STATISTICS ON STRUCTURES 3.0

Some properties of engineering structures or structural parts are usually of random nature due to manufacturing tolerances, material scatter or random loads. For the assurance of product quality, avoidance of recalls and fulfillment of safety requirements, such randomness has to be taken into account by applying correct statistical modeling. Statistics on Structures 3.0 makes random fields ready for CAE-based simulation.

Introduction:
Random variables vs. random fields

Random fields are not yet commonly used in CAE design. Probabilistic methods involving scalar random numbers, however, have experienced a success story in the recent years: They are used in a wide area from robustness evaluation, reliability analysis, robust design optimization, etc. In these areas, DYNARDO provides engineers optiSLang as one of the worldwide leading software tools. There is a wide acceptance among engineers that CAE-based stochastic analysis helps to obtain a more accurate representation of reality leading to better products with less manufacturing costs, better reliability and longer life-cycles. Nevertheless, most probabilistic procedures are currently restricted to individual random numbers, i.e. scalar parameters.

When applying CAE-based stochastic analysis to virtual prototyping, we have to learn what is the appropriate discretization level of the uncertainties, like we have learned and identified which is the appropriate accuracy level of spatial discretization (h/p/r-refinement, interpolation and integration types of finite elements) or the appropriate accuracy level of material constitutive models (linear, nonlinear, plas-

tic, creep, damage). Usually we start with rough estimations of uncertainties (for example, a uniform variation of a sheet metal thickness using one scalar stochastic parameter). Some phenomena, however, will require the introduction of spatially distributed random variations and here we will need random field parametric models in the future.

Random fields, i.e. random numbers being spatially distributed and obeying some spatial relationship, however, are currently very rarely used in CAE design and simulation. One reason is the conceptual difficulty of identifying and understanding. Another reason is the missing availability of numerically efficient and user friendly software solutions to identify and model random fields. DYNARDO's goal is to fill this gap with Statistics on Structures 3 introducing random fields in CAE design and analysis.

The assumption of scalar parameters is a very helpful view onto commonly used numerical models. Usually, the scalar output parameters are values being excerpted from scalar or even multi-dimensional fields. For example, in structural mechanics, one often extracts integral-weighted or maximum principal values (from stresses, equivalent plastic strains

or displacements) from fields being distributed among the nodes and elements of a finite element mesh. Even the input parameters are rarely truly scalar random numbers. For example, the shell thickness is never a constant for a single sample design in reality, but varies randomly among the finite element structure due to its manufacturing process. The same is true for the spatial distribution of geometric coordinates, material properties or boundary and loading conditions - all being randomly distributed among the FEM mesh. The parameterization and characterization of these random effects and, even more challenging, the identification of input-output relations are generally nearly impossible tasks when using element wise discretization of random properties since they may involve many thousands of random numbers. Spatially distributed random variables, however, can be characterized as random fields. Their description involves information on the distribution type, lower and upper bounds, mean value and standard deviation at each field position. Further, a correlation matrix between the individual field points describes the amount of fluctuation within the close neighborhood of an individual point: Usually a physical quantity (e.g. the shell thickness) does not change rapidly and the values of all nearby points are strongly correlated. The correlation matrix of a random field needs not necessarily to be computed, but can be approximated through the Karhunen-Loeve expansion.

Karhunen-Loeve expansion

The Karhunen-Loeve expansion states that an optimal choice of the basis functions is given by an eigenvalue ("spectral") decomposition of the auto-covariance function. When a scalar field is measured as a distribution on an FEM mesh (or on any other discrete space), the random field is represented by discrete values. In this case the spectral decomposition is given through the solution of a (very large scale) matrix eigenvalue problem. A significant reduction in the number of variables can be achieved when truncating the series after a few items. The field being measured in terms of a large number of values (usually in terms of single values per node or finite element) is then expressed through a small number of coefficients ("amplitudes"). The "scatter shapes" (i.e. the eigenvectors of the covariance matrix) define the transformation basis. By reducing the number of random variables with the identification of the most important scatter shapes, we are then able to eliminate noise and simplify the representation of input/output relations, which will give us the possibility to visualize and understand the relation between input and output variation. The basis functions should be orthogonal reducing the computational effort for the projection (reduction) and its inverse transformation. As a side effect, the random coefficients are uncorrelated simplifying the digital simulation of random fields. The key aspect of the Karhunen-Loeve expansion is its reduction effect. That means one computes only a few scatter shapes in practice to explain the majority of variation. If the cumulative variability associated with the computed scatter

shapes is large enough, one is able to represent most of the existing variations in a random field using only a small number of scalar random variables. The omitted scatter shapes are usually associated with small correlation length parameters. Thus, they often can be considered as noise. Hence, one can interpret the truncated Karhunen-Loeve expansion in terms of

