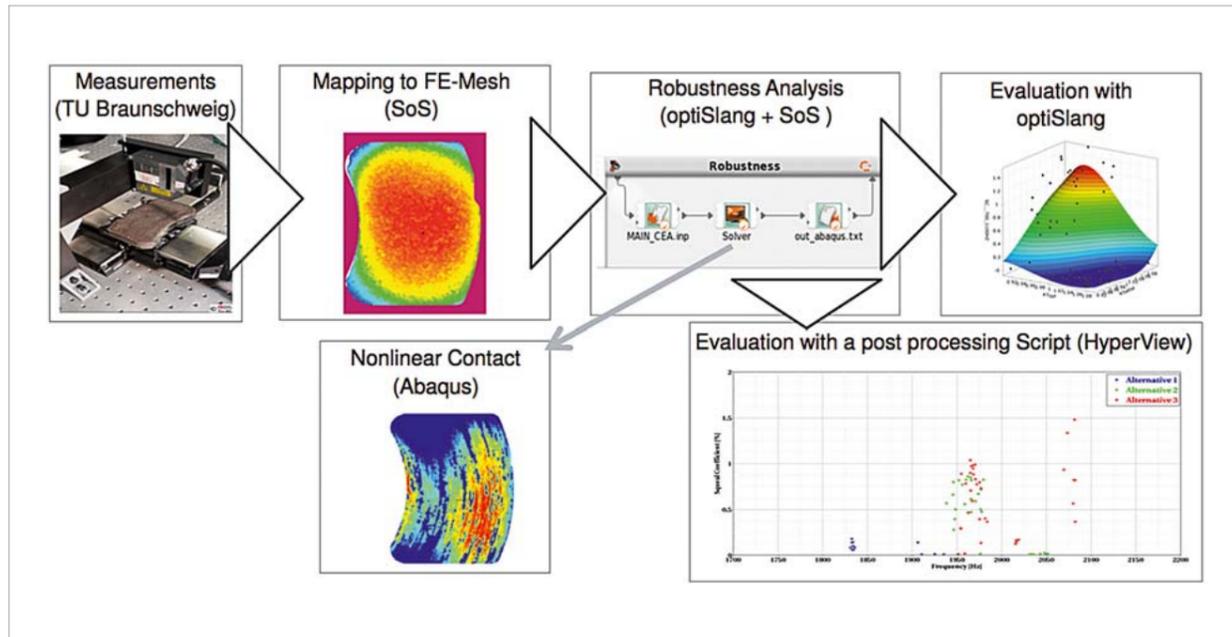
Fig. 11: Consideration workflow of measured geometries in a numerical robustness analysis

Fig. 13: Workflow of a synthetic random field model for the geometric imperfections of a knuckle

faces were done. The results were mapped onto a FEM mesh. Finally, a random field model was created which was capable of changing the FEM nodes on the surface according to the identified distribution parameters. The schematic workflow is illustrated in Figure 11. The results are illustrated in Figure 12. Therein one compares the numerically predicted noise frequencies obtained from a DoE with random scalar parameters (red) with predicted noise frequencies obtained from a DoE with additionally consideration of geometric variation patterns (blue). Obviously, the results obtained from varying the surfaces are much closer to the hardware observations.
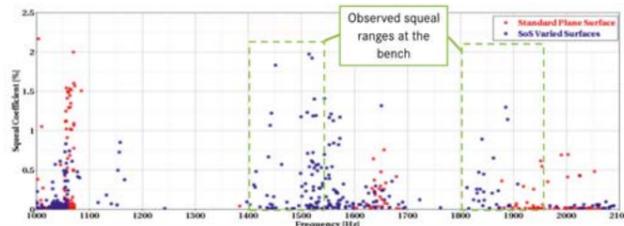

Fig. 12: Results of a robustness evaluation comparing a DoE without (red) and with (blue) varied geometry