- model order reduction, breaking down the number of random variables of a complete field to a small number of random amplitudes. These amplitudes may be visualized and directly used in optiSLang's sensitivity analysis using Metamodel of Optimal Prognosis (MoP) to determine input-output relations and the corresponding coefficient of prognosis (CoP).

- noise reduction and smoothening, eliminating random effects associated with very small correlation lengths. Such effects may, for example, be introduced by solver noise.
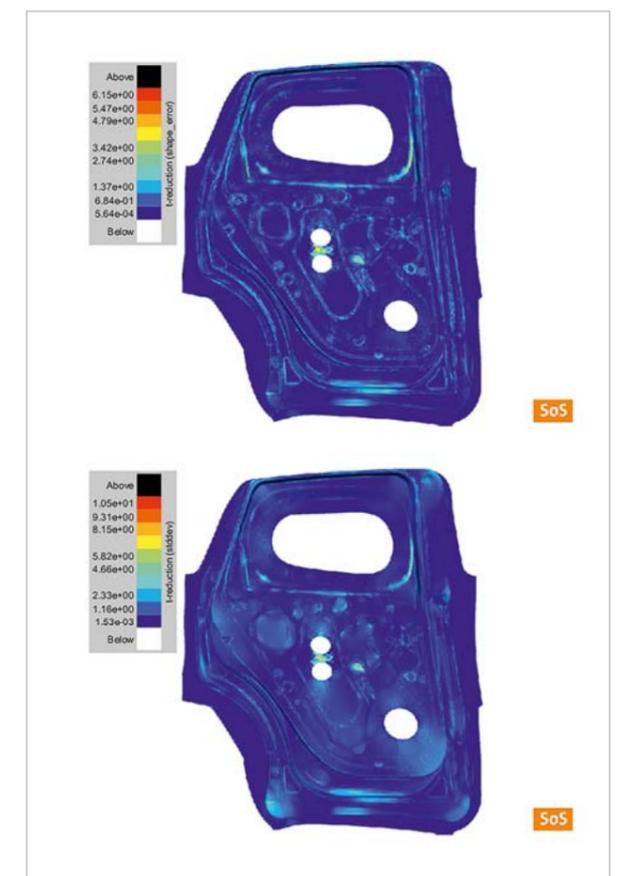
Fig. 1: New feature: Compute local error due to Karhunen-Loeve expansion (top) scaled to the dimension of the standard deviation. The respective standard deviation is shown on the bottom. The expansion uses 10 modes explaining 71% total variability. The local error can be directly related to the standard deviation enabling the engineer to predict the maximum error in variation at hot spots.

In order to validate the series truncation, one may use several error measures: A global error criterion that is related to the sum of the eigenvalues (being associated with the

actually computed scatter shapes) of the spatial covariance matrix. This global criterion provides a scalar number for the total variability percentage that can be explained by the reduced model. A local error (see Fig. 1, previous page) describes the amount of standard deviation that is lost when approximating the original data by the reduced model (see Fig. 2). If this local error is large in areas where the engineer is interested in, one must perform a detailed analysis: First, one computes the smoothened approximations to all field samples utilizing the scatter shapes. Second, one computes the minimum and maximum values and the standard deviation for all field values of the smoothened space. These quantities are then compared to their equivalents of the original data. Finally, one can determine how good the truncated Karhunen-Loeve expansion can represent extremal values and points of large variation.

Statistics on Structures (SoS)

DYNARDO developed the software Statistics on Structures (SoS) which is capable of decomposing random fields by Karhunen-Loeve expansion, visualizing the identified "scatter shapes", analyzing random properties on FEM structures, locating "hot spots" of variation and investigating correlations. So far, SoS was mainly a "post processor" for statistics on FEM structures, i.e. for visualization of the descriptive statistics on the structure, visualization of correlations between random input and structural results, visualization of quality performance (QCS). To investigate the nonlinear correlation structure between input and outputs, SoS can be coupled with optiSLang. It can read and write optiSLang binary files integrating optiSLang's MoP solver easily into the analysis of correlation.

Several successful applications of SoS are documented in the DYNARDO online library at www.dynardo.at and in the references at the end of this article. At the user conference WOST 2012, DYNARDO presented a new methodology for the analysis of random fields that was capable of analyzing very effective large finite element meshes (tested up to a range of 500.000). To meet future challenges in random field identification, the post and pre processing modes of Statistics on Structures were completely rewritten. Thus, computational efficiency and flexible work flows for user friendliness were improved opening new areas of application.

Using SoS for post processing of statistical data

Very often, engineers are interested in response value variation as a result of a robustness evaluation which must be evaluated at a mesh position of interest ("hot spot") that is not known before starting the actual analysis.

An example shall explain the problem (see Fig. 3): One wants to investigate the variation and the correlation to the maximum von Mises stress within the structure subject to pre-
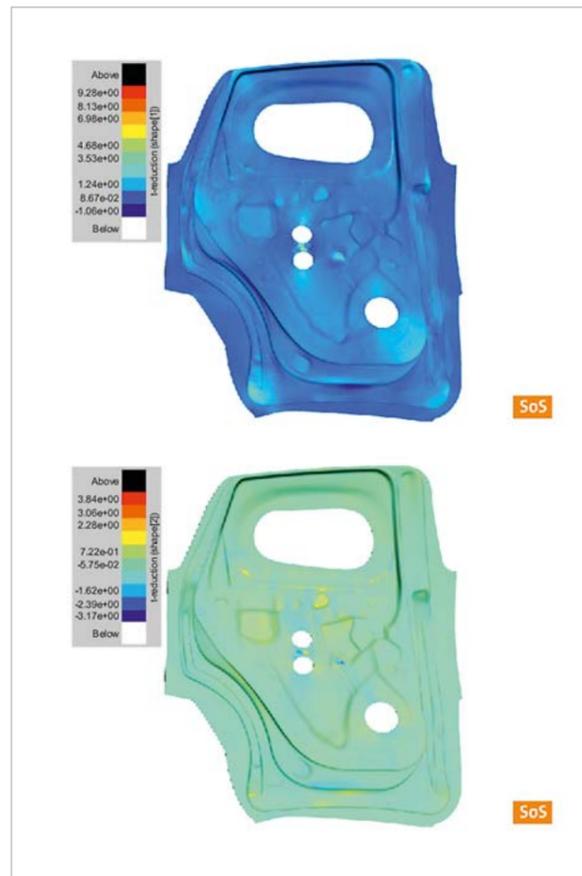
Fig. 2: The first two scatter shapes (scaled to the magnitude of the standard deviation) are shown in this figure (top: #1, bottom: #2).

defined material and process variation of a forming simulation. The engineer starts the task by performing a robustness evaluation varying all possibly influencing scattering parameters. This sampling is a necessary step to identify the hot spot of maximum stress and maximum largest stress variation. But how should the engineer express this quantity?

1. The challenge is that the position is a priori not known, where the largest stress appears or where the largest variations of the maximum stress can be expressed for the simulated designs. One choice is to perform a single simulation with representative design parameters, find the position of the maximum stress and choose this position to evaluate the variation for each design. But after performing the whole sampling, the engineers might understand that they evaluated the stress at the wrong position. This may involve a recomputation of the sampling doubling the amount of time and computational resources. The engineers may not even notice that they measured the stress at the wrong position and continues the analysis based on wrong data.

2. Another strategy is not to predefine the position of interest, but to extract the maximum stress from the complete mesh. This case may involve two types of

problems: First of all, the maximum stress may be measured at very different positions. If the maximas at different locations belong to different mechanisms the statistical significance of the correlation structure suffer (reduced coefficients of prognosis (CoP) when performing a correlation analysis including reduced ability to identify the important parameters). Even worse, one may extract the wrong values. For example, the maximum stresses may be measured near the supports. In this case one must carefully restrict the spatial domain where one extracts the maximum value from.

3. A third strategy is to evaluate integral values, e.g. one computes a weighted average of the von Mises stress. But this quantity might not represent extremal values well.