Case 2: Modeling ad-hoc variations of geometric variations
In this example from automotive, the influence of geometric imperfections of a knuckle onto the brake squeal noise frequency was analyzed, see Nunes et al [3]. Only a single measurement of the true geometry was available. It was used to estimate the principal magnitude of variation of the true geometry from the CAD model. A synthetic random field model was employed to virtually create random knuckle geometries. A schematic workflow is illustrated in Figure 13. First a random field is created after determining

the geometric deviation between a FEM mesh (based on a CAD geometry) and a measurement (STL), Figure 13 top left. Then the model is used to generate new random designs (Figure 13 top right). In the concluding sensitivity analysis, one identified that a material parameter, but not the geometric imperfections of the knuckle, was of significant influence onto the squeal noise, see [3] for more details.

Case 3: Sensitivity analysis of 3D field variations
This is an example how the field meta models can be applied to sensitivity analysis of 3D field distributed data. The example was presented by Konrad et al [4] wherein a metal forming production process was simulated. After deep drawing of a metal sheet, the resulting sheets are typically joined (joining and spring back simulation). In the considered case, the joining process is defined by the position and size of 6 clamps linking two metal sheet, see Figure 14. The main target of the analysis is a calibration of the joining parameters with the objective of minimizing the geometric deviation of the final geometry after joining with a CAD geometry created by a designer. Since the final deviation error is a different value at each spatial location, one performs a sensitivity analysis to identify the joining parameters that influence each respective position at most. The result of such a sensitivity analysis are illustrated in Figures 15 and 16: First one checks the accuracy (F-CoP) of the meta model and relates it to the magnitude of variation (e.g. standard deviation) of the geometric deviation. Locations with very small standard deviation are typically not well explained by a field meta model, while location with large variation should have a large CoP. Here the F-CoP are well distributed with an average value of 94%. After that one can identify the individual sensitivities of the input parameters (Figure 16).
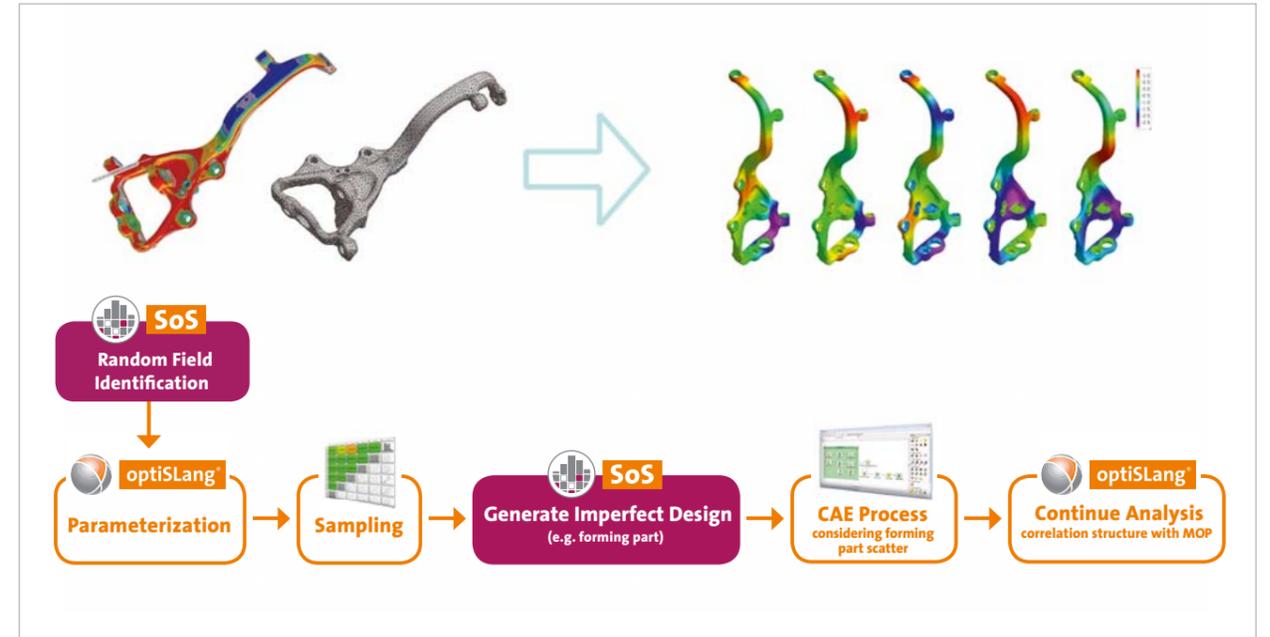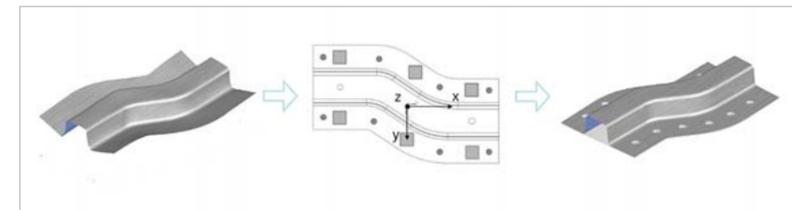