Fig. 3: Finding hot spots with optiSLang and SoS

Statistics on Structures allows to define hot spots after performing the sampling (see Fig. 3). This gives much more freedom of choice to the engineer. Thus, it can simply be checked what is the most appropriate way of extracting results which represent all important hot spots of variation and show high CoP values of the correlation between input and the response. First, the engineer creates the sampling (using optiSLang or another software) and performs the numerical analysis of the robustness evaluation of the respective designs. Then the resulting field quantities (for example, the von Mises stress field) are imported into Statistics on Structures for all designs. Depending on the actual task, one can compute the statistical properties of the response values of interest. The positions of interest can then be identified in a post processing step. The values of interest at these positions are then exported as scalar parameters and can be further analyzed, for example in a sensitivity analysis of optiSLang.

value on the mesh. By using the Karhunen-Loeve expansion to decompose the variation into the most important "scatter shapes", one can easily cumulate the related amount of variation (being distributed on the mesh) into scalar parameters. The actual amount of variability related to the total field variance can be quantified. Hence, one can safely export the new scalar quantities to continue the sensitivity analysis. By analyzing the amplitudes of a truncated Karhunen Loeve expansion, one can increase the statistical significance, identify important input parameters influencing the individual amplitudes of the scatter shapes and even localize the effect for the respective inputs on the FEM mesh.

Let the task be: Identify the input parameters that dominantly influence a field quantity. The involved steps are sketched in Fig. 4. Let optiSLang generate a sampling and

perform the numerical analysis of each design. The resulting field quantity is imported into SoS for all analyzed designs. Therein, the random field is expanded and SoS computes the most relevant amplitudes (statistically independent scalar random parameters) and the associated scatter shapes for each amplitude (quantifying and localizing the amount of scatter related to the amplitude). In many applications, the scatter shapes can be successfully computed. For example, the mechanisms of variation can be decoupled and the varia-

Fig. 4: Identify input-output relations using optiSlang and SoS

As mentioned, the selection of extreme values may lead to problems identifying the input/output correlation with large coefficient of prognosis when the extraction of the maxima involves rapid changes to the position of the considered
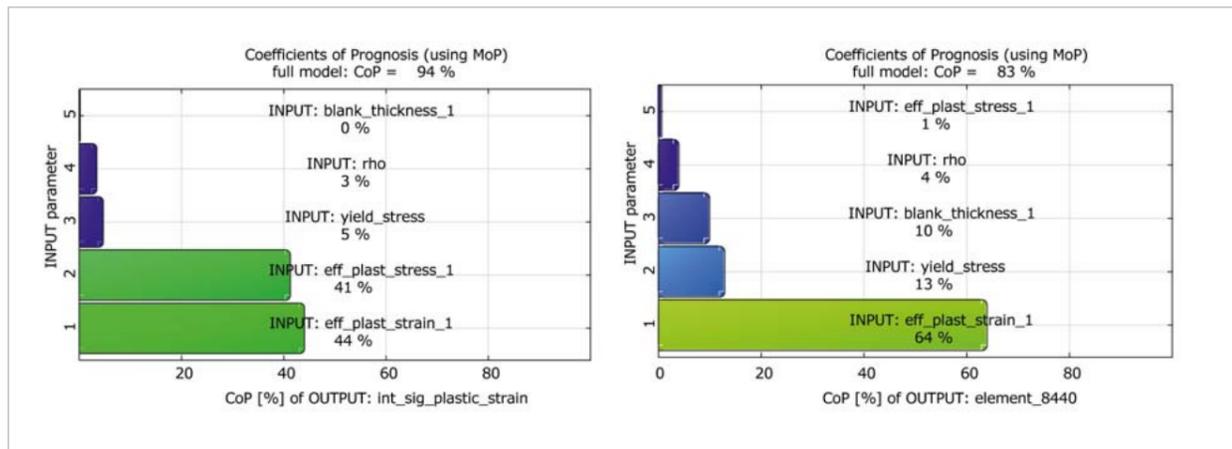
Fig. 5: CoP of the integral value of the plastic strain (left) or the CoP of an a priori specified element (element #8440 with largest plastic strain the the first design