Fig. 14: A joining process after a deep drawing step using 6 clamps
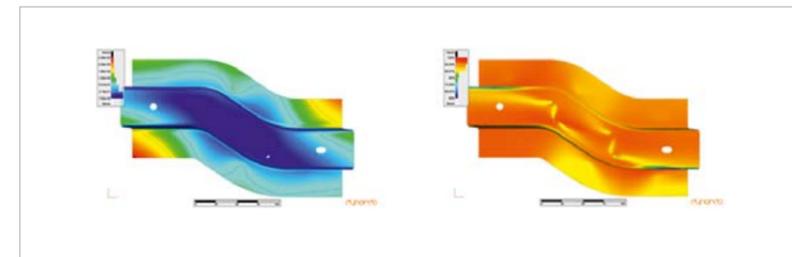

Fig. 15: Field meta model of a geometric deviation. Left: Standard deviation of the geometric deviation in vertical direction for each location on the FEM mesh (blue: small; red: large). Right: Explainable variation (F-CoP) by the field meta model
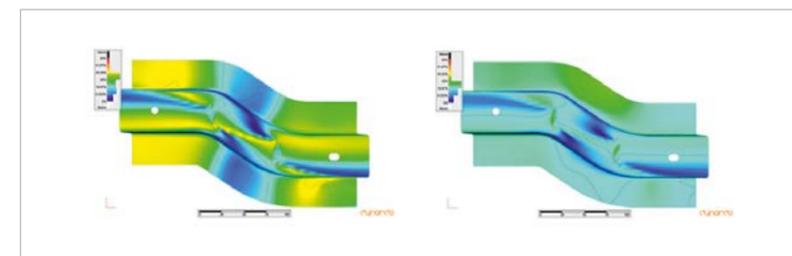

Fig. 16: Field meta model of a geometric deviation in vertical direction. Left: Distribution of the influence (F-CoP) of the position of the left-bottom clamp. Right: Distribution of the influence (F-CoP) of the position of the center-top clamp (yellow: large influence, blue: small influence)

Case 4: Pre-optimization on a 3D field meta model
Using field meta models in space, one can pre-optimize a design utilizing distributed quantities. This example was presented by Kellermeyer [5]. Therein, the draping process of a composite, as illustrated in Figure 17, is simulated. After draping the composite material onto the desired shape, a spatially varying distribution of fiber angles can be observed. Additionally, the task was to calibrate the process parameters of a draping simulation with ANSYS™ such that it matches the observed fiber angle distribution (see Figure 18). This was solved using a field meta model in SoS. First a sensitivity analysis of the draping simulation model was employed analyzing the field of fiber angles. Second, the spatial distribution of the fiber angles was measured and associated with the FEM nodes in the numerical model. The optimization objective was to minimize the average squared error between the fiber angle predicted by the numerical model and the measured fiber angle. As a result, the average deviation of the fiber direction between numerical model and measurement was reduced from initially 7.3° to 5.2°. The field meta model