tions are not too much affected by numerical errors. In these cases, the cumulative explainable variability in terms of CoP values should not be less than 90%. Ideally, only a few (3 to 5) amplitudes are needed to match this variability. The local error distribution should be validated in the post processing of SoS (see page 3). The amplitude samples are then exported to optiSLang. In optiSLang, the sensitivity analysis is carried
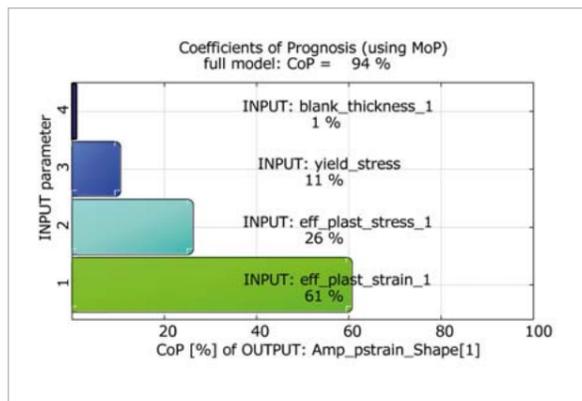
Fig. 6: CoP for the individual input parameters that influence the first amplitude of the Karhunen-Loeve expansion (45% of total variability in the model)

out for the original input parameters and the individual amplitudes (as outputs) using MoP. When interpreting the individual CoP, one must relate them to the amount of variability being associated with the respective amplitude. Further, one can quantify the position of the influence using the respective scatter shape.

The advantage of analyzing random field amplitudes compared with analyzing other scalar values is shown in figures 5 and 6: In figure 5, the overall CoP of a single element value might be too small for a useful interpretation. The integral average value (figure 5) is not sensitive to the yield stress although the response is (see the CoP of the amplitude in figure 6).

Using SoS for generation of imperfect sample sets using random fields

One of the main reasons of recoding SoS was to use the identified scatter shapes and amplitudes which represent the best possible parametric representation of spatially distributed quantities, for example shell thickness in sheet metal forming or bounding surface coordinates to generate imperfect sample sets. If the performance of a part having these spatial correlated scattering values after the forming process will be investigated in an assembly for a crashworthiness, it might become necessary to introduce the identified parameters in the robustness evaluation of the crash load case. In fact, the random field parametric model represents all information where each point on the mesh is a random variable being spatially correlated to the other mesh points. If the correlation structure and the probability distribution of the quantity are known, one can generate a sample of such a random field. Simulating a set of imperfect designs (involving multiple realizations of the same random field), thus, improves the accuracy of the representation of variations in reality.

Fig. 7 illustrates the procedure involving two steps: The first step is required to identify the random fields. Therein, one assumes that a random field quantity to be simulated, for example the shell thickness, is an output of a robustness evaluation of the forming process. A sampling is performed. For each design, the resulting field data are imported into SoS. A subsequent Karhunen-Loeve expansion analyzes the correlation structure and also identifies the scatter shapes and the related amplitudes. The random field parametric model necessary for generating imperfect geometries having the statistical representation of variation seen in the robustness evaluation of the forming process is being stored. In the following solver run (e.g. a robustness evaluation of crashworthiness simulation, or another manufacturing process like additional deep-drawing, welding, bending, folding, etc.) one then wants to introduce the scatter from the former process step, the spatially distributed shell thickness of the formed part using SoS random field parametric models.
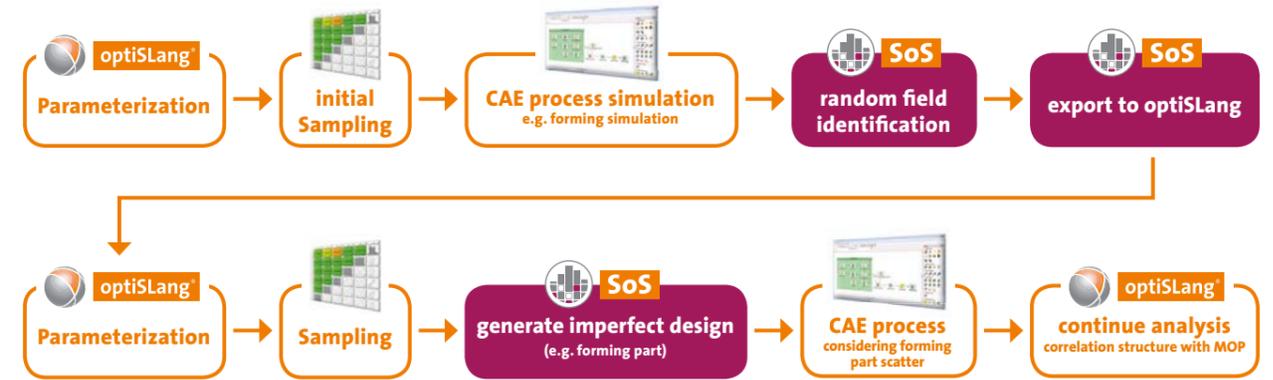
Fig. 7: Simulation of random fields using SoS and optiSLang

In optiSLang one parameterizes the problem involving both, scalar parameters and the random field parametric model imported from SoS. A sampling is created. Therein, optiSLang generates design directories for each sample. For each design directory it modifies all scalar parameters according to the parameterization. SoS is additionally called for each design reading the sample values of the amplitudes and generating imperfect realization of the formed part sheet metal thickness using the SoS random field parametric model.

Fig. 8: The top figure illustrates a single design of the original data. The bottom figure plots the smoothened design (using the Karhunen-Loeve expansion with 10 scatter shapes, i.e. 71% variability)

Using SoS to eliminate random noise in a robustness evaluation

Noise is defined as a random perturbation at each mesh position which is statistically independent and has zero mean and finite variance. An extended interpretation used in SoS is to regard noise being a random perturbation which is not or negligibly correlated to the perturbation at other mesh positions. These variations are exactly those which are not part of the truncated Karhunen-Loeve basis. Hence, one can use the Karhunen-Loeve expansion to filter the noise from the computed field data samples. This is called "smoothening" (see Fig. 8). Smoothening can be used to filter random noise of those solver outputs being used as inputs in a subsequent simulation.

Detection of geometric deviations

A typical example of random fields are geometric perturbations. The geometry of a product is usually randomly perturbated due to random effects during the manufacturing process. Sometimes, for example in metal forming, it is possible to simulate the production process and to evaluate the geometric deviations after this step to reuse them as a random input in the next phase of the analysis of the production process. SoS supports the analysis by detecting geometric deviations (given a modified mesh and a reference mesh for the closest point projection), by analyzing and post-processing the correlation structure of the detected deviations by identifying shapes and amplitudes of deviation and by exporting the deviated geometry as input for a consecutive analysis.

The procedure is illustrated in Fig. 9 (see next page). In the example, a first CAE process simulates a production process, i.e. metal forming of a car cowling. The final geometry is randomly distributed due to random production parameters. A second CAE process is set up using the random geometry of the first process, i.e. a crash analysis, robustness analysis or a second production step. One usually sets up a single geometry and mesh. In order to apply randomly distributed geometries to the second CAE process, one has two
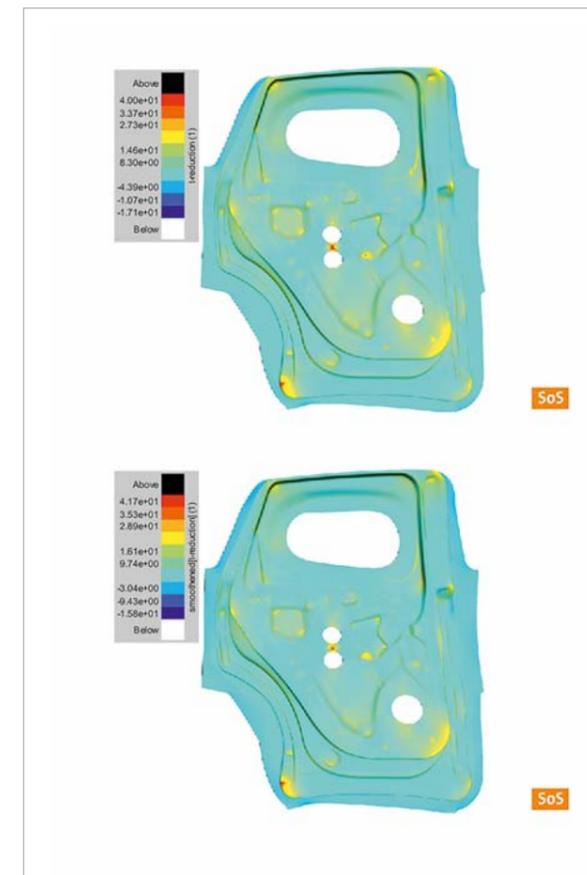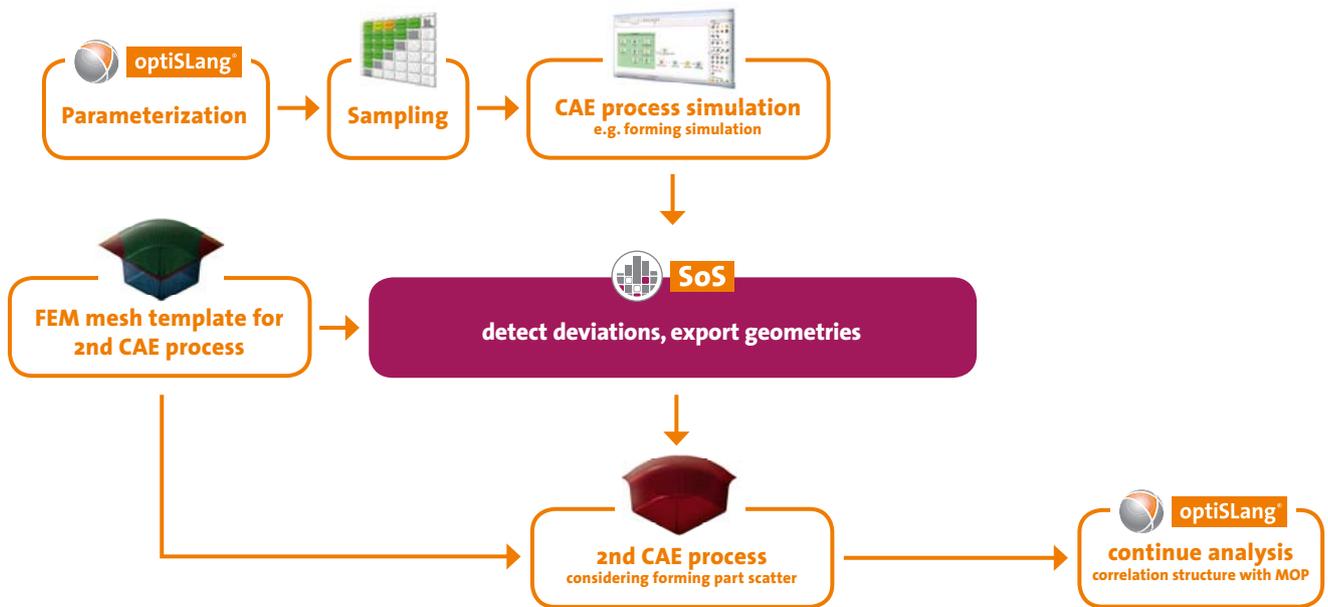
Fig. 9: Detection of geometric deviations and pre-process a consecutive analysis

choices: Either one detects the coordinate deviations between the final geometries of the first CAE process and the assumed model of the second CAE process to insert them directly as input geometries into the second solver chain. Or one detects the deviations and performs a Karhunen-Loeve expansion in order to generate the random input geometries for the second CAE process. Figure 10 illustrates the first scatter shapes of such coordinate deviations used in random field generation.

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Figures 1, 2 and 8 refer to the numerical example:
Hansjörg Lehmkuhl, Johannes Will, Vera Sturm, and Jörg Gerlach. Which discretization level for uncertainties do we need for reliable robustness evaluations in forming application? In 9th Weimar Optimization and Stochastic Days, Weimar, Germany, 2012. DYNARDO GmbH. (courtesy of AUDI AG and Thyssen AG)
Compared with this example, the images were created with SoS 3.0 demonstrating the new post processing capabilities of the new version.

Figure 10 is excerpted from:
Will, J. Integration of CAE-based optimization and robustness evaluation in virtual proto-typing processes at Daimler using optiSLang, Daimler EDM Forum, Stuttgart, 2013 (copy-right/courtesy of DAIMLER AG).

Fig. 10: Detection of geometric deviations between two incompatible meshes of a car cowling. Illustrated are the first 2 scatter shapes of the normal coordinate deviation explaining 90% of total variability.